ECE 137 A Mid-Term Exam

Tuesday February 9, 2021

Closed book: Class crib sheet and 1 page personal notes permitted.

There are 2 problems on this exam, and you have 75 minutes.

Use any and all reasonable approximations   (5% accuracy is fine.) , ***AFTER STATING and approximately Justifying them.***

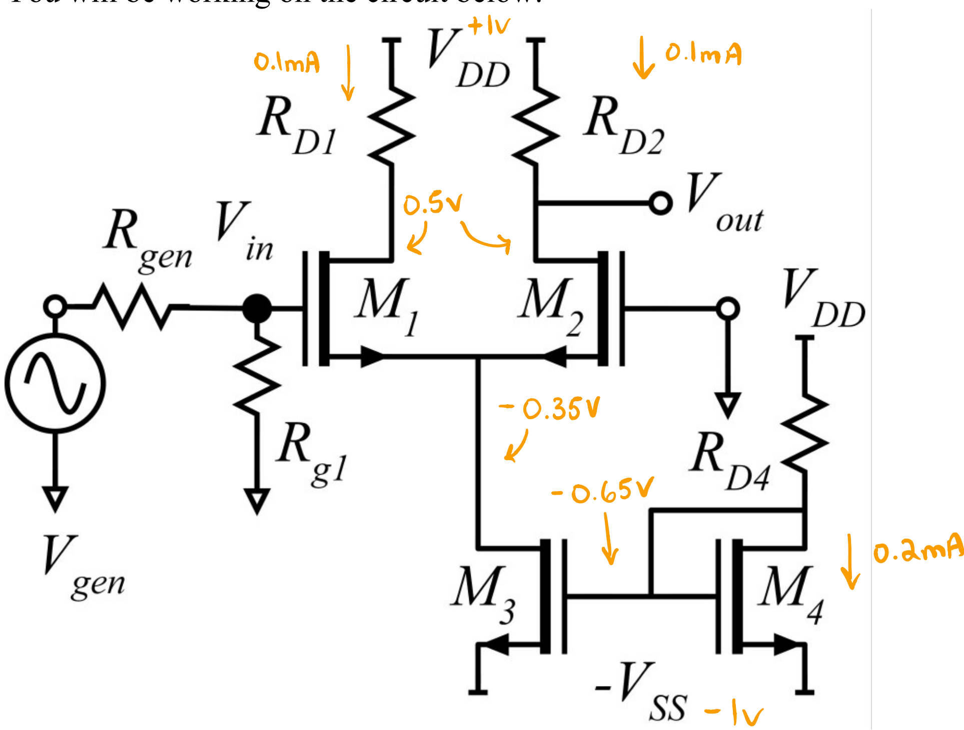
Name: SOLUTION

Part	Points Received	Points Possible
1a		9
1b		5
1c		6
1d		15
1e		15
1f		6
1g		14
2a		12
2b		13
2c		5
TOTAL		100

b

Problem 1, 70 points

You will be working on the circuit below:



$V_{gs_1} = V_{gs_2} = 0.35V$
$V_{gs_3} = V_{gs_4} = 0.35V$

The transistors all have: $K_\mu = \mu c_{gs} W_g / 2L_g = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$

$\Delta V = v_{inj} L_g / \mu = 0.1\text{V}$, $V_{th} = 0.3\text{V}$, $1/\lambda = 10\text{V}$

The supplies are +1V and -1V
Rgen=100 kOhms, Rg1=1 MOhms,

Part a, 9 points
DC bias.
The sources of M1 and M2 are to be biased at -0.35 Volts
The drains of M1 and M2 are to be biased at +0.5 volts.
The gates of M3 and M4 are to be biased at -0.65 Volts.
M1 and M2 are to be biased at 0.1 mA drain current
M4 is to be biased at 0.2 mA drain current .

Find the following:

RD1= 5KΩ RD2= 5KΩ RD4= 8.25 KΩ

Wg1= 3.69 μm Wg2= 3.69 μm Wg3= 7.51 μm Wg4= 7.73 μm

3

$R_{D_1} = (V_{DD} - V_{D_1})/I_{D_1} = (1v - 0.5v)/0.1mA = \underline{5K\Omega}$

$R_{D_2} = (V_{DD} - V_{D_2})/I_{D_2} = \underline{5K\Omega}$

$R_{D_4} = (V_{DD} - V_{G_4})/I_{D_4} = (1V + 0.65v)/0.2mA = \underline{8.25\,K\Omega}$

$V_{gs_1} = V_{gs_2} = V_{gs_3} = V_{gs_4} = 0.35v$

1

$V_{gs} - V_{th} = 0.35v - 0.3v = 50mV < \Delta V = 100mV \Rightarrow$ All FETs are Mobility-Saturated

$W_g = 1\mu m \times I_D/[K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})]$

2

$W_{g_1} = W_{g_2} = 1\,\mu m \times (0.1mA)/[10mA/V^2 (0.35v - 0.3v)^2 (1 + 1/10\,V^{-1} \cdot 0.85v)] = \underline{3.69\,\mu m}$

1.5

$W_{g_3} = 1\mu m \times (0.2mA)/[10mA/V^2 (0.36v - 0.3v)^2 (1 + 1/10\,V^{-1} \cdot 0.65v)] = \underline{7.51\mu m}$

1.5

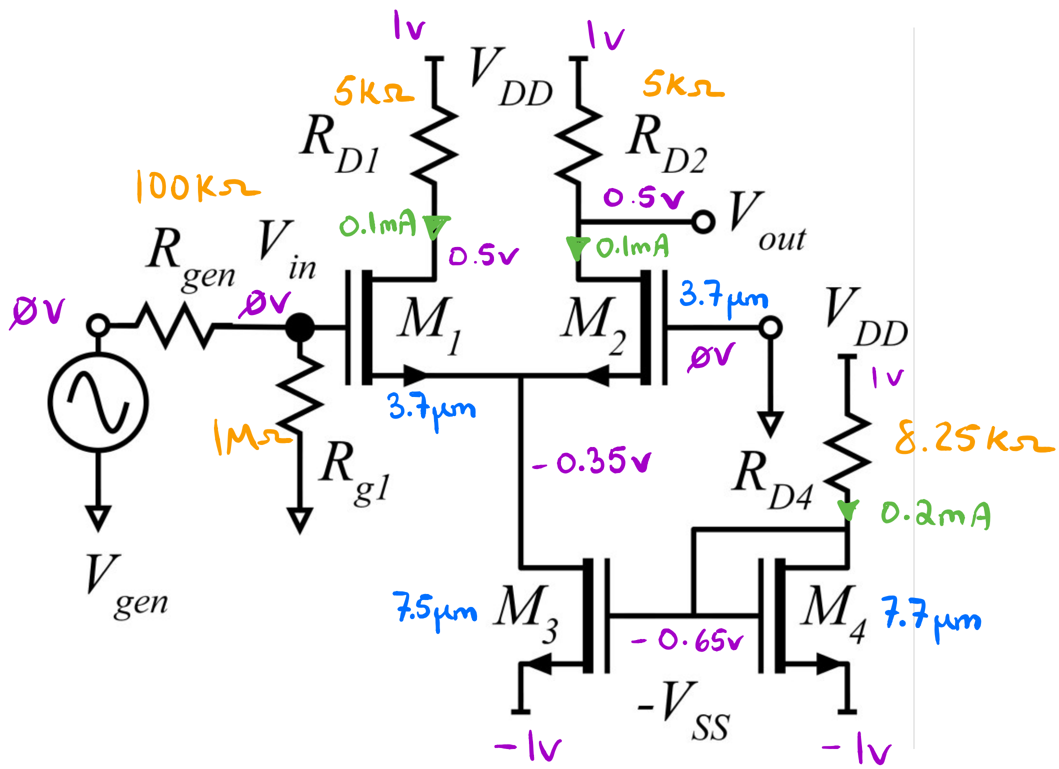
$W_{g_4} = 1\mu m \times (0.2mA)/[10mA/V^2 (0.36v - 0.3v)^2 (1 + 1/10\,V^{-1} \cdot 0.35)] = \underline{7.73\mu m}$

b

b

Part b, 5 points

DC bias



On the circuit diagram above (or on a hand-redrawing of the figure), label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors. Also label all resistor values, and the width of all MOSFETs.**

+2 Voltages

+1 Resistors

+1 Currents

+1 widths

b

Part c, 6 points

Find the small signal parameters of all FETs

Transistor	M1	M2	M3	M4
gm	4mS	4mS	8mS	8mS
R_{DS}	108.5 KΩ	108.5 KΩ	53.25 KΩ	51.75 KΩ

3

$$g_{m_1} = g_{m_2} = \frac{2 I_{D_1}}{V_{gs_1} - V_{th}} = \frac{2 \times 0.1 mA}{(0.35V - 0.3V)} = 4 mS$$

$$g_{m_3} = g_{m_4} = \frac{2 I_{D_3}}{V_{gs_3} - V_{th}} = \frac{2 \times 0.2mA}{(0.35V - 0.3V)} = 8mS$$

1

$$R_{DS_1} = R_{DS_2} = [1/\lambda + V_{DS_1}]/I_{D_1} = [10V + 850mV]/0.1mA = 108.5 K\Omega$$

1

$$R_{DS_3} = [10V + 650mV]/0.2mA = 53.25 K\Omega$$

1

$$R_{DS_4} = [10V + 350mV]/0.2mA = 51.75 K\Omega$$

b

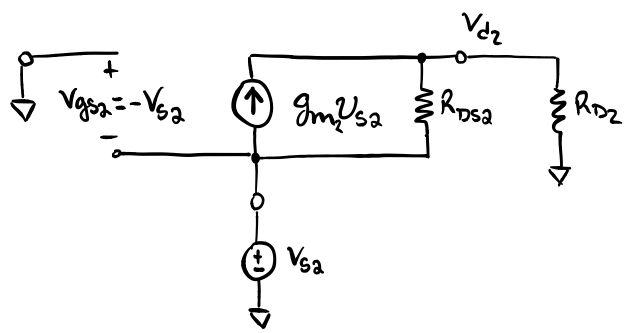
Part d, 15 points.

Find the small signal voltage gain (Vd2/Vs2) of M2 and M2's small-signal input resistance.

Vd2/Vs2= 19.16 V/V

Rin,m2= 260.9 Ω

10



$$(v_{s2} - v_{d2})/R_{ds2} + g_{m2} v_{s2} = v_{d2}/R_{D2} \quad \text{(Nodal)}$$

$$(v_{s2} - v_{d2}) R_{D2} + g_{m2} v_{s2} R_{ds2} R_{D2} = v_{d2} R_{ds2}$$

$$v_{s2}(1 + g_{m2} R_{ds2}) R_{D2} = v_{d2}(R_{D2} + R_{ds2})$$

$$v_{d2}/v_{s2} = \frac{(1 + g_m R_{ds2}) R_{D2}}{R_{D2} + R_{ds2}} = \frac{(1 + 4mS \times 108.5K\Omega)(5K\Omega)}{108.5K\Omega + 5K\Omega} = \underline{19.16 \text{ V/V}}$$

5

$$R_{in,M2} = R_{D2} \times \frac{V_{s2}}{V_{d2}} = (5K\Omega)(19.16) = \underline{260.9\,\Omega}$$

$$\text{or } R_{in,M2} = \frac{R_{ds2} + R_{D2}}{g_{m2} R_{ds2}}$$

b

Part e, 15 points

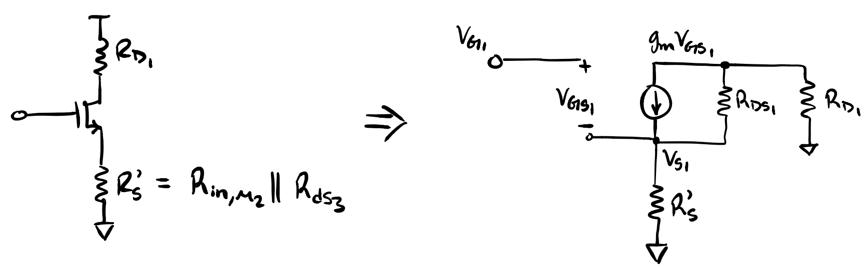
Find the small signal voltage gain (Vs1/Vg1) of M1 and the *** amplifier *** input resistance.

Vs1/Vg1= 0.51 V/V

Rin,amplifier = 1 MΩ

3 [$R_{in,amp} = R_{g_1} \| R_{in,M_1} = 1M\Omega \| \infty = \underline{1M\Omega}$

12



$$V_{S_1} = g_{m_1} V_{GS_1} (R_S' \| R_{DS_1})$$
$$= g_{m_1} (V_{G_1} - V_{S_1})(R_S' \| R_{DS_1})$$
$$= g_{m_1} (R_S' \| R_{DS_1}) V_{G_1} - g_m (R_S' \| R_{DS_1}) V_{S_1}$$

$$V_{S_1}/V_{g_1} = \frac{g_{m_1}(R_S' \| R_{DS_1})}{g_{m_1}(R_S' \| R_{DS_1}) + 1} = \frac{(R_S' \| R_{DS_1})}{(R_S' \| R_{DS_1}) + 1/g_{m_1}}$$

$$= \frac{R_{in,M_2} \| R_{ds_3} \| R_{ds_1}}{R_{in,M_2} \| R_{ds_3} \| R_{ds_1} + 1/g_{m_1}} \approx \frac{R_{in,M_2}}{R_{in,M_2} + 1/g_{m_1}} = \frac{261\Omega}{261\Omega + 1/4mS}$$

$$= \underline{0.51} \ V/V$$

Part f, 6 points

Find (Vout/Vin), (Vin/Vgen) and (Vout/Vgen)

(Vout/Vin) = 9.77 V/V

(Vin/Vgen) = 0.91 V/V

(Vout/Vgen) = 8.9 V/V

2 [$V_{out}/V_{in} = (V_{s1}/V_{g1})(V_{d2}/V_{s2}) = (19.16)(0.51) = \underline{9.77 \text{ V/V}}$

2 [$V_{in}/V_{gen} = \frac{R_{in,Amp}}{R_{in,Amp} + R_{gen}} = \frac{R_{g1}}{R_{g1} + R_{gen}} = \frac{1M\Omega}{1M\Omega + 100k\Omega} = \underline{0.91 \text{ V/V}}$

2 [$V_{out}/V_{gen} = V_{out}/V_{in} \times V_{in}/V_{gen} = (9.77)(0.91) = \underline{8.9 \text{ V/V}}$

b

Part g, 14 points

Now you must find the maximum signal swings.
Give the sign (+ or -) in your answers below.

Cutoff of M1; Maximum ΔVout resulting = -0.5 V

Knee voltage of M1; Maximum ΔVout resulting = $+15.3\text{ V}$

Cutoff of M2; Maximum ΔVout resulting = $+0.5\text{ V}$

Knee of M2; Maximum ΔVout resulting = -0.8 V

3

M_1 Cutoff $\rightarrow I_{D_1} = 0,\ I_{D_2} = 0.2\text{mA}$

$V_{out} = V_{DD} - I_{D2}R_{D2} = 1\text{V} - (0.2\text{mA})(5\text{K}\Omega) = \varnothing\text{V}$

$\Delta V_{out} = \underline{-0.5\text{ V}}$

3

M_2 Cutoff $\rightarrow I_{D_2} = \varnothing\text{mA},\ V_{out} = V_{DD} = 1\text{V}$

$\Delta V_{out} = \underline{+0.5\text{ V}}$

4

Knee Voltage, M_1: $V_{gs_1} - V_{th} = 0.35\text{v} - 0.3\text{V} = 50\text{mV}$

$V_{DS_1} = 0.5\text{v} - (-0.35\text{v}) = 0.85\text{ V}$

V_{S_1} goes up 0.8V

$\Delta V_{out} = \frac{V_{d2}}{V_{S2}} \times 0.8\text{v} = (19.16)(0.8\text{v})$

$= \underline{+15.3\text{ V}}$ Obviously you'll Saturate

4

Knee Voltage, M_2 : $V_{gs_2} - V_{th} = 50mV$

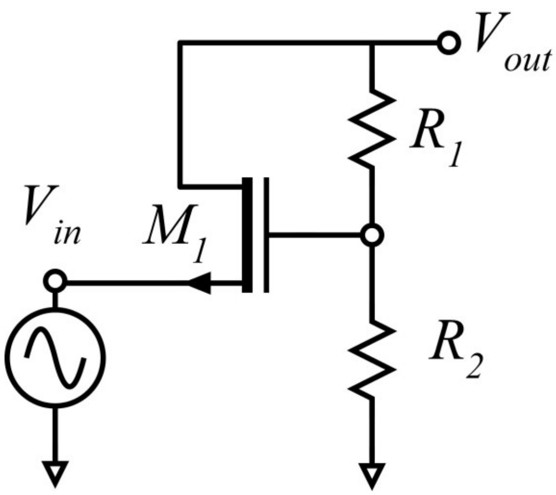
$V_{DG,min} = -V_{th} = -0.3V$

$\Delta V_{out} = -0.3V - 0.5V = \underline{-0.8V}$

b

b

Problem 2, 30 points
nodal analysis



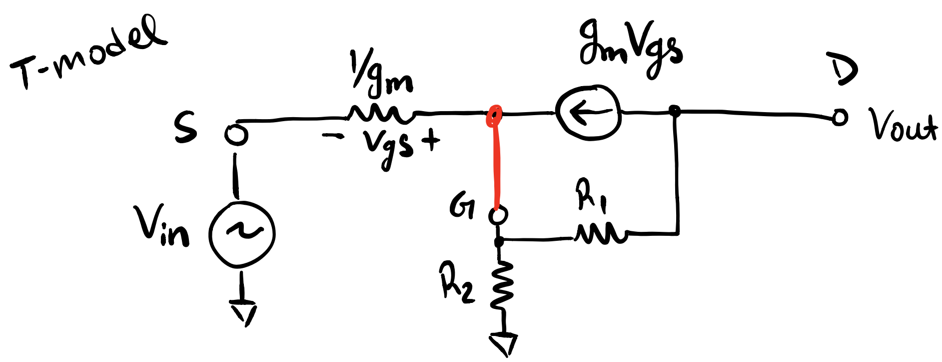
You will be working on the circuit to the left.

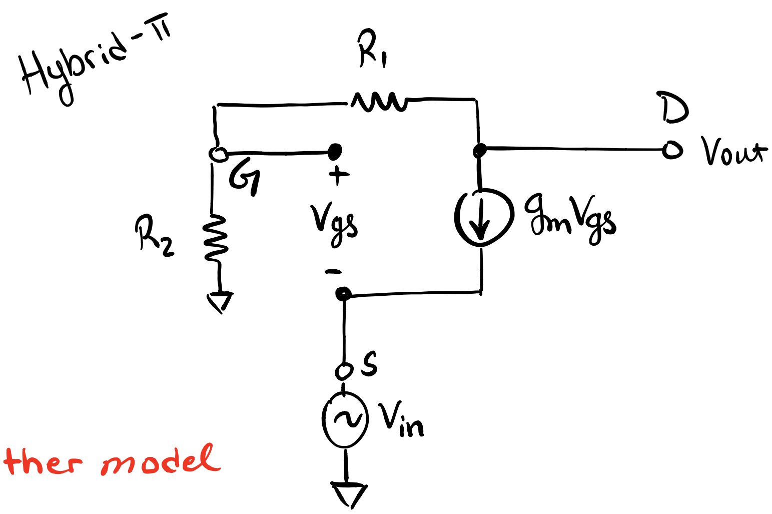
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm1.
The drain-source resistance Rds is infinity (so you don't need to draw it!)

Part a, 12 points

Draw the small-signal equivalent circuit





+12 either model

b

Part b,13 points

Find, by nodal analysis, a small-signal expression for Vout/Vin.

Vout/Vin= $g_m(R_1+R_2)/(1+g_mR_2)$

$$V_{gs} = V_g - V_{in} = \frac{R_2}{R_1+R_2}V_{out} - V_{in}$$

$$V_{out} = -g_m V_{gs}(R_1+R_2) \Rightarrow V_{gs} = \frac{-1}{g_m(R_1+R_2)}V_{out}$$

$$\left[\frac{1}{g_m(R_1+R_2)} + \frac{R_2}{R_1+R_2}\right]V_{out} = \frac{1+g_mR_2}{g_m(R_1+R_2)}V_{out} = V_{in}$$

$$V_{out}/V_{in} = \frac{g_m(R_1+R_2)}{1+g_mR_2}$$

b

Part c, 5 points

gm= 1 mS , R1=90kOhm, R2=10kOhm
Give a numerical value for Vout/Vin.

Vout/Vin= 9.09 V/V

$$V_{out}/V_{in} = \frac{g_m(R_1 + R_2)}{1 + g_m R_2}$$

$$= \frac{(1mS)(100k\Omega)}{1 + (1mS)(10k\Omega)} = \underline{9.09\ V/V}$$

b