

**ECE 137 A Mid-Term Exam**

**Thursday, February 9, 2017**

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 3 problems on this exam, and you have 75 minutes.

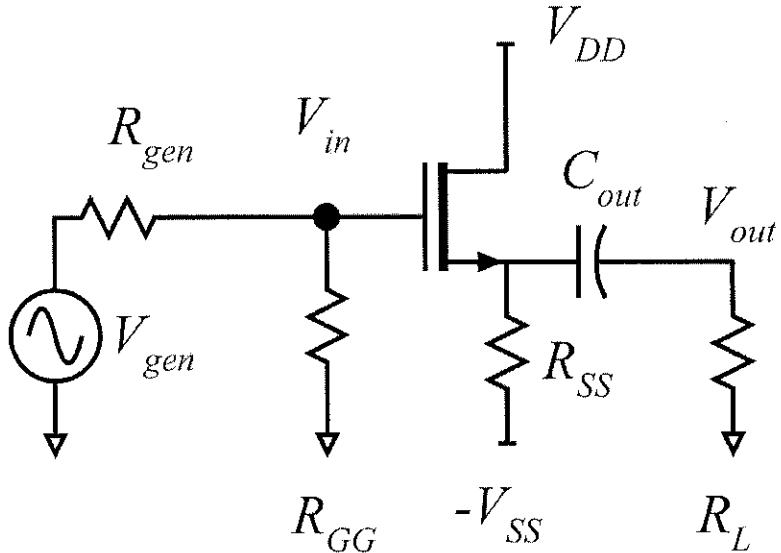
Use any and all reasonable approximations (5% accuracy is fine.) ,  
***AFTER STATING and approximately Justifying them.***

Name: Solution 2

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		10	2f		15
1b		5	3a		8
1c		5	3b		8
1d		10	3c		4
1e		15			
2a		10			
2b		5			
2c		5			
2d		10			
2e		5			
TOTAL					100

**Problem 1, 30 points**

You will be working on the circuit below:



The transistor has

$$L_g = 22 \text{ nm}, \mu = 180 \text{ cm}^2/\text{V}\cdot\text{s}, \varepsilon_{r,ox} = 3.8, T_{ox} = 1 \text{ nm}, v_{sat} = 10^7 \text{ cm/s}, V_{th} = 0.3 \text{ V}, 1/\lambda = 10 \text{ V},$$

From which we calculate:

$$c_{ox}v_{sat} = 3.36 \text{ mA/V}/\mu\text{m}, \mu c_{ox}/2L_g = 13.8 \text{ mA/V}^2/\mu\text{m}, \Delta V = L_g v_{th} / \mu = 0.122 \text{ V},$$

The supplies are +2V and -2 V

You are to bias the transistor at 2mA drain current, and with -0.5 V DC source voltage.

$$R_{GG} = 10 \text{ M}\Omega, R_{gen} = 100 \text{ k}\Omega, R_L = 1 \text{ k}\Omega$$

$C_{out}$  are very large (AC short-circuit)

Part a, 10 points

DC bias.

Use this approximation: Ignore (i.e. set to zero) the FET  $\lambda$  parameter in the DC bias calculation.

Find the following:

FET gate width  $W_g = \underline{4.3\mu m}$      $R_{SS} = \underline{750\Omega}$

2 [ Transistor is biased at  $V_g = 0$  &  $V_s = -V_{DD}$  ]

$\rightarrow V_{GS} = 11.2V$

2 [  $\Delta V = V_{GS} - V_{TH} \approx 0.122V$ , so FET is velocity-limited ]

$V_{DS} + \Delta V = 0.422V$

2 [  $\Rightarrow I_D \approx \text{Cox. } V_{DS} \cdot W_g \cdot (V_{GS} - V_{TH} - \Delta V/2)(1 + d/V_{DS})$  ]

└ ignore for DC

2 [  $I_{DS} = 3.36 \frac{mA}{V} \cdot \frac{1}{\mu m} \cdot (0.5V - 0.061V - 0.3V) \cdot W_g$  ]

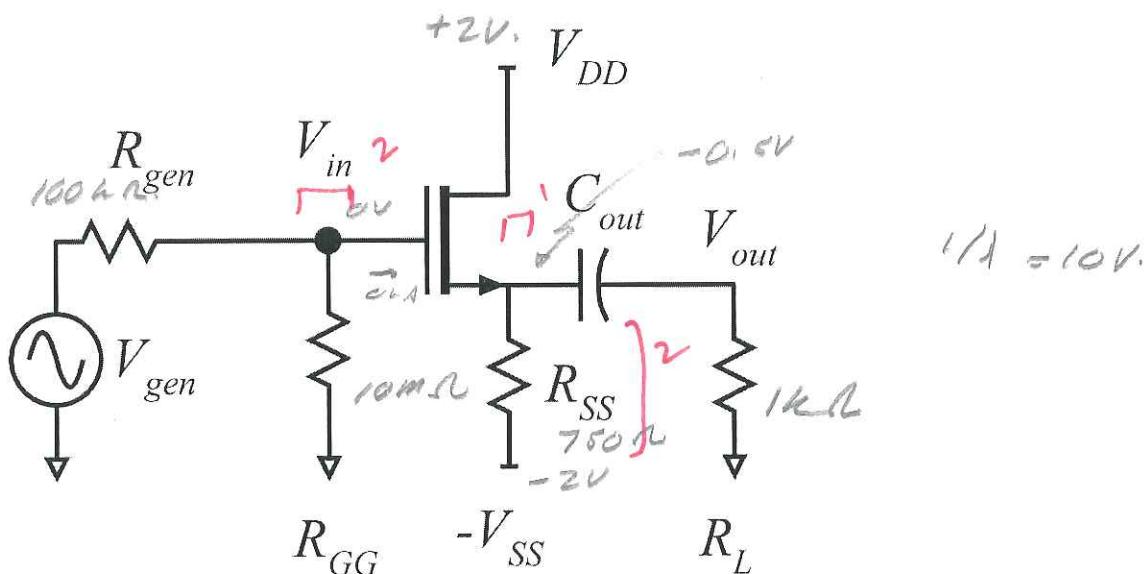
2 [  $I_{DS} = 3.36 \frac{mA}{V} \cdot \frac{W_g}{\mu m} \cdot \frac{\Delta V/2}{V_{DS}} \cdot \frac{V_{DS}}{V_{DD}}$  ]

$\rightarrow W_g = \underline{\underline{4.3\mu m}}$

2 [  $R_{SS} = \frac{2V - 0.5V}{I_{DS}} = \frac{1.5V}{2mA} = 0.75k\Omega = 750\Omega$  ]

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

$$W_g \cdot C_{ox} \cdot V_{TH} = \frac{3.36 \text{ mS}}{N} \cdot \frac{W_g}{L_{MOS}} = 19.9 \frac{\text{mA}}{\text{V}}$$

$$R_{DS} = 5 \text{ k}\Omega$$

$$g_m = 16.6 \text{ mS}$$

Part c, 5 points

Using the actual (nonzero) FET  $\lambda$  parameter, find the FET small signal parameters  
 $gm = \underline{18.1 \text{ mS}}$ ,  $Rds = \underline{565}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = C_{ox} \cdot \eta_F \cdot W \left( 1 + dV_{DS} \right)$$

$$= 3.36 \frac{\mu\text{A}}{\text{V}} \cdot 4.3 \mu\text{m} \left( 1 + \frac{2.5\text{V}}{10\text{V}} \right)$$

$$= 18.06 \text{ mS}$$

3

$$R_{DS} = \left\{ \frac{1}{k_I I_D} = \frac{10\text{V}}{2\text{mA}} = 5\text{k}\Omega \right.$$

2

$$\frac{V_{DS} + U_A}{I_D} = \frac{10\text{V} + 2.5\text{V}}{2\text{mA}} = 6.25 \text{ k}\Omega$$

either answer acceptable; I will use 5k $\Omega$  in the solution.

Part d, 10 points.

Find the small signal voltage gain  $V_{out}/V_{in}$  and the amplifier small-signal input resistance.

$$V_{out}/V_{in} = \underline{0.877}$$

$$R_{in, \text{amplifier}} = \underline{10\text{M}\Omega}$$

$$\begin{aligned} R_{eq} &= R_{S5} \parallel R_L \parallel R_{O5} \\ &= 750\Omega \parallel 1k\Omega \parallel 5k\Omega = 395\Omega. \end{aligned} \quad ]^3$$

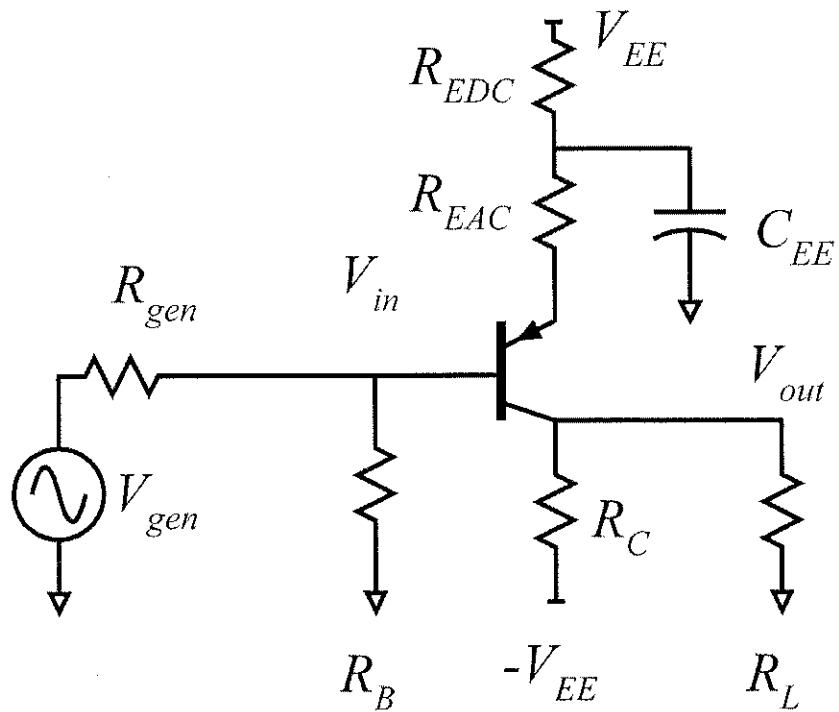
$$1/g_m = \frac{1}{18.1\text{mS}} = 55.2\Omega \quad ]^2$$

$$A_v = \frac{R_{eq}}{R_{eq} + 1/g_m} = \frac{395\Omega}{395\Omega + 55.2\Omega} = \underline{\underline{0.877}} \quad ]^3$$

$$R_{in, \text{amp}} = R_{gg} = 10\text{M}\Omega \quad ]^2$$

**Problem 2, 50 points**

You will be working on the circuit below:



Q1:  $\beta = 100$ ,  $V_A = \text{infinity V}$

The supplies are +5V and -5 V.

You will bias the transistor with 10mA collector current.

The DC collector bias voltage is 0V.

$R_L$  is  $500\Omega$ ,  $R_{gen}$  is  $100\ \Omega$ ,  $R_b$  is  $1\ k\Omega$ ,  $R_{EAC}$  is  $25\ \Omega$

$C_{EE}$  is very large. Assume that it is an AC short-circuit.

Part a, 10 points

DC bias.

Find the following:

$$R_{EE} = \cancel{5V} \quad R_C = \frac{500\Omega}{10mA} \quad R_{EDC} = \frac{405\Omega}{25mA}$$

$$R_C = \frac{5V}{10mA} = 500\Omega$$

]

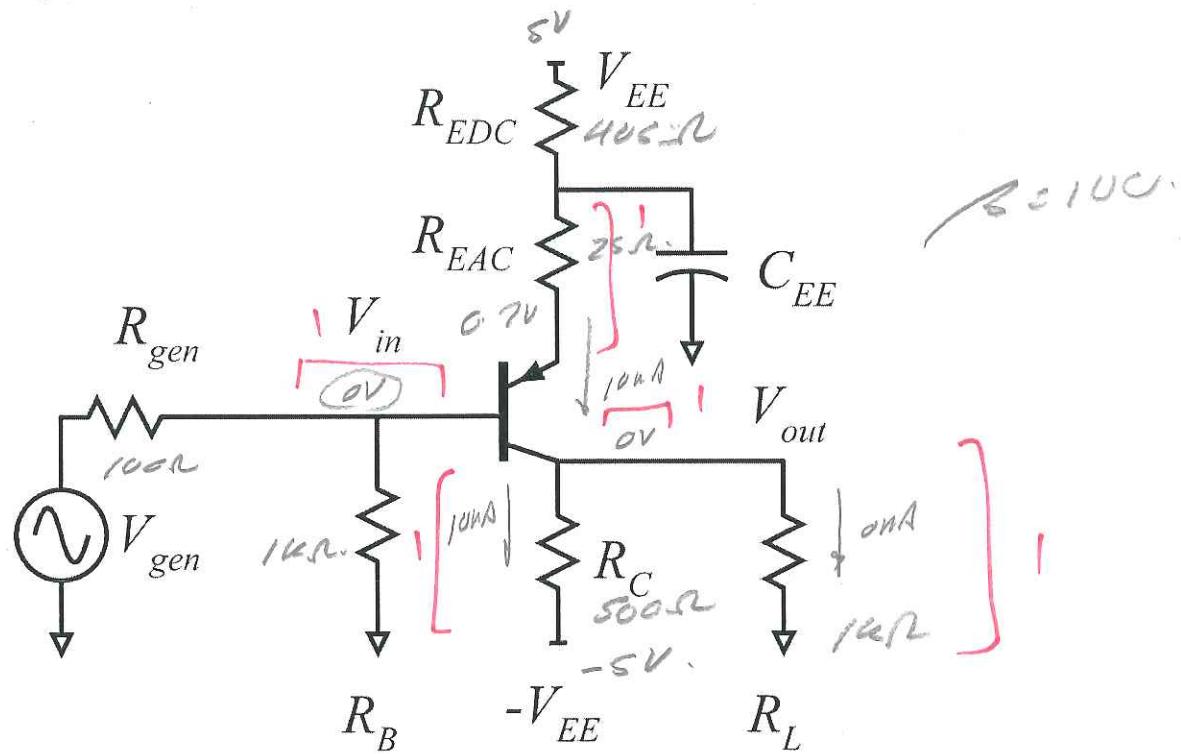
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5

$$R_{EDC} = \frac{5V - 0.7V}{10mA} - 25\Omega = 430\Omega - 25\Omega = 405\Omega.$$

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

Part c, 5 points

Find the small signal parameters of Q1.

$$gm = \underline{385 \text{ ms}}, \quad R_{ce} = \underline{260 \Omega}, \quad R_{be} = \underline{260 \Omega}$$

$$g_m = \frac{10 \text{ mA}}{26 \text{ mV}} = \frac{1}{2.6 \Omega} = 385 \text{ ms} \quad ]^1$$

$$R_{ce} = \frac{V_A + V_{CE}}{I_C} = \frac{\infty + V_{CE}}{I_C} = \infty \quad ]^2$$

$$R_{be} = \beta / g_m = 100 \cdot 2.6 = 260 \Omega \quad ]^2$$

Part d, 10 points.

Find the small signal voltage gain ( $V_{out}/V_{in}$ ) of Q1 and the amplifier small-signal input resistance.

$$V_{out}/V_{in} = \frac{-12.1}{1}$$

$$R_{in,amp} = \frac{724\Omega}{1}$$

$$R_{eq} = R_C // R_L = \frac{500\Omega // 1k\Omega}{500\Omega} = 333.3\Omega \quad [2.5]$$
$$= 250\Omega$$

$$A_v = \frac{-R_{eq}}{1/g_n + R_{eq}} = \frac{-333.3\Omega}{2.62 + 250\Omega} = -12.1 \quad [2.5]$$
$$- 9.06$$

$$R_{in,T} = \beta C (1/g_n + R_{eq}) = 100(1.33 + 250\Omega) \quad [2.5]$$
$$= 100(261.3\Omega) = 2.613k\Omega$$

$$R_{in,amp} = 2.636\Omega // \frac{1}{1/\mu_A} = \frac{724\Omega}{736\Omega} \quad [2.5]$$

Part e, 5 points

Find  $(V_{in}/V_{gen})$  and  $(V_{out}/V_{gen})$

$$(V_{in}/V_{gen}) = \underline{0.879}$$

$$(V_{out}/V_{gen}) = \underline{-10.6}$$

$$-7.97$$

$$\frac{V_{in}/V_{gen}}{V_{out}/V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}} = \frac{724\Omega}{724\Omega + 100\Omega} \quad \boxed{2.5}$$
$$= 0.871$$

$$\frac{V_{out}/V_{gen}}{V_{in}/V_{gen}} = \frac{V_{in}}{V_{gen}} \cdot \frac{V_{out}}{V_{in}} = \cancel{0.879}(-12.1) \quad \boxed{2.5}$$
$$= -10.6$$

Part f. 15 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. *Give the sign (+ or -) in your answers below.*

Cutoff of Q1; Maximum  $\Delta V_{out}$  resulting =  $3.333V\downarrow$        $2.5V\downarrow$

Saturation of Q1; Maximum  $\Delta V_{out}$  resulting = \_\_\_\_\_

Cutoff

$$\left. \begin{aligned} \Delta I_E &= 10mA \\ R_{eq} &= 333\Omega \parallel 250\Omega \\ \Delta V_{out} &= 10mA \cdot 333\Omega = 3.333V \end{aligned} \right] \quad \begin{array}{l} 3 \\ 4 \\ 7 \end{array}$$

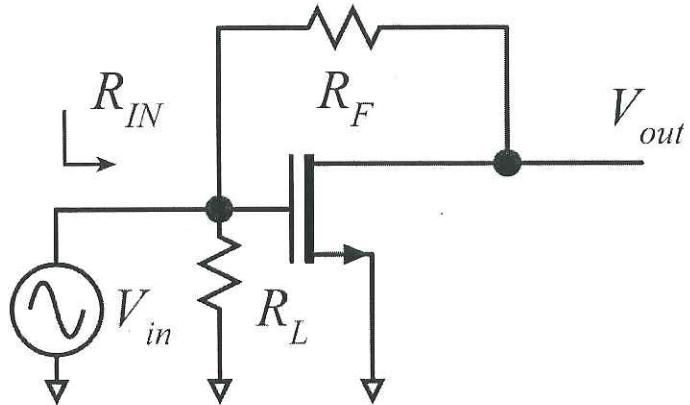
$2.5V\downarrow$

Saturation

$$\left. \begin{aligned} V_{CEsat} &= 11.2V \\ V_{CB, b.b} &= 0.7V \\ \Delta V_{CE} &= 0.2V \text{ — acceptable answer} \end{aligned} \right] \quad 8$$

$$\left. \begin{aligned} \Delta V_{out} &= \Delta V_{CE} \cdot \frac{\delta V_{out}}{\delta V_{CE}} = \Delta V_{CE} \cdot \frac{R_{eq}}{R_{eq} + R_{load}} \\ &= 0.2V \cdot \frac{333\Omega}{333\Omega + 250\Omega} = 0.2V \cdot 0.73 = \\ &\approx 0.146V \end{aligned} \right] \quad \begin{array}{l} 6 \\ 7 \\ \text{More exact answer} \\ \text{if } 6 \text{ is ok.} \end{array}$$

**Problem 3, 20 points**  
*nodal analysis*



You will be working on the circuit to the left.

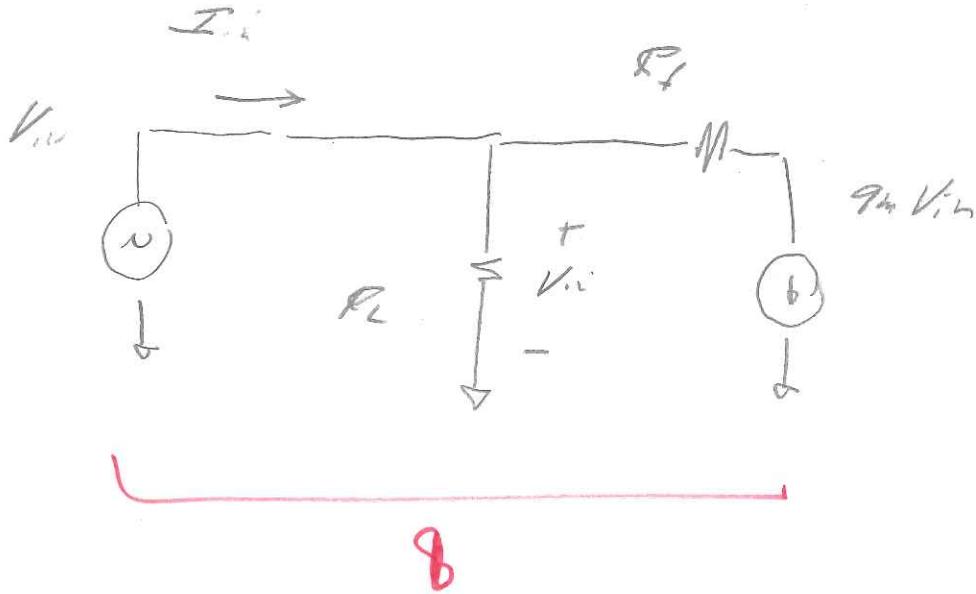
Ignore DC bias analysis. You don't need it.

The transistor has transconductance  $g_m$ .

The drain-source resistance  $R_{ds}$  of the transistor is infinity (so you don't need to draw it!)

Part a, 8 points

Draw the small-signal equivalent circuit



Part b, 8 points

Find, by nodal analysis, a small-signal expression for  $R_{in}$ .

$$R_{in} = \underline{\hspace{2cm}}$$

the currents in the 2 branches are }  
 $v_i/R_e$  and  $g_m v_o$  ] 4

$$R_{in} = R_e \parallel \frac{1}{g_m} ] 4$$

Part c, 4 points

$gm = 1 \text{ mS}$ ,  $R_L = 3\text{kOhm}$ ,  $R_f = 2 \text{ kOHm}$ .  
Give a numerical value for  $R_{in}$ .

$R_{in} = \underline{750\Omega}$

$$R_{in} = R_L \parallel \frac{1}{g_m} = 300 \parallel 1\text{m}\Omega \quad ] 4$$
$$= 750\Omega$$