to be realized is not utilized in practice when implementing filters for fixed systems, and our approach eliminates that over capacity. An error analysis was carried out and expressions derived which will allow the designer to choose the word length which will insure a specified performance.

The continuing emergence of technologies which further reduce the power requirements and increase the devity of semiconductor memory such as silicon on sapphire (SOS) appears to suggest that in the future the exchange between multiplier logic and memory bits for digital filters will only become more economic and offer even more savings than what has been pointed out by us in this paper.

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Some Considerations in the Design of Multiband Finite-Impulse-Response Digital Filters

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Abstract—Although much has been learned about the relationships between design parameters for finite impulse-response (FIR) low-pass digital filters, very little is known about the relationships between the parameters of multiband filters. Thus given a set of design specifications for a multiband FIR filter (e.g., filter band edge frequencies and desired ripples in each of the bands) it is difficult to choose a set of modified parameters which will yield an acceptable filter using a standard FIR design algorithm. By an acceptable filter we mean one with monotonic behavior of the frequency response in the DON'T-CARE or transition regions between bands and one providing at least the desired attenuation (or ripple) in each of the bands. In this paper, we examine the theoretical and practical issues of designing multiband filters and present several strategies for choosing the input parameters for the McClellan et al. filter-design algorithm to yield reasonable filters which meet arbitrary specifications.

I. INTRODUCTION

THE PROBLEM of designing optimal (in a Chebyshev or minimax sense) linear phase, finite impulse-response (FIR) digital filters has two aspects. The first part involves choosing a set of parameters (e.g., band edge frequencies, impulse response duration, desired response in the various bands, and weights on the approximation error in each of the bands) to specify the desired filter. The second part

"Fig. 8. Model for the mean squared error of a digital filter realized in cascade form."
involves solving the approximation problem to obtain the impulse response coefficients of a filter that meets the tolerance scheme specified by the filter parameters. A large amount of work has gone into devising algorithms for obtaining FIR filter coefficients. However, except for the special cases of low-pass (and high-pass) filters \([6]–[10]\), differentiators \([11]\), and Hilbert transformers \([12]\), the relationships between the various filter-design parameters and their subsequent effects on the frequency response of the resulting filter are not well understood. It is the purpose of this paper to examine the design relationships for the case of multiband filters, i.e., filters with several passbands and stopbands, and, when using a design algorithm such as the McClellan et al. \([5]\) program, to propose several strategies for modifying the values of the design parameters so as to yield filter designs with acceptable frequency responses.

The organization of this paper is as follows. In Section II we discuss the theory of optimal multiband, linear phase FIR filters. It is shown how the number of bands in the filter affects the possible number of ripples in the filter’s frequency response in a predictable manner. In Section III several examples are presented which show the inherent problems involved in choosing input parameters for various design algorithms. It is shown in this section that unless great care is taken in choosing filter parameters (i.e., making sure that certain design constraints are approximately maintained) the filter response of the resulting filter in the don’t-care or transition regions may be unacceptable to the designer. Finally, in Section IV we present several strategies for choosing the filter parameters to essentially guarantee the acceptability of the resulting filter frequency response.

II. THEORY OF MULTIBAND FIR FILTERS

Let \( h(n), 0 \leq n \leq N - 1, \) be the impulse response of an \( N \)-point linear phase FIR digital filter. For the type of design problem we are considering, the linear phase condition implies that the impulse response satisfies the symmetry condition

\[
h(n) = h(N - 1 - n), \quad n = 0, 1, \ldots, N - 1. \tag{1}
\]

Depending on whether \( N \) is odd or even, the frequency response of the filter can be written in the form

\[
H(e^{j\omega}) = e^{-j\omega(N-1)/2} \sum_{n=0}^{(N/2)-1} a(n) \cos \omega n \tag{2}
\]

when \( N \) is odd (Case 1), where

\[
a(n) = 2h \left( \frac{N - 1}{2} - n \right), \quad n = 1, 2, \ldots, (N - 1)/2,
\]

and

\[
a(0) = h \left( \frac{N - 1}{2} \right),
\]

or

\[
H(e^{j\omega}) = e^{-j\omega(N-1)/2} \sum_{n=-1}^{N/2-1} b(n) \cos \omega (n - 1/2) \cos \omega \left( \frac{N}{2} - n \right) \tag{3}
\]

when \( N \) is even (Case 2), where

\[
b(n) = 2h(\frac{N}{2} - n), \quad n = 1, 2, \ldots, N/2.
\]

To formulate the approximation problem for multiband filters, it is necessary to define \( D(e^{j\omega}) \), the desired frequency response of the filter, and \( W(e^{j\omega}) \), a weighting function on the approximation error which enables the designer to choose the relative size of the error in each of the frequency bands of interest. The weighted error of approximation \( E(e^{j\omega}) \) is, by definition,

\[
E(e^{j\omega}) = W(e^{j\omega}) \left[ D(e^{j\omega}) - G(e^{j\omega}) \right] \tag{4}
\]

where \( G(e^{j\omega}) \) is the real summation in either (2) or (3).

The optimal filter or Chebyshev approximation problem is one of finding the coefficients \( a(n) \) for Case 1 or \( b(n) \) for Case 2 to minimize the maximum absolute value of \( E(e^{j\omega}) \) over the frequency bands in which the approximation is being performed. The properties of the solution to the Chebyshev approximation problem are given in the alternation theorem which says that for the optimum filter the weighted error function \( E(e^{j\omega}) \) must have at least \( (N + 3)/2 \) (for Case 1 designs) or \( (N/2 + 1) \) (for Case 2 designs) points in the frequency bands of interest at which \( |E(e^{j\omega})| \) attains the maximum value.

Thus the alternation theorem constrains the number of equal amplitude extrema of \( E(e^{j\omega}) \) to be

\[
N_\epsilon \geq \left\lfloor \frac{(N + 3)}{2} \right\rfloor, \quad \text{Case 1}
\]

\[
N_\epsilon \geq \frac{(N/2 + 1)}{2}, \quad \text{Case 2.} \tag{5}
\]

It is important to be able to bound the maximum number of extrema of \( E(e^{j\omega}) \) in order to understand the design difficulties which are inherent in multiband filter design. By examining the behavior of \( E(e^{j\omega}) \) as given by (4), it is straightforward to show that the number of extrema of \( E(e^{j\omega}) \) is bounded above by

\[
N_\epsilon \leq \frac{(N + 1)}{2} + (2N_B - 2), \quad \text{Case 1}
\]

\[
N_\epsilon \leq \frac{N}{2} + (2N_B - 2), \quad \text{Case 2.} \tag{6}
\]

where \( N_B \) is the number of bands in the filter, and assuming that the first band begins at \( \omega = 0 \) and the last band ends at \( \omega = \pi \). Combining (5) and (6) gives the relations
\[
[(N + 3)/2] \leq N_e \leq [(N + 1)/2] + (2N_b - 2),
\]

Case 1

\[
(N/2 + 1) \leq N_e \leq (N/2) + (2N_b - 2),
\]

Case 2.

For the case of a low-pass filter (a two-band problem), (7) gives the familiar result [7]

\[
[(N + 3)/2] \leq N_e \leq [(N + 5)/2]
\]

for Case 1 designs.

Since \(N_e\) must be an integer, the optimal Case 1 low-pass filter can have either \([(N + 3)/2]\) equal amplitude extrema, or \([(N + 5)/2]\) extrema with all but possibly one of equal amplitude. Similar statements can be made for Case 2 low-pass filters.¹

When the number of bands exceeds two, the possibilities increase tremendously. Thus for a three-band problem, for Case 1 designs, (7) gives \([(N + 3)/2] \leq N_e \leq [(N + 9)/2]\), or equivalently, there can be \((N + 3)/2, (N + 9)/2, (N + 7)/2, (N + 5)/2\) extrema of the error function. Only \((N + 3)/2\) of these extrema need be equiripple and are constrained to lie within the frequency bands of the filter. The remaining ripples are free to occur anywhere, e.g., within the filter bands (in which case they must be smaller in value than the other ripples), or in the transition of non-care bands between filter bands (in which case they are totally unconstrained). Thus some of the ripples of the frequency response of a multiband filter are inherently unconstrained and can lie outside of the passbands and stopbands of an optimal filter. When such cases occur the resulting frequency response of the filter is generally unacceptable. This is the real essence of the problem. Several examples of such unacceptable behavior are given in the next section.

It should be noted that (7) shows that as the number of bands in the filter increases, the difference between the upper bound and the lower bound on \(N_e\) grows linearly with \(N\). Thus, inherently, filters with a large number of bands are more likely to require careful attention in their design parameters to give acceptable frequency response behavior than filters with a small number of bands.

III. EXAMPLES OF OPTIMAL MULTIBAND
FIR FILTERS

In this section several examples of optimal multiband FIR filters are presented which illustrate the inherent design difficulties referred to in the preceding section. Fig. 1 shows linear and log magnitude plots of the frequency response of a bandstop (three-band) filter,² Design 1, with input specifications.³

![Fig. 1. Linear and log magnitude frequency responses for initial Design 1 specifications.](image)

³ In the remainder of this paper we will consider only Case 1 designs strictly for convenience. Trivial modifications are required for Case 2 designs.

² All of the filters included in this paper were designed using the method of [5], unless otherwise noted.

³ All the input specifications for the filters of this section were selected in order to test the ideas presented in this paper.

band edge frequencies: 0, 0.14375973, 0.1653943, 0.37032451, 0.41679744, 0.5;

desired ripples: 0.07181388, 0.0032954, 0.08552212.

A value of \(N = 75\) was used as the required impulse response duration to meet the given specifications. As discussed earlier, the optimal three-band filter must have at least \((N + 3)/2\) equal amplitude extrema (of alternating sign) in the frequency range of interest. However, it was shown that a three-band filter could have up to \((N + 9)/2\) extrema, thereby permitting up to 3 extrema of the frequency response to occur outside of the bands of interest, i.e., in the transition bands. Such ripples in the transition band are completely unconstrained, and therefore the frequency response of such an optimal filter can assume any value in the transition band. If we postulate that the only reasonable or acceptable filter frequency response behavior in the transition band be monotonic, such nonmonotonic transition bands are unacceptable. In Fig. 1 we see an example of such unacceptable behavior between the stopband and the second passband of the filter where the frequency response has an extraneous maximum with a magnitude of about 2.58, or about 8.2 dB. For reference purposes we shall refer to this set of design specifications as Design 1 throughout this paper. Appendix I gives the actual input parameters to the program.
of [5] for this filter and others referred to in this paper.

Fig. 2 shows linear and log magnitude frequency responses of a bandpass filter (another three-band filter), Design 2, with input specifications:

- Band edge frequencies: 0, 0.0728033, 0.22754845, 0.29030124, 0.3445328, 0.5.
- Desired ripples: 0.00007691, 0.00047589, 0.02257863.

A value of \( N = 43 \) was used as the required impulse response duration to meet the given specifications. As seen in this figure, the frequency response exhibits nonmonotonic behavior between the first stopband and the passband where there are two extraneous extrema. Again such a filter is unacceptable for most applications even though its behavior in the bands of interest is quite acceptable.

Fig. 3 shows linear and log magnitude frequency response plots for a five-band filter, Design 3, with input specifications:

- Band edge frequencies: 0, 0.00820222, 0.11018835, 0.20585967, 0.26931373, 0.31158715, 0.37675551, 0.38929958, 0.46299174, 0.5.
- Desired ripples: 0.00055765, 0.00061921, 0.00010811, 0.00059465, 0.00050708.

A value of \( N = 57 \) was used as the required impulse response duration. Since there are five bands for this example, there can be up to seven extraneous ripples in the frequency response. In this particular example there are four extraneous extrema in the first and fourth transition bands, two in each band. Again such frequency response behavior in the transition band makes this filter unacceptable.

As a final example, Fig. 4 shows the linear and log magnitude frequency response of another five-band filter, Design 4. For this example there are five extraneous frequency response extrema in the first three transition bands. One of these extrema attains a peak value of about 36 dB, thereby effectively dominating the filter response in each of the five bands. Thus this filter, despite specifications to the contrary, effectively approximates a resonator at the frequency of the transition band extremum at which the peak response is attained.

The above examples serve to effectively illustrate the potential problems associated with designing optimal multiband FIR filters when careful attention is not paid to the design parameters. An important question which precedes any discussion of solutions to these problems is whether such unacceptable frequency response behavior is fairly typical or only occurs for special cases. A complete answer to this question cannot be readily given because any extensive exhaustive investigation of even three-band filters is totally unreasonable because of the number of parameters involved, i.e., four band edge frequencies, and two independent ripple parameters. Thus a preliminary investigation was made by selecting filter parameters to yield a wide range of filter specifications and then investigating the resulting frequency response of each filter. For such an experiment over 90 percent of the resulting filter designs were classed as unacceptable. Thus it was concluded that design methods which could circumvent such difficulties would be of great value to filter designers.

Before proceeding to a discussion of how to choose filter-design parameters to avoid unacceptable behavior, we first give some evidence that acceptable filter behavior can be obtained over a fairly wide range of input parameters provided certain relationships are maintained. Fig. 5 shows a sequence of plots of the frequency response of a bandpass filter with \( N = 25 \), and with band edge frequencies 0., FS1, 0.2, 0.3, 0.4, 0.5 and with fixed ripple weights (1, 10, 1). The stopband edge frequency FS1 is varied in the plots of Fig. 5 from 0.025 to 0.19.

Although nonmonotonic behavior in the transition bands of the frequency response is attained in the first two plots (FS1 = 0.025 and FS1 = 0.05), it is seen that in the range 0.052 \( \leq \) FS1 \( \leq \) 0.14, the resulting filter frequency response is acceptable. Beyond FS1 = 0.14, the filter frequency response again displays nonmonotonic behavior with extraneous ripples in the second transition band. The significance of this example is that there is a reasonably large range of values of FS1 for acceptable filter behavior. Thus a set of design relationships which could approxi-
mately predict acceptable parameter ranges would be expected to be useful for most cases of interest.

Fig. 6 gives additional evidence that by varying a single filter parameter acceptable frequency response behavior can be attained over a fairly wide range. This figure shows log magnitude frequency responses of four band-pass filters designed with identical parameters except for \( N \) which took on the values \( N = 15, 41, 55, 81 \). For values of \( N \) of 15, 41, and 55, the filter frequency response remains acceptable; even at \( N = 81 \) only a very small ripple occurs in the first transition band.

In summary, the examples of this section have shown that undesirable frequency response behavior is more the rule than the exception for multiband filters unless special care is taken in selecting the filter parameters. In the next section we discuss several design procedures to essentially insure desirable frequency response behavior.

IV. STRATEGIES FOR DESIGNING ACCEPTABLE MULTIBAND FIR FILTERS

We have already shown that unless special care is taken in choosing multiband filter input specifications, the resulting optimal filter will generally be unacceptable in that the frequency response will not be monotonic in the DON'T-CARE or transition region between bands. In this section we present three strategies for designing multiband FIR filters which are capable of avoiding such problems. The three strategies are the following.

1) Modify the stopband edge frequencies.
2) Modify the error weighting function.
3) Design maximal ripple filters only.

It will be shown here that although all three procedures can yield acceptable filters, in most cases the first procedure is the most reasonable one.

A. Modified Filter Specifications

In order to be able to design acceptable optimal multiband FIR filters, it is necessary to understand the mechanism which leads to the nonmonotonic frequency response in the transition band. Since there are so many parameters involved in the design of a multiband filter it is virtually impossible to give any design relationship that takes into account the effects of them all. Instead the assumption is made that one can treat the design problem as a composite of uncoupled low-pass filter designs. This approximation would appear to be valid in most cases; the exceptions being where the width of any filter band is very narrow. Thus in the example of Fig. 7 we would like to design a five-band filter with the given band-edge frequencies, and the desired ripples.
of $N$ [(8)–(11)] vary greatly depending on the actual filter specifications. In order to meet the tightest of the equivalent low-pass filter specifications requires using for $N$ the maximum of $N_1$, $N_3$, $N_5$, and $N_6$. (A weighted average of the $N_i$'s provides an alternative estimate of the value of $N$ to meet the tightest of the specifications.)

Using a value of $N$ to meet the tightest of the specifications means that the low-pass with the loosest specifications, requiring the smallest value of $N$, will be greatly over-designed. Since the band-edge frequencies are specified, and the amplitude of the ripples are controlled by the fixed weighting function, such gross discrepancies between low-pass specifications leads to the undesirable transition band ripples shown in the examples of Section III.

To verify the above hypothesis, values of the $N_i$'s were computed for a wide range of multiband FIR filters. With almost no exceptions, there was a fairly large difference between the required values of the $N_i$'s for filters with unacceptable frequency response behavior; whereas for those filters with monotonic transition bands (e.g., the maximal ripple designs of Section IV-D) the values of the $N_i$'s for all the bands were very nearly the same.

The implication of the above argument is that in order to obtain acceptable frequency response behavior for multiband filters the filter specifications should be chosen such that the condition

$$N_1 \approx N_2 \approx \cdots \approx N_L$$

(12)

holds for all the $L$ bands. Equation (12) implies that one has to modify the original filter specifications in such a way that the modified filter response would be as good or better than that initially specified. There are two obvious ways of achieving this approximate equality of (12). The first alternative is to narrow some of the filter transition bands (by broadening the stopbands or alternatively by broadening the passbands) thereby raising the required estimate of $N_i$ for those bands affected. The second way is to tighten some of the ripple specifications again raising the estimate of the $N_i$'s for those bands involved. The resulting filter would then have smaller ripples than initially specified; but such behavior is entirely acceptable. Both these procedures have been tested and results of their effectiveness will now be presented.

B. Modified Stopband Specifications

The first method of equalizing the $N_i$'s is to reduce the width of certain of the transition bands (by widening the appropriate stopbands) thereby raising the required estimate of $N_i$. This procedure has the advantage of leaving the filter passbands unaffected, and sharpening the transition bands, thereby giving a better filter. (Alternatively one could consider widening the filter passbands; however such a modification is not as intuitively pleasing to the filter designer since widening the passbands will ordinarily increase the filter output while widening the stopbands will necessarily decrease the filter output.)

Figs. 8–11 show log magnitude frequency responses for
the modified stopband method applied to Designs 1 to 4 of Section III. Each of the resulting filters has monotonous transition bands and essentially meets all the given input filter specifications.

To illustrate the use of this procedure we outline the steps used to obtain the modified specifications for Design 1. From the low-pass prediction formulas [5], values of 75 and 35 were obtained for the $N_i$'s. To equalize the $N_i$'s, the upper stopband edge frequency was raised to the value $F_{52}$ as determined by the relation

$$N_2 = 75 = \frac{D(0.0855, 0.0033)}{(0.41679744 - F_{52})} + 1. \quad (13)$$

The value of $F_{52}$ from (13) was 0.3958 where the Herrmann et al. formula [8] was used to evaluate $D(0.0855, 0.0033)$. The new frequency specifications were used as the input parameters to the design algorithm yielding a filter with the frequency response shown in Fig. 8.

C. Modified Ripple Specifications

To illustrate the ripple modification procedure we will take the data from three of the four unacceptable designs of Section III and redesign the filters. For Design 1, the
estimates of the $N_i$ were 75 and 35 from the original specifications. The second passband ripple was greatly reduced to raise the estimate of $N_2$ to be 75. The resulting log magnitude response of the filter is shown in Fig. 12. (The input parameters for all filters in this and the preceding section are given in Appendixes II and III.) The filter frequency response is now monotonic in all transition bands, as required.

For Design 2 the estimates of $N_i$ were 27 and 49. To equalize the $N_i$’s the ripple requirement in the first band was greatly reduced leading to the filter whose log magnitude frequency response is shown in Fig. 13. For Design 3 the desired ripples in the first and fifth bands were greatly reduced leading to the filter whose log magnitude frequency response is shown in Fig. 14. For all these examples the tightening of the specifications succeeded in eliminating the undesirable transition band ripples in the filter frequency response.

Although this ripple modification procedure appears to be a reasonable way of achieving an acceptable filter which meets all design specifications, there are some difficulties with this method. One problem is in many cases the discrepancies in the $N_i$’s are so great that the ripple specifications required to equalize the $N_i$’s call for ripples on the order of $10^{-7}$ or less. For example, for Design 2 (as seen in Fig. 10) the modified ripple in the first stopband was on the order of $10^{-3}$. In such cases there are two distinct problems. First, the design algorithm starts to lose accuracy (a function of the computer word-length) in trying to design filters with such small ripples. Further, the design formulas for predicting the $N_i$’s from the ripples are not very accurate for ripples much less than $10^{-6}$. Thus, for several practical reasons the method of ripple modification is not the best way of achieving the desired response.

**D. Maximal Ripple Multiband Filters**

In Section II we showed that the maximum number of ripples of the frequency response of an FIR filter was a function of the filter impulse response length $N$, and the number of bands in the filter. The unacceptable filter behavior was due to ripples (local maxima and minima) occurring in the filter transition bands. A fairly simple, direct method for preventing any ripples from occurring in transition bands is to force the filter to have the maximum possible number of ripples, and to constrain these ripples to be within the frequency bands of interest. Design procedures of this type have been proposed by Herrmann [1], and Hofstetter et al. [2], and the resulting filters have been called maximal ripple filters. This procedure, although circumventing the problem of having
ripples in the filter transition bands, presents a new design problem in that the filter band edge frequencies cannot be specified exactly as input parameters to the design algorithm, but instead are derived from the resulting frequency response. Since it is quite difficult to place the band edges where desired, this procedure is not useful for most applications.

To illustrate the maximal ripple design procedure, Fig. 15 shows the log magnitude frequency response of a three-band filter with \( N = 41 \), and the \( (N + 9)/2 = 25 \) error extrema distributed as 14 in the first stopband, 5 in the passband, and 6 in the second stopband. As noted above, the band-edge frequencies of the filter were not initially specified as design parameters, but instead were calculated from the final filter frequency response.

As another example, the specifications for Design 2 were used as input to the program of [2]. After several iterations in which the number of ripples in each band was varied, the program was unable to give a filter whose band edges were within a couple of percent of the desired band edges. Thus to use this method one must be willing to allow the filter band edges to be variable.

E. Discussion of Design Strategies

Three strategies have been proposed for choosing input parameters to the McClellan et al. algorithm for designing multiband FIR digital filters. Although any of the strategies can be used in most cases, the first strategy, i.e., the method of modified stopbands, is the one which is recommended for several reasons. First, it is a simple procedure to apply. Using any of the proposed low-pass design relations it is fairly simple to determine input parameters for the computer program. Second, the ripples remain fixed at the specified values. Thus one need not worry about trying to design filters with artificially large ranges of ripples. Third, the resulting filters have sharper transition bands than the design specifications initially called for. Thus, in some sense, the actual filter is a closer approximation to the infinitely sharp cutoff ideal multiband filter. Finally, the resulting filters passbands are left unaffected by the modifications. Since the passband-edge frequencies are generally determined by the intended application it is important that they remain fixed.

The method of modified stopbands was tested over a wide range of filter examples and yielded filter input parameters which led to acceptable filter frequency response in all cases. The method of modified ripples was also tested for these cases and for most, but not all, examples it also yielded filter input parameters which led to acceptable behavior of the frequency response.

V. SUMMARY

We have presented a wide range of data on the design of optimal multiband FIR filters in which we showed that unless proper care was taken in choosing the filter parameters, unacceptable frequency response behavior resulted in the filter don't-care or transition bands. After giving an intuitive explanation of the origin of this undesirable frequency response behavior, several techniques were proposed for choosing filter parameters to avoid this phenomenon. The most reasonable of these techniques was the method of modified stopband specifications in which the multiband filter was treated as a sequence of equivalent lowpass filters for which known parameter relations could be used to predict the required values of \( N \) to meet the given specifications. To equalize the values of \( N \) for each of the equivalent low-pass filters, certain of the stopband edge frequencies were varied, thereby sharpening

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Fig. 13. Log magnitude frequency response for Design 2 with modified ripple specifications.

Fig. 14. Log magnitude frequency response for Design 3 with modified ripple specifications.

Fig. 15. Log magnitude frequency response of a maximal ripple bandpass filter.
some of the transition bands of the filter. Using these modified filter specifications the resulting filter frequency responses were acceptable for all examples tested.

**APPENDIX I**

In this appendix we give the input specifications to the design program of [5] for each of the four designs of Section III. The format of the input is as follows.

Card 1: Filter length, filter Type (1 for multiband), number of bands, punch option, grid density.

Card 2: Band edges.

Card 3: Desired value in each band.

Card 4: Weight function in each band.

**Design 1**

Card 1: 75, 1, 3, 0, 16.
Card 2: 0.14375973, 0.16533943, 0.37032451, 0.41679744, 0.5.
Card 3: 1, 0, 1.
Card 4: 0.04588809, 1, 0.03853275.

**Design 2**

Card 1: 43, 1, 3, 0, 16.
Card 2: 0, 0.0728033, 0.22754845, 0.29030124, 0.3445328, 0.5.
Card 3: 0, 1, 0.
Card 4: 1, 0.1616032, 0.00340608.

**Design 3**

Card 1: 57, 1, 5, 0, 16.
Card 2: 0, 0.00820222, 0.11018835, 0.20585967, 0.26931373, 0.31158715, 0.37673551, 0.38929958, 0.46299174, 0.5.
Card 3: 1, 0, 1, 0, 1.
Card 4: 0.19386528, 0.17459027, 1, 0.18180258, 0.21319649.

**Design 4**

Card 1: 73, 1, 5, 0, 16.
Card 2: 0, 0.08886197, 0.13199438, 0.18550831, 0.27193968, 0.28519105, 0.35373202, 0.43737502, 0.45732656, 0.5.
Card 3: 0, 1, 0, 1, 0.
Card 4: 0.0959953, 0.11187421, 1, 0.11177379, 0.05401694.

**APPENDIX II**

In this appendix we give the input specifications for the four stopband modified designs of Section IV.

**Design 1**

Card 1: 75, 1, 3, 0, 16.
Card 2: 0, 0.14376, 0.165339, 0.3958009, 0.416797, 0.5.
Card 3: 1, 0, 1.
Card 4: 0.04588809, 1, 0.03853275.

**Design 2**

Card 1: 49, 1, 3, 0, 16.
Card 2: 0, 0.14376, 0.165339, 0.3958009, 0.416797, 0.5.
Card 3: 1, 0, 1.
Card 4: 0.00214954, 0.04684311, 1.

**Design 3**

Card 1: 57, 1, 5, 0, 16.
Card 2: 0, 0.00820222, 0.11018835, 0.20585967, 0.26931373, 0.31158715, 0.37673551, 0.38929958, 0.46299174, 0.5.
Card 3: 1, 0, 1, 0, 1.
Card 4: 0.0959953, 0.11187421, 1, 0.11177379, 0.05401694.

**Design 4**

Card 1: 65, 1, 5, 0, 16.
Card 2: 0, 0.1100416, 0.131994, 0.185508, 0.2154533, 0.3237905, 0.353732, 0.437375, 0.45732454, 0.5.
Card 3: 0, 1, 0, 1, 0.
Card 4: 0.0959953, 0.11187421, 1, 0.11177379, 0.05401694.

**APPENDIX III**

In this appendix we give the input specifications for the three ripple modified designs of Section IV.

**Design 1**

Card 1: 75, 1, 3, 0, 16.
Card 2: 0, 0.14376, 0.165339, 0.370325, 0.416797, 0.5.
Card 3: 1, 0, 1.
Card 4: 0.00214954, 0.04684311, 1.

**Design 2**

Card 1: 49, 1, 3, 0, 16.
Card 2: 0, 0.072803, 0.227548, 0.290301, 0.344533, 0.5.
Card 3: 0, 1, 0.
Card 4: 1, 0.00000125, 0.00000004.

**Design 3**

Card 1: 65, 1, 5, 0, 16.
Card 2: 0, 0.1100416, 0.131994, 0.185508, 0.2154533, 0.3237905, 0.353732, 0.437375, 0.45732454, 0.5.
Card 3: 0, 1, 0, 1, 0.
Card 4: 1, 0.00000654, 0.00003746, 0.00001144, 0.00026532.

**REFERENCES**

NECESSITY OF INSIGHT INTO BASIC PROCESS OF SPEECH PRODUCTION AND PERCEPTION

As is well known, speech phenomena involve not only physical (acoustical) processes, but also psychological processes, and even physiological processes. For example, nitrogen narcosis may have some effect on bodily and mental conditions (the author experienced such effects, as if he drank alcohol, under compressed air). Psychological adaptations to the unusual voice environment have been observed in both speech production [4], [7], [19], [22], [23] and perception [24]. Let us assume that a parameter shows heavy helium distortion. If it has no serious perceptual effect, there is, of course, no need to process it for unscrambling in the first place.

As a whole, comprehensive study covering a wide range of speech phenomena is indispensable, even from the engineering point of view, and it will lead to the following advantages: 1) providing useful data for designing a reasonable and practical un scrambler, 2) providing insight into the basic mechanism of speech production in media different from normal air, and 3) providing insight into the perceptual mechanism of speech in such media. The last two may be considered to be secondary advantages arising from the efforts made under 1). Nevertheless, 2) and 3) have some influence on 1). An example of such situation will be presented in the next section.

DISCUSSIONS

Speech spectrum is approximately described by the product of the spectrum of the excitation source and the vocal-tract transfer spectrum [27]. As to the transfer characteristics of helium speech, upward shift of formant frequencies has been reported by many investigators [1], [3]-[9]. Fant and Sonesson have given the theoretical framework of this nonlinear shift [2]. According to their indication, the relation between formant frequency in a helium environment $F_{He}$ and that in normal air $F_{N}$ is

$$F_{He} = kF_{N} + F_{low},$$

where $k$ is the sound velocity ratio ($c_{He}/c_{N}$), $b$ is a constant related to gas composition and pressure, and $F_{low}$ is the lowest resonance frequency of the vocal tract in normal air.

A large number of measurements of $F_{He}$ and $F_{N}$, and their least-square fitting to (1), shows that they are well explained by (1) and that the $F_{N}$ range around 200 Hz [22], [23], which coincides with the values in the direct measurement of the vocal-tract transfer characteristics [28]. These experimental results will lead to both 1) and 2) described in the previous section. As for 1), the un scrambler has to treat the nonlinearity of the formant shift. The necessity of treating the nonlinearity shift has been partly supported by the fact that some confusion of phonemes observed in the perceptual analysis [21] is considered to be due to this nonlinear shift. However, attention is paid to this nonlinearity only in a few attempts at unscrambling [15], [23], [26]. As for 2), further studies will make clear the effect of wall vibration of the vocal tract on the formant frequency and formant bandwidth [29].

Concerning the characteristics of the source excitation, our present knowledge is far less than that of the transfer characteristics. Excitation source of speech is voiced (glottal source) or unvoiced (turbulent noise). As for the unvoiced sound, relative reduction of the fricative noise level has been observed [2], [5], [6], [8] and has been con-