Final Exam, ECE 137A

Thursday March 23, 2023, Noon - 3 p.m.

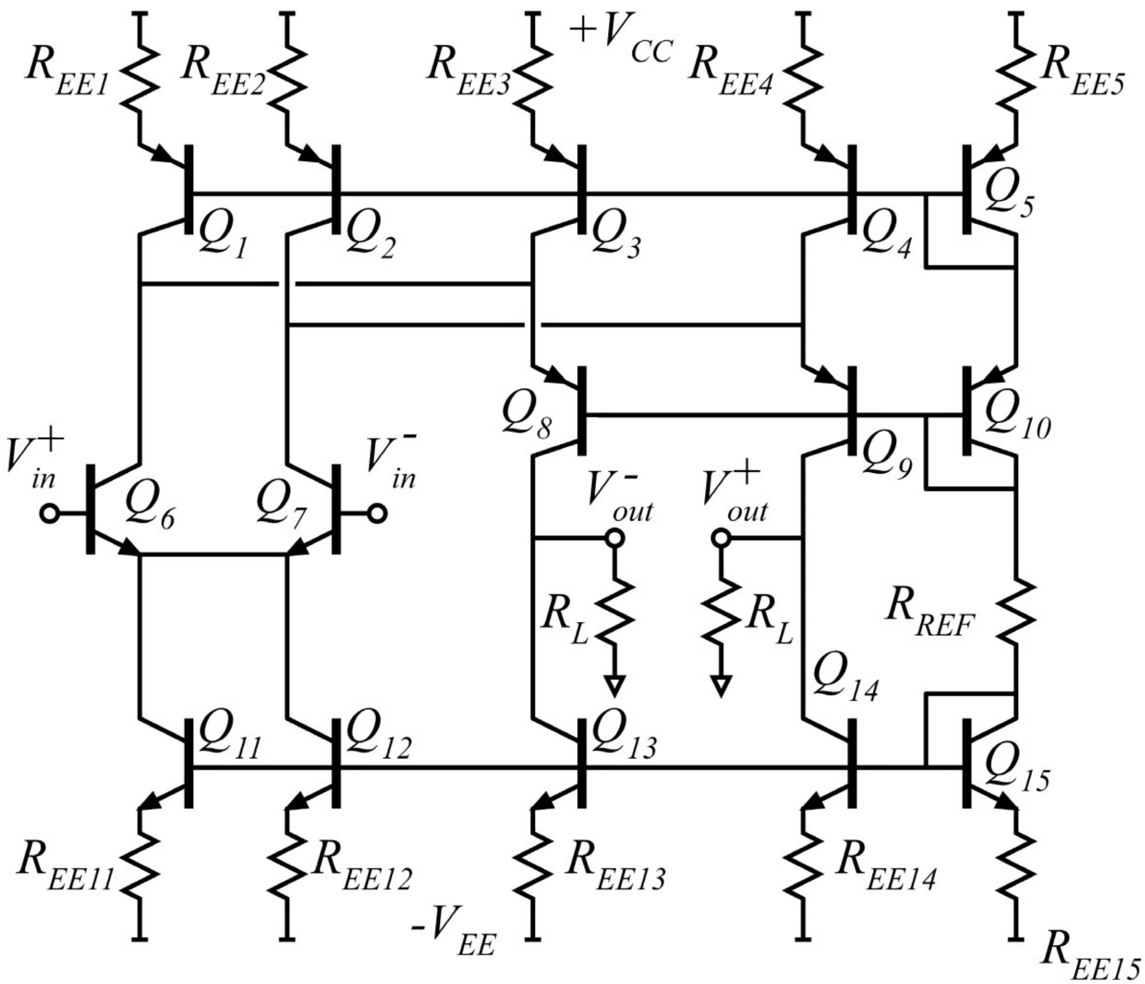
Name: Solution

Closed Book Exam:
Class Crib-Sheet and 2 pages (4 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		5
1b		5	2d		15
1c		5	2e		10
1d		10	3a		5
1e		10	3b		10
2a		10	3c		5
2b		5			
total		100			

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts.



All the transistors have the same (matched) I_S, have $\beta = \infty$, and $V_A = \infty$ Volts. $V_{CE(sat)} = 0.5\text{V}$. V_{be} is *approximately* 0.7 V, but use the *exact relationship* $V_{be} = (kT/q)\ln(I_E/I_S)$ when appropriate. The supplies are +3 Volts and -3 Volts. All transistors have the same I_S, and all are biased at 1 mA collector current.

The DC voltage drops across REE5 and REE15 are both 300 mV. R_L=1 kOhm.

a

Part a, 5 points

DC bias

Find the value of the following resistors:

Ree5= 300Ω Ree15= 300Ω R_{REF}= 3.5 kΩ

Ree11= 300Ω Ree12= 300Ω

1 all Ks: same I_E, same I_S → same V_{BE}

$R_{EE11} = R_{EE12} = R_{EE5} = R_{EE15} = 0.3V/1mA = 300\Omega$

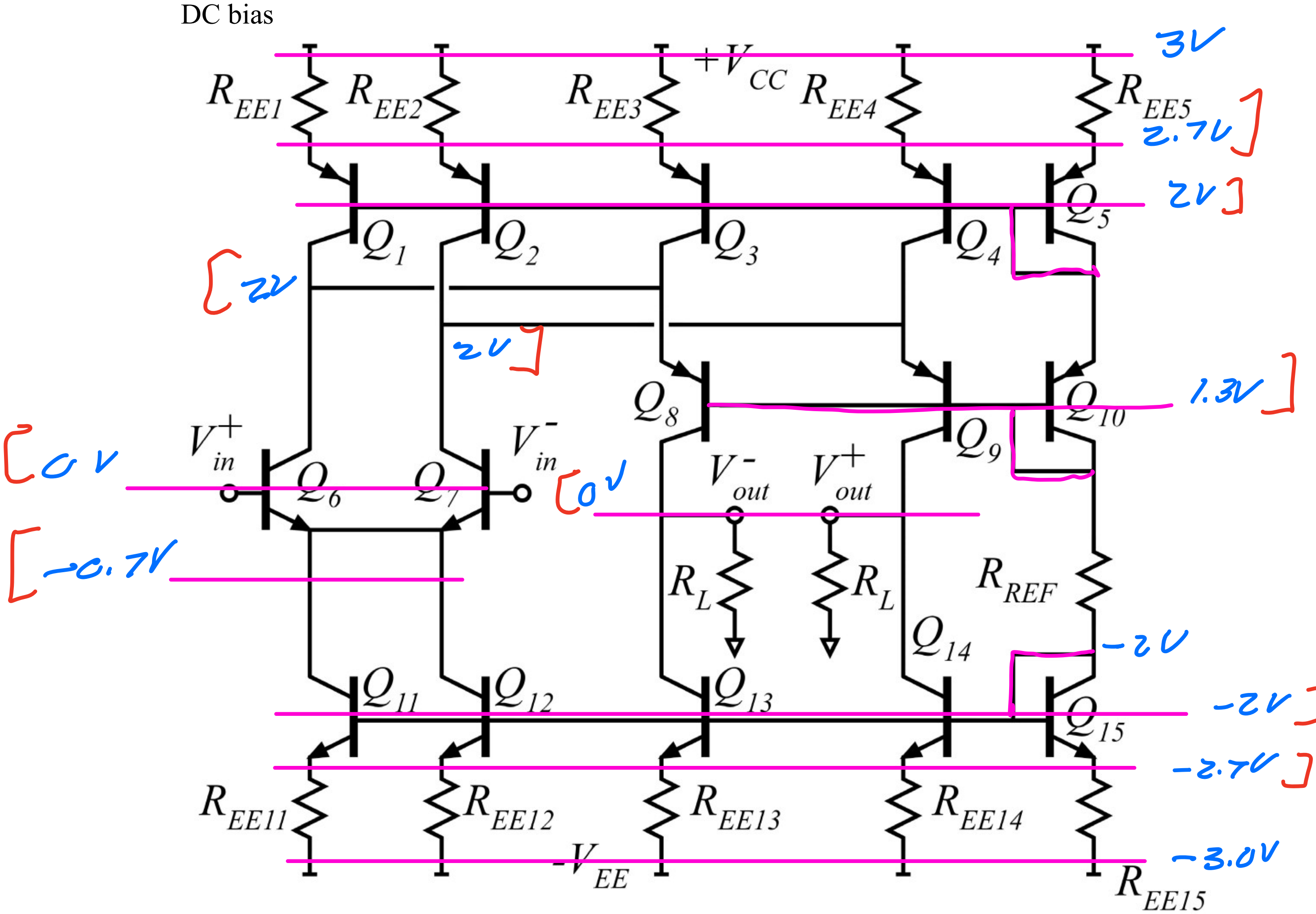
2

2 $R_{REF} = (1.5V+2V)/1mA = 3.5k\Omega$

a

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes**.

1/2 points each for the numbers indicated with "["

a

Part c, 5 points

find the following for all transistors

gm= $35.5mS$ Rbe= $\infty\ \Omega$ Rce= $\infty\ \Omega$

3 $g_m = \frac{I_c}{V_t} = \frac{1mA}{26mV} = 35.8\ mS = \frac{1}{26\Omega}$.

1 $R_{be} = \beta / g_m = \infty\ \Omega$

1 $R_{ce} = V_A / I_c = \infty\ \Omega$

a

$R_L = 1\,k\Omega$

Part d, 10 points.

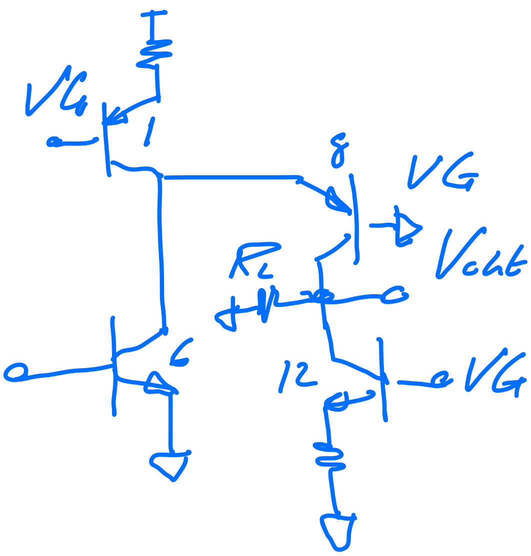
Find the following

Overall voltage gain $(V_{out}^{+} - V_{out}^{-})/(V_{in}^{+} - V_{in}^{-})$ = -38.5

Differential input impedance Rin, Diff = $2\beta/g_m = \infty\,\Omega$

first solution method

VG = virtual ground.



Q_8 $\quad R_{Leq\,8} = R_L = 1\,k\Omega$] 1

$r_{i8} = 1/g_{m8} = 26\,\Omega$] 1

$A_{v8} = R_{Leq\,8}/R_{in\,8} = 1\,k\Omega/26\,\Omega$] 2

Q_6 $\quad R_{Leq\,6} = R_{in\,8} = 1/g_{m8} = 26\,\Omega$] 2

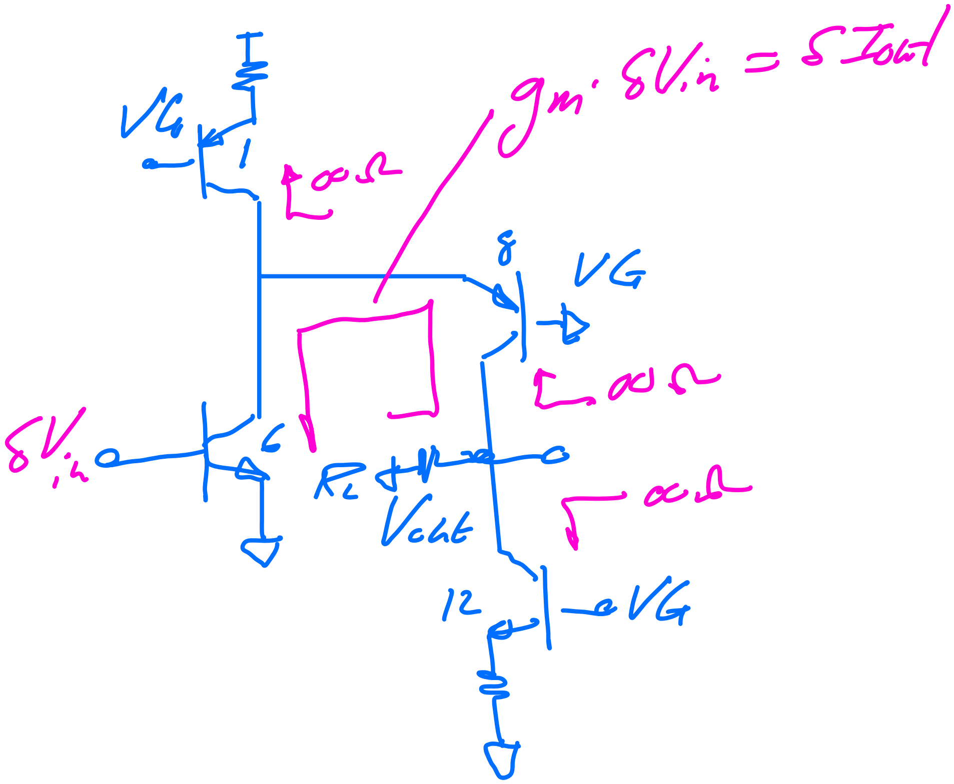
$A_{v6} = -g_{m6}R_{Leq\,6} = -26\,\Omega/26\,\Omega = -1$] 2

overall

$A_v = -A_{v8}A_{v6} = -R_{Leq\,8}\cdot g_{m6} = -\dfrac{1\,k\Omega}{26\,\Omega}$] 2

$= -38.5$

Either/or points assignment.



Second solution method

$R_{Leq8} = R_L = 1k\Omega$ — 1

$\delta I_{out} = \delta I_{c8} = \delta I_{c6} = -g_m V_{in}$ — 3

$\delta V_{out} = \delta I_{c8} \cdot R_L$ — 3

$\frac{\delta V_o}{\delta V_i} = g_{m6} \cdot R_L = \frac{1k\Omega}{26\Omega} = 38.5$ — 3

$R_L = 1k\Omega$
all K: $I_{CQ} = 1mA$

Part e, 10 points

Maximum output voltage swings ***at V_{out}^{+} (not V_{out}^{-})*** (***show all your work***).

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q6	$-1V$	Do not calculate
Transistor Q7	$+1V$	Do not calculate
Transistor Q9	-1	$+1.5V$
Transistor Q14	Do not calculate	$-2.2V$

Cutoff Q6

If $I_{C6} = 0mA$

→ $I_{C7} = 2mA$

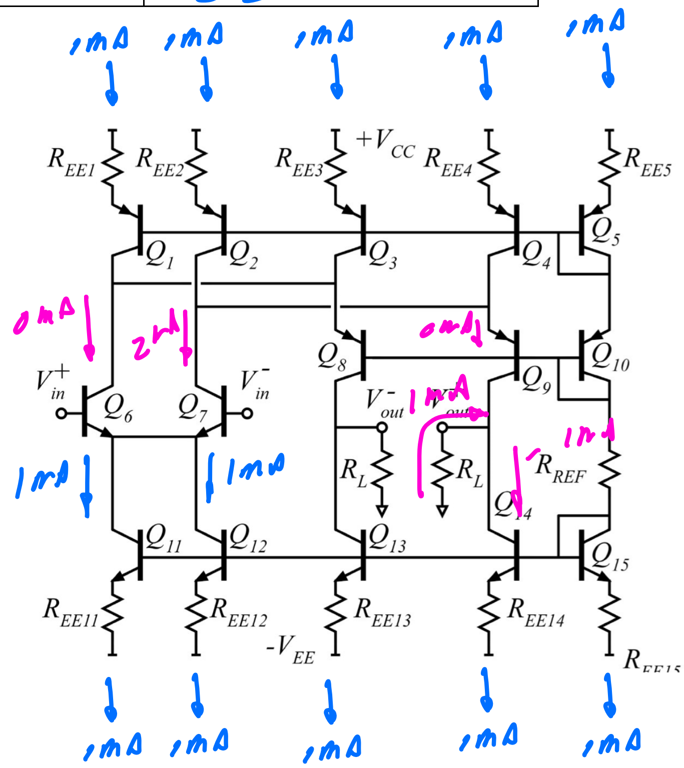
→ $I_{C9} = 0mA$

→ $I_{out}^{+} = -1mA$

1

$V_{out}^{+} = -1mA \cdot 1k\Omega$
$= -1V$

1



a

Cutoff Q7

If $I_{C7} = 0$ mA

$(\to I_{C6} = 2$ mA$)$

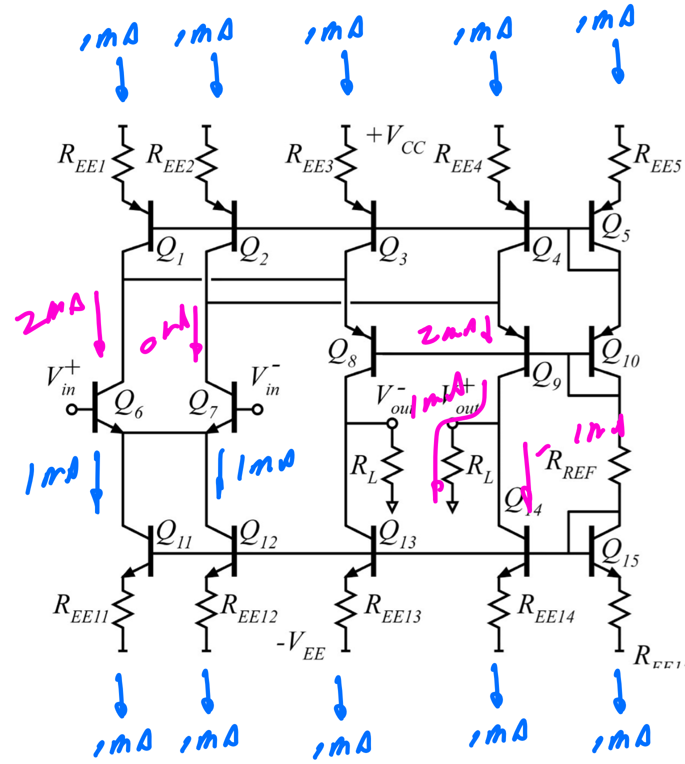
$\to I_{C9} = 2$ mA

$\to I^{+}_{out} = +1$ mA

$V^{+}_{out} = 1\,\text{mA} \cdot 1\,\text{k}\Omega = +1\,\text{V}$

1

1



Cutoff Q9

If $I_{C4} = 0$ mA

$(\to I_{C7} = 2$ mA

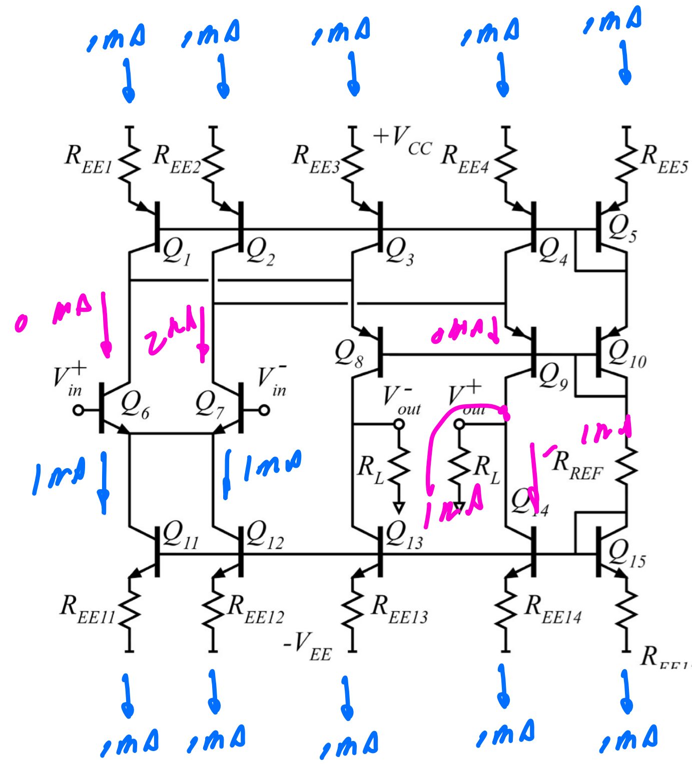
$\to I_{C6} = 0$ mA$)$

$\to I^{+}_{out} = -1$ mA

$V^{+}_{out} = -1\,\text{mA} \cdot 1\,\text{k}\Omega = -1\,\text{V}$

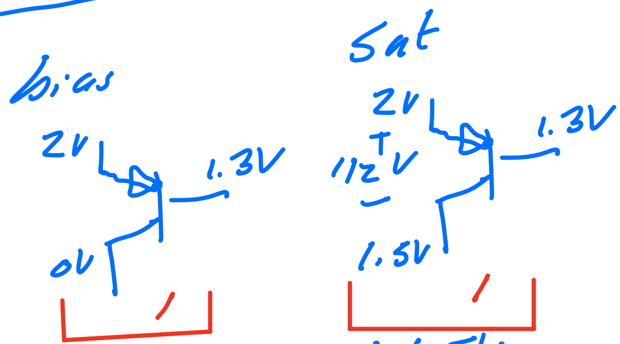
1

1

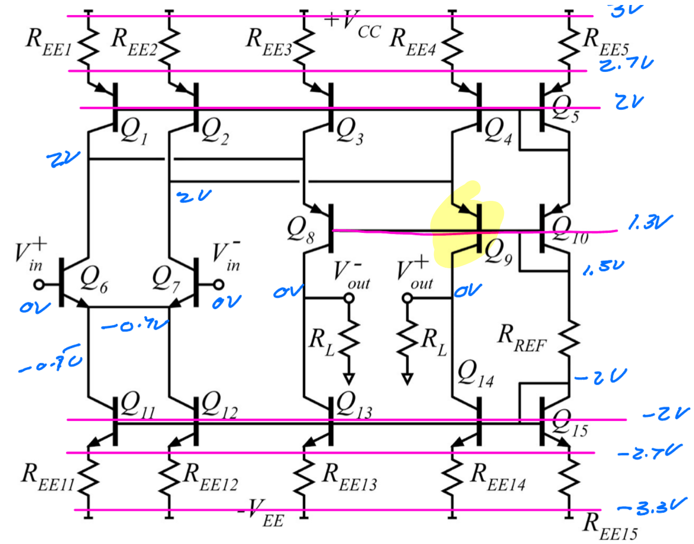


a

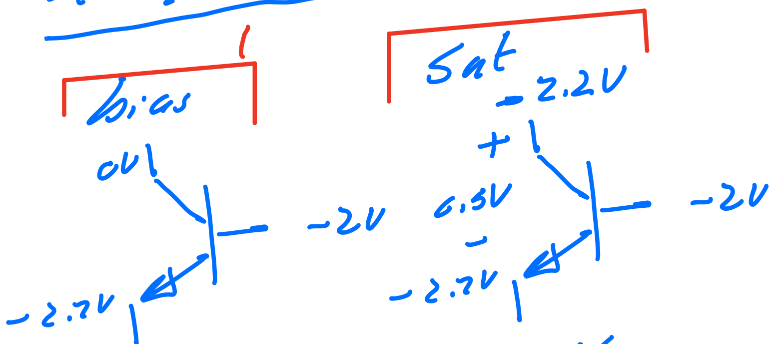
Q9 saturation



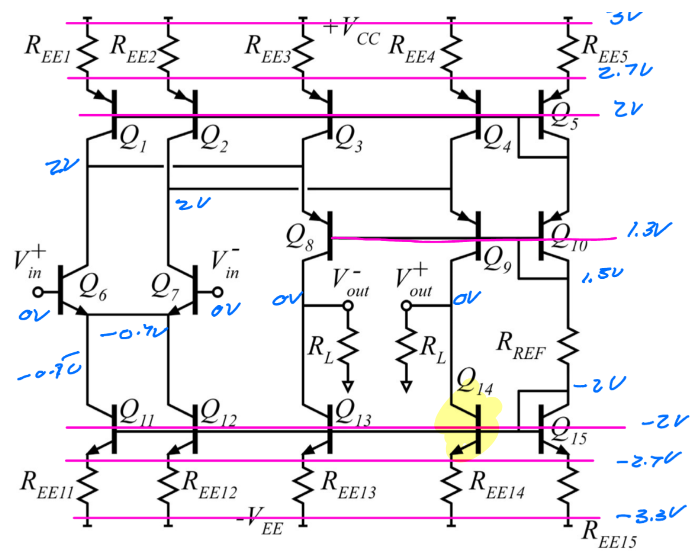
$2V_{out} = +1.5V$



Q14 saturation

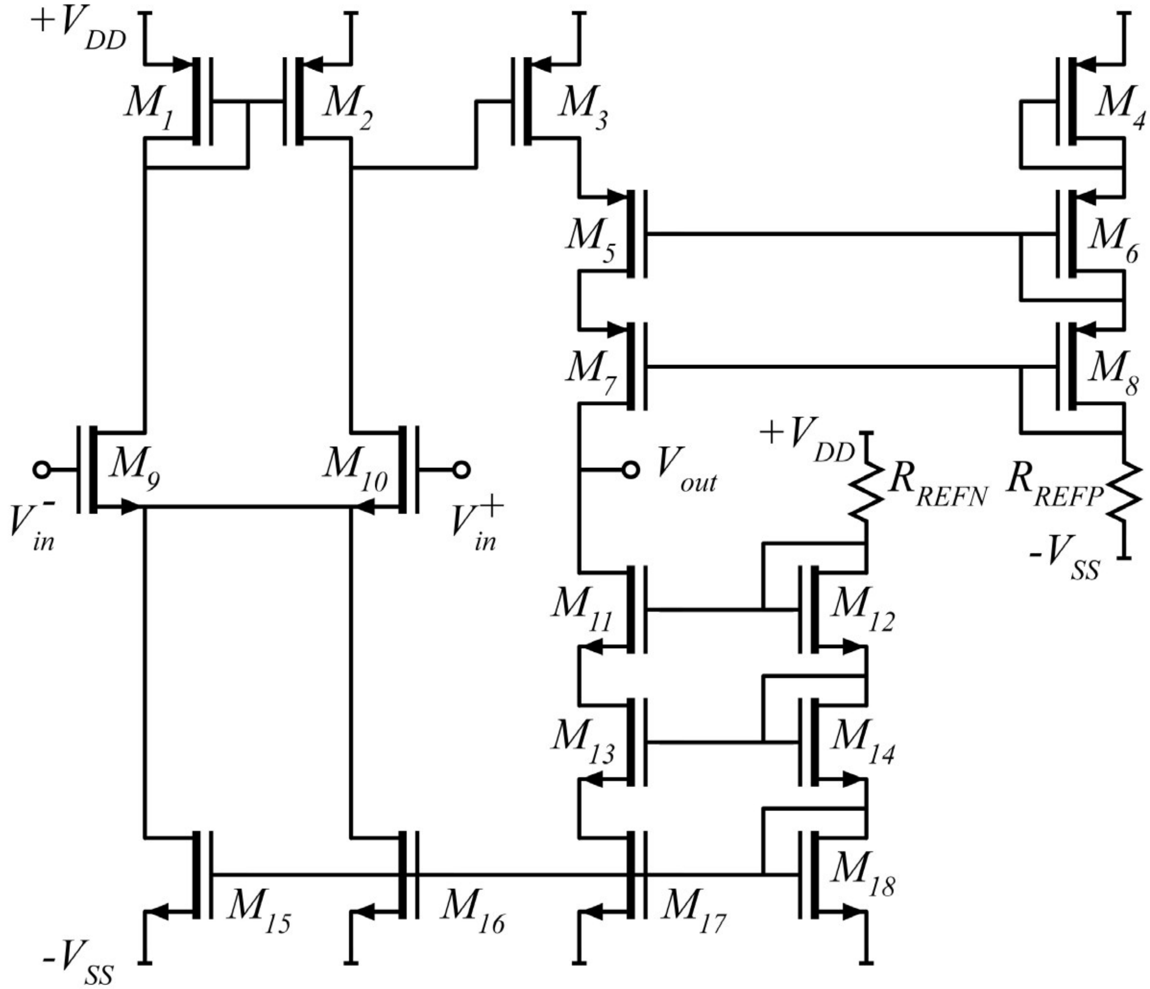


$2V_{out} = -2.2V$



Problem 2, 45 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage V_{out} is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



The NMOSFETs have $K_\mu = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$ $K_v = 2.0\text{mA/V} \cdot (W_g / 1\mu\text{m})$, $\Delta V = 0.10\text{V}$, $V_{th} = 0.2$ V, $1/\lambda = 5$V. The PMOS have identical parameters, except, of course, V_{th} is negative. V_{DD} = +0.75 V, $-V_{SS}$ = -0.75 V All transistors have |Vgs|=0.25V and are biased at I_D=50 μA.

a

Part a, 10 points

DC bias.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.
(Hint, this should give Vi- = 0V)

Find the following:

Gate widths of M9 and M10 = $1.74\mu m$

Gate width of M15 and M16 = $1.81\mu m.$

Gate width of ***all other MOSFETS*** = $1.905\mu m$

value of R_{REFN} = $15\ k\Omega$ value of R_{REFP} = $15\ k\Omega$

2

M9, M10

$V_{DS} = 0.75V$

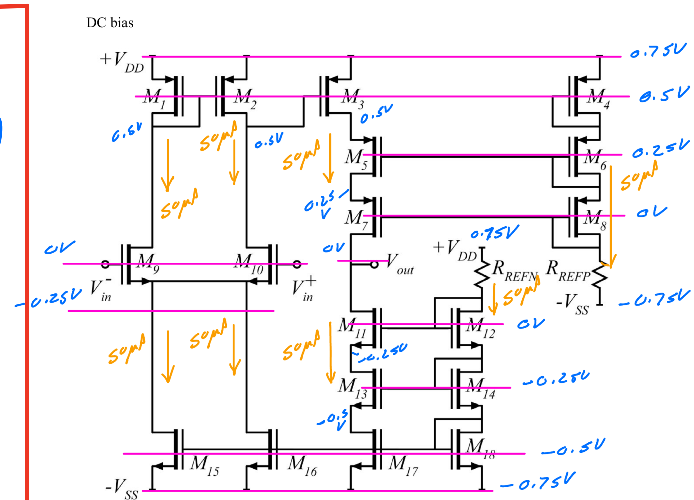
$V_{GS} = 0.25V$

$$I_D = 10\frac{mA}{V^2}\cdot\frac{W_9}{1\mu m}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$$

$$\frac{1}{20}\cancel{mA} = 10\frac{\cancel{mA}}{\cancel{V^2}}\cdot\frac{W_9}{1\mu m}\left(\frac{1}{20}\cancel{V}\right)^2\underbrace{\left(1+\frac{0.75V}{5V}\right)}_{1.15}$$

$$\frac{W_9}{1\mu m} = \frac{1}{20}\cdot\frac{1}{10}\cdot 20\cdot 20\cdot\frac{1}{1.15}$$

$$= 2\cdot\frac{1}{1.15} = 1.74$$



$W_9 = 1.74\mu m$

2

$$R_{REFN} = R_{REFP} = \frac{0.75V}{0.05mA} = \frac{75}{5}k\Omega = 15k\Omega$$

a

<u>M15, M16</u> $V_{DS} = 0.5\,V$

$V_{GS} = 0.25V$

$$I_D = 10\frac{mA}{V^2}\cdot\frac{W_g}{1\mu m}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$$

$$\frac{1}{20}\cancel{mA} = 10\frac{\cancel{mA}}{\cancel{V^2}}\cdot\frac{W_g}{1\mu m}\left(\frac{1}{20}\cancel{V}\right)^2\underbrace{\left(1+\frac{0.5V}{5V}\right)}_{1.1}$$

$$\frac{W_g}{1\mu m} = \frac{1}{20}\cdot\frac{1}{10}\cdot 20\cdot 20\cdot\frac{1}{1.1}$$

$$= 2\cdot\frac{1}{1.1} = 1.81$$

$$W_g = 1.81\,\mu m$$

2

<u>All others</u> $V_{DS} = 0.25V$

$V_{GS} = 0.25V$

$$I_D = 10\frac{mA}{V^2}\cdot\frac{W_g}{1\mu m}(V_{GS}-V_{th})^2(1+\lambda V_{DS})$$

$$\frac{1}{20}\cancel{mA} = 10\frac{\cancel{mA}}{\cancel{V^2}}\cdot\frac{W_g}{1\mu m}\left(\frac{1}{20}\cancel{V}\right)^2\underbrace{\left(1+\frac{0.25V}{5V}\right)}_{1.05}$$

$$\frac{W_g}{1\mu m} = \frac{1}{20}\cdot\frac{1}{10}\cdot 20\cdot 20\cdot\frac{1}{1.05}$$

$$= 2\cdot\frac{1}{1.05} = 1.905$$

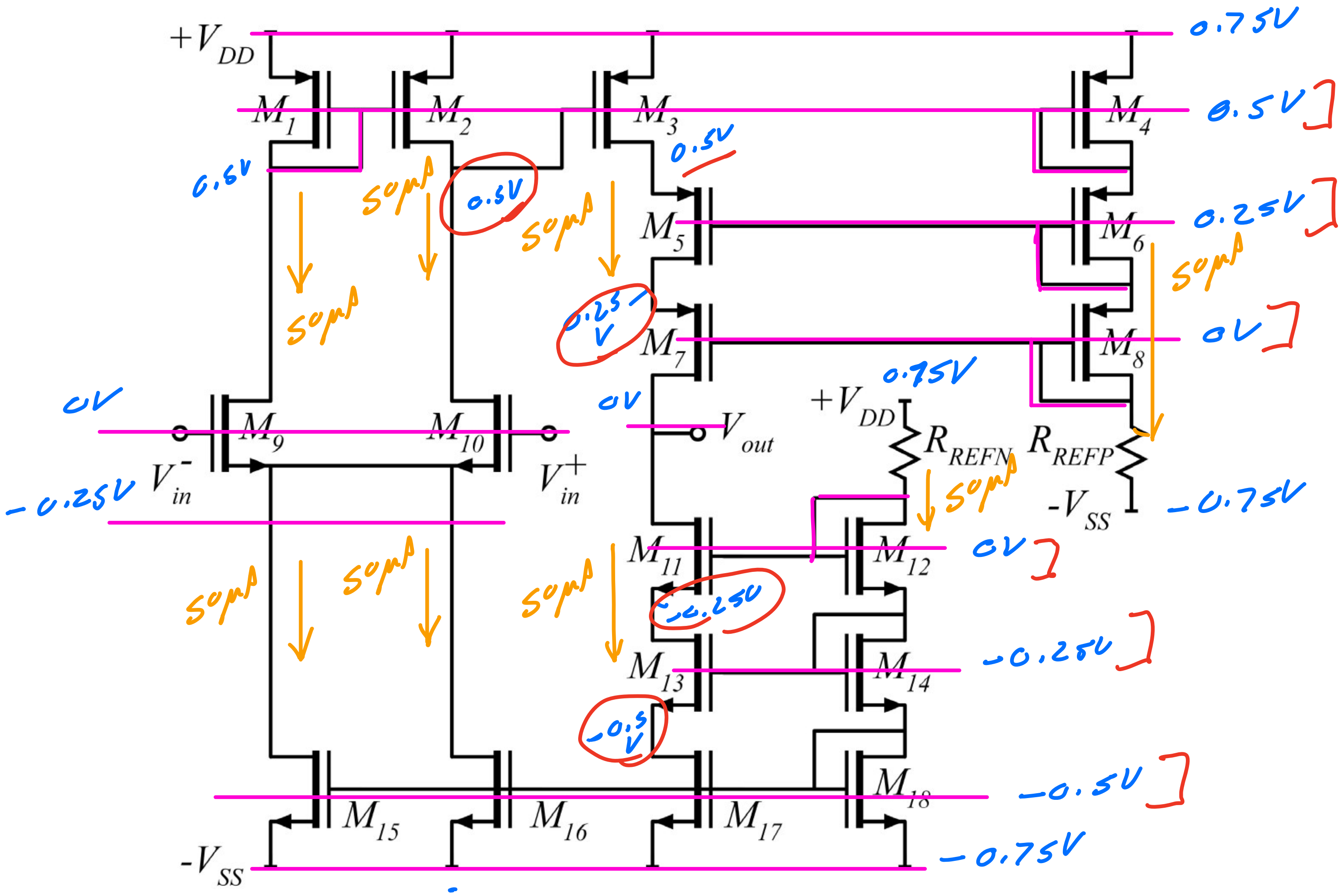
$$W_g = 1.905\,\mu m$$

2

a

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes.**

0.5 pts each for #s

in]

a

Part c, 5 points.

Find the following

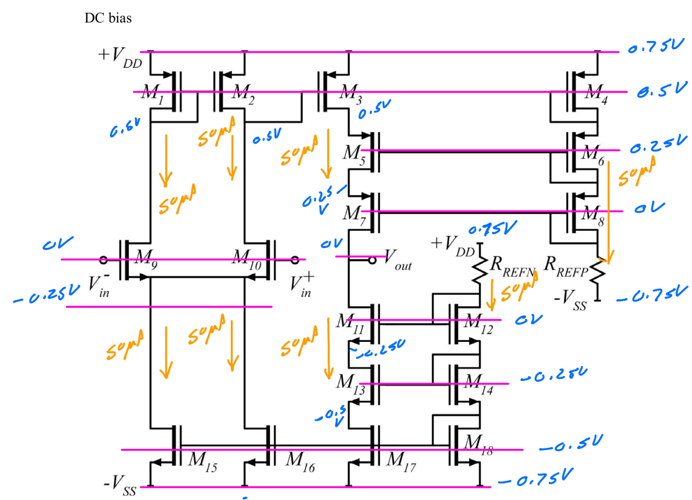
	Voltage Gain	Input impedance
Transistor combination M1,2,9, 10,15, 16	$\frac{g_m R_{ds}}{2} = 100$	$\infty\ \Omega$

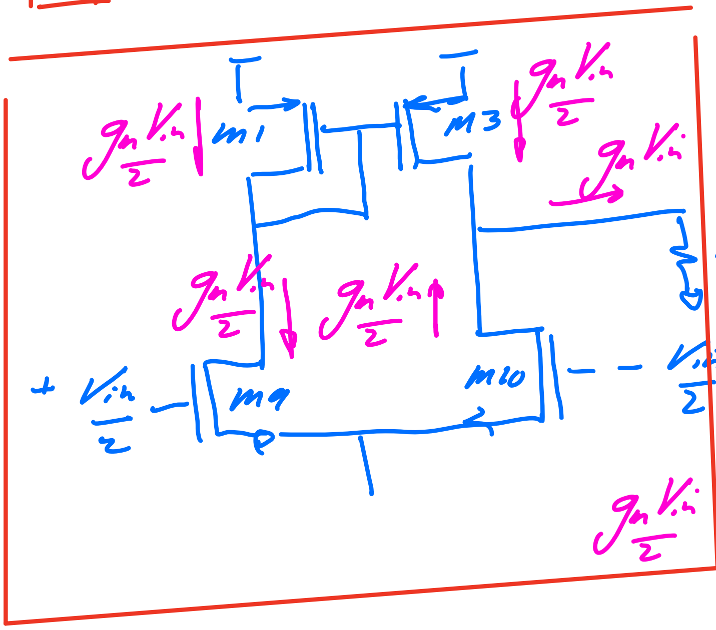
all FETs:

$$g_m = \frac{2 I_D}{V_{gs} - V_{th}} = \frac{2 \cdot 50\mu A}{50mV}$$

$$= 2mS$$

$$R_{DS} = 1/\lambda I_D = \frac{5V}{0.05mA} = 100k\Omega$$





$$R_{Leq} = R_{DS10} \parallel R_{DS16} = \frac{R_{DS}}{2} = 50k\Omega$$

$$A_v = \frac{g_m R_{DS}}{2} = 2mS \cdot 50k\Omega$$

$$= 100$$

$R_{in} = \infty\ \Omega$ because gate currents are zero.

a

$2\,mS \qquad 100\,k\Omega$

$g_m R_{ds} = 200$

Part d, 15 points.

Find the following

	Voltage Gain	Input impedance
Transistor combination M3,5,7,11,13,17	$-g_m^3 R_{DS}^3/2 = 4\cdot 10^6$	$\infty\,\Omega$
Overall op-amp $V_{out}/(V_{in}^+ - V_{in}^-)$	$\dfrac{g_m^4 R_{ds}^4}{4} = 4\cdot 10^8$	$\infty\,\Omega$

Note

1) You can analyze M3, M5, and M7 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

$R_{out17} = R_{DS} = 100\,k\Omega$ 1

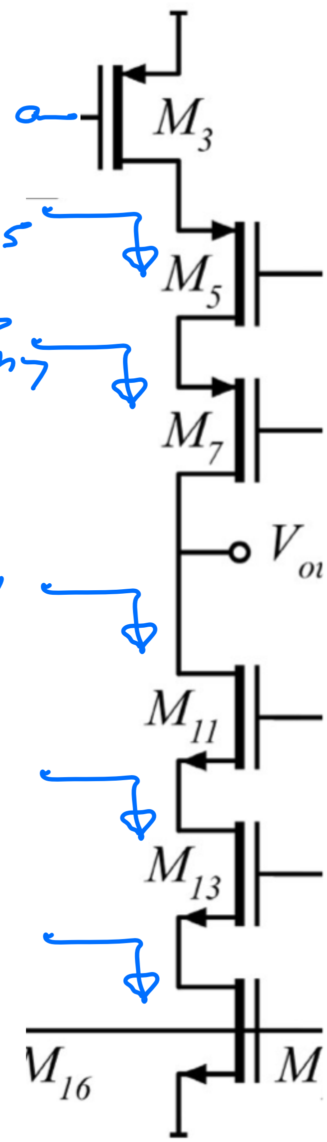
$R_{out13} = R_{DS}(1+g_m R_{DS})$ 1

$\cong g_m R_{ds}^2 = 20\,M\Omega$

$R_{out11} = R_{ds}(1+g_m^2 R_{ds}^2) = R_{Leq7}$

$\cong (g_m R_{ds})^2 \cdot R_{ds}$ 2

$= 4\cdot 10^9\,\Omega = 4\,G\Omega$



$R_{in7} = \frac{1}{g_m}\left(\frac{R_{Leq}+R_{DS}}{R_{DS}}\right)$

$\cong 1/g_m \cdot R_{Leq}/R_{DS}$

$= g_m^{-1}\, g_m^2 R_{DS}^3 / R_{DS}$ 2

$= g_m R_{DS}^2 = 20\,M\Omega$

$= R_{Leq5}$

$A_{v7} = R_{Leq7}/R_{in7}$ 2

$$= g_m^2 R_{DS}^3 / (g_m R_{DS}^2)$$

$$= g_m R_{D3} = 200$$

$$R_{in5} = 1/g_m \left(\frac{R_{DS} + R_{Leq5}}{R_{DS}} \right) \quad 2$$

$$\cong 1/g_m \cdot R_{Leq5} / R_{DS}$$

$$= 1/g_m \cdot g_m R_{D3}^2 / R_{DS}$$

$$= R_{DS} = 100k\Omega$$

$$A_{v5} = R_{Leq5} / R_{in5}$$

1 $$= g_m R_{DS}^2 / R_{DS}$$

$$= g_m R_{D3} = 200$$

$$R_{Leq3} = R_{DS3} \parallel R_{in5} = R_{DS}/2 = 50k\Omega$$

$$A_{v3} = -g_m R_{Leq3}$$

1 $$= -g_m R_{D3}/2$$

$$= -100$$

1 $$A_{v3} A_{v5} A_{v7} = -(g_m R_{DS})^3 / 2 = -4 \cdot 10^6$$

1 $R_{in3} = \infty\Omega$ because $I_g = 0A$

$$A_{v,opamp} = A_{v1,2,9,10} \cdot A_{v3} A_{v5} A_{v7} = (g_m R_{DS})^4 / 4$$

1 $$= 4 \cdot 10^8$$

a

Norton method - first find I-short-circuit.

Alternate Solution.

$R_{in7} = 1/g_m$

$$R_{in5} = \frac{1}{g_m}\left(\frac{R_{DS}+1/g_m}{R_{DS}}\right)$$

$$= \frac{1}{g_m}\left(1+\frac{1}{g_m R_{DS}}\right)$$

$$\simeq \frac{1}{g_m}$$

$\delta I_{D3} = g_m \delta V_g$

$$\delta I_{D5} = \delta I_{D3}\left(\frac{R_{DS}+1/g_m}{R_{DS}}\right)$$

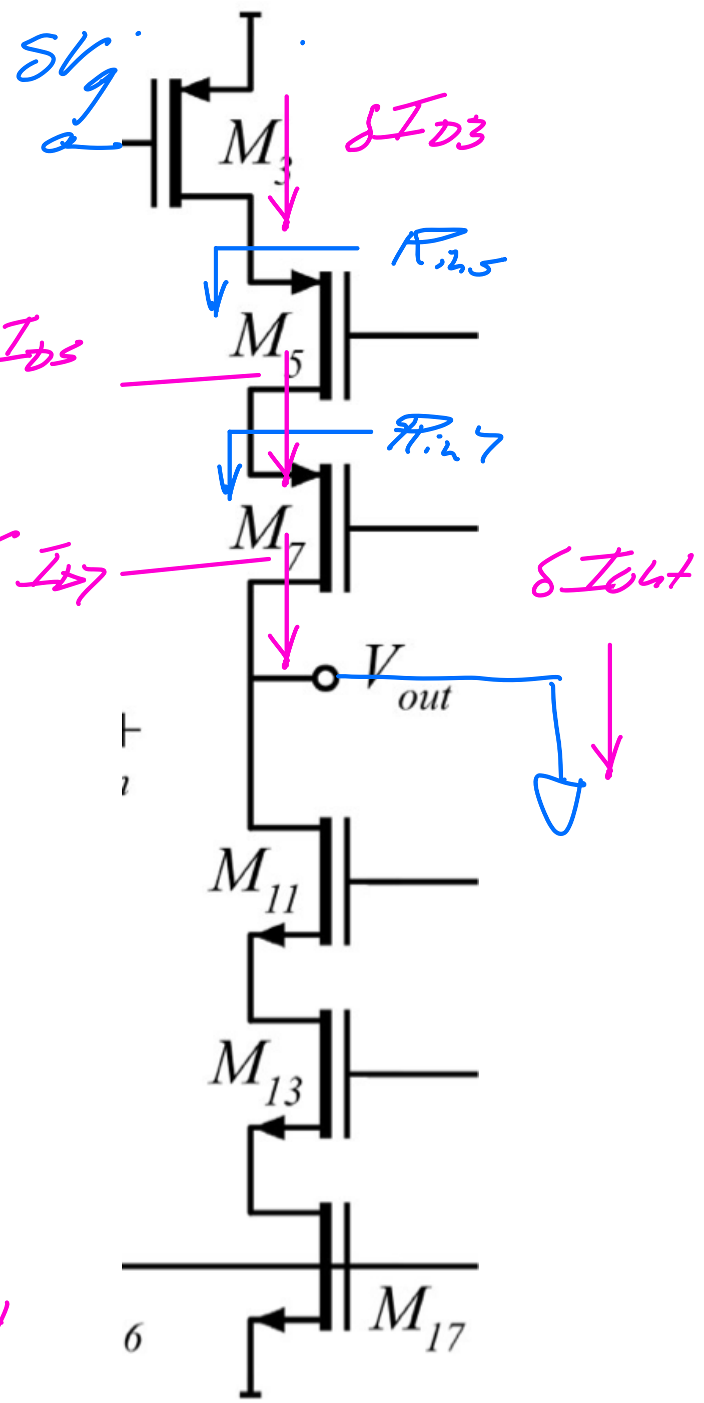
$$\doteq g_m \cdot \delta V_g \cdot \left(1 + 1/g_m R_{ds}\right)$$

$$\simeq g_m \delta V_g.$$

$$\delta I_{D7} = \delta I_{D5}\left(\frac{R_{DS}+1/g_m}{R_{DS}}\right)$$

$$\doteq g_m \cdot \delta V_g \cdot \left(1 + 1/g_m R_{ds}\right)$$

$$\simeq g_m \delta V_g. = \delta I_{out}$$



a

Norton method - output impedance

$$R_{out17} = R_{DS}$$

$$R_{out13} = R_{out17}(1 + g_m R_{DS}) = R_{DS}(1 + g_m R_{DS}) \cong g_m R_{DS}^2$$

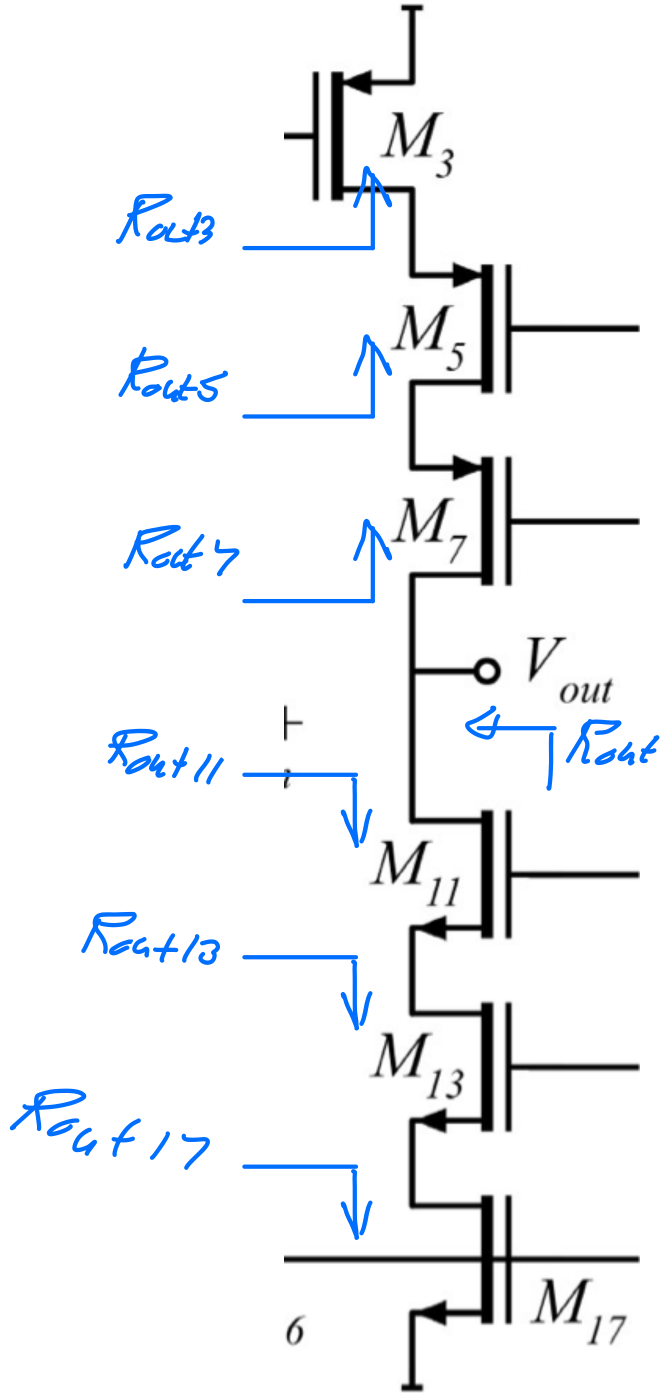
$$R_{out11} = R_{out13}(1 + g_m R_{DS}) = g_m R_{DS}^2 (1 + g_m R_{DS}) \cong g_m^2 R_{DS}^3 = 4\,G\Omega$$

$$R_{out3} = R_{DS}$$

$$R_{out5} = R_{out3}(1 + g_m R_{DS}) = R_{DS}(1 + g_m R_{DS}) \cong g_m R_{DS}^2$$

$$R_{out7} = R_{out5}(1 + g_m R_{DS}) = g_m R_{DS}^2 (1 + g_m R_{DS}) \cong g_m^2 R_{DS}^3 = 4\,G\Omega$$

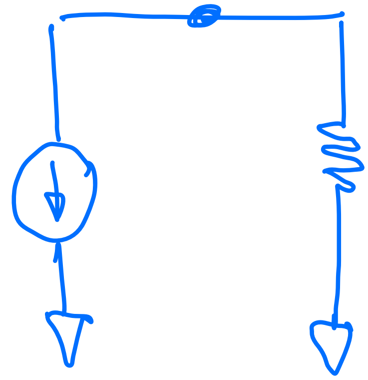
$$R_{out} = R_{out7} \,//\, R_{out11} = g_m^2 R_{DS}^3 / 2 = 2\,G\Omega$$



a

δV_{out}

$I_{out} = g_m \delta V_g$



$R_{out} = g_m^2 r_{DS}^3/2 = 2\,G\Omega$

2

$$\delta V_{out} = -I_{out} \cdot R_{out}$$

$$= -g_m \delta V_g \cdot g_m^2 r_{DS}^3/2$$

$$= -\delta V_g \cdot g_m^3 r_{DS}^3/2$$

$$\frac{\delta V_{out}}{\delta V_g} = A_{v3} A_{v5} A_{v7} = -g_m^3 r_{DS}^3/2$$

$$= -4 \cdot 10^6$$

1

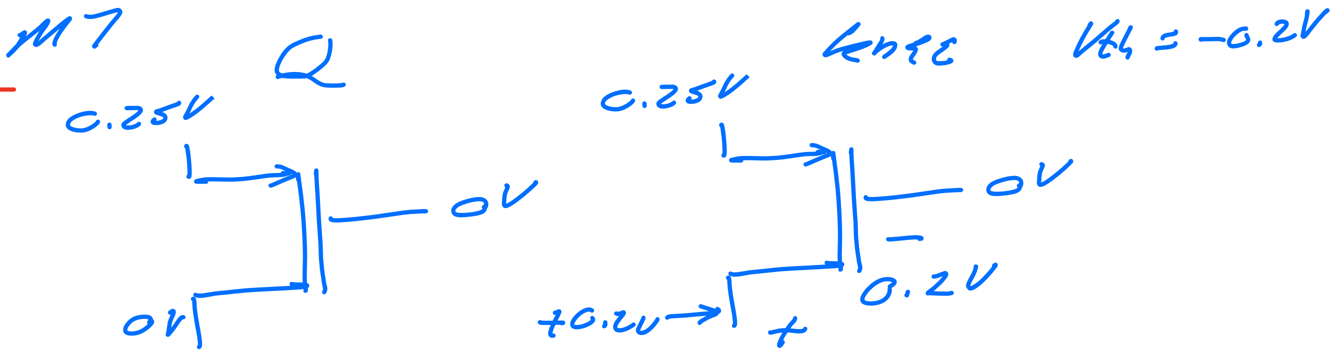
a

Part e, 10 points

Maximum output voltage swings ***at V_{out}^{+} (not V_{out}^{-})*** (***show all your work***).

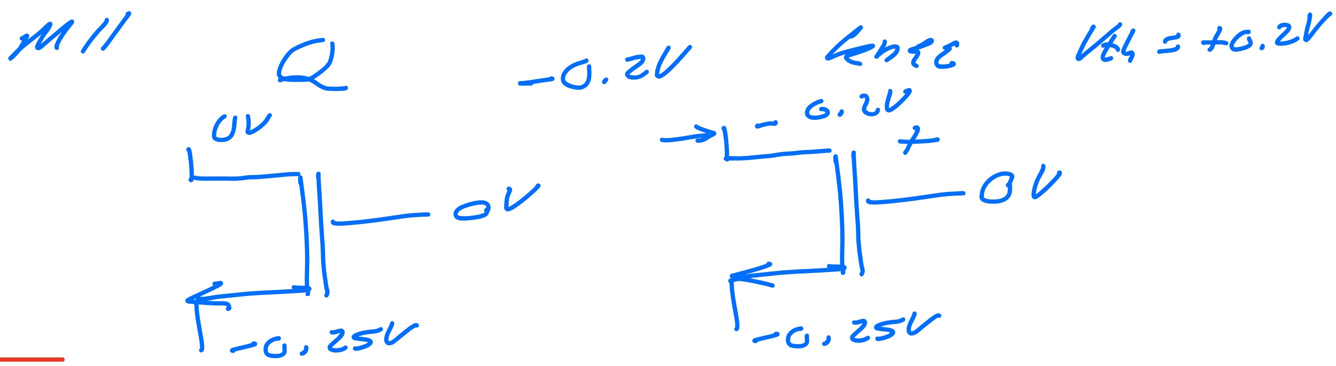
	magnitude and sign of maximum output signal swing due to ***knee voltage*** (saturation)
Transistor M7	+0.2V
Transistor M11	−0.2V
Transistor M2	$-8\cdot10^{5}V$

3



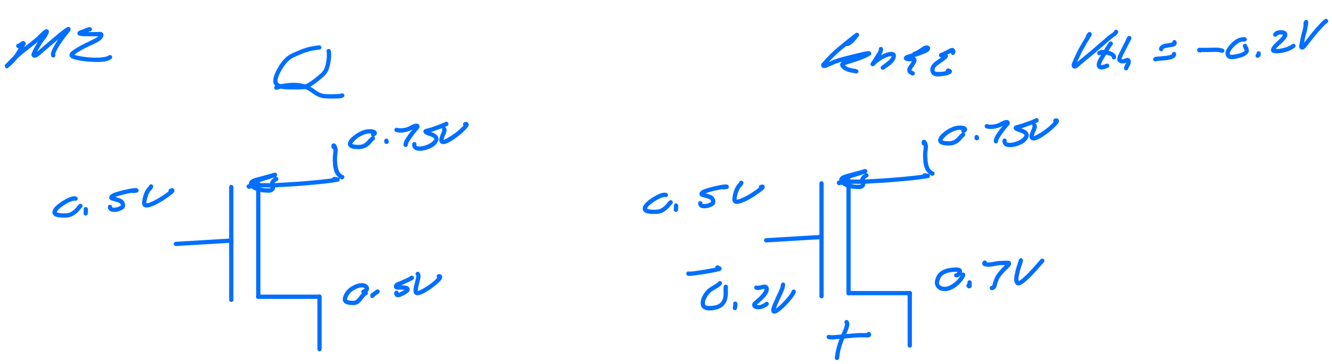
Vout = 0.2V

3



Vout = −0.2V

3

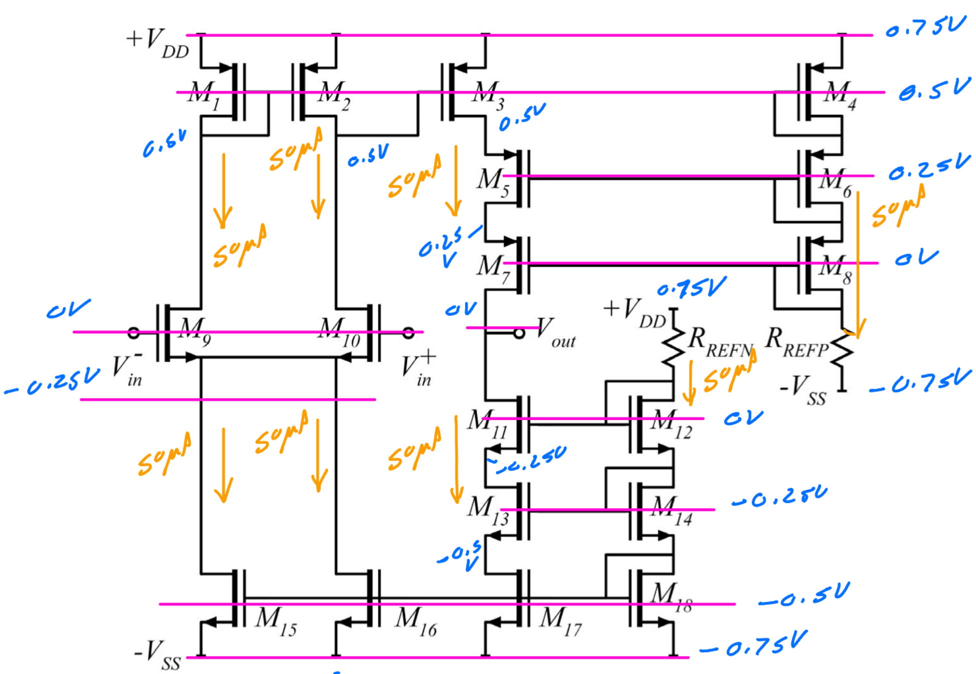


a

ΔVdrain2 = +0.2V

$$\Delta V_{out} = \Delta V_{drain2} \cdot A_{v3} A_{v5} A_{v7}$$
$$= 0.2V\,(-4 \cdot 10^{6})$$
$$= -8 \cdot 10^{5}\,V$$

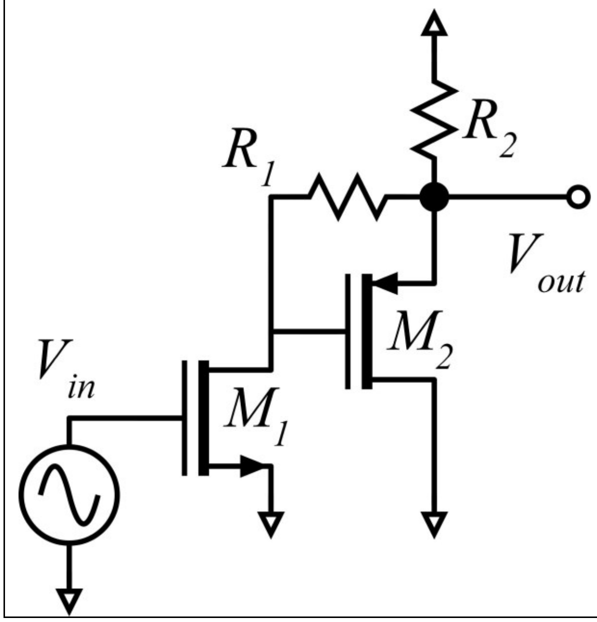
(obviously not the dominant clipping limit)



On the circuit diagram above, label the DC voltages at **ALL nodes.**

a

Problem 3, 20 points



You will be working on the circuit to the left

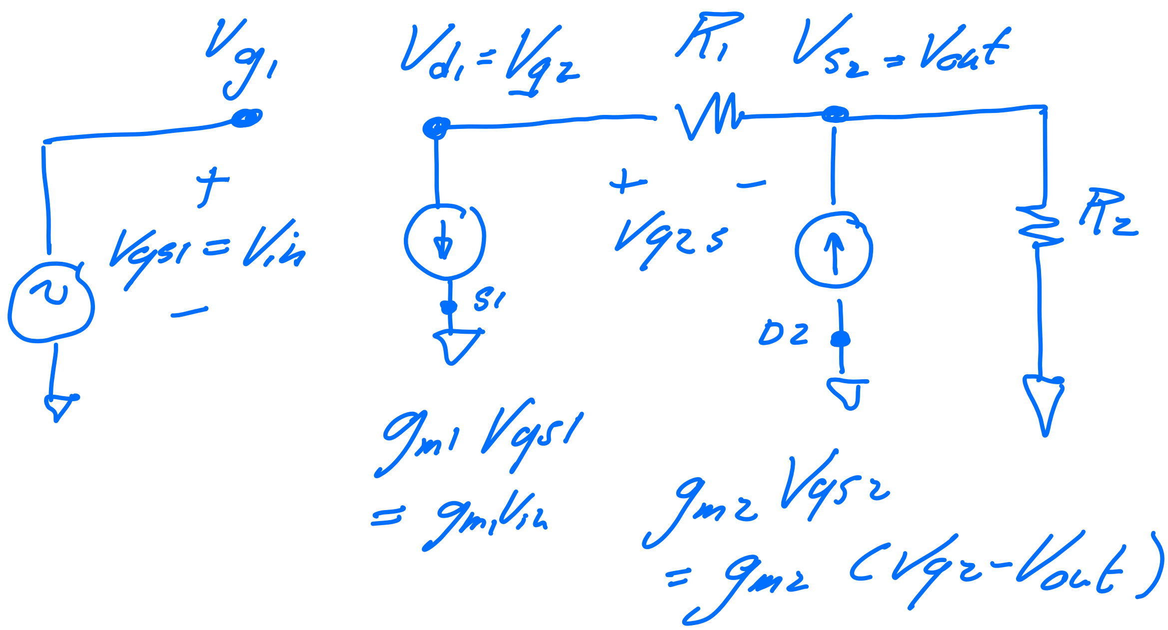
Ignore DC bias analysis. You don't need it.

M_1 and M_2 have transconductances g_{m1} and g_{m2}.

Both transistors have **infinite** R_{DS},
...so you don't need to include these elements in the circuit diagram !

Part a, 5 points

Draw a small-signal equivalent circuit of the circuit.



0 pts if topology is wrong
−2 pts each if Vgs controlling gmVgs
is not correctly indicated.

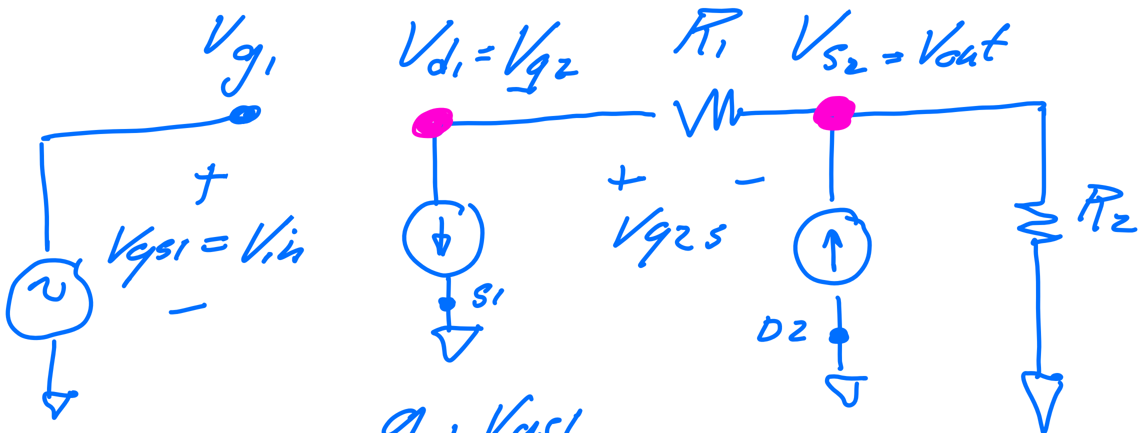
a

Part b, 10 points

Derive an algebraic expression for V_{out} / V_{in} in terms of g_{m1}, g_{m2}, R_1, and R_2

$V_{out} / V_{in} = -g_{m1} R_2 (1 + g_{m1} R_1)$

4



$g_{m1} V_{gs1} = g_{m1} V_{in}$

$g_{m2} V_{gs2} = g_{m2} (V_{d1} - V_{out})$

$\Sigma I = 0$ @ V_{d1}

$g_{m1} V_{in} = (V_{out} - V_{d1}) / R_1$ 3

$g_{m1} R_1 V_{in} = V_{out} - V_{d1}$

3

$\Sigma I = 0$ @ V_{out}

$V_{out} / R_2 + (V_{out} - V_{d1}) / R_1 = g_{m2} (V_{d1} - V_{out})$

$V_{out} / R_2 + g_{m1} V_{in} + g_{m2} (V_{out} - V_{d1}) = 0$

$V_{out} / R_2 + g_{m1} V_{in} + g_{m2} g_{m1} R_1 \cdot V_{in} = 0$

a

4

$$V_{out} + g_{m1} R_2 V_{in} + g_{m2}\, g_{m1} R_1 R_2 V_{in} = 0$$

$$V_{out} + g_{m1} R_2 (1 + g_{m1} R_1) V_{in} = 0$$

$$\frac{V_{out}}{V_{in}} = -g_{m1} R_2 (1 + g_{m1} R_1)$$

a

Part c, 5 points

Set g_{m1}=1 mS, g_{m2}=2 mS , R_1= 1 kΩ, and R_2 = 2 kΩ.
Find the numerical value of V_{out} / V_{in}

V_{out} / V_{in} = -6

5

$$\frac{V_{out}}{V_{in}} = -g_{m1} R_2 (1 + g_{m1} R_1)$$

$$= -1\,mS \cdot 2\,k\Omega\,(1 + 2\,mS \cdot 1\,k\Omega)$$

$$= -2\,(1+2)$$

$$= -6$$

a