ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 6, 2013

Do not open exam until instructed to.

Open notes, open books, etc

You have 1 hr and 15 minutes.

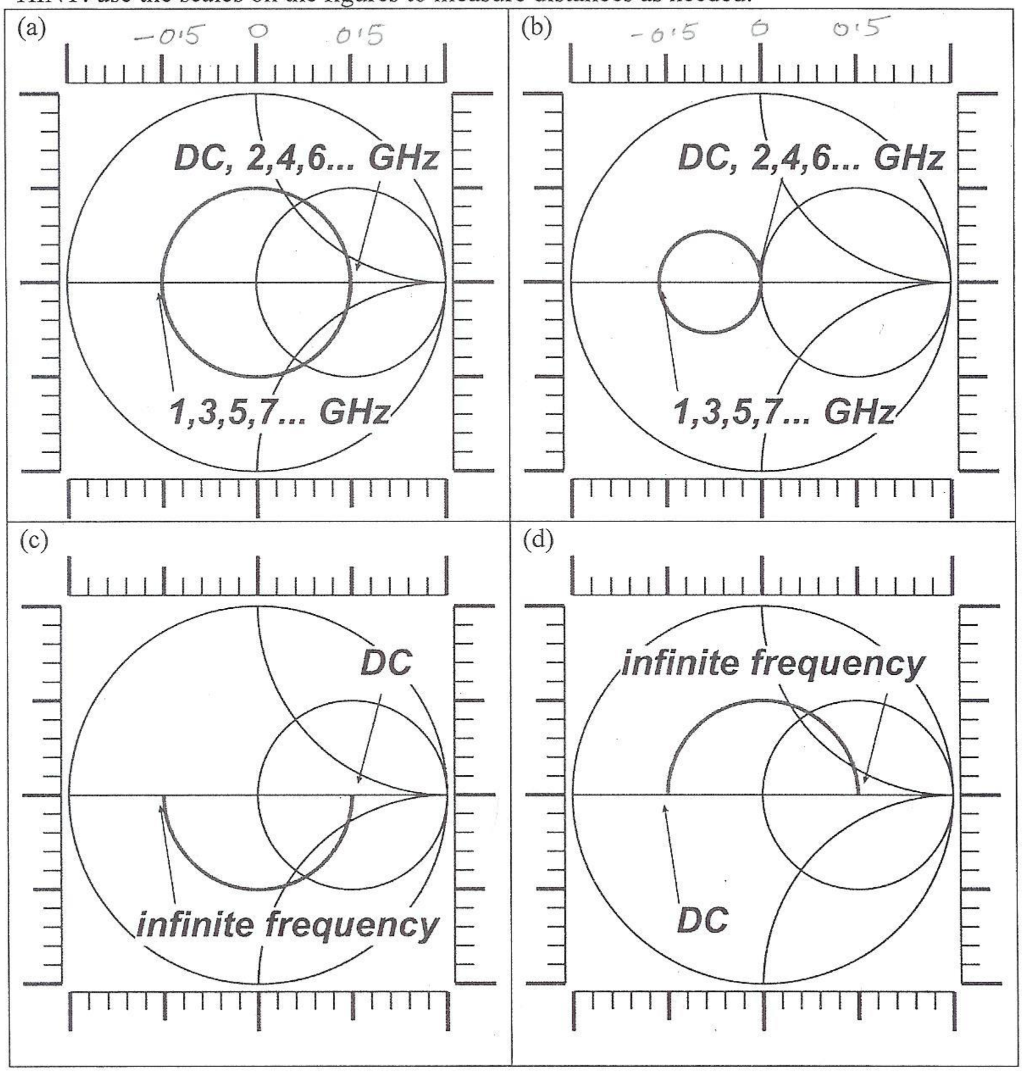
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

Problem	Points Received	Points Possible
1		15
2a		10
2b		15
2c		10
3a		10
3b		10
3c		15
4		15
total		100

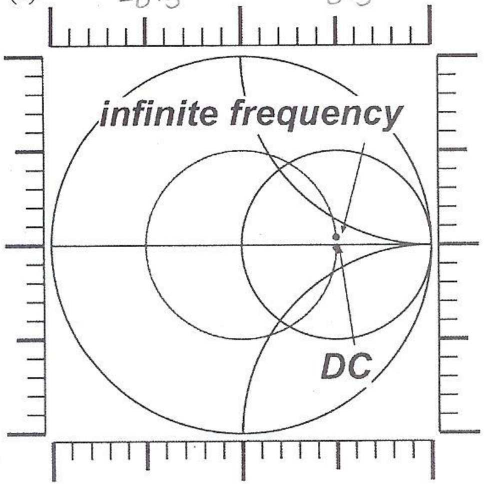
Name: ________________________

Problem 1, 15 points
The Smith Chart and Frequency-Dependent Impedances.

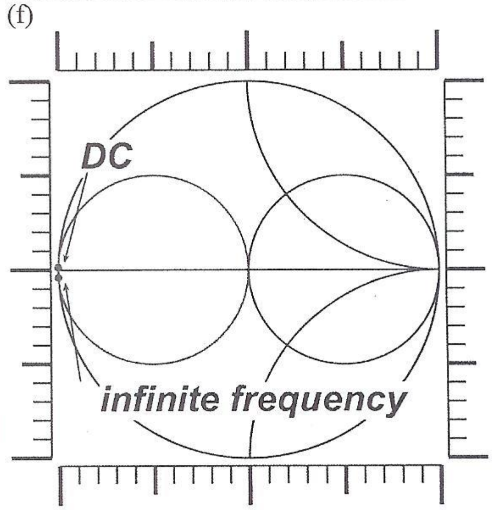
HINT: use the scales on the figures to measure distances as needed.



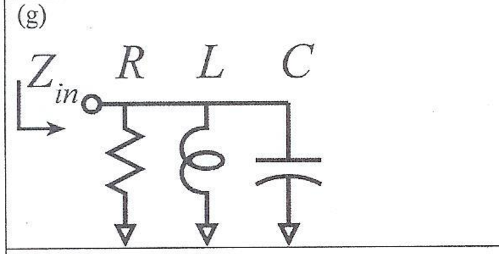
(e)


-0'5
0'5
infinite frequency
DC

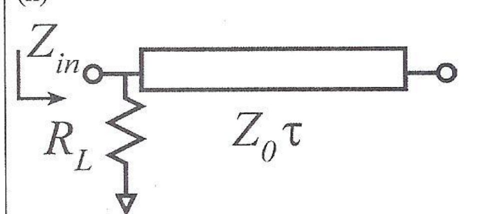
(f)


DC
infinite frequency

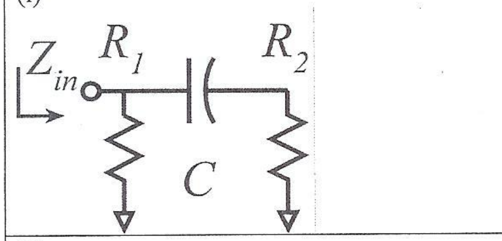
(g)


Z_{in}
R
L
C

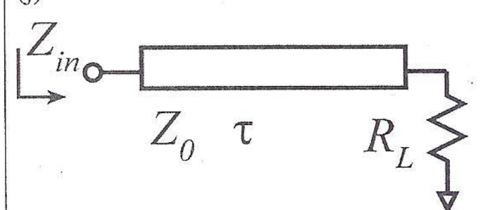
(h)


Z_{in}
R_L
$Z_0 \tau$

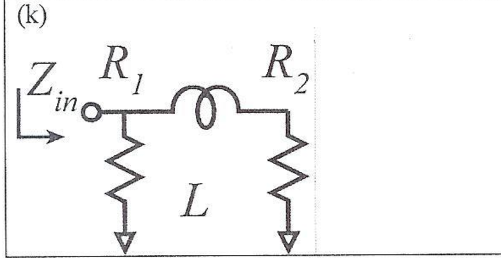
(i)


Z_{in}
R_1
R_2
C

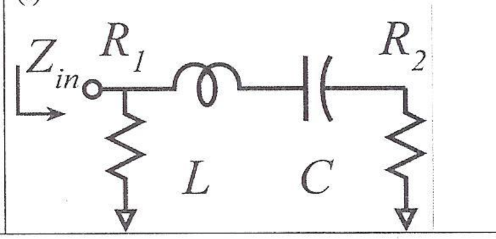
(j)


Z_{in}
Z_0
τ
R_L

(k)


Z_{in}
R_1
R_2
L

(l)


Z_{in}
R_1
R_2
L
C

First match each Smith Chart with each circuit. Then determine as many component values as is possible (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

POINTS			
2	Smith chart (a).	Circuit= (J)	Component values= $Z_0 = 50\Omega$, $\tau = 1/4$ ns, $R_L = 150\Omega$
2	Smith chart (b).	Circuit= (J)	Component values= $Z_0 = 28.86\Omega$, $\tau = 1/4$ ns, $R_L = 50\Omega$
3	Smith chart (c).	Circuit= (I)	Component values= $R_1 = 150\Omega$, $R_2 = 18.75\Omega$
3	Smith chart (d).	Circuit= (K)	Component values= $R_1 = 150\Omega$, $R_2 = 18.75\Omega$
3	Smith chart (e).	Circuit= (L)	Component values= $R_1 = 150\Omega$, $R_2 = 18.75\Omega$
2	Smith chart (f).	Circuit= (G)	Component values= $R = 50\Omega$

(a) The circuit is (J)

@ DC, 2, 4, 6 .. GHz, $Z_{in} = 50\Omega\,(1.5)/0.5 = 150\Omega$

@ 1, 3, 5 .. GHz, $Z_{in} = 50\Omega\,(1-0.5)/1.5 = 16.67\Omega$

→ Quarter Wave Line $Z_{in} \cdot Z_{load} = Z_{line}^2$

$$Z_{line} = [150 \times 16.67]^{1/2} = 50\Omega$$

50Ω Line, Loaded by 150Ω, $\tau = 1/4$ ns

(b) The circuit is (J)

@ DC, 2, 4, 6 .. GHz, $\Gamma_{in} = 0 \Rightarrow Z_{in} = 50\Omega$

@ 1, 3, 5 .. GHz $Z_{in} = 16.67\Omega$

→ Quarter wave Line. $Z_{line} = \sqrt{50 \cdot (16.67\Omega)} = \frac{50}{\sqrt{3}} = 28.86\Omega$

28.86Ω Line, $\tau = 1/4$ ns, Loaded by 50Ω

(c) The circuit is (I)

@ DC → $\Gamma = 0.5 \Rightarrow Z_{in} = 50\Omega(1.5)/0.5 = 150\Omega$

∵ capacitor behaves like an open circuit @ DC, $\boxed{R_1 = 150\Omega}$

@ $f \to \infty$, $\Gamma = -0.5 \Rightarrow Z_{in} = \frac{50}{3} = 16.67\Omega$

∵ cap behaves like a short circuit as $f \to \infty$,

$$R_1 \parallel R_2 = 16.67\Omega$$

$$\left[\frac{150 \cdot R_2}{150 + R_2}\right]\Omega = \frac{50}{3}\Omega$$, solving, $\boxed{R_2 = 18.75\Omega}$

(d) The circuit is (k)

@ DC, $\Gamma = -0.5$, $Z_{in} = \frac{50}{3}\,\Omega = 16.67\,\Omega$

$\because$ Inductor behaves like a short ckt @ DC,

$R_1 \| R_2 \rightarrow 16.67\,\Omega$

@ $f \to \infty$, $\Gamma = 0.5$, $Z_{in} = 150\,\Omega$

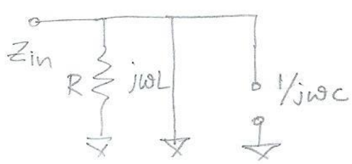
$\because$ Inductor behaves like an open ckt @ $f \to \infty$,

$\boxed{R_1 = 150\,\Omega}$

$\Rightarrow [150\,\Omega] \| R_2 = 16.67\,\Omega$, solving $\boxed{R_2 = 18.75\,\Omega}$

(f) The circuit is (g)

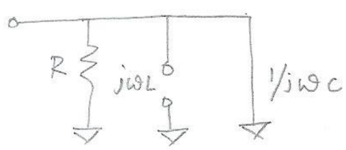
@ DC



$\Rightarrow$ $Z_{in} = 0 \Rightarrow$ Short ckt

$\Gamma_{in} = -1$

@ $f \to \infty$

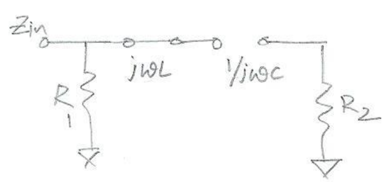


$\Rightarrow Z_{in} = 0$, $\Gamma_{in} = -1$

@ $\omega = \frac{1}{\sqrt{LC}}$, $Y_L + Y_C = 0 \Rightarrow Y_{in} = Y_R = \frac{1}{50}\,\Omega \Rightarrow R = 50\,\Omega$

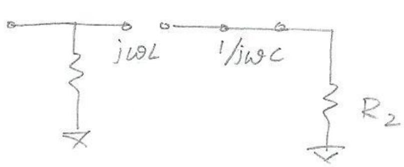
(e) The circuit is (l)

@ DC



$Z_{in} = R_1$

@ $f \to \infty$



$Z_{in} = R_1$

$\Gamma_{in} = 0.5$

$\Rightarrow Z_{in} = 150\,\Omega$

$\Rightarrow R_1 = 150\,\Omega$

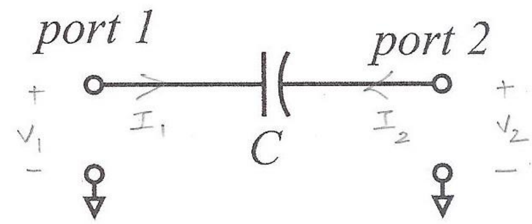
@ $\omega = \frac{1}{\sqrt{LC}}$, $X_L + X_C = 0 \Rightarrow R_1 \| R_2 = \frac{50}{3}\,\Omega$ $[\Gamma = -0.5]$

(At $\Gamma = -0.5$, $Z_{in} = \frac{50}{3}\,\Omega$, from prev parts of problem)

$\Rightarrow R_2 = 18.75\,\Omega$

Problem 2, 35 points
2-port parameters and Transistor models

Part a, 10 points
For the network at the right, give algebraic expressions for the four Z-parameters and for the four S-parameters.



This might be a DC blocking capacitor. What value would you need for C if you wanted S21 = -3 dB at 100MHz in a 50 Ohm system ?

(2)

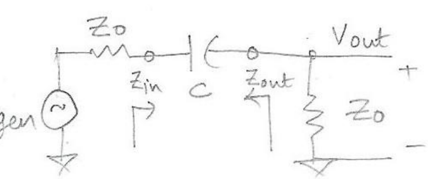
Z-Parameters ; $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $\quad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ $\quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$

If either I_1 or I_2 is zero i.e., one of the ports is open, current does not flow. So, accordingly I_2 or I_1 is zero. Implying, all z-parameters $\rightarrow \infty$.

S-parameters :

(1)



$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}\Big|_{Z_L = Z_0}$$

$$Z_{in} = Z_0 + 1/j\omega C$$

(2)

$$S_{11} = \frac{1/j\omega C}{2Z_0 + 1/j\omega C} = \frac{1}{1 + 2j\omega Z_0 C}$$

$$S_{22} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}\Big|_{Z_{gen} = Z_0} = \frac{1}{1 + 2j\omega Z_0 C}$$

(1)

Similarly, S_{12} and S_{21} would be equal as the circuit is symmetric.
To compute S_{21},

$$S_{21} = \frac{2V_{out}}{V_{gen}}\Big|_{Z_0 = Z_L = Z_{gen}}$$

(2)

$$V_{out} = \frac{Z_0}{2Z_0 + 1/j\omega C} \cdot V_{gen}$$

$$= \frac{j\omega Z_0 C}{1 + 2j\omega Z_0 C} \cdot V_{gen}$$

$$S_{21} = \frac{2j\omega Z_0 C}{1 + 2j\omega Z_0 C} = S_{12} \quad (\text{or}) \quad \frac{1}{1 + (1/2j\omega Z_0 C)}$$

(2)

$$dB(S_{21})\Big|_{\substack{f = 100\,MHz \\ Z_0 = 50\,\Omega}} = -3\,dB = 20 \log_{10}(|S_{21}|)$$

$$\Rightarrow |S_{21}| = \frac{1}{\sqrt{2}} = \left| \frac{1}{1 + \frac{1}{j\omega(2Z_0)C}} \right|$$

$$\Rightarrow \sqrt{1 + \left(\frac{1}{\omega(2Z_0)C}\right)^2} = \sqrt{2}$$

$$\Rightarrow \frac{1}{\omega(2Z_0)C} = 1 \quad \Rightarrow C = \frac{1}{2\omega Z_0} = \frac{1}{4\pi f Z_0}$$

substituting values,

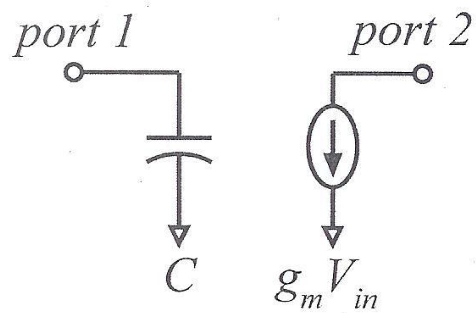
$$C = 1.59 \times 10^{-11}\ F$$

$$\boxed{C = 15.9\ pF}$$ ✓

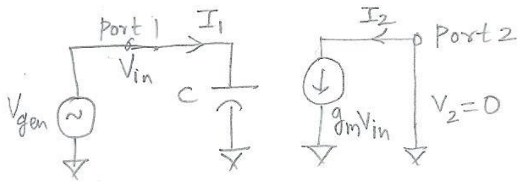
Part b, 15 points

First, compute H21 and S21, both as a function of frequency, for this network.

Second, after assuming that gmZo>>1, find the frequency at which S21 has a magnitude of 1 and compare this to the current-gain cutoff frequency. Please then comment.



$$h_{21} \overset{def}{=} \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



$V_{in} = V_{gen}$

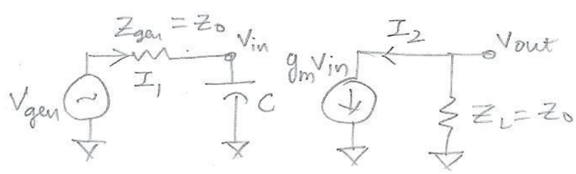
$I_1 = j\omega C V_{in}$

$I_2 = g_m V_{in}$

3

$$\boxed{h_{21} = \left[\frac{g_m}{j\omega C}\right]}$$

$$S_{21} : \left.\frac{2V_{out}}{V_{gen}}\right|_{Z_L=Z_0=Z_{gen}}$$



3

$$V_{in} = \frac{V_{gen}\cdot(1/j\omega C)}{Z_0 + (1/j\omega C)} \Rightarrow \frac{V_{gen}}{1+j\omega Z_0 C}$$

$$V_{out} = -g_m V_{in} Z_0 = \frac{-g_m V_{gen} Z_0}{1+j\omega Z_0 C} \Rightarrow S_{21} = \frac{-2 g_m V_{gen} Z_0}{V_{gen}(1+j\omega Z_0 C)}$$

$$\boxed{S_{21} = \frac{-2 g_m Z_0}{1+j\omega Z_0 C}}$$

4

$$S_{21} = \frac{-1}{(1/2 g_m Z_0) + j\omega\left(\frac{C}{2g_m}\right)}$$

$|S_{21}| = 1$ with $g_m Z_0 >> 1$

$$\Rightarrow \omega^2\left(\frac{C}{2g_m}\right)^2 = 1$$

$\because 1/2g_m Z_0 \to 0$

CONTD--

$$\omega = \frac{2 g_m}{C}$$

3 $$f\Big|_{S_{21}=1} = \frac{2 g_m}{2\pi C} = \boxed{\frac{g_m}{\pi C} = f\Big|_{S_{21}=1}}$$

$$f\Big|_{h_{21}=1} \Rightarrow 1 = \left|\frac{g_m}{j\omega C}\right| \Rightarrow$$ current - Gain - cut-off frequency

$$f_\tau = \boxed{f\Big|_{h_{21}=1} = \frac{g_m}{2\pi C}}$$

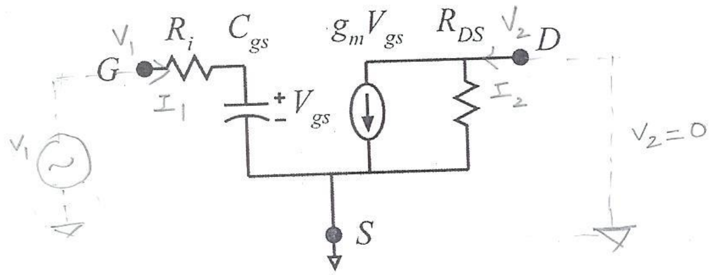
Comparing, the current gain cut off frequency is about half of

2 frequency at which $|S_{21}| = 1$. for $g_m Z_0 >> 1$

Comment: For high gain amplifiers ($g_m Z_0 >> 1$), $f\Big|_{S_{21}=1} \approx 2 \cdot f_\tau$

Part c, 10 points
Ri=10 Ohms, Cgs=1pF, gm=100 mS, Rds=100Ohms.

Calculate Y11 and Y21 at 1 GHz.



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

4 $$V_1 = I_1\left(R_i + \frac{1}{j\omega C_{gs}}\right)$$

$$\boxed{Y_{11} = \frac{j\omega C_{gs}}{1 + j\omega R_i C_{gs}}}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$I_2 = g_m V_{gs}$$

4 $$V_{gs} = \frac{V_1\left(1/j\omega C_{gs}\right)}{R_i + \left(1/j\omega C_{gs}\right)} = \frac{V_1}{1 + j\omega R_i C_{gs}}$$

$$Y_{21} = \frac{g_m V_1}{V_1(1 + j\omega R_i C_{gs})} = \frac{g_m}{1 + j\omega R_i C_{gs}}$$

@ 1GHz $\rightarrow$ $j\omega C_{gs} \rightarrow j(0.0063)$ S
$j\omega R_i C_{gs} \rightarrow j(0.063)$

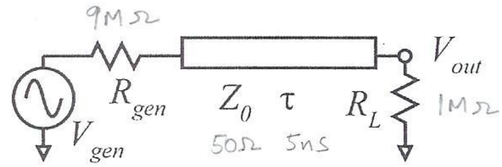
2 $$\left.Y_{11}\right|_{f=1GHz} = \frac{j(0.0063)}{1 + j(0.063)} S \qquad \left.Y_{21}\right|_{f=1GHz} = \frac{0.1}{1 + j(0.063)} S$$

Problem 3, 35 points
Transmission-line theory

Hint: we are testing here your understanding of transmission-lines and their relationships to lumped elements. If the calculation appears to be extremely difficult, you may possibly be missing some key insight.

<u>Part a, 10 points</u>

You are probing a circuit using a 9MOhm oscilloscope probe (Rgen) connected to a 1MOhm oscilloscope through 1 meter of coaxial cable. The cable has 50Ohms characteristic impedance and uses Polyethylene, with a dielectric constant of 2.25, to separate the conductors.

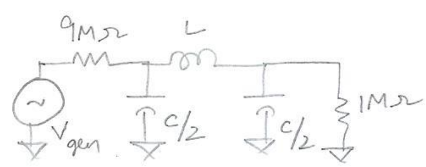


What is the -3dB frequency of this test setup ?

2 ① $v = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8\ m/s$; $\tau = \frac{l}{v} = \frac{1m}{2\times10^8\ m/s} = 0.5\times10^{-8}\ s$

$\boxed{\tau = 5ns}$

② $L = \tau Z_0$ $\qquad$ $C = \frac{\tau}{Z_0}$



$= (5ns)(50\Omega)$ $\qquad$ $= \frac{5ns}{50\Omega}$

$\boxed{L = 250nH}$ $\qquad$ $\boxed{C = 100pF}$

5

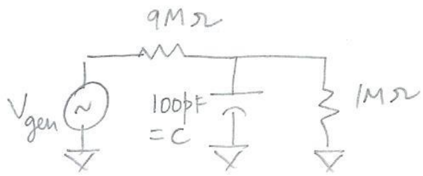
$\tau_{L/R} = \frac{L}{R_{gen} + R_L}$ $\qquad$ $\tau_{RC} = C\left[R_{gen} \| R_L\right]$

$= \frac{250nH}{9M\Omega + 1M\Omega}$ $\qquad$ $= 100pF\left[1M\Omega \| 9M\Omega\right]$

$= 100pF\left[0.9M\Omega\right]$

$\boxed{\tau_{L/R} = 25fs}$ $\qquad$ $\boxed{\tau_{RC} = 90\mu s}$

Comparing, $\tau_{RC} >> \tau_{L/R}$, effect of inductors can be neglected.



(3) This is an RC circuit with $f_{3dB} = \dfrac{1}{2\pi \tau_{RC}}$,

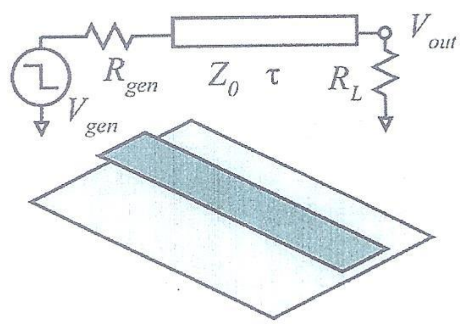
$$f_{3dB} = \frac{1}{2\pi(90\,\mu s)}$$

$$\boxed{f_{3dB} = 1.768\ \text{KHZ}}$$

3

Part b, 10 points

You are connecting a high-current driver to pulse a solid-state laser. The generator is a 1V step-function with 0.1 Ohm output impedance. The load is 0.9 Ohm. Generator and load are connected with a signal conductor (dark grey) of 1cm width and 10 cm length, separated by 1 mm from a ground-plane/ground-return conductor. Find the pulse-response 10%-90% rise time.



Approximate the effect of fringing fields at the edges of the conductors by assuming that the effective microstrip line width is the physical width plus twice the conductor-ground spacing.

① $l = 10cm$; $W = 1cm$; $H = 1mm$; Assuming the same dielectric, $\epsilon_r = 2.25$

② characteristic Impedance of this microstrip Line, $Z_0 = \frac{\eta_0}{\sqrt{\epsilon_r}}\left[\frac{H}{W+2H}\right]$

Given that $W_{eff} = W + 2H$

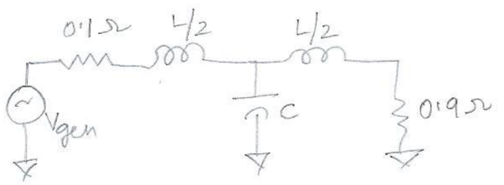
$$Z_0 = \frac{120\pi}{\sqrt{2.25}}\left[\frac{1mm}{1cm+2mm}\right] = \boxed{20.944\,\Omega = Z_0}$$

3

$$\tau = \frac{l}{v} = \frac{10cm}{(c/\sqrt{\epsilon_r})} = 0.1m \times \frac{1.5}{3\times10^8 m/s} = 0.5ns$$

$$\boxed{\tau = 0.5ns}$$

③



3

$$C = \frac{\tau}{Z_0} = \frac{0.5ns}{20.944\Omega} = 23.87pF$$

$$\boxed{C = 23.87pF}$$

$$L = \tau Z_0 = 0.5ns \times 20.944\Omega = 104.72nH$$

$$\boxed{L = 10.472nH}$$

④ $\tau_{RC} = 23.8\,pF\,[0.1 \,||\, 0.9]\,\Omega$; $\tau_{L/R} = \left(\dfrac{10.472\,nH}{1\,\Omega}\right)$

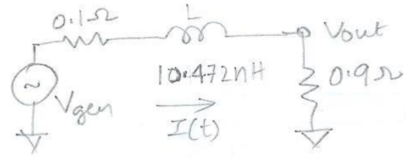
$\boxed{\tau_{RC} = 2.142\ ps}$ $\boxed{\tau_{L/R} = 10.472\ ns}$

$\tau_{L/R} \gg \tau_{RC}$

Effect of capacitors can be neglected.

2

$\Rightarrow$



$I(t) = (1 - e^{-t/\tau_{L/R}})$

$V_{out}(t) = (0.9\Omega)\, I(t)$

$V_{out} = (0.9\Omega)\left[1 - e^{-t/\tau_{L/R}}\right]$

10% $\rightarrow$ 90% Rise Time $\rightarrow$ Time taken for response from $0.09V \rightarrow 0.81V$
(t_1) (t_2)

$0.09 = 0.9\,(1 - e^{-t_1/\tau_{L/R}})$

$0.81 = 0.9\,(1 - e^{-t_2/\tau_{L/R}})$

$\Rightarrow$ $0.9\,e^{-t_1/\tau_{L/R}} = 0.81$

$0.9\,e^{-t_2/\tau_{L/R}} = 0.09$

$e^{-(t_1 - t_2)/\tau_{L/R}} = 9$

2

Solving $\boxed{\text{Rise Time} \approx 23.0\ ns}$

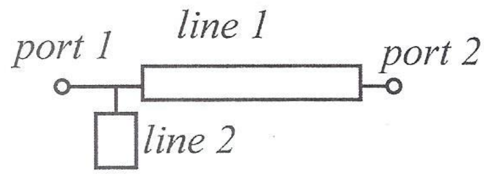
Alternately, Rise Time $= 2.2\,\tau_{L/R}$

Part c, 15 points

You are working with a Duriod board (dielectric constant of 2.4), 0.5 mm thick. Line 1 is 2 mm wide and 2 cm long. LIne 2 is 2 mm wide and 5 mm long.

First: find the characteristic impedance and propagation delay of both lines.

Second: assuming that the lines are both short in comparison with a quarter-wavelenght, draw a lumped-element equivalent circuit, calculating all element values.



Approximate the effect of fringing fields at the edges of the conductors by assuming that the effective microstrip line width is the physical width plus twice the conductor-ground spacing.

FIRST; Line 1 → $W = 2mm$; $L = 2cm$; $H = 0.5mm$; $\epsilon_r = 2.4$

Again, $$Z_0 = \frac{\eta_0}{\sqrt{\epsilon_r}}\left[\frac{H}{W+2H}\right] = \frac{120\pi}{\sqrt{2.4}}\left[\frac{0.5\,mm}{2mm+1mm}\right]$$

$$Z_0 = 40.55\,\Omega$$

$$\tau_1 = \frac{l_1}{v} = \frac{2cm \cdot \sqrt{\epsilon_r}}{c} = \frac{2cm \cdot \sqrt{2.4}}{3\times10^8\,m/s} = 103.28\,ps$$

$$\tau_1 = 103.28\,ps$$

2

(b) Line 2; similar to Line 1, $Z_0 = 40.55\,\Omega$

$$\tau_2 = \frac{l_2}{v} = \frac{5mm \cdot \sqrt{2.4}}{3\times10^8\,m/s} = 25.81\,ps$$

2

SECOND; Line 1 → $L_1 = \tau_1 Z_0$

$$= (103.28\,ps)(40.55\,\Omega)$$

$$\boxed{L_1 \approx 4.19\,nH}$$

$$C_1 = \frac{\tau_1}{Z_0} = \frac{103.28\,ps}{40.55\,\Omega}$$

$$\boxed{C_1 = 2.55\,pF}$$

4

Line 2; $L_2 = \tau_2 Z_0$

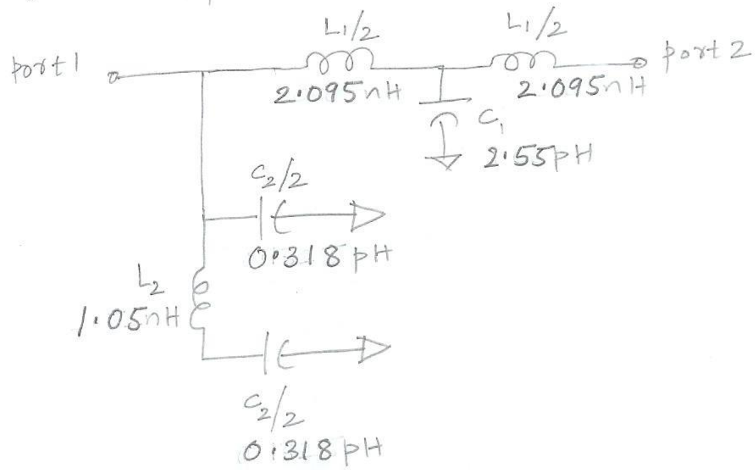
$$= (25.81\,ps)(40.55\,\Omega)$$

$$\boxed{L_2 = 1.05\,nH}$$

$$C_2 = \frac{\tau_2}{Z_0} = \frac{25.81\,ps}{40.55\,\Omega}$$

$$\boxed{C_2 = 0.636\,pF}$$

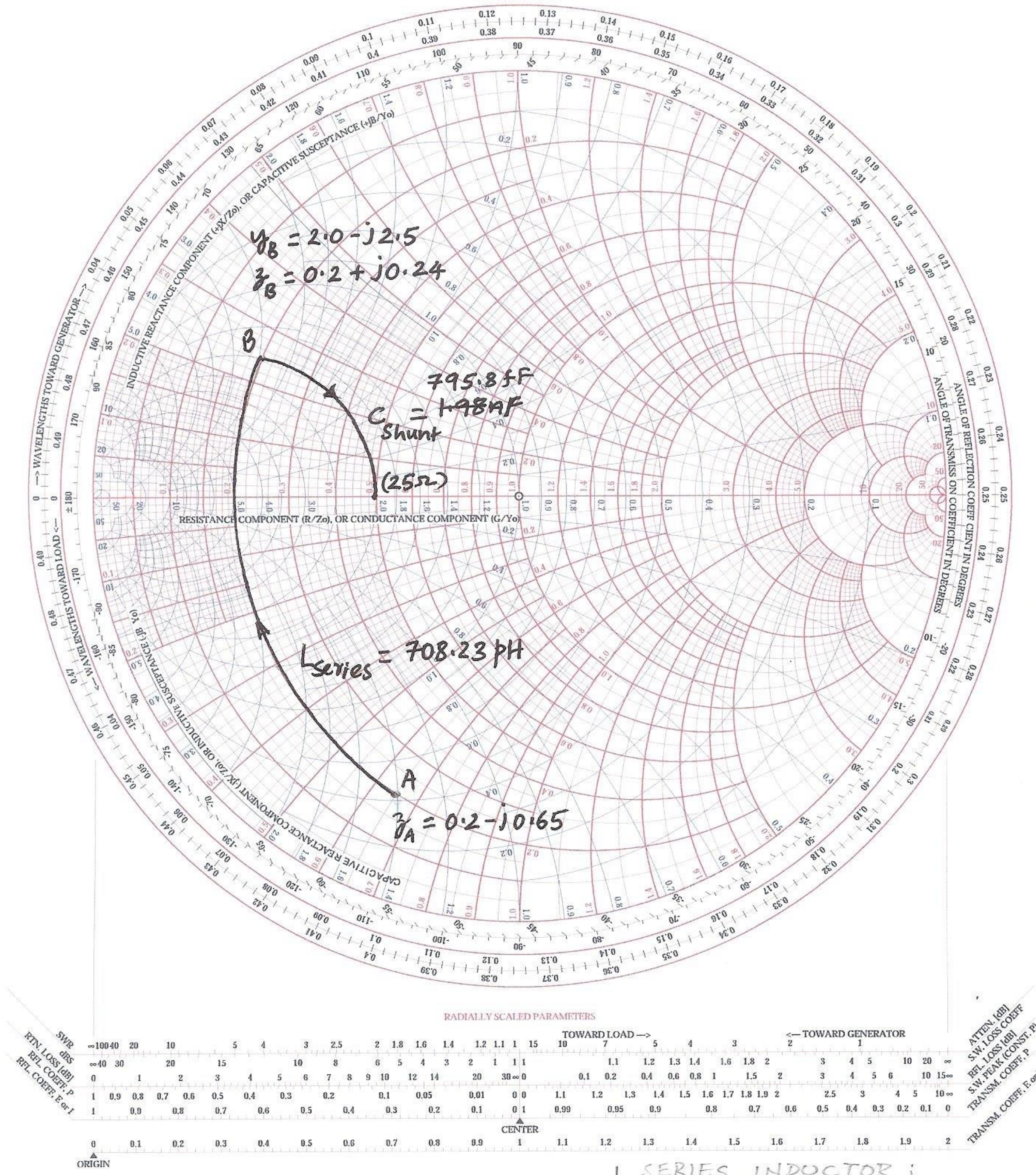
Drawing the lumped equivalent,



2

Problem 3, 15 points
Impedance-matching exercise.

The (50 OHm normalization) Smith chart gives the input impedance of a circuit at 10 GHz signal frequency. Design a lumped-element matching network which converts this impedance to **25Ohms** at 10 GHz. Give all elment values.



8 ~~10~~ PTS FOR CHART

$z_A = 0.2 - j0.65$

$Z_A = 50\Omega\,(0.2 - j0.65)$

$= [10 - j32.5]\ \Omega$

SERIES INDUCTOR :

$z_B = 0.2 + j0.24$

$z_L = j0.89$

$Z_L = j44.5 = j\omega L$

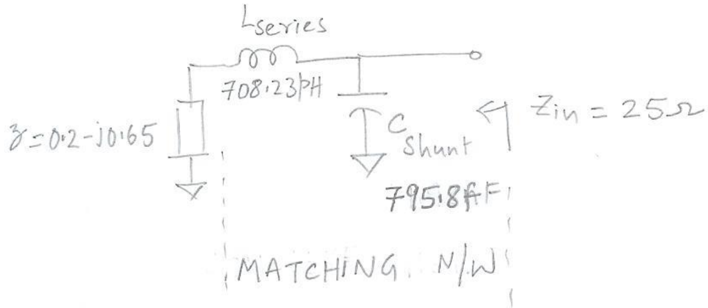
with $\omega = 2\pi(10\,GHz)$, $\boxed{L_{ser} = 708.23\,pH}$

3 (FIRST ELEMENT)

$y_B = 2.0 - j2.5$

Shunt capacitor $\quad y_c = j2.5 \Rightarrow z_c = \frac{1}{j2.5} \Rightarrow Z_c = \frac{50}{j2.5} = \frac{1}{j\omega C_{shunt}}$ 3 (SECOND ELEMENT)

$$C_{shunt} = \frac{2.5 \times 50}{2\pi(10GHz)\times 50} = 795.8\, fF$$



1 (CIRCUIT)