ECE ECE145A (undergrad) and ECE218A (graduate)

Mid-Term Exam. November 8, 2022

Do not open exam until instructed to.

Open notes, open books, etc.

You have 1 hour and 15 minutes.

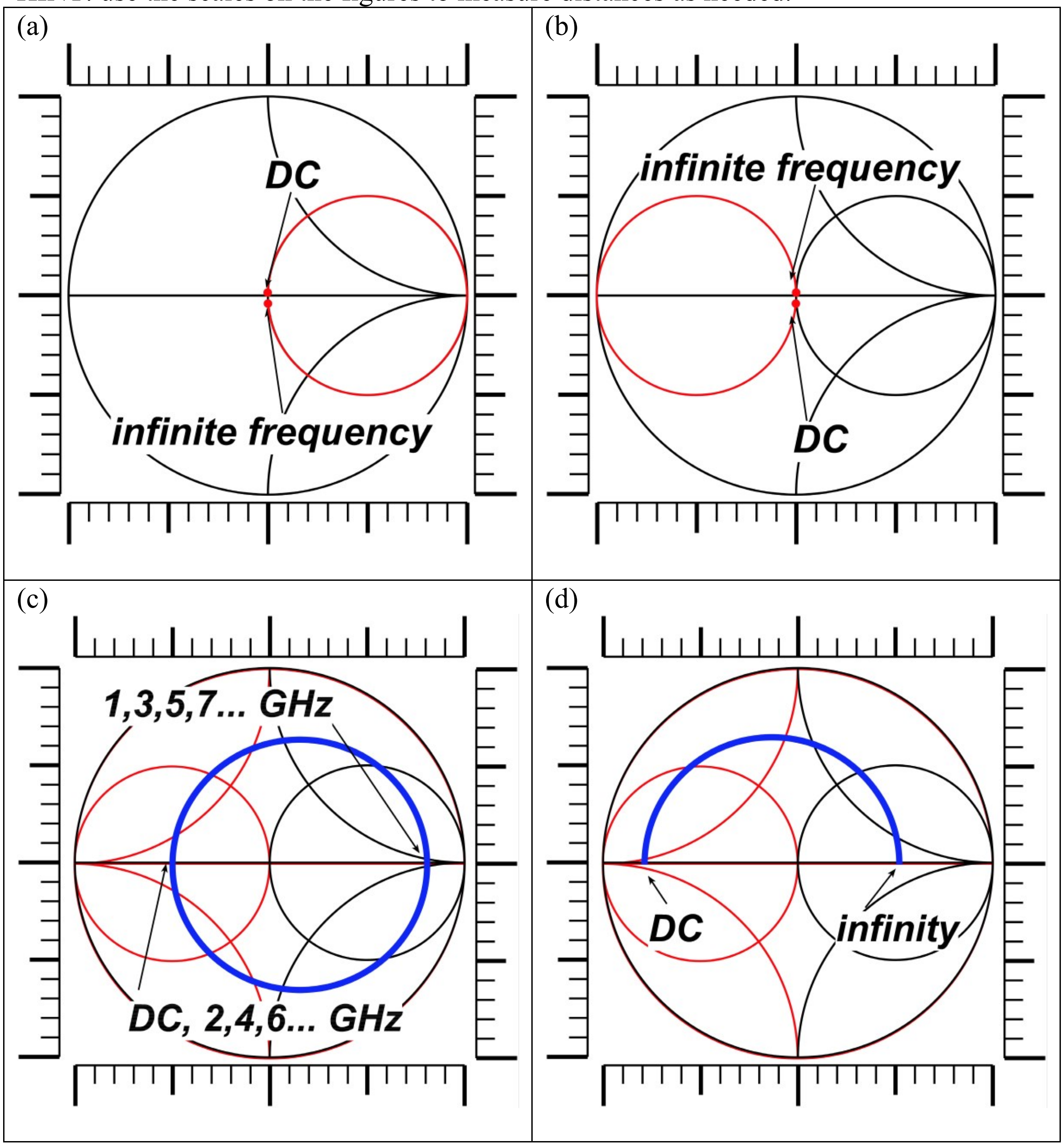
Use any and all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***

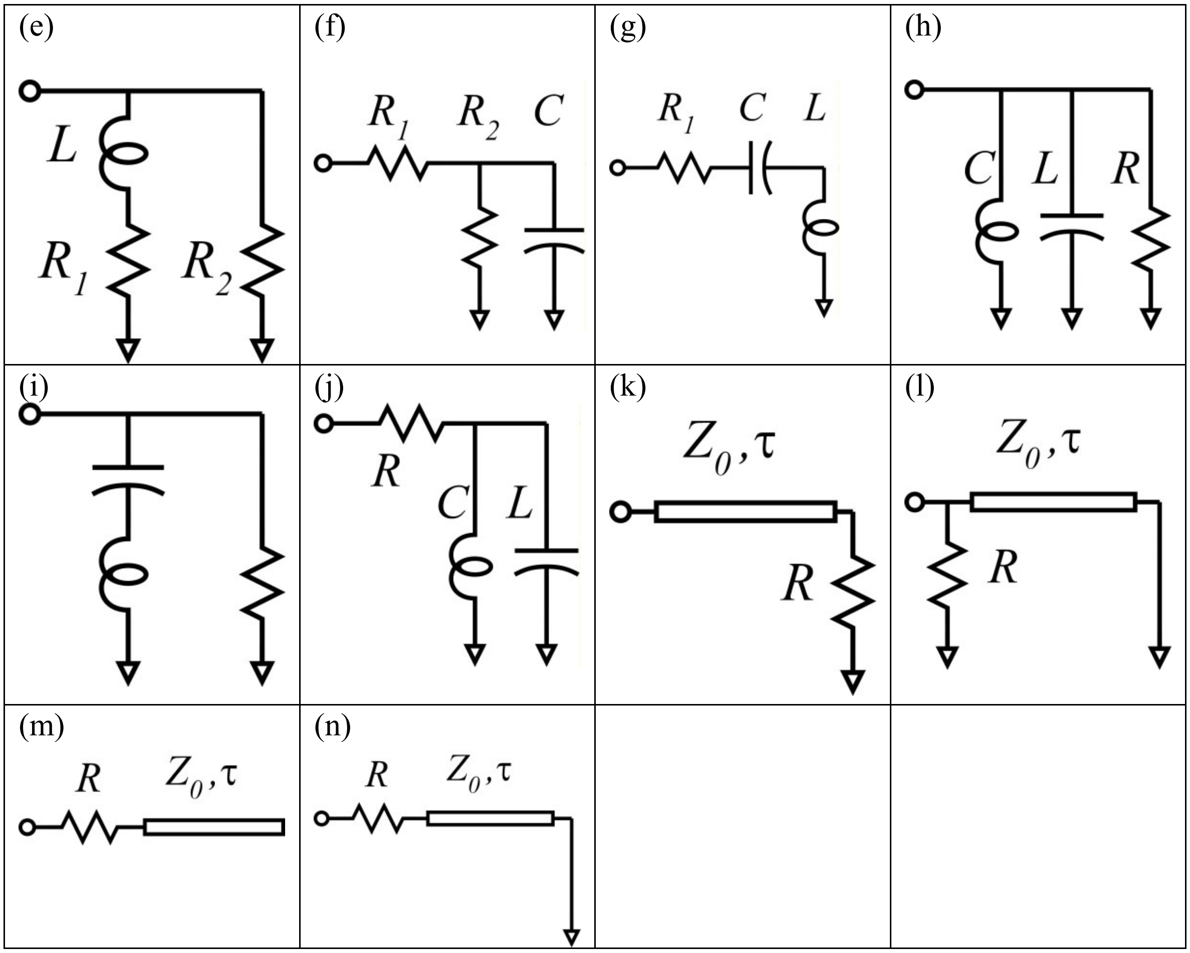
Problem	Points Received	Points Possible
1		15
2a		10
2b		7
2c		8
2d (218 only)		10 (218A only)
3a		5
3b		5
3c		7.5
3d		7.5
4		15
5a		10
5b (218 only)		15 (218A only)
6		10
total		100 (145), 125 (218A)

Name: Solution

Problem 1, 15 points
The Smith Chart and Frequency-Dependent Impedances.

HINT: use the scales on the figures to measure distances as needed.




(e)
L
R1
R2
(f)
R1
R2
C
(g)
R1
C
L
(h)
C
L
R
(i)
(j)
R
C
L
(k)
Z0, τ
R
(l)
Z0, τ
R
(m)
R
Z0, τ
(n)
R
Z0, τ

.

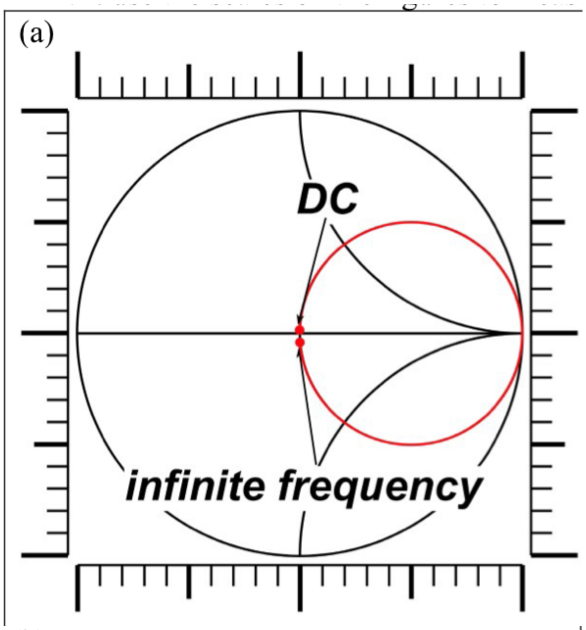
First match each Smith Chart with each circuit. ***Then determine as many component values as is possible*** (RLC values, transmission line delays and characteristic impedances)...note that some values cannot be determined with the information given. The charts all use 50 Ohm normalization:

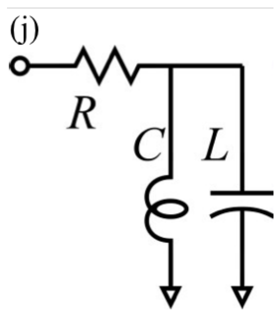
Smith chart (a). Circuit= j .
Component values: $R = 50\Omega$, __________ , __________ ,

Smith chart (b). Circuit= i .
Component values: $R = 50\Omega$, __________ , __________ ,

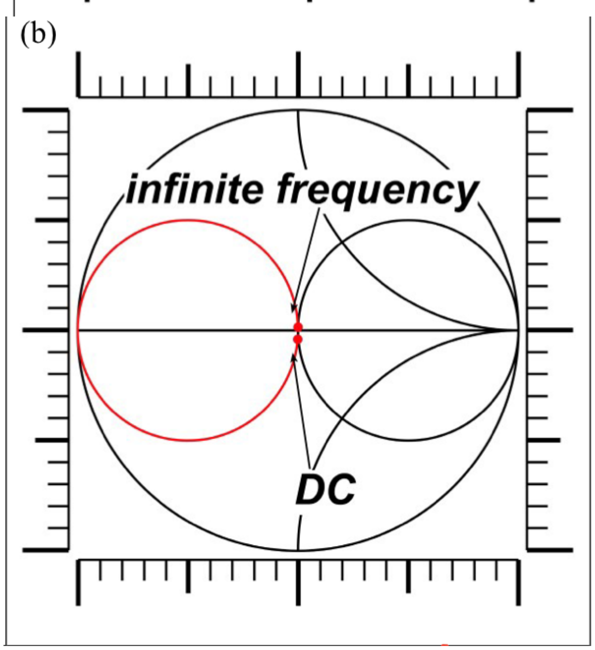
Smith chart (c). Circuit= k .
Component values: $R = 450\Omega$, $\tau = 250ps$, $Z_{line} = 50\Omega \cdot \sqrt{3}$,

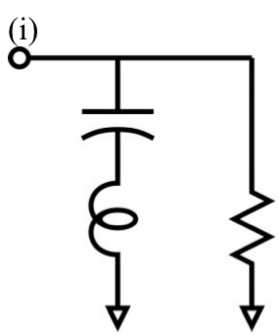
Smith chart (d). Circuit= d .
Component values: $R_1 \approx 22.5\Omega$ $R_2 = 450\Omega$, __________ ,





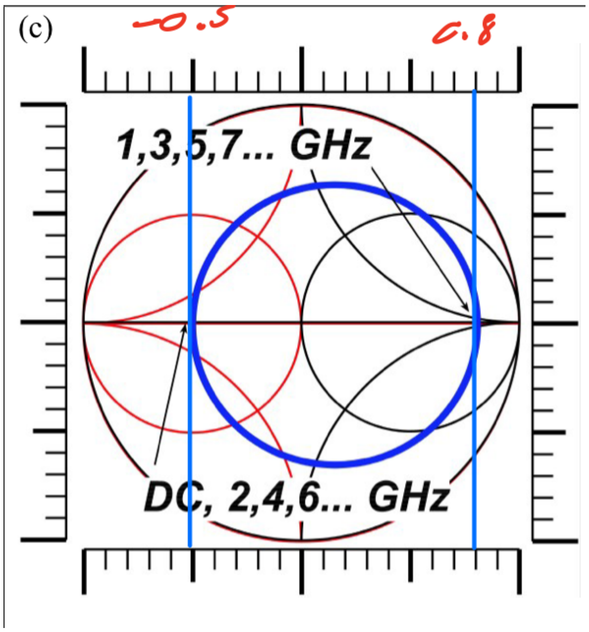
a) 5pts for correctly recognizing the circuit.
2.5pts for resistor value.





b) 5pts for correctly recognizing the circuit.
2.5pts for resistor value.

1/2 pt.

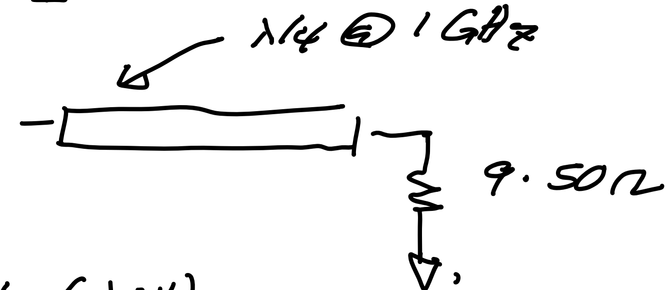


$\Gamma = -0.5 \rightarrow$

$z = \frac{1+\Gamma}{1-\Gamma} = \frac{0.5}{1.5} = 1/3$

$\Gamma = 0.8$

$z = \frac{1+\Gamma}{1-\Gamma} = \frac{1.8}{0.2} = 9$



Z_{in} @ 1 GHz ($\lambda/4$) $= \frac{50\Omega}{3}$

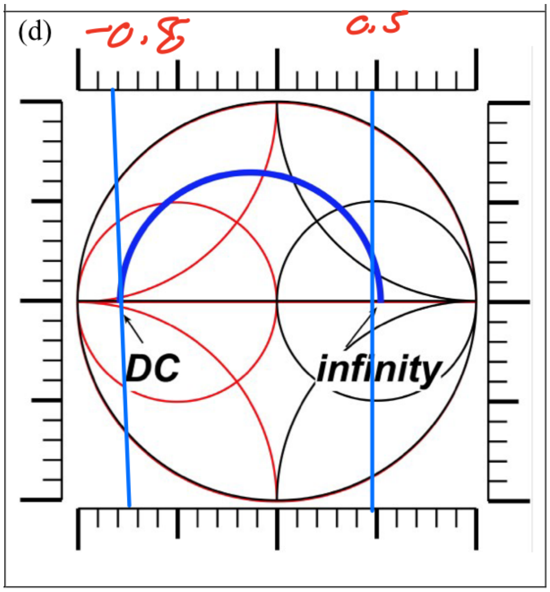
2 $\rightarrow$ $Z_0|_{line} = \sqrt{Z_{in} Z_L} = \sqrt{\frac{50\Omega}{3} \cdot 9 \cdot 50\Omega}$

$= 50\Omega \cdot \sqrt{3}$

1 line is $\lambda/4$ @ 1 GHz

line is λ @ 4 GHz $\longrightarrow$ $\tau = 250\text{ps}$

1

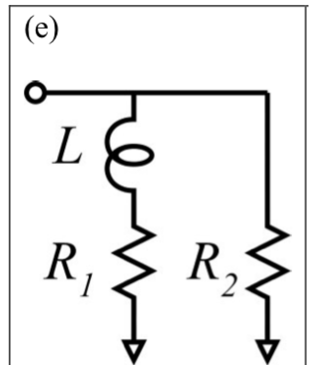


$\Gamma = 0.5$

$\rightarrow z = \frac{1+\Gamma}{1-\Gamma} = \frac{1.5}{0.5} = 3 \rightarrow 150\Omega$

$\Gamma = -0.8$

$z = \frac{1+\Gamma}{1-\Gamma} = \frac{1.8}{0.2} = 9 \rightarrow 450\Omega.$



1.5

$\rightarrow R_2 = 450\Omega$

$R_1 \parallel R_2 = 150\Omega$

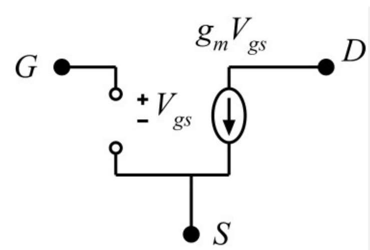
1

$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{450\Omega} = \frac{1}{150\Omega}$

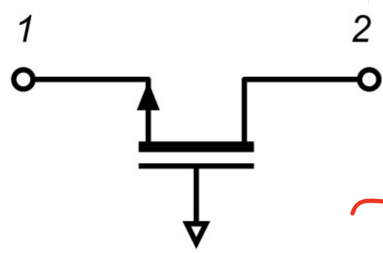
$\frac{1}{R_1} = \frac{1}{150\Omega} - \frac{1}{450\Omega} \rightarrow R_1 = 225\Omega$

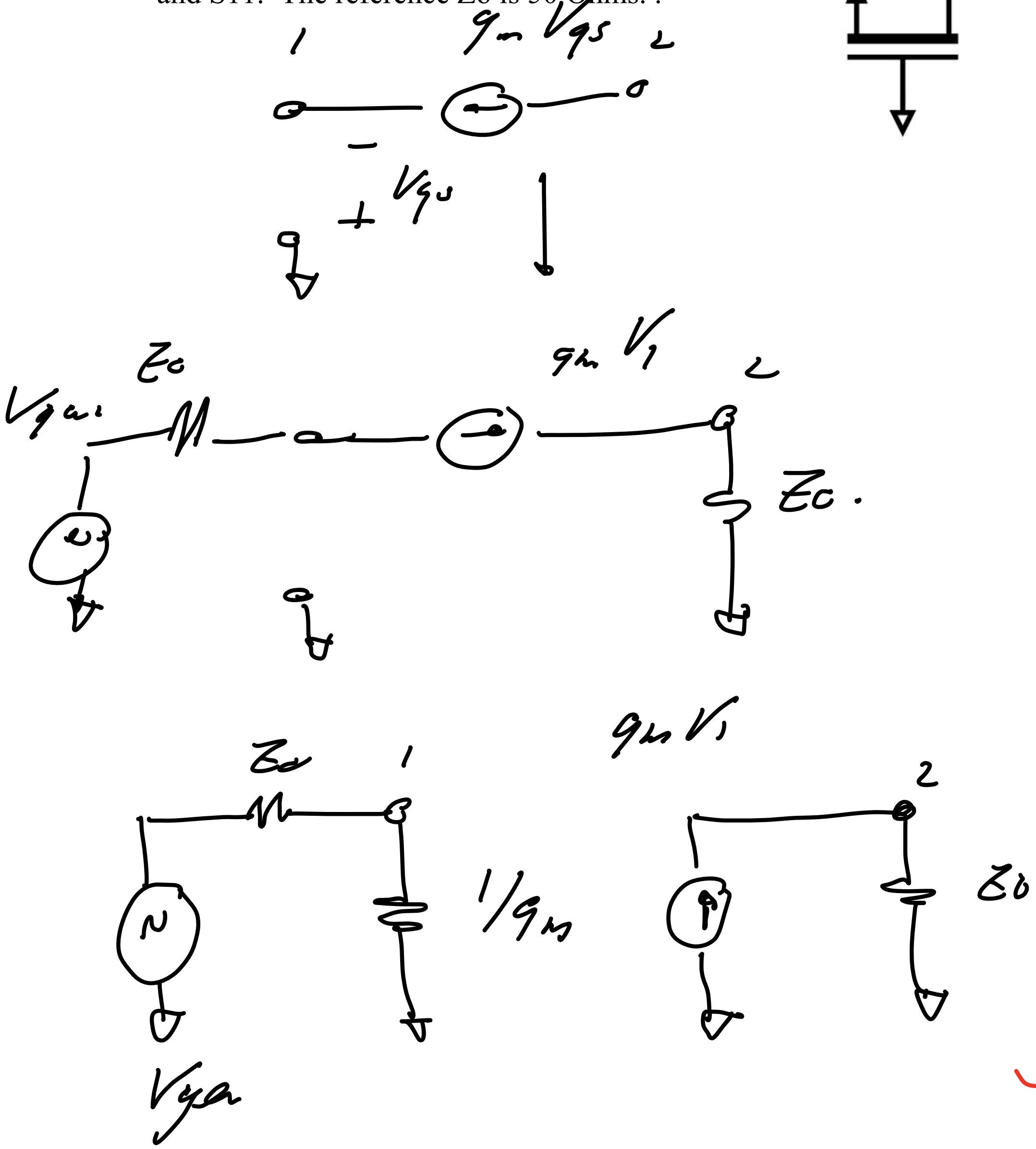
Problem 2, 25 points (ece145A), 35 points (ece218A)
2-port parameters and Transistor models

Part a, 10 points
At the right is the equivalent circuit for a FET. The transconductance gm is 1 mS.



Now, given this model, for the network at the right, give the numerical values of S21 and S11. The reference Zo is 50 Ohms. .





4

2 $Z_{in} = 1/g_m = 1000\Omega \quad 50$

2 $S_{11} = \dfrac{1000/50 - 1}{1000/50 + 1} = \dfrac{20-1}{20+1} = 19/21 = S_{11}$

$V_1 = V_{gen} \cdot \dfrac{1/g_m}{1/g_m + Z_0}$

$V_2 = g_m Z_0 \cdot V_1$

$\dfrac{V_2}{V_{gen}} = \dfrac{Z_0}{1/g_m + Z_0}$

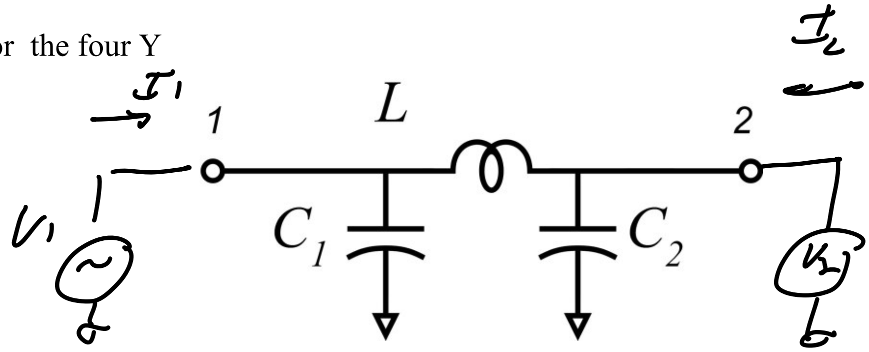
2 $S_{21} = \dfrac{2V_0}{V_g}\Big|_{Z_g = Z_L = Z_0} = \dfrac{2Z_0}{1/g_m + Z_0}$

$= \dfrac{100\,\Omega}{1000\,\Omega + 50\,\Omega}$

$S_{21} = \dfrac{100}{1050}$

Part b, 7 points

Derive algebaic expressions for the four Y parameters for this network

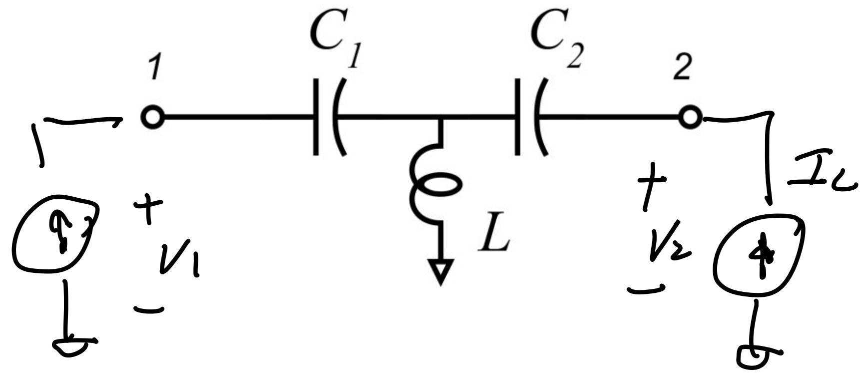


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} j\omega C_1 + \frac{1}{j\omega L} & -1/j\omega L \\ -1/j\omega L & j\omega C_2 + \frac{1}{j\omega L} \end{bmatrix}}_{Y_{ij}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

1.75 pts each

Part c, 8 points

Derive algebaic expressions for the four Z parameters for this network



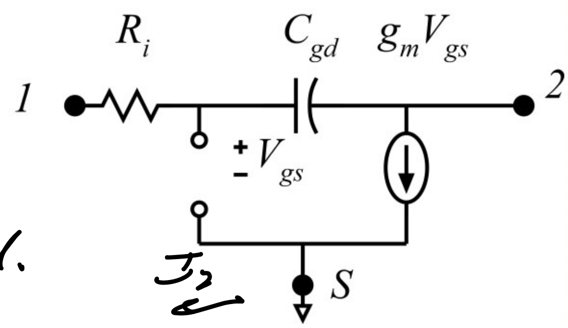
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega C_1} + j\omega L & j\omega L \\ j\omega L & \frac{1}{j\omega C_2} + j\omega L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

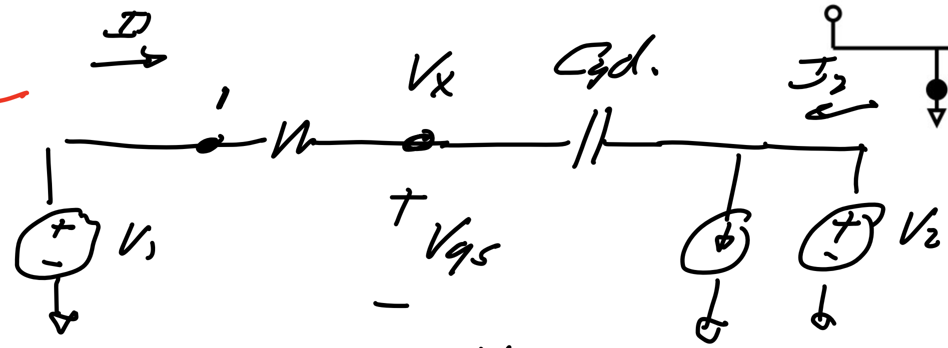
Z_{ij}

2 pts each

Part d, ***ECE218A students only*** 10 points

Compute the Y parameters for network, to second order in $j\omega C_{gd} R_i$. The Taylor series expansion $(1+\varepsilon)^{-1} = 1 - \varepsilon + \varepsilon^2 + O(\varepsilon^3)$ may be useful





2

$$V_x = V_1 \frac{1/j\omega C}{R + 1/j\omega C} + V_2 \frac{R}{R + 1/j\omega C}$$

2

$$= \frac{V_1}{1 + j\omega RC} + \frac{V_2 \, j\omega RC}{1 + j\omega RC}$$

$$= \frac{V_1}{1+\alpha} + \frac{V_2 \, \alpha}{1+\alpha} \qquad \text{where } \alpha = j\omega RC$$

2

$$\cong V_1(1 - \alpha + \alpha^2) + V_2 \alpha (1 - \alpha + \alpha^2)$$

$$\cong V_1(1 - \alpha + \alpha^2) + V_2(\alpha - \alpha^2)$$

$$I_1 = (V_1 - V_x) G \qquad \text{where } G = 1/R_i$$

$$R I_1 = V_1 - V_1(1 - \alpha + \alpha^2) - V_2(\alpha - \alpha^2)$$

$$= V_1(\alpha - \alpha^2) - V_2(\alpha - \alpha^2)$$

$$I_1 = V_1 \cdot \frac{1}{R}\left(j\omega RC + \omega^2 R^2 C^2\right) - V_2 \frac{1}{R}\left(j\omega RC + \omega^2 R^2 C^2\right)$$

$$= V_1 \underbrace{\left[j\omega C + \omega^2 R C^2\right]}_{Y_{11}} + V_2 \underbrace{\left[-j\omega C - \omega^2 R C^2\right]}_{Y_{12}}$$

2

$$I_2 = g_m V_x + (V_2 - V_x) j\omega C$$

$$= j\omega C V_2 + (g_m - j\omega C) V_x$$

$$I_2 - j\omega C V_2 = (g_m - j\omega C)\cdot\left(V_1(1-\alpha+\alpha^2) + V_2(\alpha - \alpha^2)\right)$$

$$I_2 = V_1 g_m (1 - j\omega C/g_m)\cdot(1 - j\omega RC + \omega^2 R^2 C^2) + V_2\left[j\omega C + (g_m - j\omega C)(j\omega RC + \omega^2 R^2 C^2)\right]$$

$$= V_1 \underbrace{g_m (1 - j\omega C/g_m)\cdot(1 - j\omega RC + \omega^2 R^2 C^2)}_{Y_{21}} + V_2 \underbrace{\left[j\omega C + j\omega RC g_m + \omega^2 R^2 C^2 g_m + \omega^2 C^2 R\right]}_{Y_{22}}$$

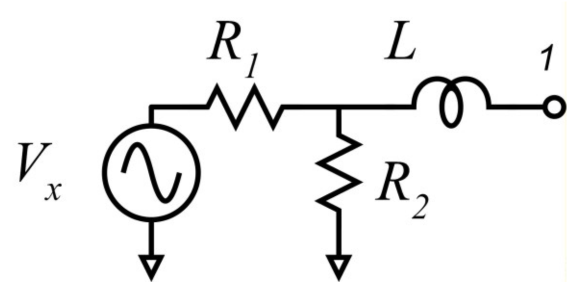
2

* note I have dropped $j\omega C(\omega^2 R^2 C^2)$ term, as this is 3rd order

Problem 3, 25 points (ECE145A), 25 points (ECE 218A)
Available source power relationships, lumped/distributed relationships.

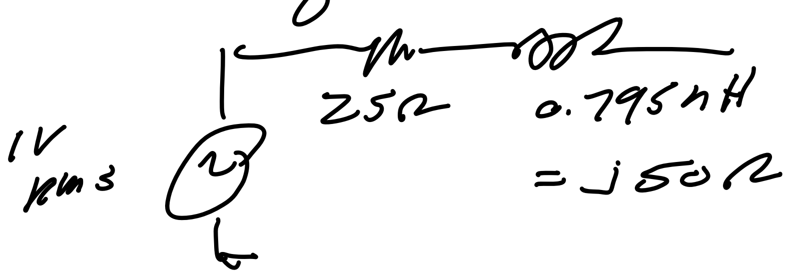
Part a, 5 points
Vs is 2V RMS at 10GHz. R1 and R2 are both 50 Ohms, L is 0.795 nH.

At 10GHz, what is the available signal power ? Draw the circuit diagram of a load network, with element values specified, that would, when connected to the source, absorb this amount of power from the generator.



Thevenin equivalent:

2



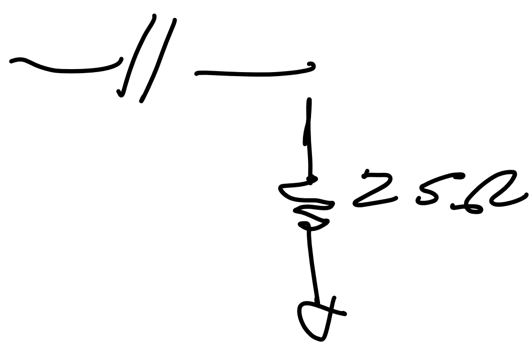
1

$$P_{avg} = \frac{(1V)^2}{4(25\Omega)} = \frac{(1V)^2}{100\Omega} = 10\,mW.$$

optimum load

$$-j50\Omega = \frac{1}{j\omega C} \rightarrow C = 0.32\ pF$$

2



Part b, 5 points

A coaxial cable has 50 Ohms characteristic impedance, is 10 meters long, and the insulating dielectric has a dielectric constant of 2.0.

a) What is the total capacitance of the cable ?

b) What is the total inductance of the cable ?

$$v = \frac{3 \cdot 10^8 \, m/s}{\sqrt{2}}$$

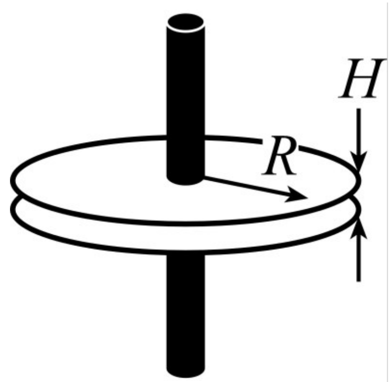
1 $$\tau = \frac{10\,m}{3 \cdot 10^8\,m/s} \cdot \sqrt{2} = 47.1\,ns$$

2 $$L = Z_0 \cdot \tau = 2.36\,\mu H$$

2 $$C = \tau / Z_0 = 943\,pF$$

Part c, 7.5 points

A capacitor has round plates of radius R=1 cm, and separation H = 0.1 mm. Between the plates is an insulator whose dielectric constant is 100.



a) What is the capacitor's impedance at 1 kHz ?

b) At what frequencies is the impedance infinity Ohms ?

c) At what frequencies is the impedance zero Ohms ?

Area = A = πR^2 100 1 cm

2 a) $C = \frac{\varepsilon_r \varepsilon_0 \cdot A}{D} = \frac{\varepsilon_r \varepsilon_0}{D} \cdot \pi R^2 = 2.78\ nF$

0.1 mm

B) speed of light delay, center–edge =

0.5 $\tau = \frac{1\ cm}{3 \cdot 10^8\ m/s} \cdot \sqrt{100} = \frac{1}{3}\ ns = \frac{1}{3\ GHz}$

$Z \to 0$ when $\ell = 0\lambda, \lambda/2, 2\lambda/2, 3\lambda/2, \ldots = n(\lambda/2)$

2.5 DC, $\frac{1}{2} \cdot 3GHz, \frac{2}{2} \cdot 3GHz, \frac{3}{2} \cdot 3GHz \ldots$

c) $Z \to 0$ when $\lambda/4, 3\lambda/4, \ldots$

2.5 $\to \frac{3}{4}\ GHz, 3 \cdot \frac{3}{4}\ GHz, 5 \cdot \frac{3}{4}\ GHz \ldots$

1

$v \approx c = 3 \cdot 10^8$ m/s

$Z_0 = 377\Omega \cdot \dfrac{1\,mm}{1\,mm + 5\,mm} = \dfrac{377}{6}\,\Omega$

$L = 10\,cm$

$\tau = \dfrac{10\,cm}{3\cdot 10^8\,m/s} = \dfrac{10^{-2}\,m}{3\cdot 10^8\,m/s} = \dfrac{1}{3}\cdot 10^{-10}\,sec$

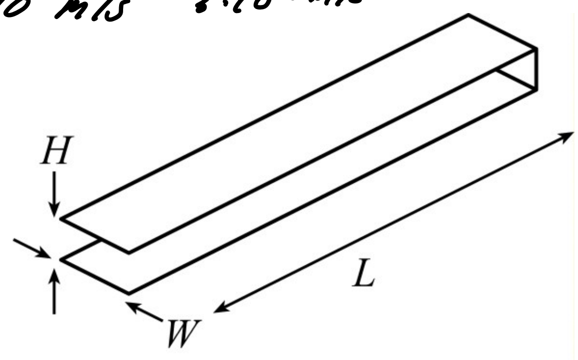
Part d, 7.5 points

Using the approximate formula for transmission-line characteristic impedance, $Z_0 \simeq \dfrac{377\Omega}{\sqrt{\varepsilon_r}} \dfrac{H}{H+W}$, if H=1 mm, W=5 mm, and L=10 cm, we have two metal plates that are short-circuited at a distance L from the drive point.

a) what is the approximate inductance between the two ends of the wire ?

b) At what frequencies is the impedance infinity Ohms ?

c) At what frequencies is the impedance zero Ohms ?



a) $L = Z_0 \cdot \tau = \dfrac{377\Omega}{6} \cdot \dfrac{1}{3} \cdot 10^{-10}\,sec = \dfrac{377}{18} \cdot 10^{-10}\,H$

1

$= \dfrac{377}{180}\,nH = 2.09\,nH$

1 $1/\tau = \dfrac{10^{10}}{3}\,Hz = \dfrac{10}{3}\,GHz.$

c) $Z \rightarrow 0$ when $L = 0\cdot\frac{\lambda}{2}, 2\cdot\frac{\lambda}{2}, 3\cdot\frac{\lambda}{2}, \ldots n\cdot\frac{\lambda}{2}$

2

$f = dc, \frac{5}{3}\,GHz, 2\cdot\frac{5}{3}\,GHz, 3\cdot\frac{5}{3}\,GHz, \ldots$

d) $Z \rightarrow \infty$ when $L = 1\cdot\frac{\lambda}{4}, 3\cdot\frac{\lambda}{4}, 5\cdot\frac{\lambda}{4}, \ldots$

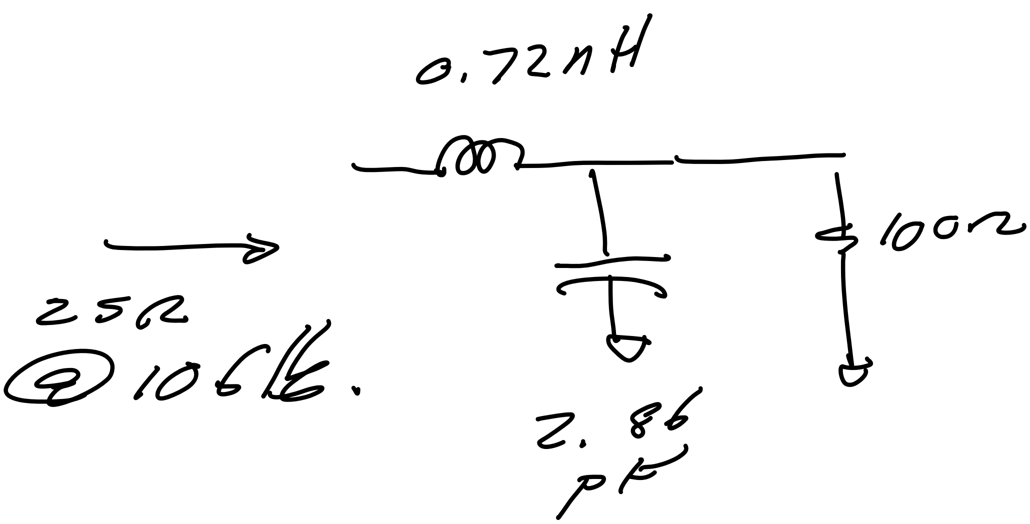
2.5

$f = \frac{5}{6}\,GHz, 3\cdot\frac{5}{6}\,GHz, 5\cdot\frac{5}{6}\,GHz \ldots$

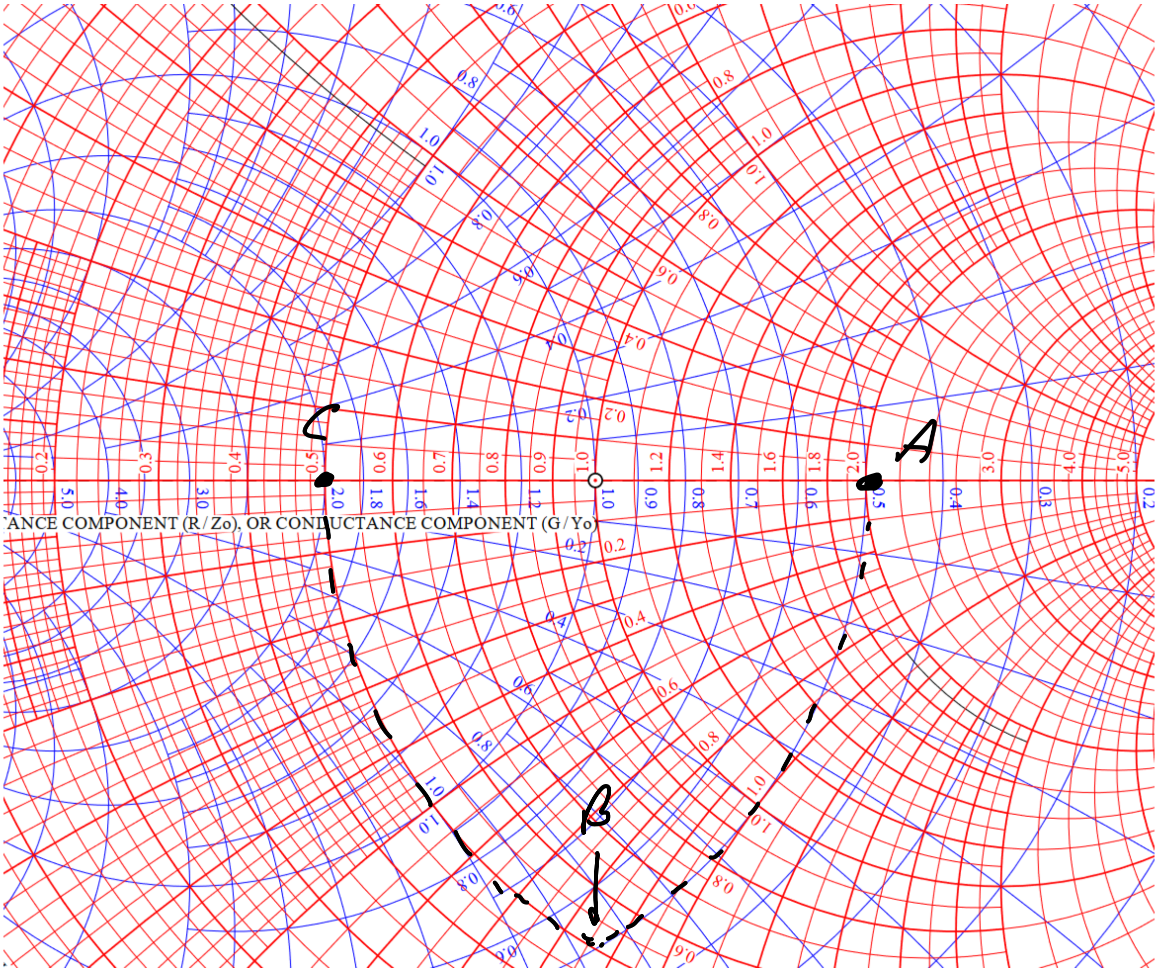
Problem 4, 15 points
Impedance-matching exercise.

At 10GHz signal frequency, an antenna has an input impedance of 100+j0 Ohms. Design a matching network, using a series inductor and a shunt capacitor, which matches this impedance to 25 Ohms. Use a Smith chart with 50 Ohms impedance normalization

Give all element values. Either use a separate impedance-admittance chart , or use the attached one below..



50 Ω chart



2 [A) $z = 2 + j0$; $y = \frac{1}{2} + j0$

2 [b) $y = \frac{1}{2} + j0.9$

4 [$\Delta y = j0.9$

$\Delta Y = \frac{j0.9}{50\Omega} = j\omega C$

$C = \frac{0.9}{50\Omega \cdot 2\pi \cdot 10\,GHz} = 2{,}86\,pF$

2 [b) $z = 0.5 - j0.9$

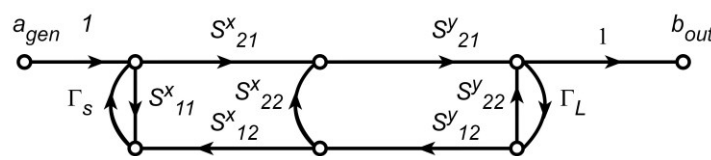
2 [c) $z = 0.5 + j0$ $\quad \Delta z = j0.9$

3 [$\Delta Z = j0.9 \cdot 50\Omega = j\omega L \rightarrow L = \frac{0.9 \cdot 50\Omega}{2\pi(10\,GHz)}$

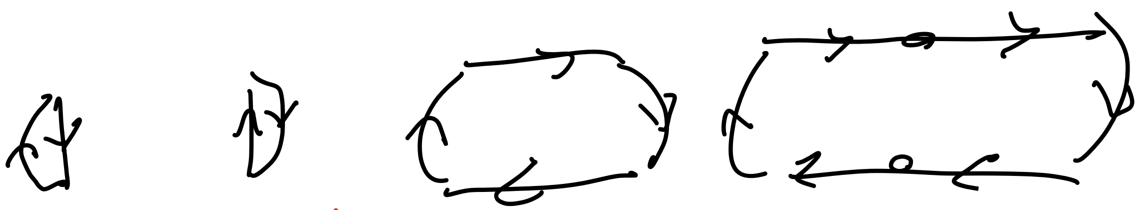
$= 0.72\,nH.$

Problem 5, 10 points (ece145A), 25 points (218A)
Signal flow graphs

Part a, 10 points
Find b_{out}/a_{gen} for this network.



Loops



$$T = \frac{S^x_{21} S^y_{21}}{D} \quad \text{where}$$

$$D = 1 - \Gamma_S S^x_{11} - \Gamma_L S^y_{22} - \Gamma_S S^x_{21} S^y_{21} \Gamma_L S^y_{12} S^x_{12} - S^x_{22} S^y_{21} \Gamma_L S^y_{12}$$

1st order

$$+ \Gamma_S S^x_{11} S^y_{22} \Gamma_L + \Gamma_S S^x_{11} S^x_{22} S^y_{21} \Gamma_L S^y_{12}$$

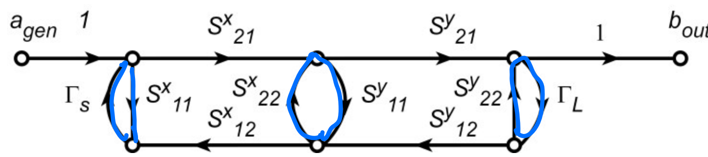
2nd order

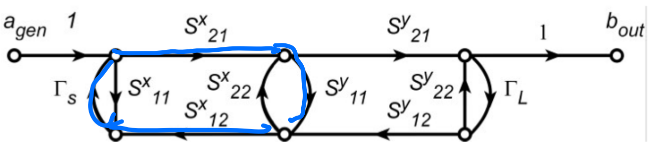
$$T = \frac{S_{21}^{X} S_{21}^{Y}}{D} \quad \text{where } D = \ldots$$

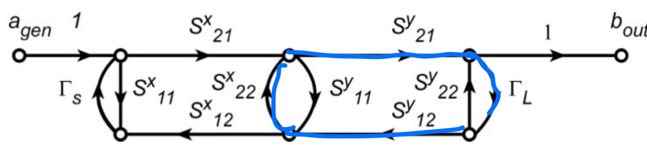
1

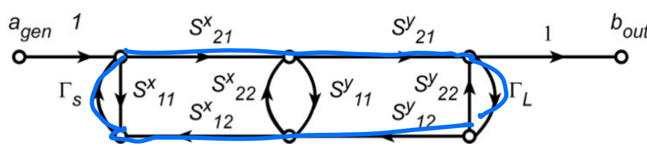
Part b, (218A only) 15 points

Find b_{out}/a_{gen} for this network.









$$D = 1$$

$$- \Gamma_s S_{11}^{X} - S_{22}^{X} S_{11}^{Y} - S_{22}^{Y} \Gamma_L$$
$$- \Gamma_s S_{21}^{X} S_{11}^{Y} S_{12}^{X} - S_{22}^{X} S_{21}^{Y} \Gamma_L S_{12}^{Y}$$
$$- S_{21}^{X} S_{21}^{Y} \Gamma_L S_{12}^{Y} S_{12}^{X} \Gamma_s$$

1st order
1pt each.

2nd

$$+ \Gamma_s S_{11}^{X} S_{22}^{X} S_{11}^{Y} + \Gamma_s S_{11}^{X} S_{22}^{Y} \Gamma_L + S_{22}^{Y} \Gamma_L S_{22}^{X} S_{11}^{Y}$$
$$+ \Gamma_s S_{11}^{X} S_{22}^{X} S_{21}^{Y} \Gamma_L S_{12}^{Y} + S_{22}^{Y} \Gamma_L \Gamma_s S_{21}^{X} S_{11}^{Y} S_{12}^{X}$$

1pt each

$$- \Gamma_s S_{11}^{X} S_{22}^{X} S_{11}^{Y} S_{22}^{Y} \Gamma_L$$

3rd
3pts

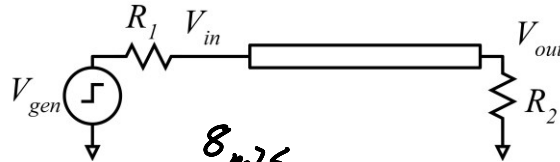
2 $\left[T_s = 1/2 \quad \Gamma_s = 0 \right.$

$\Gamma_L = -1$

Problem 6, 10 points

Transmission lines in the time domain.

Vgen is a 1V step-function occurring at t=0 seconds. Zline is 50 Ohms. The line is 2 meters long and has a diectric constant of 4.0.



R2 is zero Ohms
R1 is 50 Ohms.

Plot Vin (t) on the graph below.

2 $\left[v = \frac{3 \cdot 10^8 \, m/s}{2} \right.$

$\tau = \frac{2m}{v} = \frac{2 \, m}{1.5 \cdot 10^8 \, m/s} = \frac{4}{3} \cdot 10^{-8} \, sec = \underline{\underline{13.3 \, ns}}$