

# Fault-Tolerant Computing

Hardware  
Design  
Methods

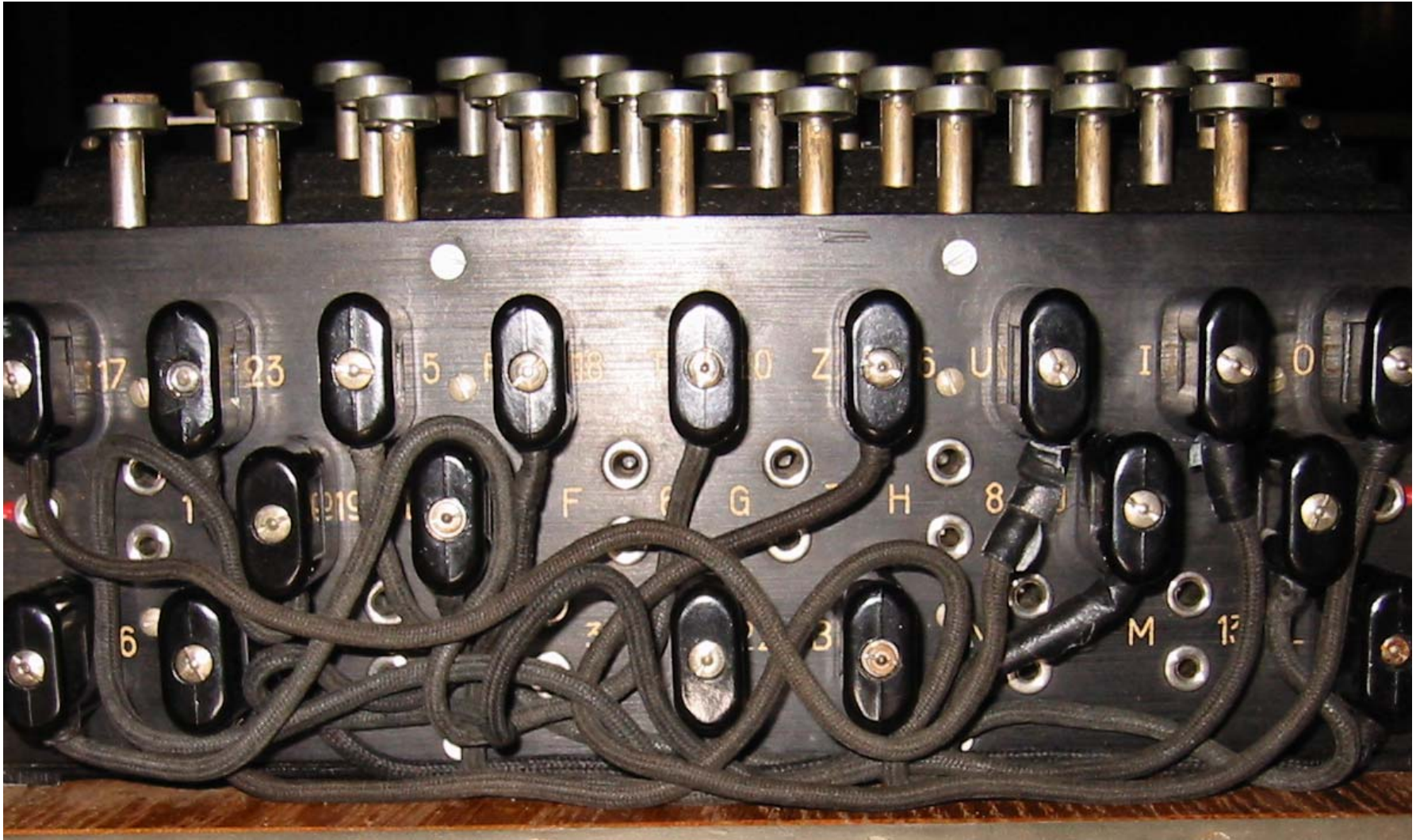


# About This Presentation

This presentation has been prepared for the graduate course ECE 257A (Fault-Tolerant Computing) by Behrooz Parhami, Professor of Electrical and Computer Engineering at University of California, Santa Barbara. The material contained herein can be used freely in classroom teaching or any other educational setting. Unauthorized uses are prohibited. © Behrooz Parhami

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# Reconfiguration and Voting





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"AFTER MONTHS OF SPEECHES, PROMISES, AND ACCUSATIONS, I'VE DECIDED TO JUST VOTE FOR THE GUY WITH THE COOLEST WEB SITE."

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"...and who's been messing with my desktop configuration?"



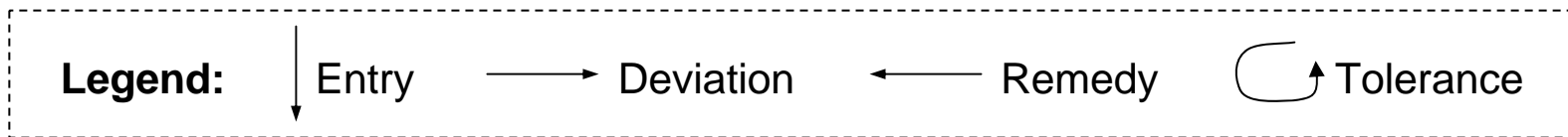
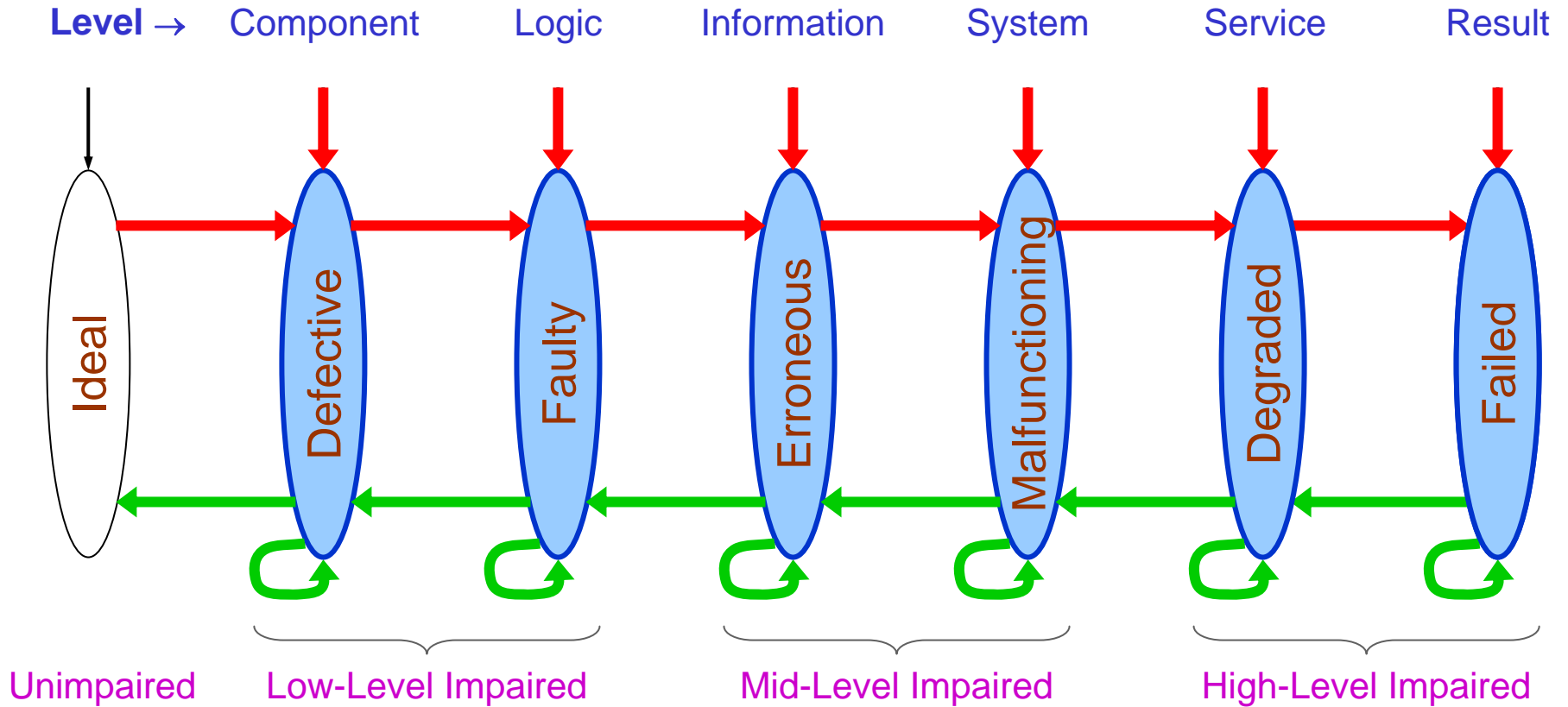
Wow! It works! My personalized configuration is up and running!



"Let's try voting for the greater of the two evils this time and see what happens."



# Multilevel Model of Dependable Computing



# Requirements for Reconfiguration

A system consists of modular resources (processors, memory banks, disk storage, . . . ) and interconnects

Redundant resources can mitigate the effect of module failures

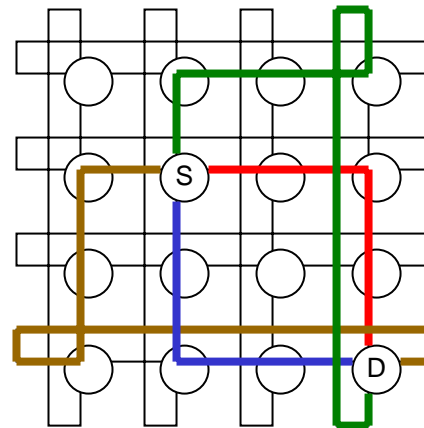
The main challenge of reconfiguration is dealing with interconnects

Assumption: Module and interconnect failures are promptly diagnosed

Overcoming the effect of interconnect failures requires the availability of multiple paths from each source to every possible destination

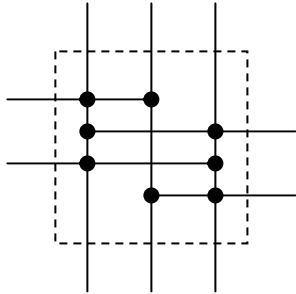
In graph-theoretic terms, we need “edge-disjoint” paths

Existence of  $k$  edge-disjoint paths between two nodes provides the means for tolerating  $k - 1$  link failures



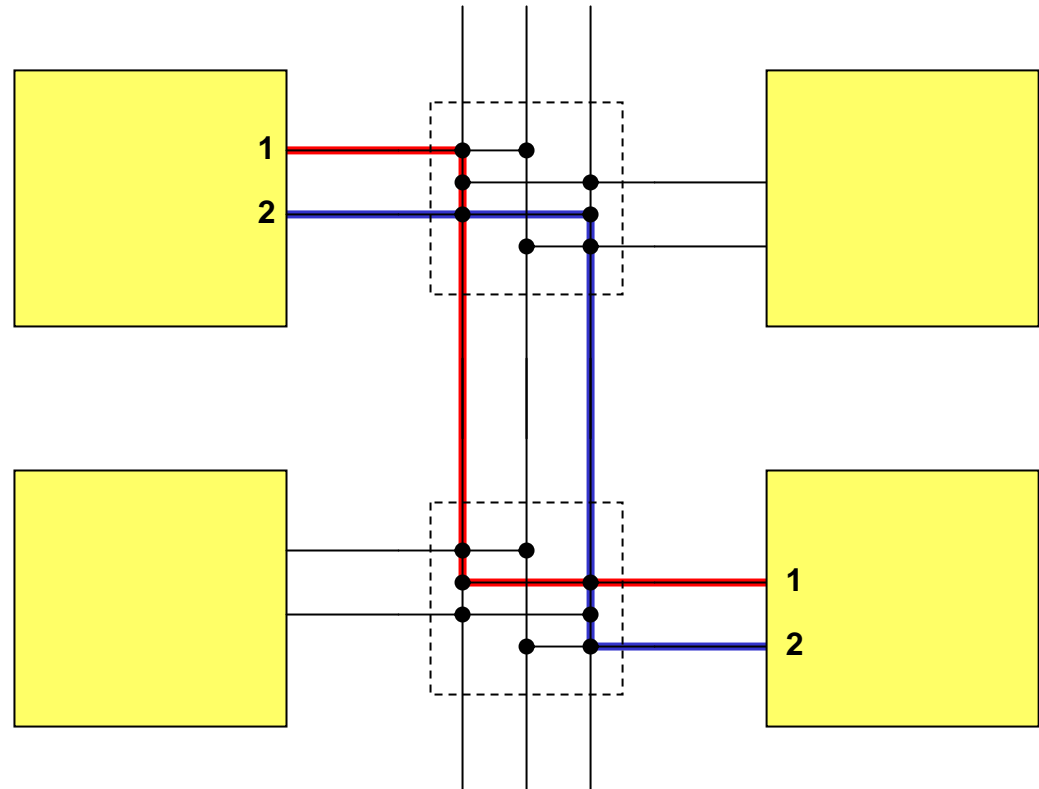
This particular interconnection scheme (torus) is 4-connected and tolerates 3 link/node losses without becoming disconnected

# Reconfiguration via Programmable Connections



Interconnection switch with 4 ports (horizontal lines) and 3 channels (vertical lines)

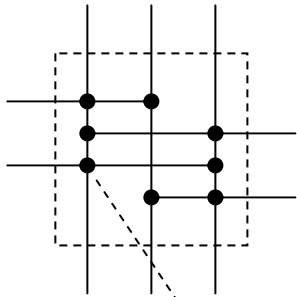
Each port can be connected to 2 of the 3 channels



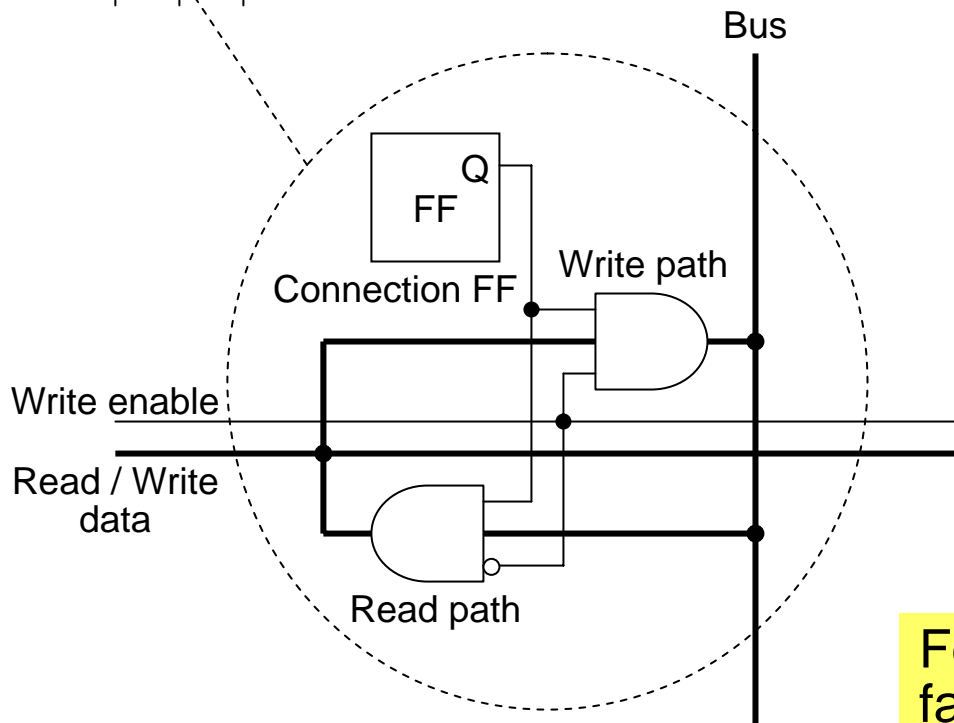
If each module port were connected to every channel, the maximum flexibility would result (leads to complex hardware & control, though)

The challenge lies in using more limited connections effectively

# Multiple-Bus Multiprocessors



The vertical channels may be viewed as buses and the heavy dots as controllable bus connections, making this method applicable to fault-tolerant multiprocessors



Failed units can be isolated from the buses

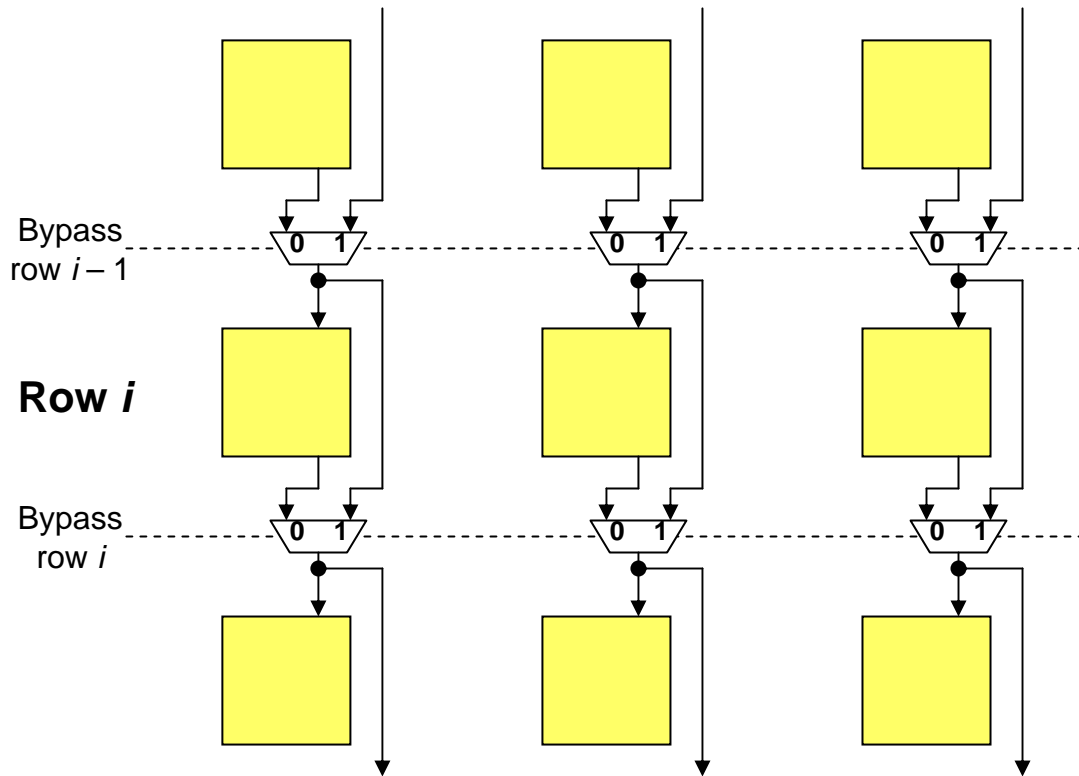
No single bus failure can isolate a module from the rest of the system

If we have extra buses, then failure in the bus connection logic can be tolerated by avoiding the particular bus

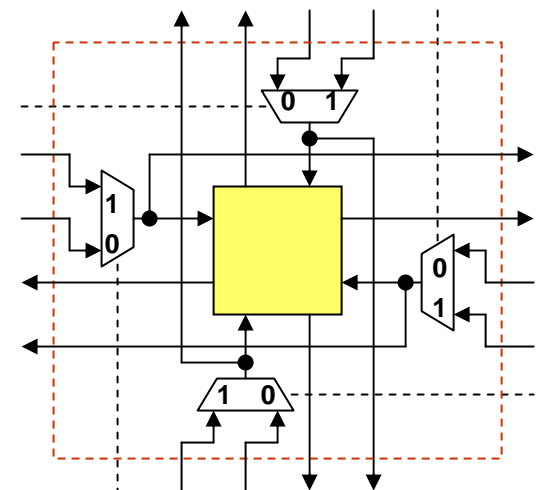
For reliability analysis, lump the failure rate of reconfiguration logic with that of its associated bus



# Row/Column Bypassing in 2D Arrays

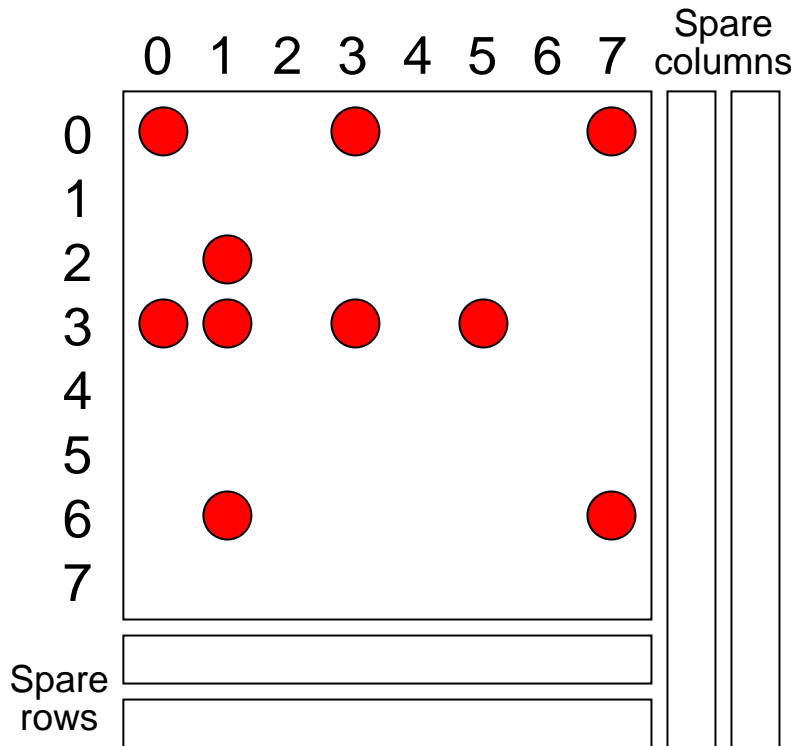


Similar mechanisms needed for northward links in columns and for the eastward and westward links in rows



**Question:** What types of mechanisms do we need at the edges of this array to allow the row and column edge connections to be directed to the appropriate (nonbypassed) rows and columns?

# Choosing the Rows/Columns to Bypass

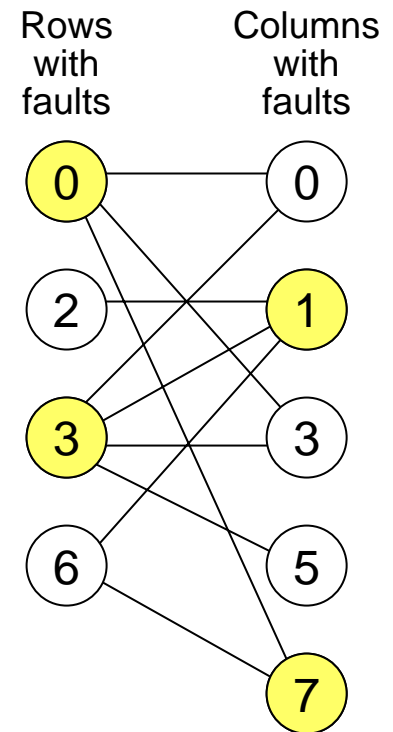


In the adjacent diagram, can we choose up to 2 rows and 2 columns so that they contain all the faults?

Convert to graph problem (Kuo-Fuchs):

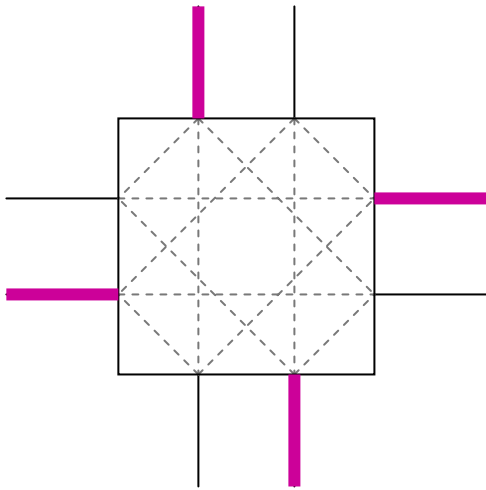
Form bipartite graph, with nodes corresponding to faulty rows and columns

Find a cover for the bipartite graph (set of nodes that touch every edge)



In a large array, with  $r$  spare rows and  $c$  spare columns, what is the smallest number of faults that cannot be reconfigured around with row/column bypassing?

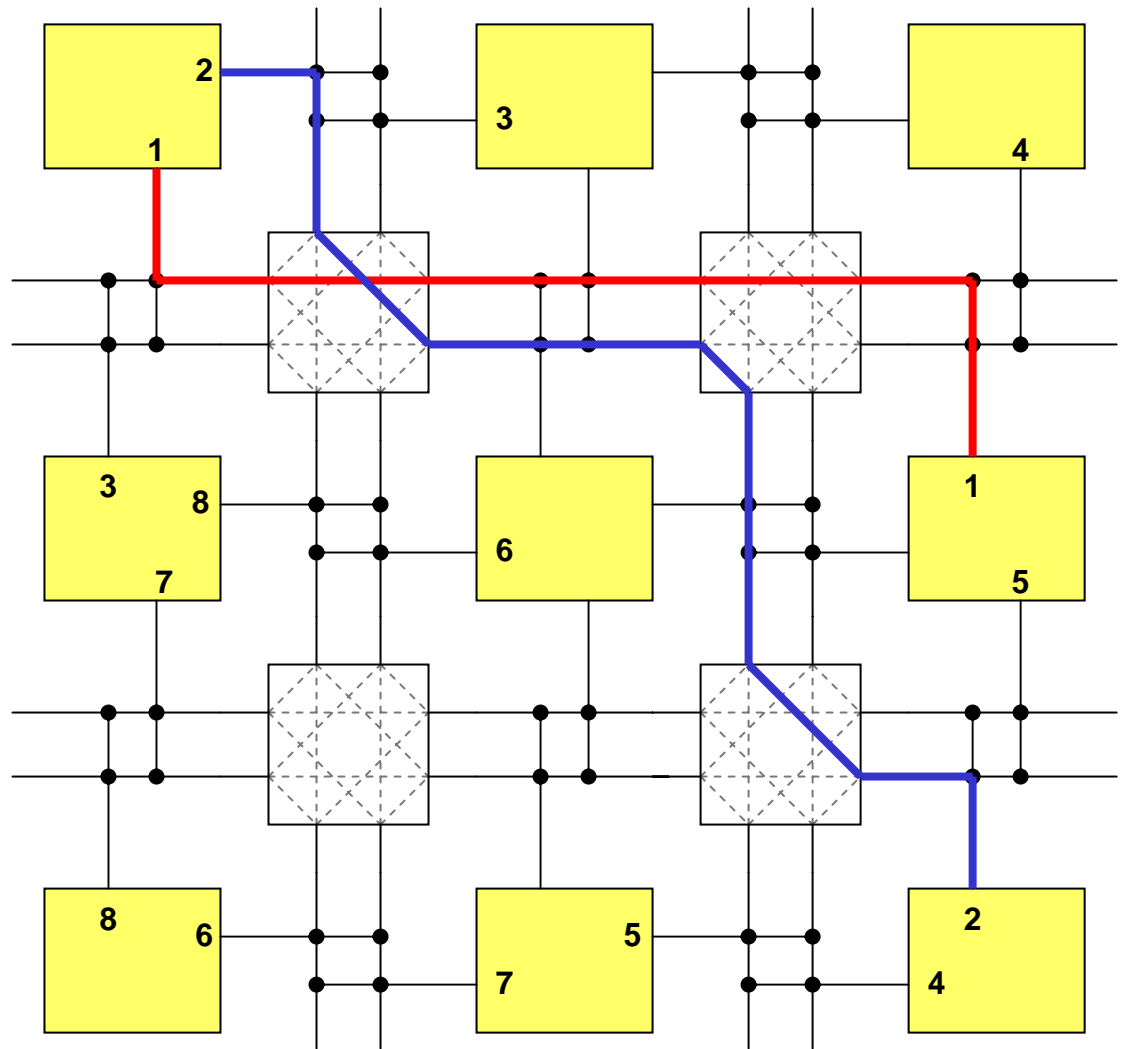
# Switch Modules in FPGAs



Interconnection switch with 8 ports and four connection choices for each port:

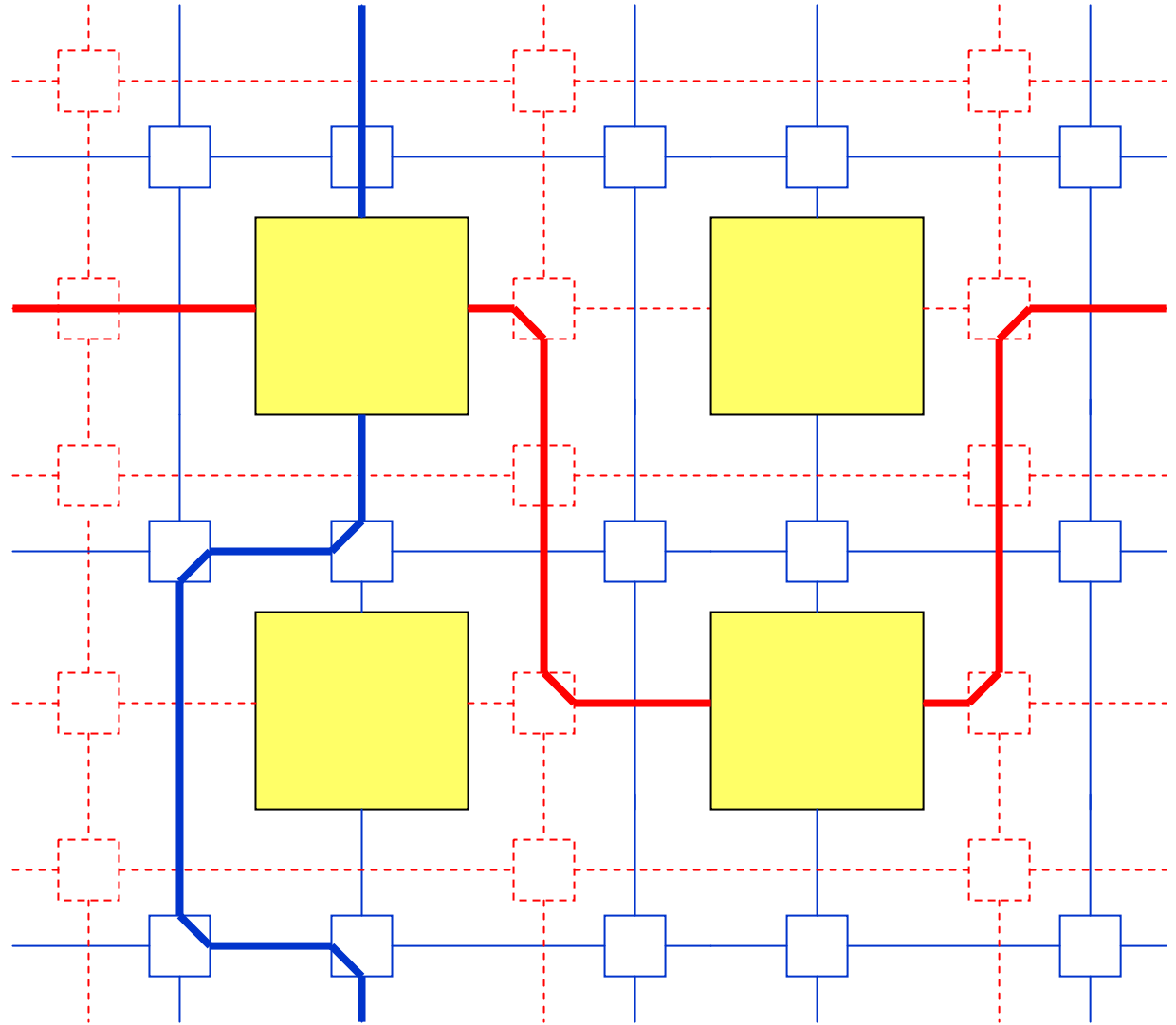
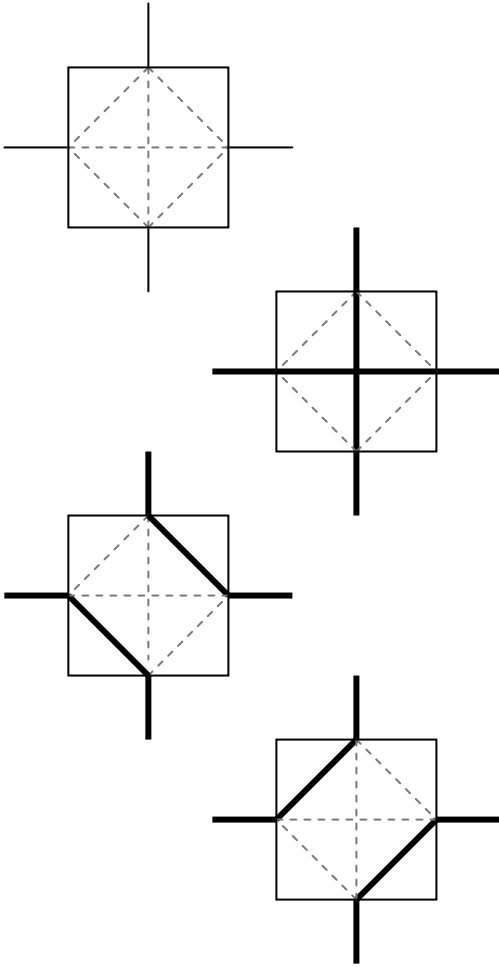
- 0 – No connection
- 1 – Straight through
- 2 – Right turn
- 3 – Left turn

8 control bits (why?)

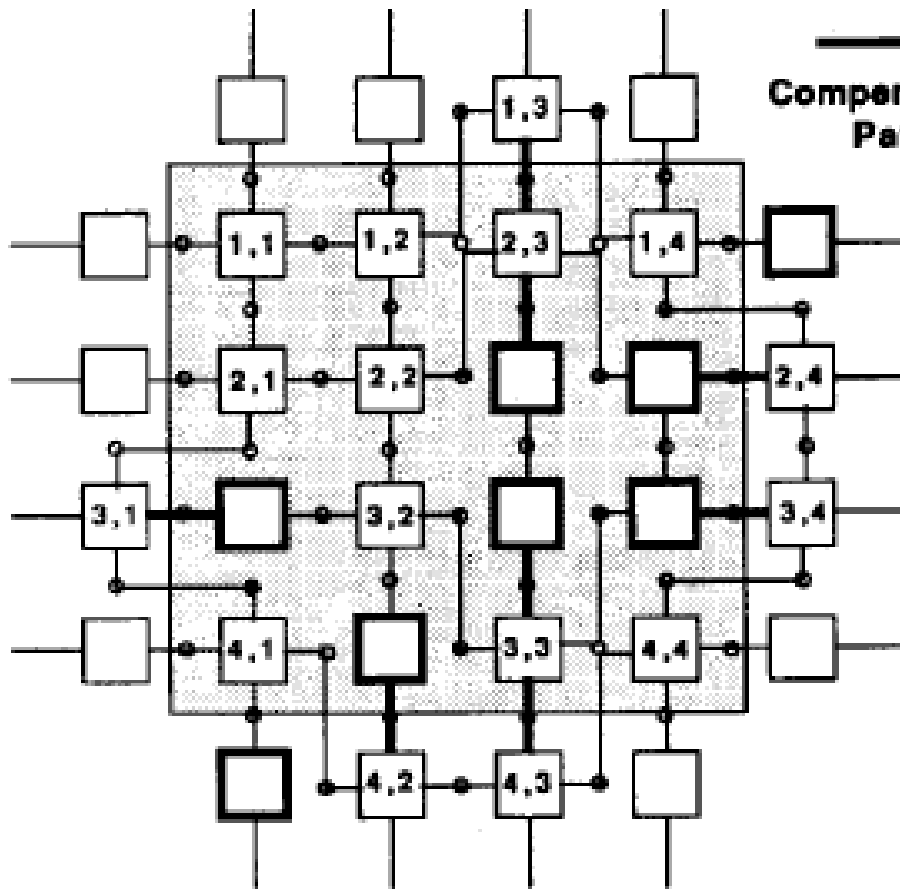


# An Array Reconfiguration Scheme

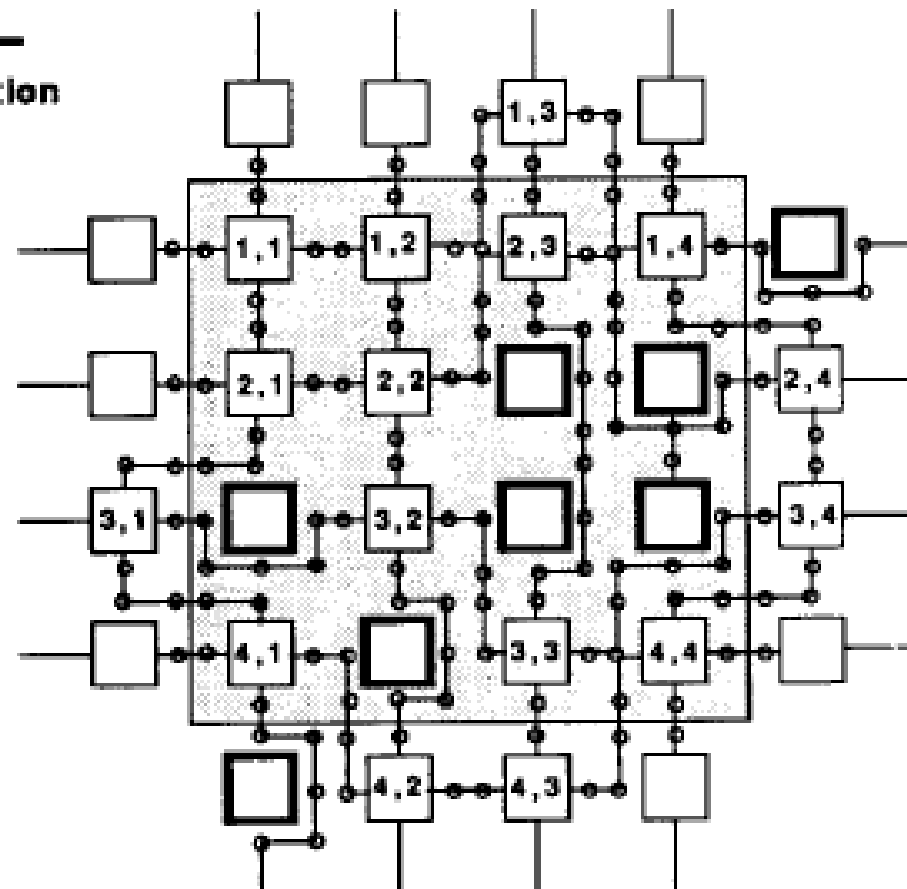
Three-state switch



# One-Track and Two-Track Switching Schemes



One-track switching model



Two-track switching model

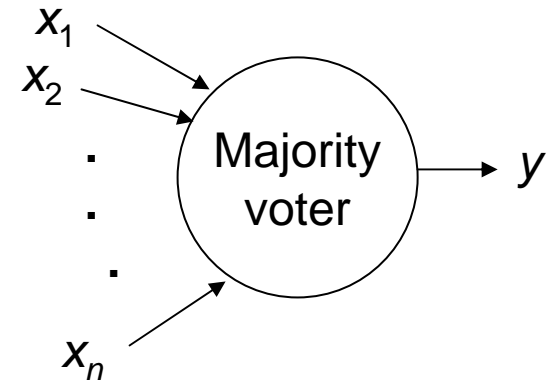
Source: S.-Y. Kung, S.-N. Jean, C.-W. Chang, *IEEE TC*, Vol. 38, pp. 501-514, April 1989

# What We Already Know About Voting

Majority voting: Select the value that appears on at least  $\lfloor n/2 \rfloor + 1$  of the  $n$  inputs

Number  $n$  of inputs is usually odd, but does not have to be

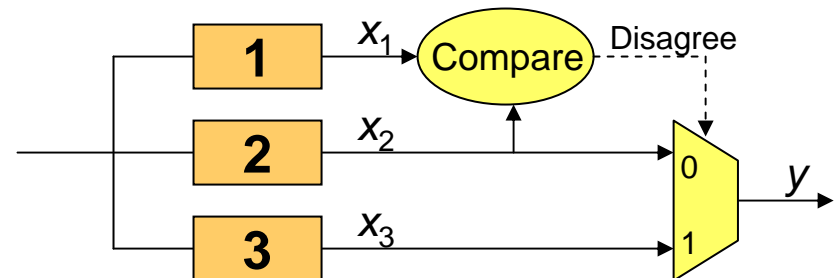
**Example:**  $\text{vote}(1, 2, 3, 2, 2) = 2$



Majority voters can be realized by means of comparators and muxes

This design assumes that in the case of 3-way disagreement any one of the inputs can be chosen

Can add logic to this voter so that it signals three-way disagreement



# There is More to Voting than Simple Majority

Plurality voting: Select the value that appears on the largest number of inputs

**Example:**  $\text{vote}(1, 3, 2, 3, 4) = 3$

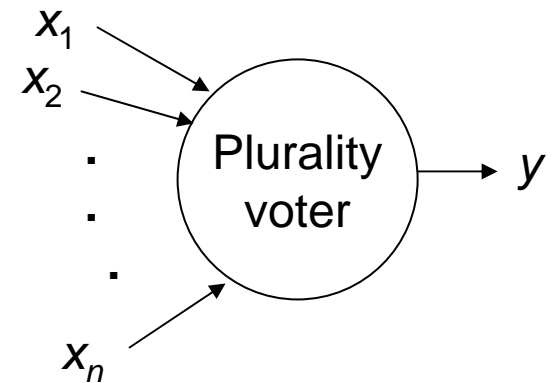
What should we take as the result of  $\text{vote}(1.00, 3.00, 0.99, 3.00, 1.01)$ ?

It would be reasonable to take 1.00 as the result, because 3 inputs agree or approximately agree with 1.00, while only 2 agree with 3.00

Will discuss approximate voting and a number of other sophisticated voting schemes under software design topics

Median voting: one way to deal with approximate values  
 $\text{median}(1.00, 3.00, 0.99, 3.00, 1.01) = 1.01$

Median value is equal to the majority value when we have majority



# Threshold Voting and Its Generalizations

Simple threshold ( $m$ -out-of- $n$ ) voting:

Output is 1 if at least  $m$  of the  $n$  inputs are 1s

Majority voting is a special case of threshold voting:  $(\lfloor n/2 \rfloor + 1)$ -out-of- $n$  voting

Weighted threshold ( $w$ -out-of- $\sum v_i$ ) voting:

Output is 1 if at least  $\sum v_i x_i$  is  $w$  or more

Agreement or quorum sets

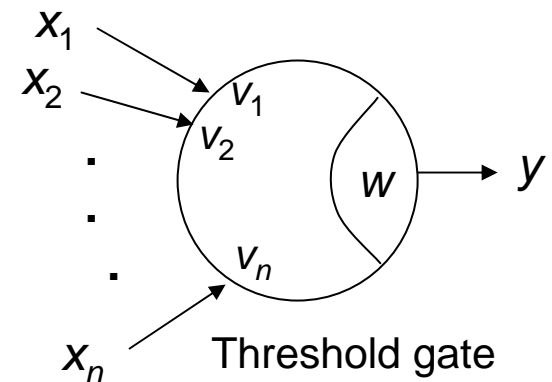
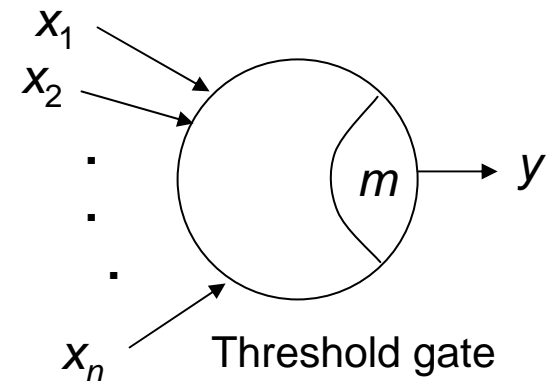
$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_1\}$  – same as 2-out-of-3

$\{x_1, x_2\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}$

The 2<sup>nd</sup> example above is weighted voting with

$v_1 = v_2 = 2, v_3 = v_4 = 1$ , and threshold  $w = 4$

Agreement sets are more general than weighted voting in the sense of some agreement sets not being realizable as weighted voting





# Usefulness of Weighted Threshold Voting

Unequal weights allow us to take different levels of input reliabilities into account

Zero weights can be used to disable or purge some of the inputs (combined switch-voter)

Maximum-likelihood voting

$$\text{prob}\{x_1 \text{ correct}\} = 0.9$$

$$\text{prob}\{x_2 \text{ correct}\} = 0.9$$

$$\text{prob}\{x_3 \text{ correct}\} = 0.8$$

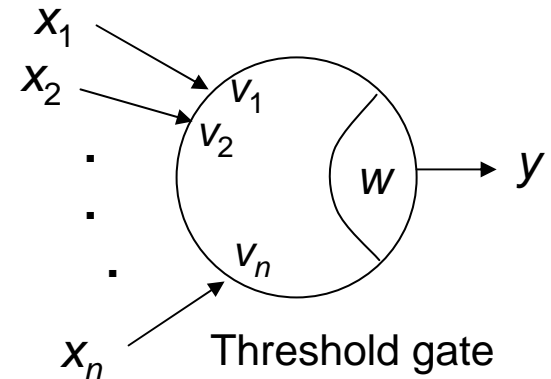
$$\text{prob}\{x_4 \text{ correct}\} = 0.7$$

$$\text{prob}\{x_5 \text{ correct}\} = 0.6$$

Assume  $x_1 = x_3 = a$ ,  $x_2 = x_4 = x_5 = b$

$$\text{prob}\{a \text{ correct}\} = 0.9 \times 0.1 \times 0.8 \times 0.3 \times 0.4 = 0.00864$$

$$\text{prob}\{b \text{ correct}\} = 0.1 \times 0.9 \times 0.2 \times 0.7 \times 0.6 = 0.00756$$

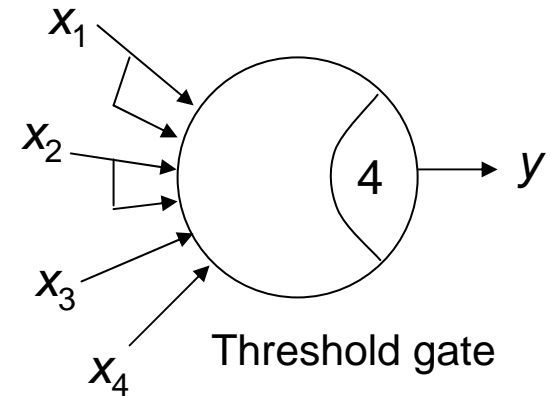


Max-likelihood voting can be implemented in hardware via table lookup or approximated by weighted threshold voting (otherwise, we need software)

# Implementing Weighted Threshold Voters

**Example:** Implement a 4-input threshold voter with  $v_1 = v_2 = 2$ ,  $v_3 = v_4 = 1$ , and threshold  $w = 4$

**Strategy 1:** If weights are small integers, fan-out each input an appropriate number of times and use a simple threshold voter



**Strategy 2:** Use table lookup based on comparison results

$x_1=x_2$	$x_1=x_3$	$x_2=x_3$	$x_3=x_4$	Result
1	x	x	x	$x_1$
0	0	0	1	Error
0	1	0	1	$x_1$
0	0	1	1	$x_2$
0	x	x	0	Error

Is this table complete?  
Why, or why not?

**Strategy 3:** Convert the problem to agreement sets (discussed next)

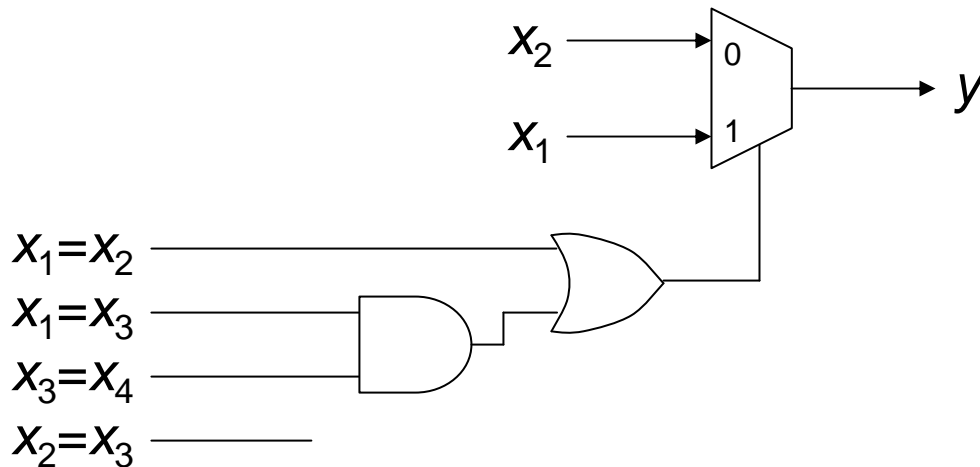
# Implementing Agreement-Set Voters

**Example:** Implement a voter corresponding to the agreement sets  $\{x_1, x_2\}$ ,  $\{x_1, x_3, x_4\}$ ,  $\{x_2, x_3, x_4\}$

**Strategy 1:** Implement as weighted threshold voter, if possible

**Strategy 2:** Implement directly

Find a minimal set of comparators that determine the agreement set

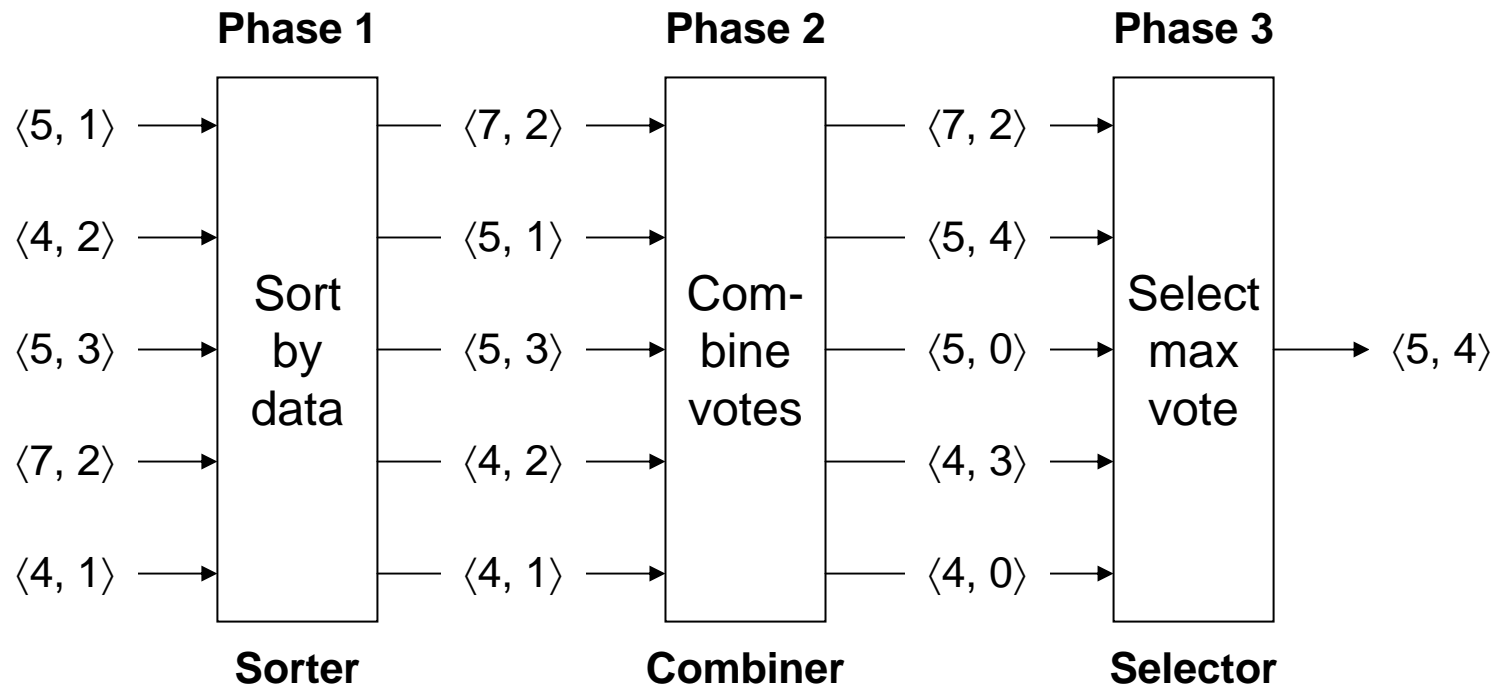


Complete this design by producing the “no agreement” signal

# Implementing Weighted Plurality Voters

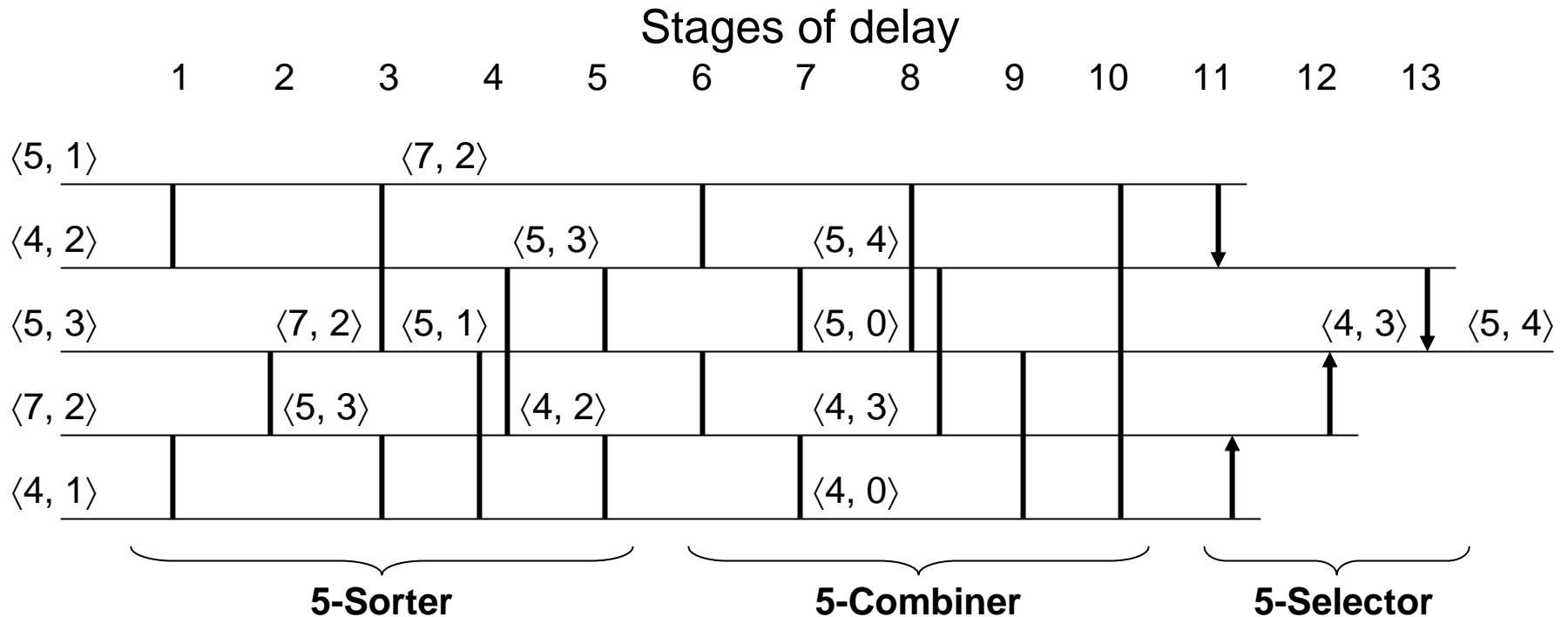
Inputs: Data, vote-weight pairs

Output: Data with maximal vote and its associated vote tally



Source: B. Parhami, *IEEE Trans. Reliability*, Vol. 40, No. 3, pp. 380-394, August 1991

# Details of a Sorting-Based Plurality Voter



The first two phases (sorting and combining) can be merged, producing a 2-phase design – fewer, more complex cells lead to tradeoff

Source: B. Parhami, *IEEE Trans. Reliability*, Vol. 40, No. 3, pp. 380-394, August 1991