

# ECE 137 A Mid-Term Exam

Thursday February 5, 2015

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 3 problems on this exam, and you have 75 minutes.

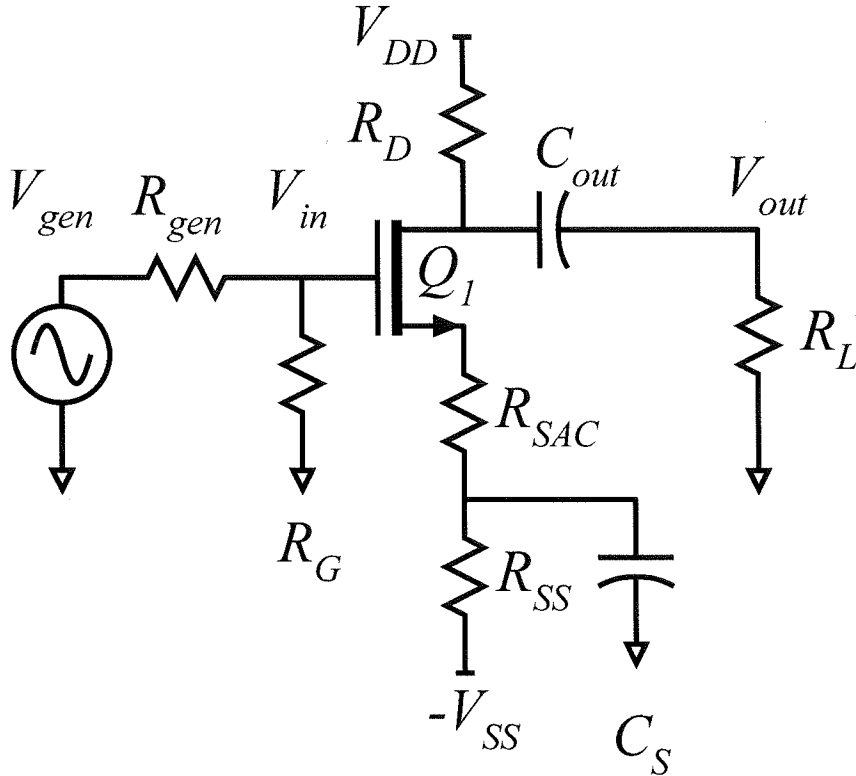
Use any and all reasonable approximations (5% accuracy is fine. ),  
***AFTER STATING and approximately Justifying them.***

Name: Solution B

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		10	2f		15
1b		5	3a		8
1c		5	3b		8
1d		10	3c		4
1e		15			
2a		10			
2b		5			
2c		5			
2d		10			
2e		5			
TOTAL					100

**Problem 1, 30 points**

You will be working on the circuit below:



The transistor has

$$L_g = 45 \text{ nm}, \quad \mu = 400 \text{ cm}^2/\text{V}\cdot\text{s}, \quad \varepsilon_{r,ox} = 3.8, \quad T_{ox} = 1 \text{ nm}, \quad v_{sat} = 10^7 \text{ cm/s}, \quad V_{th} = 0.284 \text{ V},$$

$$1/\lambda = 10 \text{ V},$$

From which we calculate:

$$c_{ox} v_{sat} = 3.36 \text{ mA/V}/\mu\text{m}, \quad \mu c_{ox} / 2L_g = 15 \text{ mA/V}^2/\mu\text{m}, \quad \Delta V = L_g v_{th} / \mu = 0.113 \text{ V},$$

The supplies are +1V and -1 V

You are to bias the transistor at 1mA drain current,  
with 0.5V DC drain voltage, and with -0.35 V DC source voltage.

$$R_{SAC} = 10 \Omega, \quad R_G = 1 \text{ M}\Omega, \quad R_{gen} = 100 \text{ k}\Omega, \quad R_L = 10 \text{ k}\Omega.$$

$C_S$  and  $C_{out}$  are very large (AC short-circuit)

Part a, 10 points

DC bias.

Use this approximation: Ignore (i.e. set to zero) the FET  $\lambda$  parameter in the DC bias calculation.

Find the following:

FET gate width  $W_g = 15.3 \mu\text{m}$   $R_{SS} = 640 \Omega$   $R_D = 500 \Omega$ .

1 = 5T.  $V_g = 0V$  so  $V_{gs} = 0.35V$  1

1 [ note  $V_{th} + \Delta V = 0.284V + 0.113V = 0.397V$ .

1 [ so, we are mobility-limited.

2 [  $I_D = \frac{15 \text{ mA}}{V^2 \cdot \mu\text{m}} \cdot W_g \cdot (V_{gs} - V_{th})^2$   
 $0.35V - 0.284V$

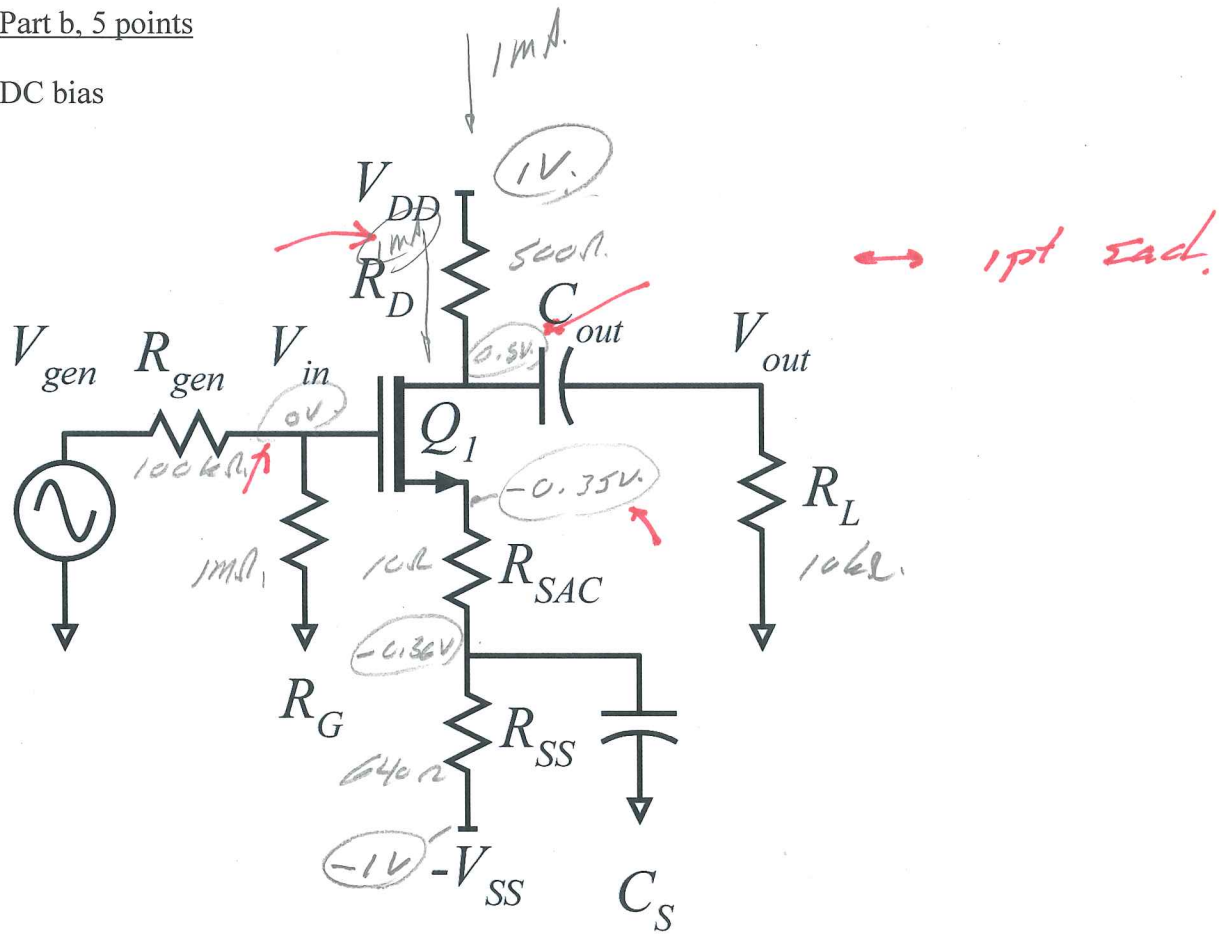
1 [  $\rightarrow W_g = \frac{I_D}{\frac{15 \text{ mA}}{V^2 \cdot \mu\text{m}} (V_{gs} - V_{th})^2} = 15.3 \mu\text{m}$ .

2 [  $R_{SS} + R_{SAC} = \frac{1V - 0.35V}{1 \text{ mA}} = \frac{0.65V}{1 \text{ mA}} = 650 \Omega$   
 $R_{SS} = 650 \Omega - R_{SAC} = 640 \Omega$

2 [  $R_D = \frac{V_{DD} - V_D}{I_D} = \frac{1V - 0.5V}{1 \text{ mA}} = 500 \Omega$ .

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors

Part c. 5 points

Using the actual (nonzero) FET  $\lambda$  parameter, find the FET small signal parameters

$g_m =$  \_\_\_\_\_  $R_{ds} =$  \_\_\_\_\_

We are mobility-limited, so

3

$$\begin{aligned} g_m &= \frac{Wq}{Lq} \mu C_{ox} (V_{gs} - V_{th}) (1 + \lambda V_{DS}) \\ &= 2 \cdot \frac{15 \text{ mA}}{\text{V}^2 \cdot \mu\text{m}} \cdot 15.3 \mu\text{m} \cdot (0.35\text{V} - 0.284\text{V}) \left(1 + \frac{0.85\text{V}}{10\text{V}}\right) \\ &= 30.3 \text{ mS} \cdot (1 + \lambda V_{DS}) = 32.9 \text{ mS} \\ & \quad \text{OK to omit } (1 + \lambda V_{DS} \text{ term}) \\ &= 1/30.4 \Omega \end{aligned}$$

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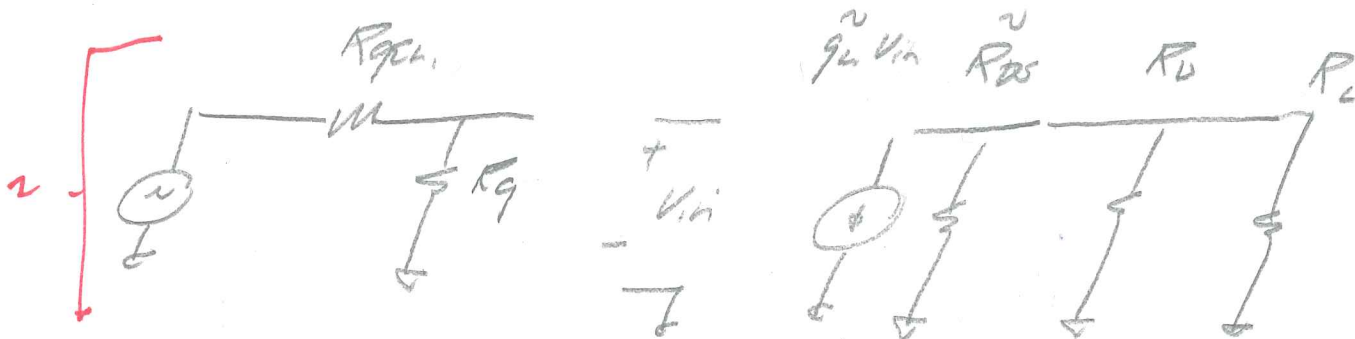
$$R_{ds} = \frac{V_{DS} + \frac{1}{\lambda}}{I_D} \approx \frac{1}{\lambda I_D} = \frac{10\text{V}}{1 \text{ mA}} = 10 \text{ k}\Omega$$

Part d, 10 points.

Find the small signal voltage gain  $V_{out}/V_{in}$  and the amplifier small-signal input resistance.

$V_{out}/V_{in} =$  \_\_\_\_\_

$R_{in, \text{ amplifier}} =$  \_\_\_\_\_



$$g_m = \frac{g_m}{1 + g_m R_{SAC}} = \frac{1}{1/g_m + R_{SAC}} = \frac{1}{30.4\Omega + 10\Omega} = \frac{1}{40.4\Omega}$$

$$R_{DS} = R_{DS}(1 + g_m R_{SAC}) = 10k\Omega \left(1 + \frac{10\Omega}{40.4\Omega}\right) = 12.5k\Omega$$

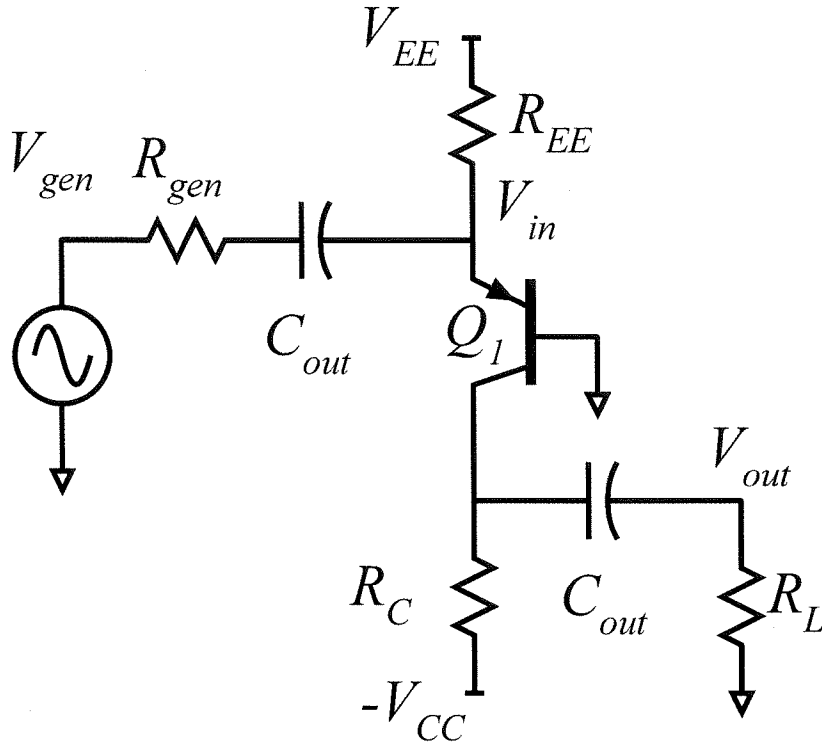
$$R_{eq} = R_{DS} \parallel R_D \parallel R_L = 12.5k\Omega \parallel 500\Omega \parallel 10k\Omega = 459\Omega$$

$$V_o/V_{in} = -g_m R_{eq} = -\frac{459\Omega}{40.4} = -11.4$$

$$R_{in, \text{ amp}} = R_g = 1M\Omega$$

**Problem 2, 50 points**

You will be working on the circuit below:



Q1:  $\beta = 100$ ,  $V_A = \text{infinity V}$

The supplies are +7.5V and -7.5 V.

You will bias the transistor with 1mA collector current.

The DC collector bias voltage is -4V.

$R_L$  is 10 k $\Omega$ ,  $R_{gen}$  is 75  $\Omega$

Part a, 10 points

DC bias.

Find the following:

$$R_{EE} = \underline{\hspace{2cm}} \quad R_C = \underline{\hspace{2cm}}$$

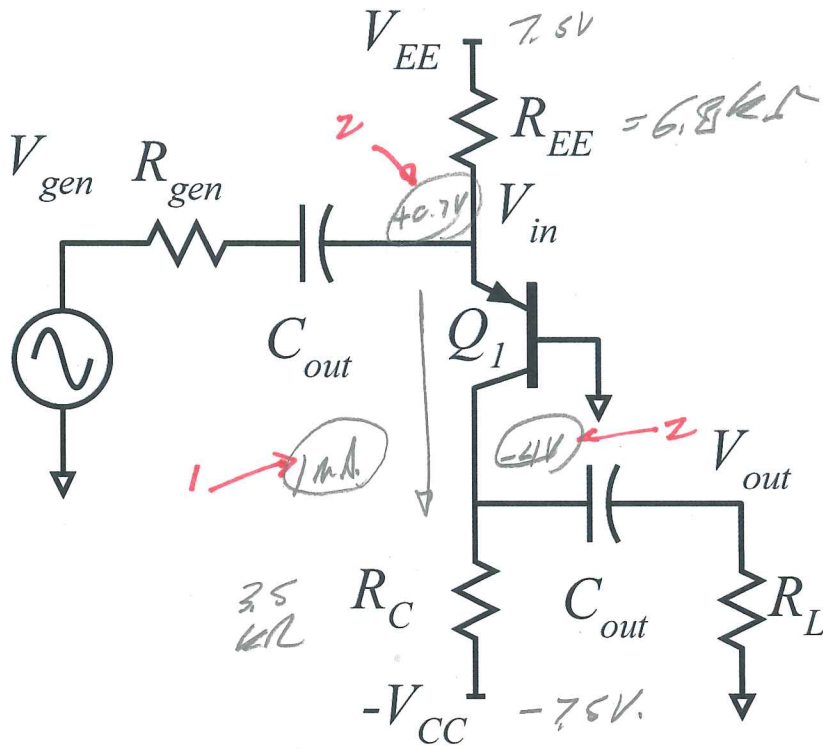
$$5 \left[ R_{EE} = \frac{7.5V - 0.7V}{1mA} = \frac{6.8V}{1mA} = 6.8k\Omega \right]$$

$$5 \left[ R_C = \frac{3.5V}{1mA} = 3.5k\Omega \right]$$



Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors

Part c, 5 points

Find the small signal parameters of Q1.

$g_m =$  \_\_\_\_\_  $R_{ce} =$  \_\_\_\_\_  $R_{be} =$  \_\_\_\_\_

2. 
$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = \frac{1}{26 \Omega} = 38 \text{ mS.}$$

1. 
$$R_{be} = \beta / g_m = 100 \cdot 26 \Omega = 2.6 \text{ k}\Omega$$

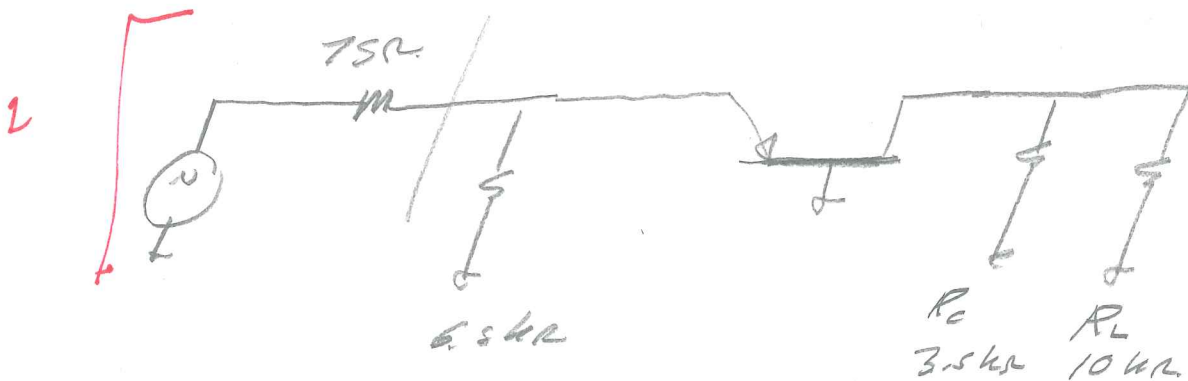
2. 
$$R_{ce} = \frac{V_{CE} + V_D}{I_C} \rightarrow \infty$$

Part d, 10 points.

Find the small signal voltage gain ( $V_{out}/V_{in}$ ) of Q1 and the amplifier small-signal input resistance.

$V_{out}/V_{in} =$  \_\_\_\_\_

$R_{in,amp} =$  \_\_\_\_\_



2  $\left[ R_{reg} = 10k\Omega \parallel 3.5k\Omega = 2.6k\Omega \right]$

2  $\left[ R_{in,T} = \left( \frac{1}{g_m} + \frac{R_C}{\beta} \right) \left( \frac{R_C + R_{reg}}{R_C} \right) = \frac{1}{g_m} = 26\Omega \right]$

2  $\left[ v_o/v_{in} = g_m R_{reg} = \frac{2.6k\Omega}{26\Omega} = 100 \right]$

2  $\left[ R_{in,amp} = R_{in,T} \parallel 6.8k\Omega = 26\Omega \parallel 6.8k\Omega \right]$   
 $= 25.8\Omega \approx 26\Omega$

Part e, 5 points

Find  $(V_{in}/V_{gen})$  and  $(V_{out}/V_{gen})$

$(V_{in}/V_{gen}) =$  \_\_\_\_\_

$(V_{out}/V_{gen}) =$  \_\_\_\_\_

3  $\left[ \frac{V_{in}}{V_{gen}} = \frac{26\Omega}{26\Omega + 75\Omega} = \frac{26}{101} = 0,257 \right]$

2  $\left[ \frac{V_o}{V_{gen}} = 0,257 \cdot 100 = 25,7 \right]$

Part f, 15 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. *Give the sign (+ or -) in your answers below.*

Cutoff of Q1; Maximum  $\Delta V_{out}$  resulting = \_\_\_\_\_

Saturation of Q1; Maximum  $\Delta V_{out}$  resulting = \_\_\_\_\_

Cutoff.

$$I_{CQ} = 1 \text{ mA}$$

$$I_{Cmin} = 0 \text{ mA}$$

$$\Delta I_C = 1 \text{ mA max}$$

$$R_{eq} = 2.6 \text{ k}\Omega$$

$$\Delta V_{out} = 2.6 \text{ k}\Omega \cdot 1 \text{ mA} = 2.6 \text{ V, } \underline{\text{negative}}$$

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4.

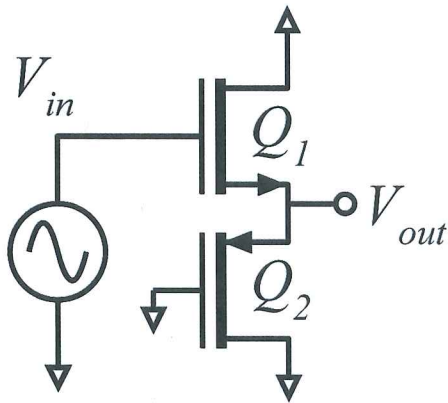
Saturation.

$$3 \quad [ V_{CE, Q} = 4.7 \text{ V}$$

$$3 \quad [ V_{CE, min} = V_{CE, sat} = 1/2 \text{ V}$$

$$1 \quad [ \Delta V_{out} = 4.2 \text{ V, } \text{positive}$$

**Problem 3, 20 points**  
nodal analysis



You will be working on the circuit to the left.

Ignore DC bias analysis. You don't need it.

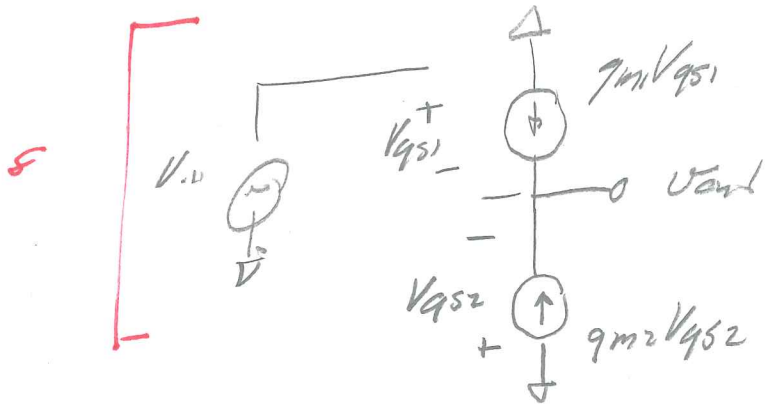
Transistor 1 has transconductance  $g_{m1}$ .

Transistor 2 has transconductance  $g_{m2}$ .

The drain-source resistances  $R_{ds}$  of both transistors are infinity (so you don't need to draw it!)

Part a, 8 points

Draw the small-signal equivalent circuit



*takes off points \* if controlling voltages not shown*

*\* 3 points for each missing control voltage*

Part b, 8 points

Find, by nodal analysis, a small-signal expression for  $V_{out}/V_{in}$ .

$V_{out}/V_{in} =$  \_\_\_\_\_

$$4 \quad \left[ \sum I = 0 \quad @ \quad V_{out} \right]$$

$$g_{m1} V_{gs1} + g_{m2} V_{gs2} = 0$$

$$\text{but } V_{gs1} = V_{in} - V_{out}$$

$$V_{gs2} = -V_{out}$$

$$g_{m1} (V_{in} - V_{out}) + g_{m2} (-V_{out}) = 0$$

$$g_{m1} \cdot V_{in} = (g_{m1} + g_{m2}) V_{out}$$

$$4. \quad \left[ \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m1} + g_{m2}} \right]$$

Part c, 4 points

$g_{m1} = 1 \text{ mS}$   $g_{m2} = 2 \text{ mS}$

Give a numerical value for  $V_{out}/V_{in}$ .

$V_{out}/V_{in} =$  \_\_\_\_\_

$$4. \quad \frac{v_o}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{m2}} = \frac{1}{3}$$