Department of Electrical and Computer Engineering
Digital Speech Processing
Homework No. 8

Problem 1 – The uniform probability density function is defined as

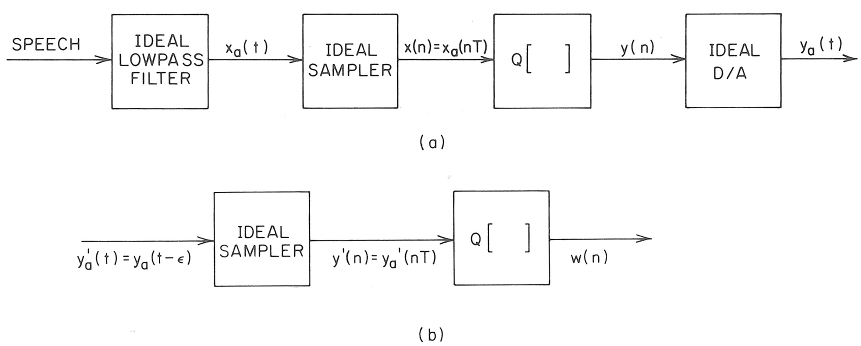
$$p(x) = \begin{cases} \dfrac{1}{\Delta} & |x| < \Delta/2 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and variance of the uniform distribution.

Problem 2 – Consider the Laplacian probability density function

$$p(x) = \frac{1}{\sqrt{2}\sigma_x} e^{-\sqrt{2}|x|/\sigma_x}$$

Find the probability that $|x| > 4\sigma_x$.



Problem 3 – A speech signal is bandlimited by an ideal lowpass filter, sampled at the Nyquist rate, quantized by a uniform B-bit quantizer, and converted back to an analog signal by an ideal D/A converter, as shown in the figure above. Define $y[n] = x[n] + e_1[n]$ where $e_1[n]$ is the quantization error. Assume that the quantization step is $\Delta = 8\sigma_x/2^B$ and that B is large enough so that we can assume:

1. $e_1[n]$ is stationary
2. $e_1[n]$ is uncorrelated with $x[n]$
3. $e_1[n]$ is a uniformly distributed white noise sequence

We have seen that, under these conditions, the signal-to-quantizing noise ratio is:

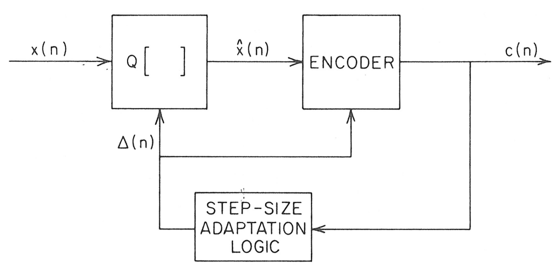
$$SNR_1 = \frac{\sigma_x^2}{\sigma_{e_1}^2} = \frac{12}{64} \cdot 2^{2B}$$

Now assume that the analog signal $y_a(t)$ is sampled again at the Nyquist rate and quantized by an identical B-bit quantizer, as shown in part (b) of the above figure. (Assume that $0 < \epsilon < T$; i.e., the two sampling systems are not exactly synchronized in time.) Assume that $w[n] = y'[n] + e_2[n]$ where $e_2[n]$ has identical properties to $e_1[n]$.

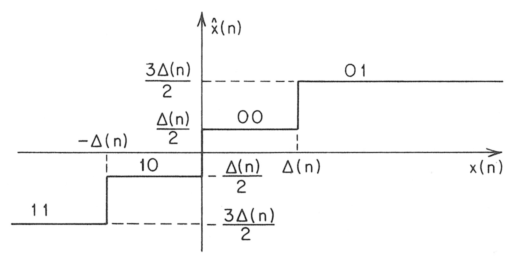
(a) Show that the overall signal-to-noise ratio is

$$SNR_2 = \frac{SNR_1}{2}$$

(b) Generalize the result of (a) to N stages of A/D and D/A conversion.



(a)



(b)

Problem 4 – Consider the adaptive quantization system shown in part (a) of the figure above. The 2-bit quantizer characteristic and code word assignment is shown in part (b) of the above figure. Suppose the step size is adapted according to the following rule:

$$\Delta[n] = M\ \Delta[n-1]$$

where M is a function of the previous codeword $c[n-1]$ and

$$\Delta_{\min} \leq \Delta[n] \leq \Delta_{\max}$$

Furthermore suppose that

$$M = \begin{cases} P & \text{if } c[n-1] = 01 \text{ or } 11 \\ 1/P & \text{if } c[n-1] = 00 \text{ or } 10 \end{cases}$$

(a) Draw a block diagram of the step-size adaptation system.
(b) Suppose that

$$x[n] = \begin{cases} 0 & n < 5 \\ 20 & 5 \leq n \leq 13 \\ 0 & 13 < n \end{cases}$$

Assume that $\Delta_{\min} = 2$ and $\Delta_{\max} = 30$ and $P = 2$. Make a table of values of $x[n]$, $\Delta[n]$, $c[n]$ and $\hat{x}[n]$ for $0 \leq n \leq 25$. (Assume that at $n = 0$, $\Delta[n] = \Delta_{\min} = 2$, and $c[n] = 00$).
(c) Plot the samples $x[n]$ and $\hat{x}[n]$ on the same coordinate scale.

Problem 5 – Write a MATLAB program to quantize a speech file using a uniform quantizer with B-bits/sample, where $B = 12, 10, 8, 4, 2, 1$.

(a) For each of the above bit rates, B, determine the quantization error sequence (consider using the supplied routine fxquant.m for quantization with rmode=round and lmode=sat), and plot a histogram of the quantization noise values (using the matlab hist command - think carefully about how many bins should be used)
(b) Plot the spectrum of the original waveform and the quantization error at each of the bit rates on a single plot. (Consider using the supplied code pspect.m to plot the long-time average power spectrum using the method of modified periodograms).
(c) Listen to the waveform of both the quantized speech file and the quantization noise. When can you hear structure in the quantization noise.
(d) Determine the SNR of the quantized signal at each value of B.

Use the speech file s5.wav to test your program. Only process 19000 samples of s5.wav beginning at sample 1300 (since the first 1300 samples are numerically equal to zero and will distort the statistics of the quantization error signal if not removed from the calculations.