Further Results on Binaural Unmasking and the EC Model

LAWRENCE R. RABINER*
Bell Telephone Laboratories, Inc., Murray Hill, New Jersey

C. L. LAURENCE AND N. I. DURLACH
Center for Communications Sciences, Research Laboratory of Electronics
Massachusetts Institute of Technology, Cambridge, Massachusetts

This paper reports the results of further experiments on the binaural unmasking of tones masked by broad-band Gaussian noise and further theoretical work on the EC model of binaural unmasking. Data are presented on binaural unmasking for interaural time delays and/or phase shifts in the noise, and for statistically independent noise, at a variety of tone frequencies. Many aspects of these data cannot be interpreted by the preliminary version of the model, and consideration is given to some possible revisions of the model.

INTRODUCTION

This paper reports the results of further experiments on the binaural unmasking of tones masked by broad-band Gaussian noise and further theoretical work on the equalization and cancellation (EC) model of binaural unmasking. It is assumed throughout that the reader is familiar with previous work related to the model.1-6

In each of the experiments, the interaural amplitude ratios of the tone and noise were unity, and the noise was maintained at a high power level. Variations in the binaural-masked threshold were studied as a function of the interaural time delay and interaural phase shift of the noise, as well as the frequency of the tone. (In some cases, the noise signals differed only by a time delay or a phase shift; in others, by both a time delay and a phase shift.) In addition, masked thresholds were measured for the case in which the two noise signals were statistically independent. For a given stimulus configuration, the quantity considered was the binaural masking-level difference (BMLD) between the masked threshold for the given configuration and the masked threshold for the configuration in which the tone and noise were identical in the two ears (referred to as "homophase"). Since the homophase threshold is known to be equal to the monaural threshold,7 this BMLD describes the extent to which the threshold is altered by the use of the two ears (i.e., the amount of binaural unmasking).

Since all BMLD's are referred to a common reference, in describing a particular BMLD, only the variable stimulus need be specified explicitly. In general, the interaural configuration for the variable stimulus is specified by a symbol of the form ( | ), the space before the bar being used for the tone and the space after the bar for the noise. These spaces will contain the interaural parameters (or the values of such parameters) that describe the interaural differences in the stimulus. If a particular parameter is omitted, it will denote the fact that there is no interaural difference with respect

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7 The equating of these two thresholds is valid provided that the noise level is not too low. Some data on these thresholds for a noise level that is comparable to the levels used in the present experiments can be found in I. J. Hirsh and M. Burgeat, "Binaural Effects in Remote Masking," J. Acoust. Soc. Am. 30, 827-832 (1958).
TABLE I. Measured BMILD's.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$B(\pi_0)$</th>
<th>$\omega_d/2\pi=200$, 400 Hz</th>
<th>$0 \leq \phi_a \leq \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>$B(\psi_{\phi_0})$</td>
<td>$\omega_d/2\pi=200$, 400 Hz</td>
<td>$0 \leq \phi_a \leq \pi$</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>$B(\psi_\pi)$</td>
<td>$\omega_d/2\pi=167$, 297, 500, 694, 1040 Hz</td>
<td></td>
</tr>
<tr>
<td>Experiment 3</td>
<td>$B(\psi_{\tau_{\phi_0}}-\omega_d\tau_{\phi_0})$</td>
<td>$0 \leq \tau_{\phi_0} \leq 0.6$ msec</td>
<td></td>
</tr>
</tbody>
</table>

The precise choice of frequencies was dictated by the requirement that the delays $\tau = q\pi/\omega_d$ (a integer) be available with the equipment used to produce the interaural time delay in the noise.

In the stimulus $(\xi_\tau \tau_{\phi_0} - \omega_d\tau_{\phi_0})$, the noise is delayed by $\tau_{\phi_0}$ and then phase-shifted by $-\omega_d\tau_{\phi_0}$ (radians). Thus, the ratio of the complex spectra of the two noise signals is given by $e^{i\omega_d\tau_{\phi_0}}$.

The BMILD's measured in the present set of experiments are shown in Table I. Experiments 2 and 3 were performed by the same experimenter in the same laboratory with the same subjects and equipment (see Acknowledgment). The primary motivation for performing these experiments was to clarify certain problems encountered in the attempt to apply the EC model to previous data. The data sources have been coded as follows: H--Hirsh, HB--Hirsh and Burgert, HW--Hirsh and Webster, JBD--Jeffress, Blodgett, and Deathager, JBSW--Jeffress, Blodgett, Sandel, and Wood, L.J. Langford and Jeffress, BJT--Blodgett, Jeffress, and Taylor, R.J. Robinson and Jeffress, E. Egan, SG--Sondhi and Gutman. Data from the present experiments are denoted RLD.) Some of the relevant previous data are shown in Figs. 1 and 2. The principal characteristics of these data that are of interest in the present context are the divergence of $B(\pi_0)$, $B'(\pi_\phi)$ and $B(M)$ from unity at frequencies below approximately 500 Hz, the "flattening" of the JBD data on $B(0_\psi r_{\tau_{\phi_0}}$ at 17 Hz beyond $\tau_{\phi_0}=0.6$ msec, and the damping of successive cycles in the J and JBD data on $B(0_\psi r_{\tau_{\phi_0}}$ at 300 Hz. An additional motivation for measuring $B(\pi_0, \tau_{\phi_0} - \omega_d\tau_{\phi_0})$ was to explore further certain aspects of the binaural critical band. The data on $B(\psi_\pi r_{\tau_{\phi_0}}$ and $B(0_\psi r_{\tau_{\phi_0}}$ were obtained in order to extend previous results on these BMILD's to frequencies other than 500 Hz. For an understanding of why the BMILD's considered in the present paper were thought to be important determinants for future theoretical developments, the reader is referred to Sec. II.

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I. EXPERIMENTS

In all three experiments, the stimuli were continuous and were presented to the subjects through earphones in a soundproof room. TDH-39 earphones were used in Expt. 1, and specially designed condenser earphones in Expts. 2 and 3. The thresholds were determined by forms of the method of adjustment, and all subjects (of which there were four to five for each experiment) were found to have normal hearing as measured by Bekesy audiometry. In Expt. 1, the noise spanned the region 100-1000 Hz and had a constant spectral level of 60 dB SPL. The subjects in this experiment were instructed to set a step attenuator to a value that made the tone "just audible." In order to decorrelate the trials, a second attenuator (in series with the subject's) was varied from trial to trial in a manner unknown to the subject. Each subject was tested with each stimulus configuration between 4 and 8 times (usually 4). In Expts. 2 and 3, the noise spectrum spanned the region 60 Hz to 600 Hz. In Expt. 2, the spectral level was tapered so that the noise power per cycle was at a constant sensation level (SL), independent of frequency. The spectral level was 64 dB SPL at 100 Hz and declined monotonically to 54 dB SPL at 1000 Hz. In Expt. 3, the spectral level was 57 dB SPL, independent of frequency. In both Expts. 2 and 3, the thresholds were determined by a technique similar to the Bekesy audiometer method, except that the subject had the option of holding the signal level constant as well as causing it to increase or decrease. The estimate of the threshold for a single test was defined as the average of 3 to 4 settings of the attenuator at which the tone was "just inaudible." The number of tests per subject per stimulus varied considerably, depending upon the availability of the subjects. On the average, it was between two and three. The results of all the experiments (averaged over tests and subjects) are shown in Figs. 3–6. The square root of the average variance around the sample means in these experiments was between 1 and 2 dB.

The results on $B(\pi | 0) / B(0 | 0)$ obtained in Expts. 1–3 are shown in Figs. 3–5. The results on $B(0 | \phi_0)$, $B(0 | \tau_n)$, and $B(\pi | \tau_n - \omega n \tau_n)$ obtained in Expts. 1–3, on $B(0 | \tau_n)$ obtained in Expt. 1 and 2, and on $B(0 | \tau_n)$ and $B(\pi | \tau_n)$ obtained in Expt. 3, are shown in Fig. 6. The values of $B(\pi | 0) / B(0 | 0)$ obtained in Expts. 1 and 2 are shown in Fig. 1.

In considering the results in Figs. 1–6, the following points should be noted—

- $B(\pi | 0)$ decreases at very low frequencies. This result is consistent with the results of Hirsh, Schenkel,19


and Jeffress, Blodgett, and Deatherage, but is at variance with the results of Webster and Durlach. The decrease of $B(0|\pi)$ at low frequencies, and the decrease of both $B(\pi|0)$ and $B(0|\pi)$ at high frequencies, is consistent with all previous data.

- $B(0|u) = B(\pi|u) = 4$ dB, independent of frequency. Previous results on these BMLD's at 500 Hz vary between 2 and 6 dB (Ref. 22).

- The damping of $B(0|\tau_n)$ and $B(\pi|\tau_n)$ and the decrease of $B(\pi|\tau_n - \omega_0\tau_n)$ as $\tau_n$ increases are reflections of the imperfect frequency resolution of the auditory system.

- $B(0|\tau_n)$ and $B(\pi|\tau_n)$ must be independent of $\tau_n$ and approach $B(0|u)$ and $B(\pi|u)$ as $\tau_n$ becomes large. Similarly, $B(\pi|\tau_n - \omega_0\tau_n)$ must approach $B(\pi|u)$ as $\tau_n$ becomes large.  

The flattening in each cycle of $B(0|\tau_n)$ begins at approximately 0.5–0.8 msec after the previous minimum.

- The flattening that occurs in $B(\phi_1|\phi_0)$ and $B(\phi_1|\tau_n)$ when $\phi_0 = 0$ must be substantially altered when $\phi_0$ takes on values other than zero.

- The deviation of $B(\pi|0)/B(0|\pi)$ from unity is explained by the occurrence of flattening in $B(0|\tau_n)$. Presumably, the deviation of $B(M|0)/B(M|\pi)$ from unity (which has approximately the same magnitude) is due to a similar flattening in $B(M|\tau_n)$.

- For $\omega_0/2\pi < 500$ Hz, the damping in $B(0|\tau_n)$ and $B(\pi|\tau_n)$ occurs unevenly. Specifically, there is more local symmetry about points of the form $\tau_n = 2\pi q/\omega_0$ than about those of the form $\tau_n = (2q-1)\pi/\omega_0$ ($q$ an integer).

- $B(0|\tau_n) = B(\pi|\tau_n)$ for $\omega_0/2\pi = 500$ Hz and $\tau_n = (2q-1)\pi/2\omega_0$.

- The subjective experience on the lateralization of the noise in Expt. 2 was consistent with the BMLD.

The amount of flattening in $B(0|\tau_n)$ and $B(0|\phi_0)$ increases as $\omega_0$ decreases and becomes appreciable only when $\omega_0/2\pi < 500$ Hz.

- The flattening in $B(0|\phi_0)$ for $|\phi_0| < \pi$ is similar to the flattening in $B(0|\tau_n)$ for $|\tau_n| < \pi/\omega_0$ (where the two functions are compared by means of the transformation $\phi_0 = \omega_0\tau_n$).

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data, both with respect to the cyclic behavior and with respect to the flattening.20

- No satisfactory explanation has been found to explain the difference between the values of $B(\pi r)$ obtained in Expts. 2 and 3 or the difference between the magnitudes of the various BMLD's obtained in the present experiments and those obtained by Jeffress and his associates.

II. THEORY

In the original version of the model, it was assumed that when the two noise signals were identical, except for time delays or phase shifts, all the processing errors could be represented by random jitter in the amplitude and time alignment of the signals at the input to the subtractor, and that the statistics of these errors were independent of the interaural relations of the noise. In other words, the ability to suppress the noise was assumed to be independent of the interaural transformation required to equalize the noise components before the subtraction. Thus, the only effect of inserting interaural time delays or phase shifts in the noise was to modify the interaural phase of the tone at the input to the subtractor. This assumption, combined with the other assumptions of the model, leads to the predictions

$$B(\pi r_n - \omega_0 r_n) = B(\pi 0) = B(0 \pi) = (k + 1)/(k - 1),$$

$$B(M 0) = B(M \pi) = k/2(k - 1), \text{ when } k \leq 2,$$

$$B(M 0) = B(M \pi) = 1, \text{ when } k > 2,$$

$$B(0 \phi_n) = (k - \cos \phi_n)/(k - 1),$$

$$B(0 \tau_n) = [k - \cos(\omega_0 r_n)]/(k - 1),$$

$$B(\pi \tau_n) = [k + \cos(\omega_0 r_n)]/(k - 1),$$

where $k = (1 + \sigma^2) \exp(\omega_0^2 \sigma^2)$, and $\sigma^2$ and $\sigma^2$ are the variances of the amplitude and time jitter. To a first approximation, these equations are consistent with the data with respect to the following features: (1) $B(\pi 0)/B(0 \pi) = B(M 0)/B(M \pi)$; (2) periodicity in $B(0 \tau_n)$ and $B(\pi \tau_n)$; (3) $B(0 \tau_n) = B(\pi \tau_n)$ at $\tau_n = (2k - 1)\pi/2\omega_0$; (4) shape of $B(0 \tau_n)$, $B(\pi \tau_n)$, and $B(0 \phi_n)$ within a single cycle (provided that there is no flattening). They are inconsistent with the data, however, in that they do not predict: (1) damping in $B(0 \tau_n)$ and $B(\pi \tau_n)$; (2) deviation of $B(\pi 0)/B(0 \pi) = B(M 0)/B(M \pi)$ from unity; (3) flattening in $B(0 \tau_n)$ and $B(0 \phi_n)$ for $\omega_0/2\pi < 500$ Hz; (4) decrease in $B(\pi \tau_n)$ as $\tau_n$ is increased. In addition, they are inconsistent with the data on $B(\pi 0)$ and $B(0 \pi)$ at frequencies above approximately 1500 Hz (Refs. 3, 4) and with those data which indicate that $B(\pi 0)$ decreases at very low frequencies. Finally, it should be noted that the preliminary model is useless for interpreting data on BMLD's [such as $B(0 \omega)$ and $B(\omega \pi)$] in which the noise signals are decorrelated by the introduction of independent noise sources.

These defects in the preliminary model (together with certain other defects that appear in connection with other types of data23) require that the model be revised. Although no set of revisions has yet been found that solves all of the difficulties, there are a few that appear to eliminate or reduce at least some of them and that provide a structure for expressing certain of the remaining ones in a more useful form. One such set is the following. First, in place of the assumption that the system has an arbitrarily large store of interaural transformations for use in the equalization process, assume that the repertoire of such transformations is limited. Second, assume that, for a given stimulus, the system selects that transformation in the repertoire that maximizes the signal-to-noise (S/N) ratio at the output of the subtractor.23 Third, assume that, for each of the stimuli to be considered, the transformation in the repertoire that maximizes this S/N ratio has the form of a time delay. Fourth, assume that for each frequency $\omega_0$, the maximization over the time delay is restricted to delays whose magnitudes do not exceed $\pi/\omega_0$ (i.e., that the delays tested are confined to a single cycle of the tone). For stimuli in which the two noise signals are identical, these revisions leave the predictions unaltered. For the stimuli $(\pi \tau_n - \omega_0 \tau_n)$, $(0 \pi)$, $(M \pi)$, $(0 \phi_n)$, $(0 \tau_n)$, and $(\pi \tau_n)$, however, the predictions must be recomputed. Letting $\Delta$ denote the time delay inserted by the system and $\gamma$ the envelope of the normalized autocorrelation function of the effective noise

20 In addition to the defects mentioned in this paper, the preliminary model is inadequate for describing the dependence of binaural unmasking on the over-all power level of the noise and on the interaural amplitude ratio of the noise.

23 In the original simplified model, the S/N ratio is maximized by the transformation that minimizes the noise. When the repertoire is restricted, however, the transformation that minimizes the noise may not maximize the S/N ratio.
Solving Eq. 13 for \( \gamma(\tau_n) \) as a function of \( B(\pi | \tau_n, -\omega_0 \tau_n) \) and using the data in Fig. 5, one obtains the results shown in Fig. 7. (The fact that these functions are all precisely equal to unity at \( \tau_n = 0 \) corresponds to the fact that, for each of the frequencies tested, \( k \) was chosen to fit the data in Fig. 5 at \( \tau_n = 0 \).) Assuming, for illustrative purposes, that the gain characteristic of the peripheral filter is an isosceles triangle centered on \( \omega_0 \) with a width \( \beta \) (between the 3-dB points) and fitting the corresponding transform \( \sin^2(\beta \tau_n/2)/\beta \tau_n^2 \) to the data on \( \gamma(\tau_n) \) by eye, one obtains the values of \( \beta \) and the curves shown in Fig. 7. The curves shown in Fig. 5 are obtained by inserting these functions back into Eq. 13 and computing \( B(\pi | \tau_n, -\omega_0 \tau_n) \). According to previous results on the critical bandwidth, the values \( \beta/2\pi = 60 \) and 85 Hz for \( \omega_0/2\pi = 297 \) and 500 Hz are the envelope \( \gamma(x) \) of \( \Psi(x) \) is given by

\[
\gamma(x) = \int_0^\infty \langle \Psi(x) \rangle \exp(\angle(x, \omega)) \, d\omega / \int_0^\infty \langle \Psi(x) \rangle^2 \, d\omega,
\]

and \( \langle \cdot \rangle \) is related by the equation \( \langle \Psi(x) \rangle = \gamma(x) \cos(\omega_0 \tau_n) \). Since the noise spectrum is approximately flat across the passband of the peripheral filter, the function \( \langle \Psi(x) \rangle \) (which can be obtained up to a multiplicative constraint by taking the Fourier transform of \( \langle \Psi(x) \rangle \)) is approximately equal to the gain characteristic of the peripheral filter. As in previous work, the EC factor \( f_j \) equals the S/N ratio at the output of the subtractor divided by the S/N ratio at the output of the filter in ear \( j \). When \( j = 1 \), the subscript \( j \) is omitted. In the monaural presentation, it is assumed that the tone is presented to Ear 1.
not unreasonable. The value \( \beta/2\pi = 200 \) Hz for \( \omega_0/2\pi = 694 \) Hz, however, seems rather large. Note also that (1) \( B(\pi | r_n - \omega_0 r_n) \) becomes increasingly less sensitive to variations in \( \gamma(r_n) \) as \( \gamma(r_n) \) decreases (i.e., \( r_n \) increases); (2) in the region \( 0 \leq r_n \leq 3 \) msec (where the sensitivity is greatest), the curves become increasingly less adequate as the frequency decreases; (3) if the widths \( \beta \) for the 297- and 500-Hz data had been chosen to provide a better fit in the region \( 0 \leq r_n \leq 3 \) msec, these widths would be greater than 60 and 85 Hz; (4) in order to obtain a good fit to the data at 295 and 500 Hz, it is necessary to choose a very different shape (or shapes) for the gain characteristics (i.e., the characteristics must have relatively large values in some frequency regions relatively far removed from \( \omega = \omega_0 \)); (5) in order to fit the 694-Hz data at 5.0 and 8.7 msec, an asymmetric character must be chosen so that \( \gamma(r_n) \) can assume negative values. Thus far, no effort has been made to estimate the gamma characteristics directly by computing the Fourier transforms of the curves determined by the data on \( \gamma(r_n) \).

The predictions of the revised model for the ratios \( B(\pi | 0)/B(0 | \pi) \) and \( B(0 | 0)/B(0 | \pi) \) in the frequency region \( 297 \leq \omega_0/2\pi \leq 694 \) Hz are shown by the curves in Fig. 1. These curves (which are identical) were obtained by plotting points at the frequencies 297, 500, and 694 Hz and then drawing a smooth curve through the points. The values of \( \beta \) used in computing the points were obtained by employing the values \( \sigma_1 = 0.25 \) and \( \sigma_2 = 105 \) \( \mu \)sec (used in previous work) and the values of \( \gamma(\pi/\omega_0) \) were obtained from the data in Fig. 7.

The prediction of the revised model for the data on \( B(0 | \phi_0) \) at 400 Hz is shown in Fig. 3 by the dashed curve. The prediction of the preliminary model (the solid curve) is included for purposes of comparison. The same value of \( \beta \) is used for both curves. This value was computed from the value \( B(\pi | 0) = 10.0 \) dB obtained in the same experiment. The values of \( \gamma(r_n) \) used for the dashed curve were obtained by averaging the data in Fig. 7 for \( \omega_0/2\pi = 297 \) and 500 Hz. Since the value of \( B(\pi | 0) \) obtained in this experiment for 200 Hz differed from that obtained for 400 Hz by only 0.2 dB, the solid curve also represents the prediction of the preliminary model for the data at 200 Hz. According to previous work, the solid curve would provide an excellent description of the data on \( B(\phi_0 | 0) \) for both 200 and 400 Hz had such data been taken in this experiment.

The predictions of the revised model for the data on \( B(0 | r_n) \) and \( B(\pi | r_n) \) constitute an improvement over those of the preliminary model with respect to the damping of the local maxima and with respect to the local symmetry structure [symmetry about the points \( r_n = 2\pi\omega_0/\omega_0 \) and asymmetry about the points \( r_n = (2q-1)\pi/\omega_0 \)]. They are grossly inadequate, however, in that the damping is predicted only for the maxima (i.e., the local minima are predicted to be unity, as in the preliminary model). Also, the local minima are predicted to occur over intervals rather than just at single points, the size of the intervals decreasing with each successive cycle. The points \( r_n = (2q-1)\pi/\omega_0 \) (at which \( \gamma \) changes value and the damping and asymmetry are created) occur within the minima of \( B(\pi | r_n) \) and at the maxima of \( B(0 | r_n) \). The local asymmetry in \( B(\pi | r_n) \) appears in an asymmetric position of the interval minimum about the point \( r_n = (2q-1)\pi/\omega_0 \). In \( B(0 | r_n) \), the local asymmetry takes the form of a discontinuity. The prediction for \( B(0 | r_n) \) is also grossly inadequate in that it does not predict the flattening observed at frequencies below 500 Hz. The general characteristics of the predictions for \( B(\pi | r_n) \) and \( B(0 | r_n) \) are illustrated graphically by the curves in Fig. 8. In computing these curves, the values of \( \omega_0 \) and \( k \) were 500 Hz and 1.083 [corresponding to \( B(\pi | 0) = 14 \) dB] and the values of \( \gamma(r_n) \) were obtained from the points at \( r_n = 2, 4, \) and 6 msec for 500 Hz in Fig. 7. [The curve for \( B(\pi | r_n) \) provides an excellent fit to the LJ data in Fig. 2. However, if the density of points in the LJ data were greater, or if the comparison were continued to much larger values of \( r_n \), the fit would degenerate because of the obviously false predictions concerning the minima.]

In considering these results, it should be noted that the revisions not only leave invariant the successful predictions for stimuli in which the two noise signals are identical but they are also consistent with the revisions discussed in a previous paper in connection with the RJ data on decorrelation. For the stimuli considered in the RJ experiment (in which the two noise signals were decorrelated by the introduction of inde-
pendent noise sources), one obtains
\begin{equation}
    f(\pi \rho_0)(\Delta) = \frac{k + \cos(\omega_0 \Delta)}{k - \rho_0 \gamma(\Delta) \cos(\omega_0 \Delta)},
\end{equation}
(27)
\begin{equation}
    f(\pi \rho_0)(\Delta) = \frac{k - \cos(\omega_0 \Delta)}{k + \rho_0 \gamma(\Delta) \cos(\omega_0 \Delta)},
\end{equation}
(28)
where \( \rho_0 \) is the normalized crosscorrelation coefficient of the two noise signals. Applying the maximization approximation, one obtains the choices \( \Delta = 0 \) and \( \pi / \omega_0 \) and the equations
\begin{equation}
    B(\pi \rho_0) = (k + 1)/(k - \rho_0),
\end{equation}
(29)
\begin{equation}
    B(\pi \rho_0) = (k + 1)/(k - \rho_0 \gamma(\pi / \omega_0)).
\end{equation}
(30)

The Rf data and the curve obtained from Eq. 29 for \( B(\pi \rho_0) \) [with \( k \) chosen so that \( B(\pi \rho_0) = 1.3 \) dB] are shown in Fig. 9. If the results in Fig. 7 were used to evaluate \( \gamma(\pi / \omega_0) \) at 500 Hz, the curve for \( B(\pi \rho_0) \) would be lower at \( \rho_0 = 1 \) by approximately 3 dB and would be within 1 dB of the curve plotted for all \( \rho_0 \leq 0.8 \).

When \( \rho_0 = 0 \), one obtains the predictions \( B(\pi \rho_0) = B(\pi \rho_0) = B(\pi \rho_0) = (k + 1)/k \). If one uses the “standard” values \( \sigma_0 = 0.25 \) and \( \sigma_3 = 105 \) usec to evaluate \( k \) as a function of \( \omega_0 \), the theoretical curve for \( B(\pi \rho_0) = B(\pi \rho_0) \) has a value of approximately 3 dB at the lower frequencies (about 1 dB less than the data in Fig. 6) and declines monotonically to 1.8 dB at 1200 Hz (2–3 dB less than the data in Fig. 6).

Although the results of eliminating the maximization approximation have not yet been studied in detail, it appears that the predictions for \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) could be substantially improved if \( k \) and \( \gamma \) were assigned their proper values in the maximization process. For example, the local minima of \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) would then be achieved only at single points (rather than over extended intervals) and their values would be approximately \((k + 1)/(k + \gamma(\pi / \omega_0)) \) (rather than unity). For large \( \rho_0 \), both the maxima and minima of \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) would approach the value \((k + 1)/k \) predicted for \( B(\pi \rho_0) = B(\pi \rho_0) \). Note, however, that these changes would not eliminate the discontinuities at \( \tau_n = (2q - 1)\pi / \omega_0 \) nor solve the flattening problem in \( B(\pi \rho_0) \). One could eliminate the discontinuities by omitting the constraint \(| \Delta | \leq \pi / \omega_0 \); however, this would lead to even greater absurdities. In particular, the model would then no longer predict any damping. Also, the prediction for all of the local minima [including those at \( (0,0) \) and \( (\pi/\pi) \)] would be \((k + 1)/k \). Presumably, one could retain the constraint \(| \Delta | \leq \pi / \omega_0 \) and still avoid the discontinuities by employing some sort of “smoothing” in the neighborhood of \( \tau_n = (2q - 1)\pi / \omega_0 \); however, no reasonable logic for this smoothing has yet been established.\(^{20}\)

\(^{20}\)In Ref. 3, an excellent fit was obtained to the J100 data on \( B(\pi \rho_0) \) at 500 Hz by assuming that \( \Delta \) was confined to the interval \(| \Delta | \leq 0.9 \) msec (0.1 msec less than \( \pi / \omega_0 \) at 500 Hz), and that

The most serious failure of these revisions in interpreting the data considered in Figs. 1–5 is that they leave the flattening in \( B(\pi \rho_0) \) completely unexplained and treat it as though it had no relation to the flattening in \( B(\pi \rho_0) \). Although the cause of the flattening in \( B(\pi \rho_0) \) is far from obvious, the idea that the flattening in \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) are unrelated is clearly absurd. In a previous paper,\(^{3} \) it was suggested that both the flattening in \( B(\pi \rho_0) \) and the deviation of \( B(\pi \rho_0) / B(\pi \rho_0) \) from unity [which can be identified with the flattening in \( B(\pi \rho_0) \)] might be caused by difficulties encountered in providing delays \( \Delta \) for which \(| \Delta | > H \), where \( H \) is the time width of the head (approximately 0.7 msec). This modification is preferable to the present ones in that it relates the flattening in \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \). Also, it is consistent with the following observations: (1) in a natural environment, the class of \( \Delta \)'s normally required lie in the interval \(| \Delta | \leq H \) (Ref. 31); (2) the frequencies \( \omega_0 \) for which flattening occurs all have a half period \( \pi / \omega_0 \) that exceeds \( H \); (3) at those frequencies for which flattening occurs, the flattening in the \( q \)th cycle of \( B(\pi \rho_0) \) begins in the neighborhood of \( \tau_n = 2q \pi / \omega_0 + H \) and the flattening in \( B(\pi \rho_0) \) begins (very roughly) in the neighborhood of \( \phi_n = \omega_0 H \). When one attempts to apply this “headwidth constraint” in a systematic fashion, however, one is faced with the following difficulties. First, although the headwidth constraint is consistent with the existence of critical points at \( \tau_n = 2q \pi / \omega_0 + H \) and \( \phi_n = \omega_0 \), it does not explain the form of \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) in the intervals \( \pi / \omega_0 - H < \tau_n < (2q + 1) \pi / \omega_0 \) and \( \omega_0 H < \phi_n < \pi / \omega_0 \). If it is assumed that \( A = -H \) in these intervals, then \( B(\pi \rho_0) \) and \( B(\pi \rho_0) \) should decrease sharply as \( \tau_n \) and \( \phi_n \) increase beyond the critical points.\(^{31}\) It is possible that this prob

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lem could be solved by assuming that $|\Delta|$ can exceed $H$, but that, when it does so, the value of $\alpha$ increases. How useful this approach would be depends upon the extent to which the function $\alpha_0(\Delta)$ required to fit the data on $B(0|\phi_0)$ and $B(0|\tau_0)$ is independent of $\omega_0$ and $q$. Second, it is not obvious how the headwidth constraint can be integrated with the fact that the interval of $\tau_n$ between the termination of the flattening and the local minimum in $B(0|\tau_n)$ is considerably larger than the interval between the initiation of the flattening and the previous local minimum (a fact that is implied by the damping and the local symmetry of the slopes about $\tau_n = 2\pi q/\omega_0$). Third, the headwidth constraint is inconsistent with the lack of flattening at 500 Hz. Even if $H$ is as large as 0.8 msec, one would expect the flattening to be apparent at 500 Hz. Fourth, the headwidth constraint is inconsistent with the fact that $B(0|u)$ is no smaller than $B(\pi|u)$ for frequencies at which flattening occurs in $B(0|\tau_n)$ and $B(0|\phi_0)$. Fifth, it is unclear how the headwidth constraint can be integrated with the disappearance of the flattening in $B(0|\phi_0)$ and $B(0|\tau_n)$ that must take place when $\phi$ is assigned values different from zero. Sixth, and finally, the headwidth constraint is inconsistent with the relations among the data on $B(0|\phi_0)$, $B(0|\tau_n)$, $B(\pi|0)$, $B(0|\pi)$, and $B(\pi|\tau_n - \omega_0\tau_n)$. According to the proposed modifications, the flattening in $B(0|\phi_0)$ [which is responsible for the deviation of $B(\pi|0)/B(0|\pi)$ from unity] is caused by both the decorrelation factor $\gamma$ and the headwidth constraint, whereas the flattening in $B(0|\tau_n)$ is caused only by the headwidth constraint. Thus, in order to explain the similarity of the flattening in $B(0|\phi_0)$ to the flattening in $B(0|\tau_n)$ over the first half-cycle, it must be assumed that the flattening in $B(0|\phi_0)$ caused by the decorrelation factor is negligible as compared with that caused by the headwidth constraint. This assumption, however, is contradicted by the data on $B(\pi|\tau_n - \omega_0\tau_n)$. According to these data (or more precisely, according to the values of $\gamma$ derived from these data), the decorrelation factor is not only not negligible, but it can account for most of the flattening in $B(0|\phi_0)$ [and most of the deviation of $B(\pi|0)/B(0|\pi)$ from unity] by itself.  

In studying the problems related to the flattening phenomenon, the following auxiliary findings should be noted. First, the existence of the flattening is consistent with the subjective lateralization experience: the “detection flattening” is mirrored by a corresponding “lateralization flattening.” Second, there is the possibility that this same detection flattening occurs in other senses. Results that are similar to the lateralization effects in hearing have been reported for the senses of touch, smell, and taste. Also, recent data on “bilateral-cutaneous unmasking” indicate that many phenomena that appear in binaural unmasking can be reproduced on the skin. To what extent the detection flattening occurs in these other senses has not been determined. Finally, it is worth noting that no one has yet been able to shift the critical point $\tau_n = H$ in $B(0|\tau_n)$ through training. A preliminary attempt to achieve such a shift has been made by using pseudophones that simulate a headwidth of 3 ft and applying modern adaptation techniques. The results of this experiment showed no indication that one could modify $B(0|\tau_n)$ so that it would continue to increase beyond its value at $\tau_n = H$.

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