Final Exam, ECE 137A

Wednesday March 22, 2017, 7:30 - 10:30pm

Name: solution.

Closed Book Exam:
Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

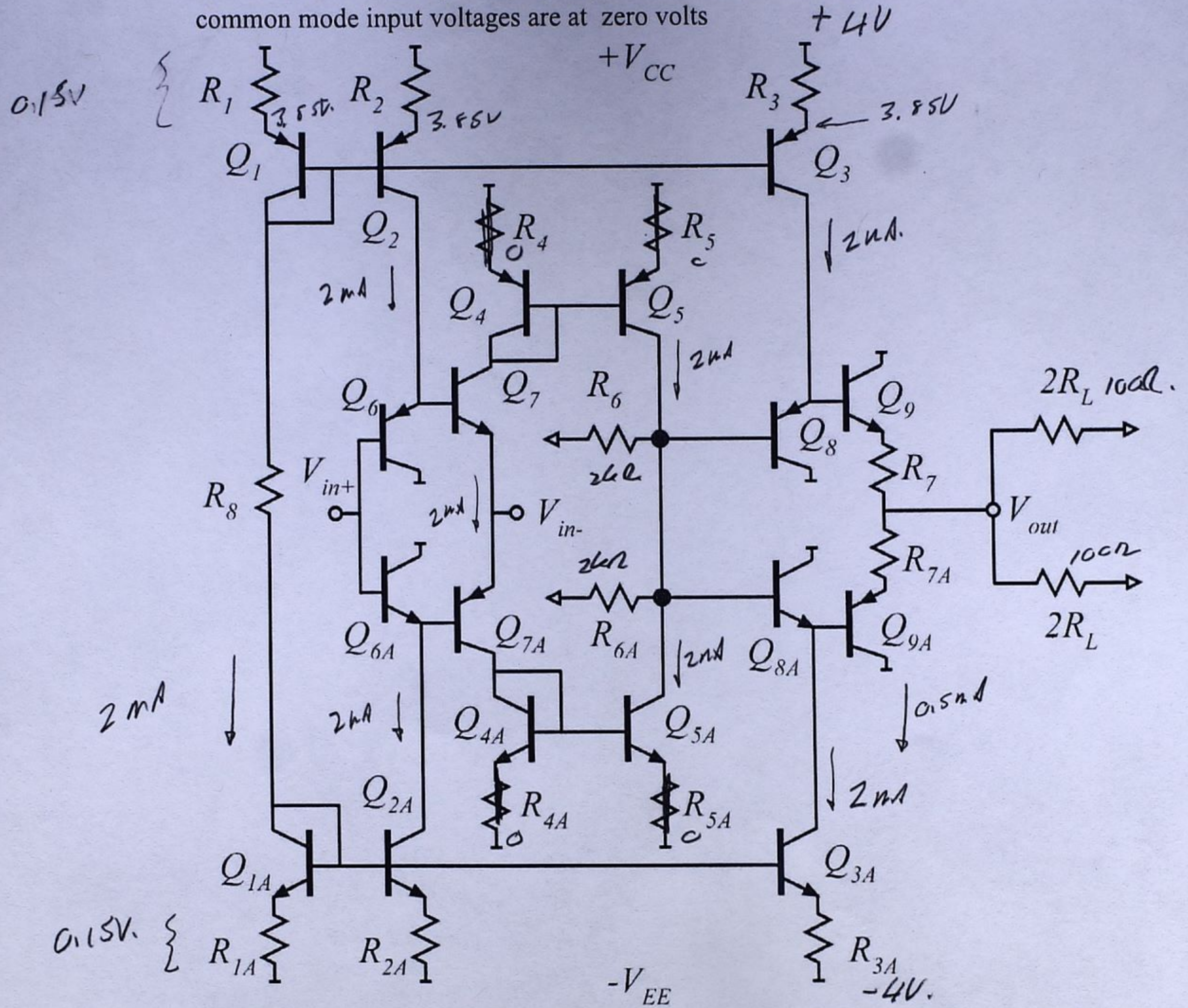
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

a

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



All the transistors have the same (matched) I_S, have $\beta = 100$, and $V_A = \infty$ Volts .

$V_{CE(sat)} = 0.5\text{V}$.

V_{be} is roughly 0.7 V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary and appropriate.

The supplies are +4 Volts and -4 Volts.

R1=R1A, R2=R2A, R3=R3A, R6=R6A, R7=R7A.

The voltage drops across R1 and R1A are both 150mV.

Q1, Q1A, Q6, Q6A, Q7, Q7A, Q5, Q5A, Q8, Q8A : I_C =2mA.

Q9, Q9A: I_C =0.5 mA.

RL=50 Ohms, R6=R6A=2000 Ohms.   R4= R4A=R5=R5A=0 Ohms.

$\frac{kT}{q} \ln 4 = R_7 \cdot 0.5\text{mA} -$

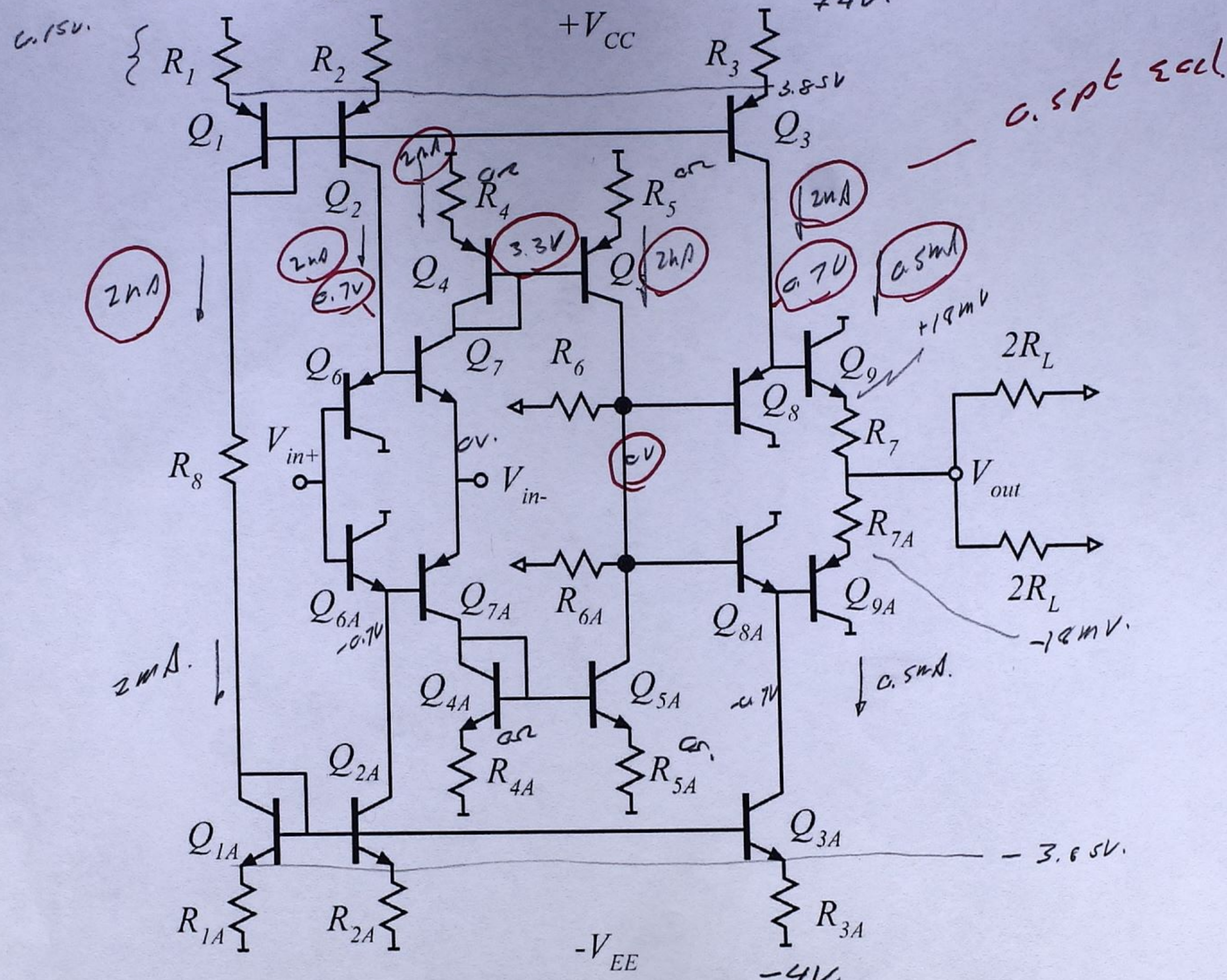
$R_7 = 36\Omega.$

a

$\beta = 100,\ V_A = \infty$

Part a, 5 points

DC bias—to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors,** and the DC collector currents of **all transistors**.

a

Part b, 6 points
DC bias:

Find the value of all resistors.
R1/R1A= 75 R2/R2A = 75 R3/R3A = 75 R4/R4A = 0Ω
R5/R5A = 0Ω R6/R6A = 2kΩ R7/R7A = ~~36Ω~~ R8 = 3.84kΩ.

72Ω.

2

$R_1/1A/2/2A/3/3A$: $R = \frac{0.15V}{2mA} = 75\Omega$.

2

R7, R7A: $\underbrace{V_{be8} + V_{be8A} - V_{be9} - V_{be9A}}$ $= 2 \cdot R_7 \cdot 0.5mA$.

$2\frac{kT}{q}\ln(4) = 2R_7 \cdot 0.5mA$.

$R_7 = 72\Omega$.

2

R_8: V drop is $(4V - 0.15V) \cdot 2$, current is 2mA.

$R = \frac{(4V - 0.15V)2}{2mA} = 3.85k\Omega$

a

76.9mS 19mS.

Part c, 4 points

find the following

device	Q1/1A	2/2A	3/3A	4/4a	5/5A	6/6A	7/7A	8/8A	9/9A
gm, S									
Rce, Ω	∞	∞	∞	∞	∞	∞	∞	∞	∞

1 $R_{ce} = \dfrac{V_A + V_{CE}}{I_C} \rightarrow \infty\,\Omega$, $V_A \rightarrow \infty V$.

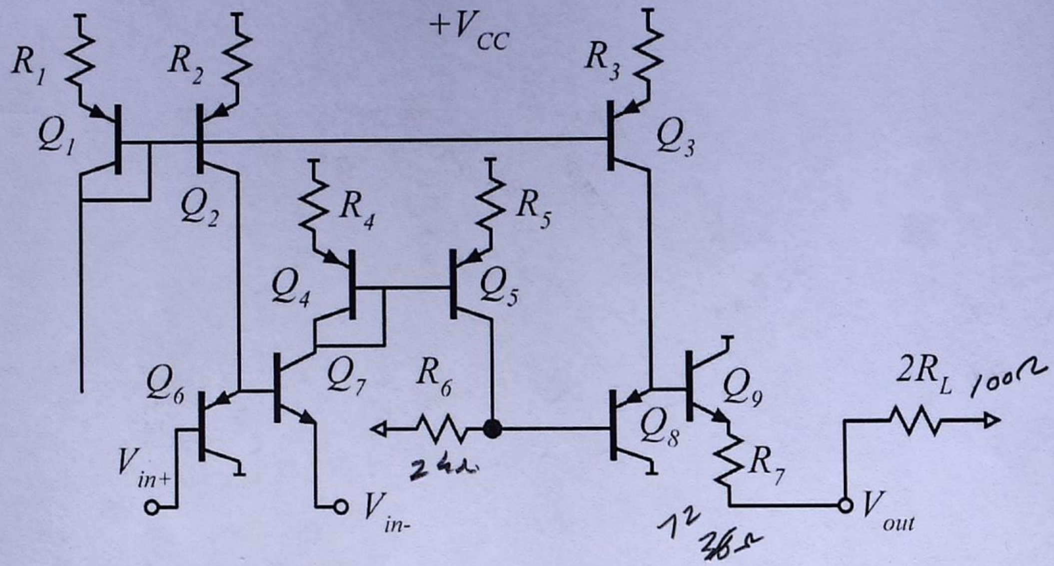
1.5 All I_C except Q9/Q9A:
$I_C = 2mA \rightarrow 1/g_m = 13\Omega \rightarrow g_m = 76.9mS$

1.5 Q9/9A
$I_C = 1/2\ mA \rightarrow 1/g_m = 52\Omega \rightarrow g_m = 19.2mS$

a

Part d, 10 points.

The circuit is 100% symmetric, and can be represented by the simpler small-signal diagram below;



Find the following, ***using the actual value of β, i.e. β=100***

	Voltage Gain	Input impedance
Q9	0.45	22kΩ.
Q8	1	2.2 MΩ
Q5	−154	1.3kΩ
Q7	−1	1.3kΩ
Q6	1	130kΩ
Overall differential Vout/Vin	64	130kΩ.

Note: with some insight, you can find the combined gain of Q7/Q4/Q5 in a single step. If would would like to do so, omit the separate answers for Q5 and Q7 in the table above, and instead fill in the table below,

	Voltage Gain	Input impedance
Q7/Q4/Q5 combination.	154	1.3 kΩ.

a

Q9/

$R_7 : 2R_L$ divider

$V_{out}/V_{eq} = \frac{100\Omega}{172\Omega}$

1 Q9: $A_v = \frac{172\Omega}{172\Omega + 52\Omega} = 0.77$ } 0.45 overall

1/2 $R_{in} = \beta(172\Omega + 52\Omega) = 22.4\ k\Omega$

Q8: 1/2 $R_{Leq} = R_{in}9 = 22.4\ k\Omega$.

$1/g_m = 13\Omega$

1/2 $A_v = \frac{22.4\ k\Omega}{22.4\ k\Omega + 13\Omega} = 0.999 \cong 1$

1/2 $R_{in} = \beta(22.4\ k\Omega) = 2.24\ M\Omega$.

Q5 1/2 $R_{Leq5} = R_6 \| R_{in8} = 2k\Omega \| 2.24 M\Omega = 1.995\ M\Omega$
$\cong 2k\Omega$.

1/2 $A_v = -g_{m5} \cdot R_{Leq5} = -\frac{2k\Omega}{13\Omega} = -153.8$

1/2 $R_{in5} = \beta/g_m = 100 \cdot 13\Omega = 1.3\ k\Omega$

Q7 1/2 $R_{Leq} = 1/g_{m4} \| R_{in5} = 13\Omega \| 1.3k\Omega = 0.99 \cdot 13$
$\cong 13\Omega$

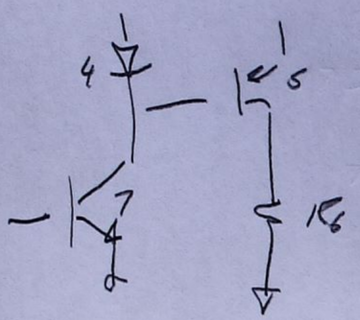
1 $A_{v7} = -g_m R_{Leq7} = -13\Omega / 13\Omega = -1$

1 $R_{in7} = \beta/g_m = 1.3\ k\Omega$

Q6/ 1 $R_{Leq} = R_{in7} = 1.3k\Omega$ 1 $A_v = \frac{1.3k\Omega}{1.3k\Omega + 13\Omega} = 0.99 \cong 1$

1 $R_{in} = \beta \cdot R_{Leq} = 130k\Omega$

Alternative: for the Q7/4/5 combination:



$$A_v = +g_{m7} R_6 = \frac{2k\Omega}{13\Omega}$$

$$= \underline{\underline{153.8}}$$

Note: QxA answers are the negative of Qx answer.

Part e, 10 points

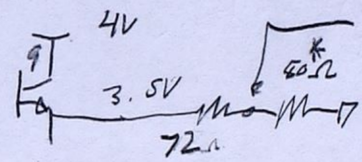
Maximum peak-peak output voltage (***show all your work***)
For this, you must use the full circuit diagram, not the half circuit diagram.

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q9	1.43V	N/A
Transistor Q9A	−1.43V	N/A
Transistor Q8	−1.84V	+10V
Transistor Q8A	+1.84V	−10V
Transistor Q5	1.43V	N/A
Transistor Q5A	−1.43V	N/A
Transistor Q7	N/A	N/A
Transistor Q7A	N/A	N/A

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q9/9A form a push pull stage, so be careful about your answer there. ***Hint***: There is, effectively, another push-pull stage in the circuit, which will affect two of the other answers.

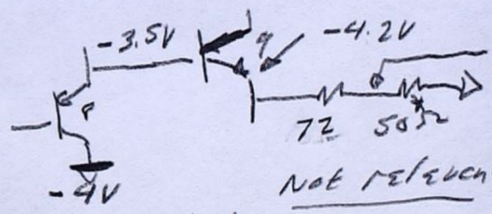
1 [Q9 cutoff - not relevant - push pull

1 [Q9 saturation



$3.5V \cdot \frac{50\Omega}{122\Omega} = 1.48V$

1 [Q8 saturation

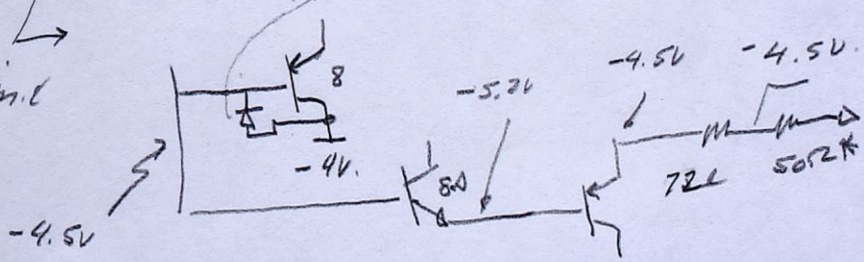


$-4.2V \cdot \frac{50}{122\Omega} = \ldots$

Not relevant, as Q9 will be off.

be diode

1 [actual limit



$-4.5V \cdot \frac{50\Omega}{122\Omega} = -1.84V.$

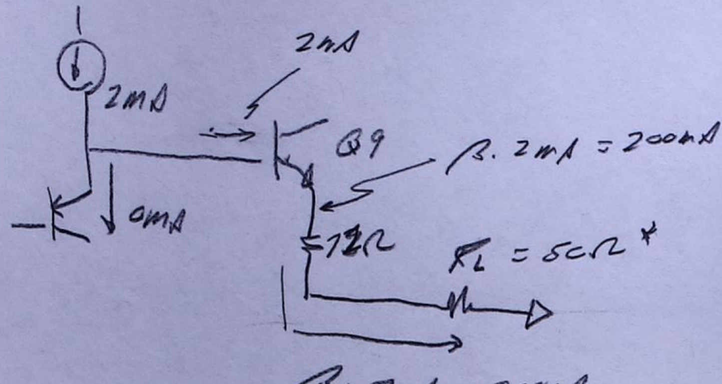
*note - we will accept $\frac{100\Omega}{100\Omega + 72\Omega}$ as acceptable.

and will accept use of 100Ω for load.

a

1

Cutoff Q9

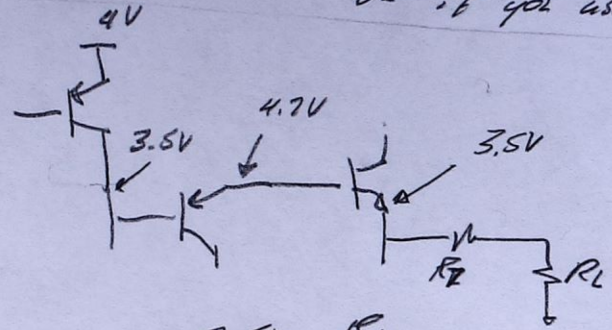


$V_o^+ = R_L \cdot 200mA = 50\Omega \cdot 200mA = \underline{+10V}$

*ok if you used 100Ω.

2

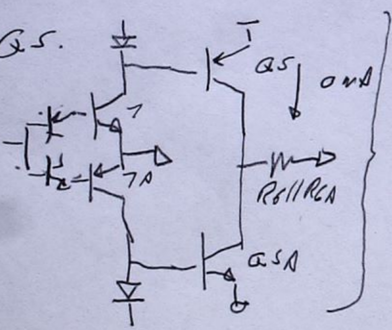
Sat Q5:



$$3.5V \cdot \frac{R_L}{R_L + R_2} = 3.5V \cdot \left(\frac{50\Omega}{122\Omega}\right)^{*} = 1.43\,V.$$

1

Cutoff Q5.

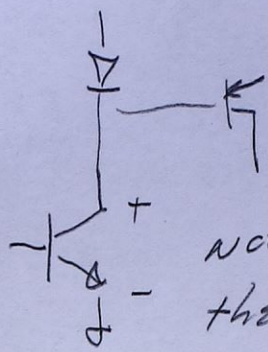


Cutoff of Q5 is <u>not</u> <u>relevant</u> as Q5/Q5A form another push-pull pair

a

Sat Q7

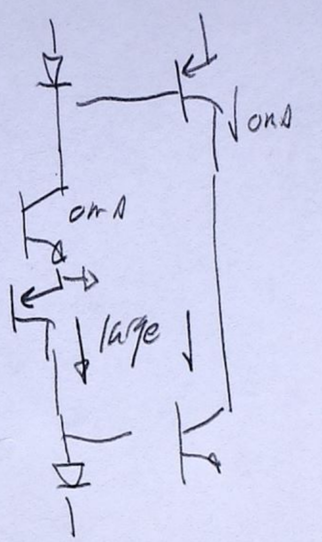
1



not relevent - if $V_{CE7} = 0.8V$
then I_{CS} would be VERY large.

Cutoff Q7

1

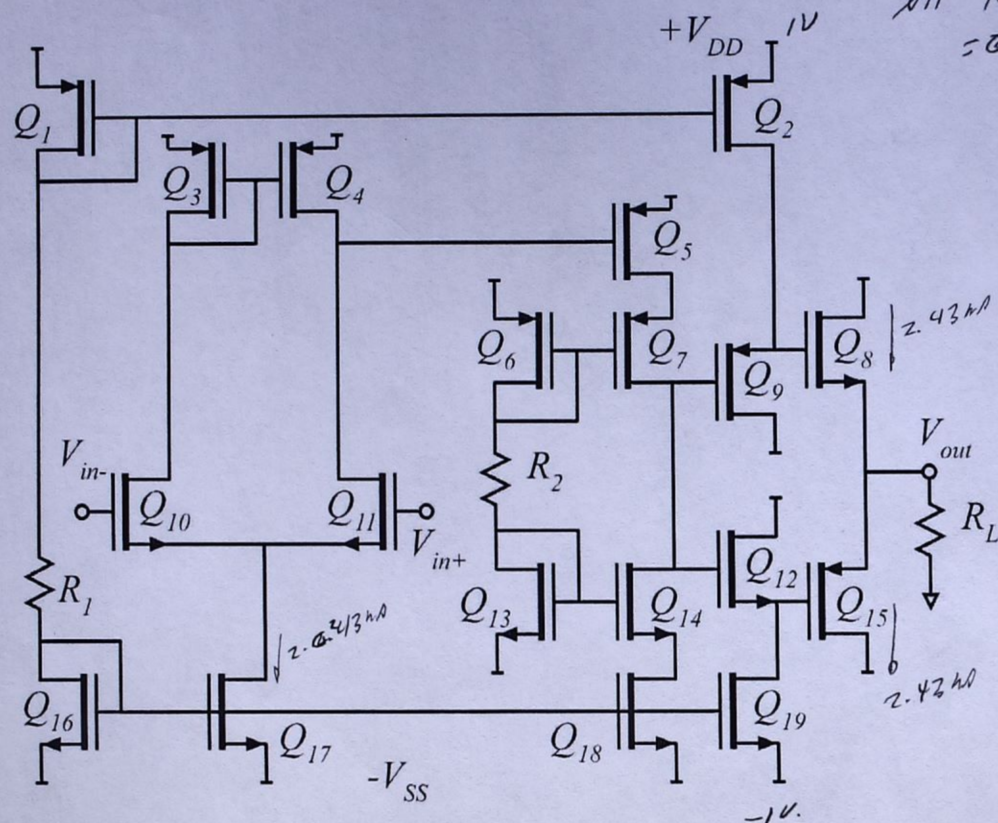


Again - not relevent
as Q7/7A
& Q5/5A
form push pull pair

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

All fed Io = 2.243 mA.



The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22nm gate length, 200 cm^2/Vs mobility, a 10^7cm/s injection velocity, and $1/\lambda$=4 Volts. The gate oxide thickness is 0.8nm and the dielectric constant is 3.8. This gives

$\mu c_{ox} W_g / 2L_g$=19.1 mA/V$^2 \cdot (W_g / 1\mu\text{m})$ and

$v_{sat} c_{ox} W_g$=4.21 mA/V$\cdot(W_g / 1\mu\text{m})$ (both are a bit unrealistic for a real technology).

and $v_{sat} L_g / \mu$=0.110 V

$V_{DD}=+1$ V, $-V_{SS}=$-1 V, R_L=10 kOhm

a

Part a, 10 points

DC bias.

Approximation: ignore the term $(1+\lambda V_{DS})$ ***in DC bias analysis.***

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

All transisistors ***except Q6, Q13*** have $|V_{gs}|$=0.25V.
Q6 and Q13 have Vgs=0.30V.
All transisistors ***except Q8, Q15, Q17*** have $|I_D|$=0.243mA.
Q8 and Q15 have $|I_D|$=2.43mA.
You can figure out $|I_D|$ for Q17.

Find the gate widths of all transistors, plus R1 and R2.

Find:

Wg1=______	Wg2 =______	Wg3=______	Wg4 =______
Wg5=______	Wg6 =______	Wg7=______	Wg8 =______
Wg9=______	Wg10=______	Wg11=______	Wg12 =______
Wg13=______	Wg14 =______	Wg15=______	Wg16=______
Wg17=______	Wg18=______	Wg19______	
R1=______	R1=______		

1 [$|V_{gs}| = 0.25V$ → less than $V_{th} + \Delta V$ → mobility limited.

2 [$0.243mA = 19.1\frac{mA}{V^2}\frac{W_g}{1\mu m}(0.25V-0.2V)^2 \rightarrow W_g = 5.1\mu m \cong 5\mu m.$

[$W_g = 5\mu m$ for all Fets except Q8, 15, 17, 6, 13.

1 [For Q8/15 → $W_g = 51\mu m$ (10:1 more Id, same Vgs).

2 [For Q17 $0.486mA = 19.1\frac{mA}{V^2}\cdot\frac{W_g}{1\mu m}(0.25V-0.2V)^2 \rightarrow W_g = 10.1$

1 [R1 // $I = 0.243mA$, $V = 2(1V-0.25V) \rightarrow R = 6.17k\Omega$.

1 [R2 // $I = 0.243mA$, $V = 2(1V-0.3V) \Rightarrow R = 5.76k\Omega$

a

2

Q5, Q13 $V_{gs} = 0.3V$

$V_{th} + \Delta V = 0.2V + 0.11V$

still mobility limited.

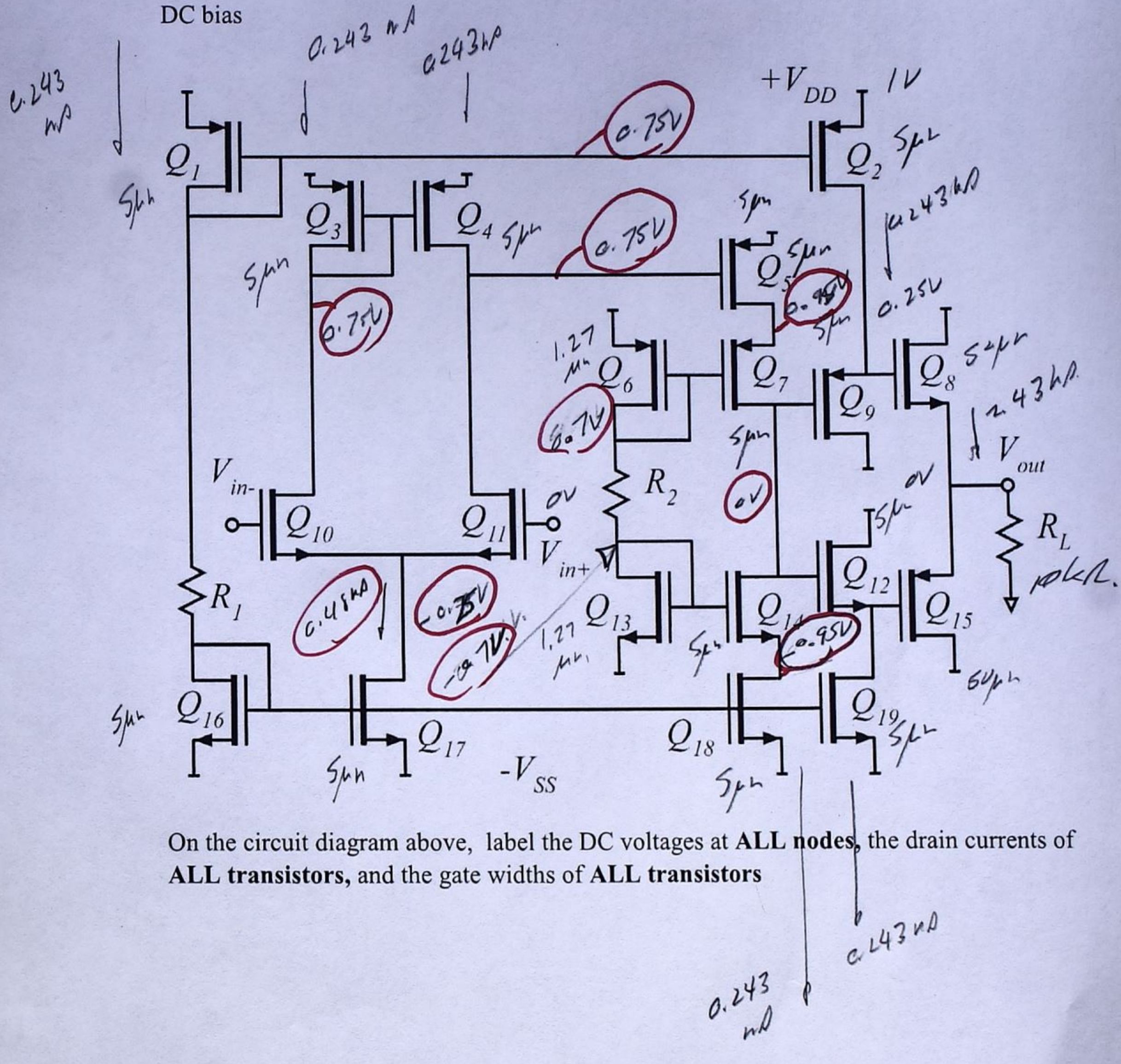
$0.243\,mA = 19.1\,\frac{mA}{V^2} \cdot \frac{W_g}{1\mu m}\,(0.3V - 0.2V)^2$

$W_g = 1.27\,\mu m$

a

Part b, 10 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes**, the drain currents of **ALL transistors,** and the gate widths of **ALL transistors**

a

1/x =4V

Part c, 15 points.

You will now compute the op-amp differential gain. ***You must consider the*** $(1+\lambda V_{DS})$ ***term in the FET IV characteristics when you do this.***

Find the following

	Voltage Gain	Input impedance
Transistor combination Q3,4,10,11	157.6	$\infty\,\Omega$
Q5,7	13,300	$\infty\,\Omega$
Q9 or Q12.	0.996	$\infty\,\Omega$
Q8 or Q15	0.99	$\infty\,\Omega$
Overall differential Vout/Vin	$1.94\cdot 10^6$	$\infty\,\Omega$

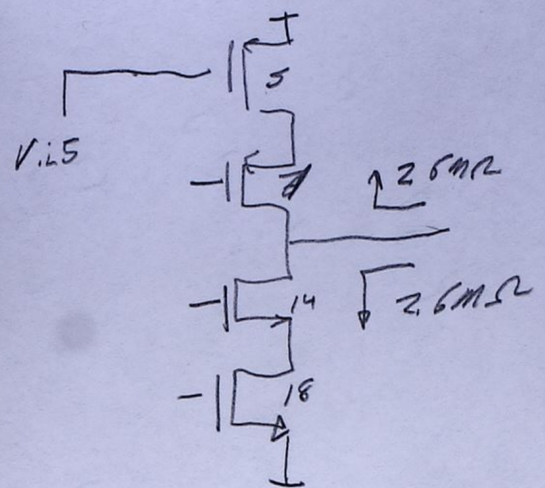
Notes:

1) You can analye Q5 and Q7 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

2) For Q9/12 and for Q8/15, you can assume that Q9 and Q12 are on for the positive signal swing and Q8 and Q15 are on for the negative signal swing. More accurately, you can assume, for the signal swing near zero volts, that all are on. If you take the latter approach (and do it correctly), you will receive a couple of extra credit points. One hint (don't ask for any other hints): use symmetry.

a

let's analyze this by ~~Norther~~ Norton method



Node impedance is

$2.6M\Omega \parallel 2.6M\Omega = \underline{\underline{1.3M\Omega}}$ 1

short-circuit current is

$g_{m5} v_{in5} \cdot \dfrac{R_{DS5}}{R_{DS5} + 1/g_{m7}}$ 1

$\rightarrow$ Voltage gain $= g_{m5} \cdot 1.3M\Omega \cdot \dfrac{R_{DS5}}{R_{DS5} + 1/g_{m7}}$ 1

or work as 2 stages.

voltage gain of Q_5 & Q_7 $= \underset{13,400}{\dfrac{1.3M\Omega}{104.7\Omega}} \cdot \underbrace{\dfrac{16.5k\Omega}{16.5k\Omega + 104.7\Omega}}_{0.994} = \underline{\underline{12,300}}$ 1

$Q_{10/11/13/14}$ –

1 $R_{Leq} = R_{DS4} \parallel R_{DS11}$

$R_{DS} = 1/\lambda I_b = 16.5k\Omega$

1 $A_v = g_m R_{Leq} = 16.5k\Omega\,(104.7\Omega)^{-1} = 157.6$

a

2

$$g_m = 2(\overbrace{V_{gs} - V_{th}}^{0.05V}) \cdot 19.1\frac{mA}{V^2} \cdot W_g$$

$= 9.55\,mS$ for all Tr except Q8/15 $= 1/104.7\Omega$.

$= 95.5\,mS$ for Q8/15 $= 1/10.47\Omega$

Q8 assume Q15 off

1 $$A_{v8} = \frac{10k\Omega \parallel R_{DS8}}{R_{DS8} \parallel 10k\Omega + \underbrace{1/g_m}_{10.47\Omega}} = 0.993 \cong 1$$

1 $R_i = \infty\,\Omega$

1/2 $R_{DS8} = 1/5\lambda I_D = 1.65k\Omega$.

Q9 1 $R_{Leq} = R_{DS2} \parallel R_{DS9} = 8.25k\Omega$ $= R_{DS2} = 1/\lambda I_D = 4V/0.243mA = 16.5k\Omega$ 1/2

1 $$A_v = \frac{8.25k\Omega}{8.25k\Omega + \underbrace{1/g_m}_{104.7\Omega}} = 0.988 \cong 1$$

1 $R_i = \infty\,\Omega$

Q7 and Q5.

1 $R_{Leq} = R_{in\ drain14} = (g_{m14} R_{DS14} + 1) R_{DS19}$

$R_{DS19} = R_{DS14} = 1/\lambda I_D = 4V/0.243mA = 16.5k\Omega$

$= (16.5k\Omega / 104.7\Omega + 1)\, 16.5k\Omega = 2.6M\Omega$.

two options to work

1) stage by stage.

2) Norton analysis of Q5/7 combination.

Part d, 10 points

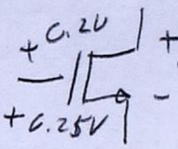
Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q8	not relevant - push-pull	+0.95V
Transistor Q15	" " " "	-0.95V.
Transistor Q9	+4V (or large/ignore)	+1.2V
Transistor Q12	-4V (or "large")	-1.2V.
Transistor Q2	N/A	0.7V
Transistor Q19	N/A	-0.7V.
Transistor Q7	-630V or irrelevant.	+0.9V
Transistor Q14	N/A	-0.9V.

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant"..

1 [Q8 - cutoff - not relevant - push-pull
Q15 - " " " " "

1 [Q8 knee voltage



knee is @ $V_{DS} = 0.05V$.

$V_{out} = 1V - 0.05 = 0.95V$

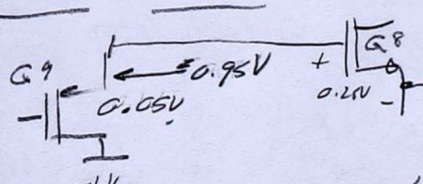
1/2 [Q15 knee voltage - identical calculation.

1/2 [Q9 cutoff $\Delta I_D = 0.243mA$ but R_{Leq} is R_{DS2} - large

$\Delta V = 0.243mA \cdot R_{DS2} = +4V \rightarrow$ large $\rightarrow$ ignore.

1/2 [Q12 cutoff identical calculation

1 [Q9 knee voltage

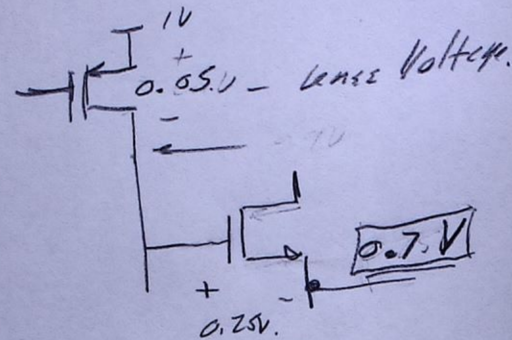


-1.2V - beyond supply - not relevant

1/2 [Q12 knee voltage] identical calculation $\rightarrow$ +1.2V.

a

105. 1 [Q2 knee Voltage

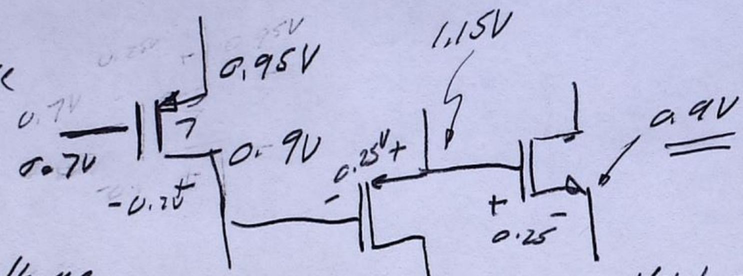


Vout = 0.65V

1/2 [Q19 knee voltage → identical calculation
→ Vout = –0.70V.

1/2 [Q2/Q19 cutoff – not relevant – IC doesn't turn off.

1 [Q7 knee Voltage



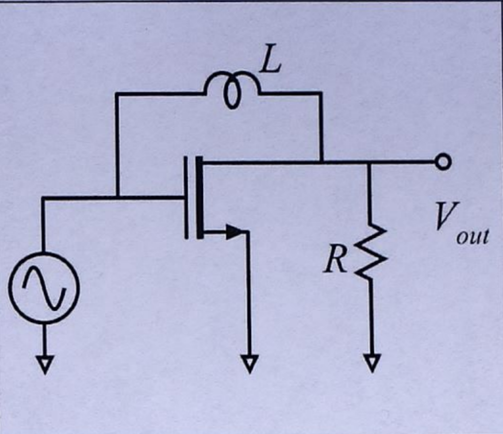
1/2 [Q14 knee ~~Voltage~~ voltage
↳ identical Calculation – Vout = –0.9V.

1 [Q7 cutoff: $\Delta I = 0.243$ mA, $R_{Eq} = 2.6M\Omega$] dV = product
= –630V
Very large
(not relevant)

1/2 [Q14 cutoff – not relevant – IO does not vary

a

Problem 3, 30 points



You will be working on the circuit to the left

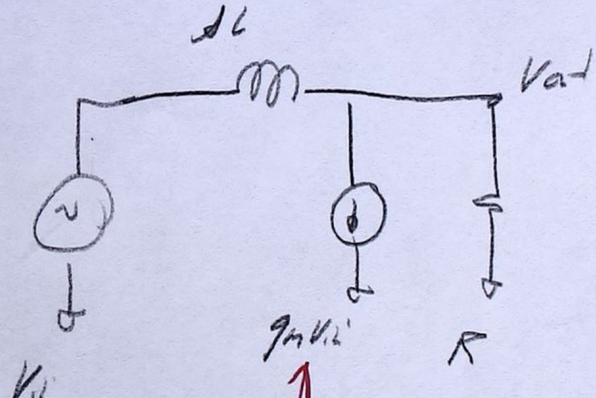
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.



control must be identified!

all the element must be labelled

a

Part b, 8 points

gm=20 mS. L=1nH. R= 1000 Ohms
Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensionally wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ____________________

ΣI @ $V_{out} = 0$.

$$\rightarrow \frac{(V_{out} - V_{in})}{sL} + g_m V_{in} + V_{out}/R = 0. \quad 4$$

$$V_{in}(1/sL - g_m) = V_{out}(1/sL + 1/R)$$

$$\frac{V_{out}}{V_{in}} = \frac{1/sL - g_m}{1/sL + 1/R} = \frac{1 - sLg_m}{1 + sL/R}$$

4

$$= \frac{1 - s(20\,ps)}{1 + s(1\,ps)}$$

a

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

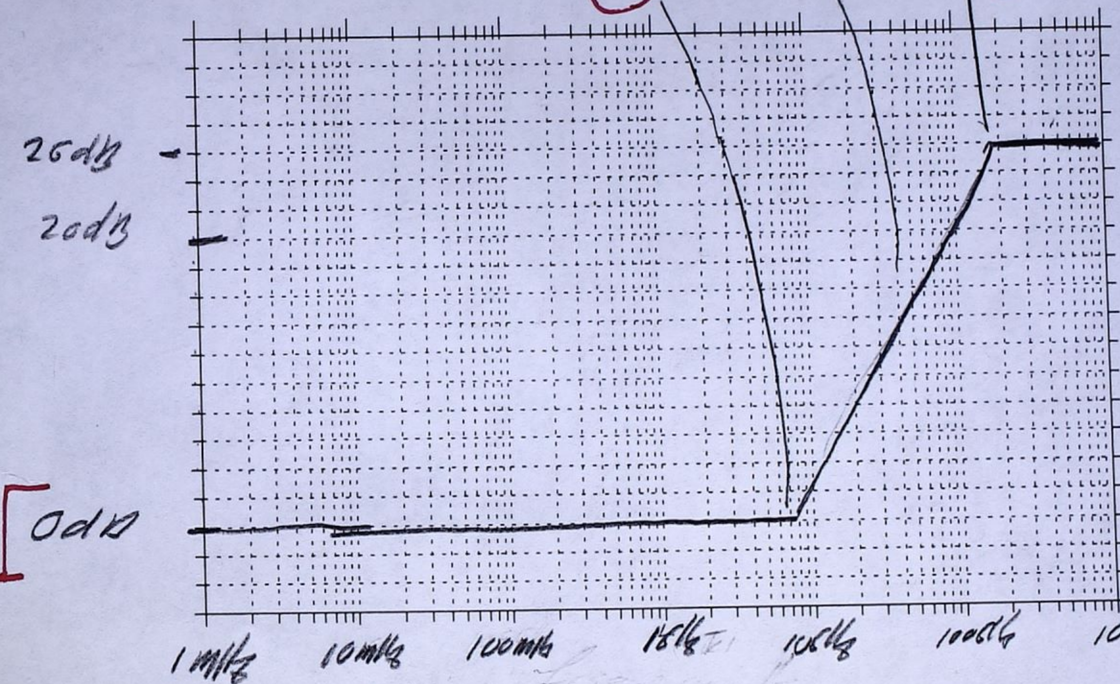
______________,______________,______________,______________

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes

1 8 GHz

+20 dB/decade 1

160 GHz 1



points deducted if these are not correctly located

$$f_p = \frac{1}{2\pi(1ps)} = 160\ \text{GHz}.$$

$$f_z = \frac{1}{2\pi(24ps)} = 8\ \text{GHz}.$$

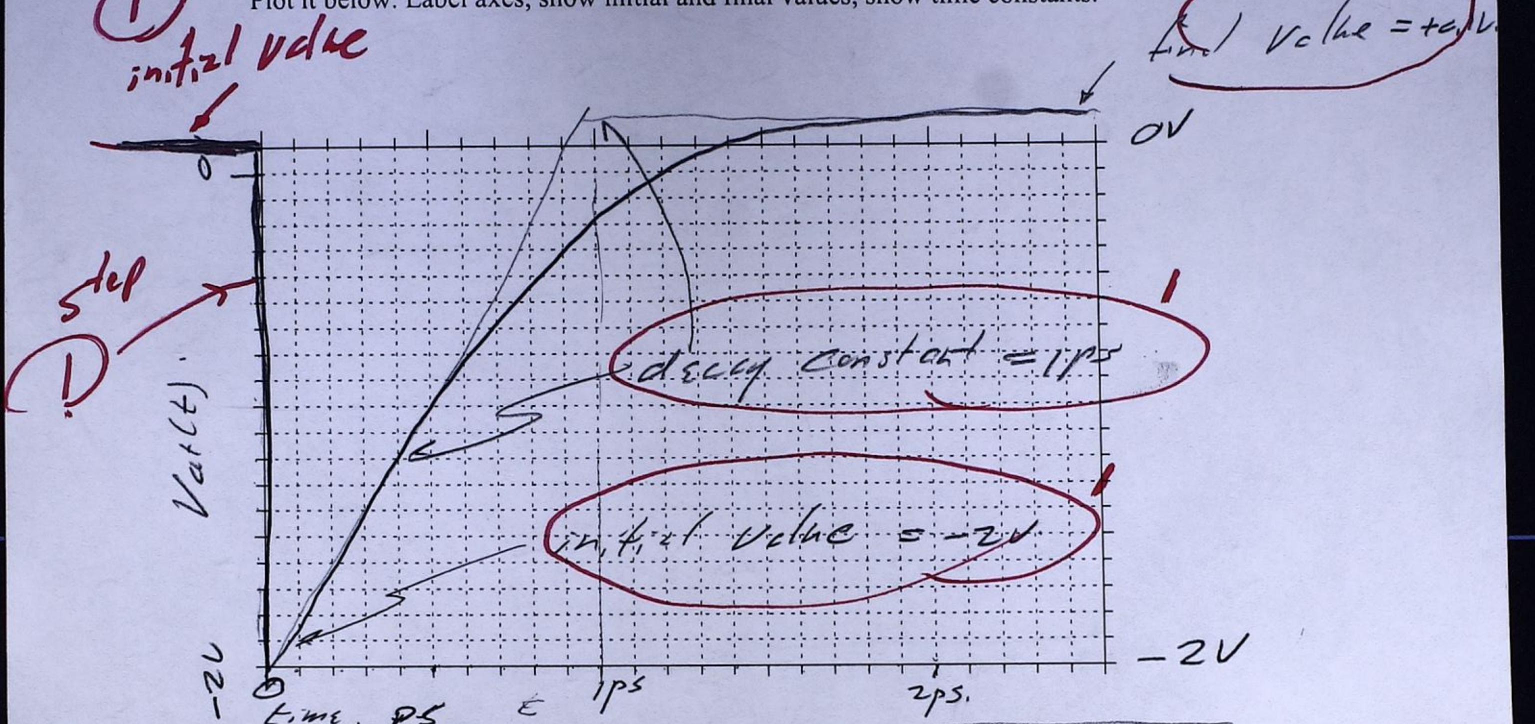
a

Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)=____________________________

Plot it below. Label axes, show initial and final values, show time constants.



$V_i(s) = 0.1V/s$

$$V_o(s) = \frac{0.1V}{s}\,\frac{1-s\tau_z}{1+s\tau_p} = \frac{0.1V}{s}\left[1+\frac{s(-\tau_z-\tau_p)}{1+s\tau_p}\right]$$

$$= \frac{0.1V}{s} + 0.1V\,\frac{(-\tau_z-\tau_p)}{\tau_p}\,\frac{\tau_p}{1+s\tau_p} = \boxed{\frac{0.1V}{s} + 0.1V(-21)\,\frac{1ps}{1+s(1ps)}}$$

1

$$\boxed{V_o(t) = 0.1V\cdot u(t) - 2.1V\,u(t)\cdot e^{-t/1ps}}$$

2.

a