

ECE 137 A Mid-Term Exam

Wednesday, February 8, 2023

Do not open exam until instructed to.

Closed book: Class crib sheet and 1 page (both sides) personal notes permitted

There are 3 problems on this exam, and you have 75 minutes.

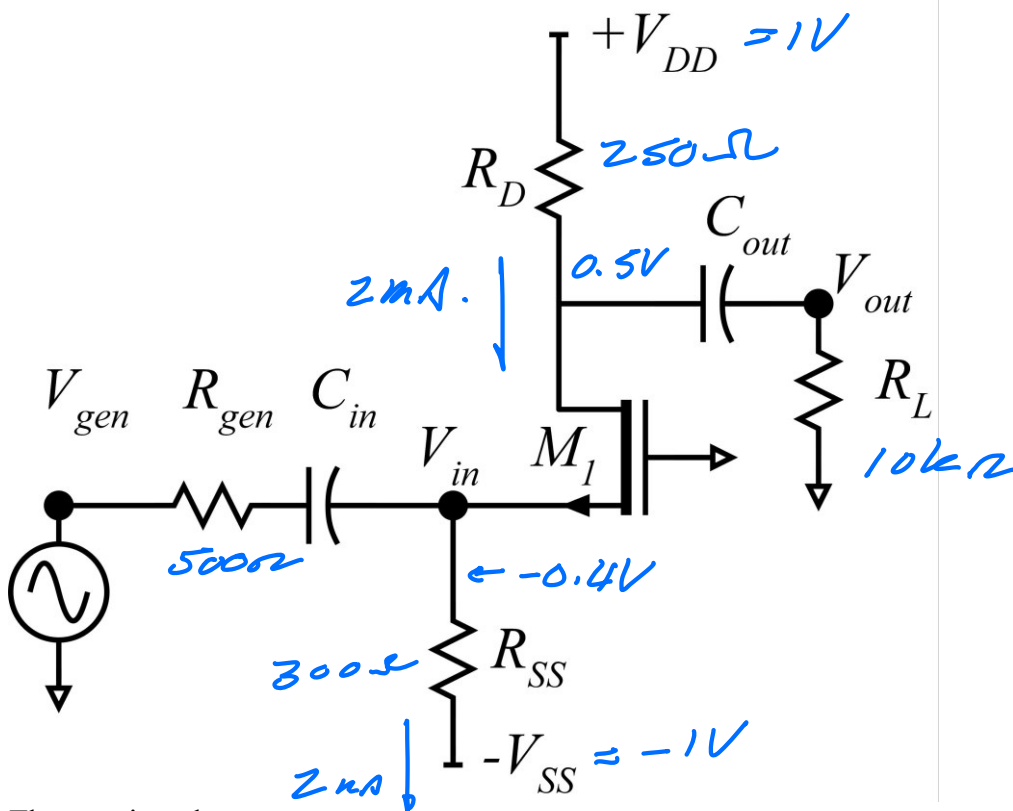
Use any and all reasonable approximations (5% accuracy is fine.),
AFTER STATING and approximately Justifying them.

Name: Solution

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		10	2e		3
1b		5	3a		8
1c		5	3b		8
1d		10	3c		4
1e		2			
1f		15			
2a		10			
2b		5			
2c		5			
2d		10			
TOTAL					100

Problem 1, 47 points

You will be working on the circuit below:



The transistor has

$$M1: K_{\mu} = \mu c_{gs} W_g / 2L_g = 10 \text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 2 \text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = v_{inj} L_g / \mu = 0.1 \text{V}, V_{th} = 0.3 \text{V}, 1/\lambda = 4 \text{V}$$

The supplies are +1.0V and -1.0 V

You are to bias the transistor at 2mA drain current, with -0.4 V DC source voltage, and with +0.5 V drain voltage

$$R_{gen} = 500 \Omega, R_L = 10 \text{k}\Omega$$

C_{in} and C_{out} are very large (AC short-circuits at the signal frequency)

Part a, 10 points

DC bias.

Find the following:

FET gate width $W_g = 16.3 \mu\text{m}$ $R_{SS} = 300 \Omega$ $R_{DD} = 250 \Omega$

2 $\left[V_{GS} - V_{th} = 0.4V - 0.3V = 0.1V \leftarrow \text{mobility or velocity limited.} \right]$

5 $\left[I_D = 10 \frac{\mu\text{A}}{V^2} \cdot \frac{W_g}{1 \mu\text{m}} \cdot (V_{GS} - V_{th})^2 \left(1 + \frac{V_{DS}}{4V} \right) \right]$

4 $2 \mu\text{A} = 10 \frac{\mu\text{A}}{V^2} \cdot \frac{W_g}{1 \mu\text{m}} \cdot (0.1V)^2 \left(1 + \frac{0.9V}{4V} \right)$

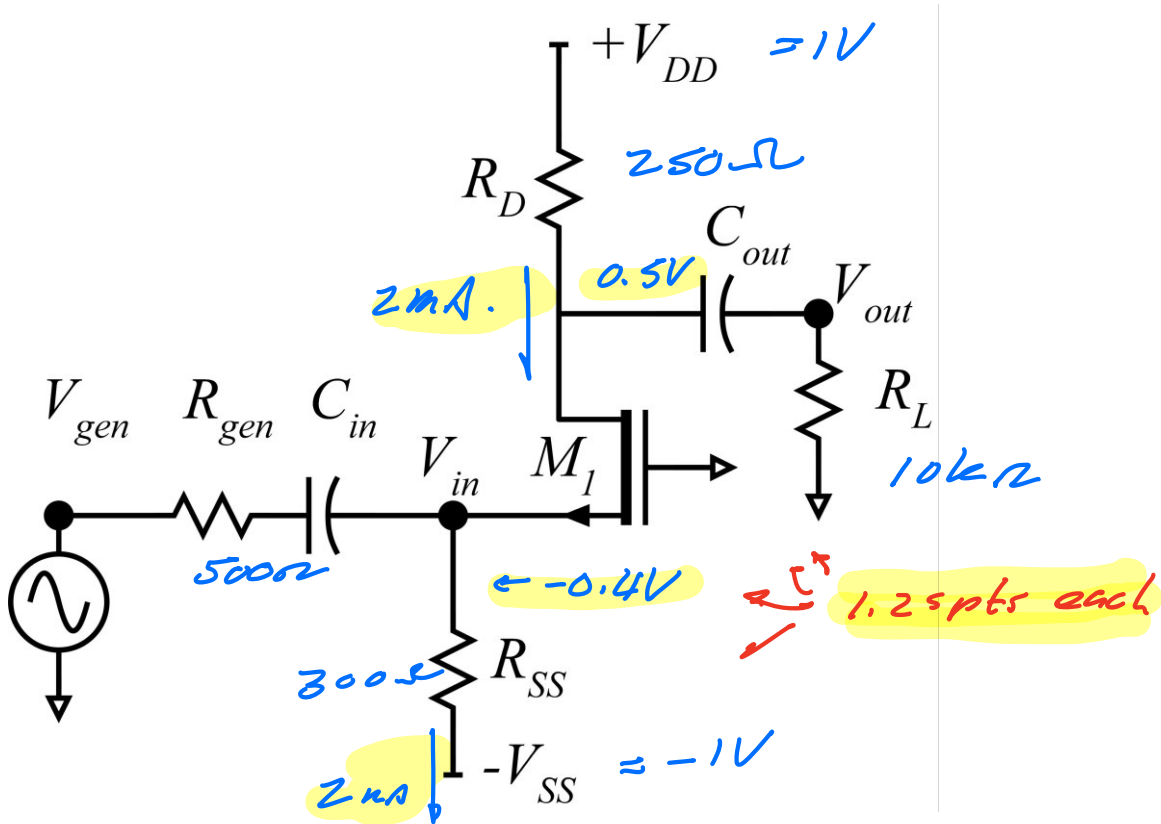
$W_g = 1 \mu\text{m} \cdot \frac{2 \mu\text{A}}{0.1 \mu\text{A}} \cdot \frac{1}{1 + 0.9/4} = 16.3 \mu\text{m}$

2 $\left[R_{SS} = \frac{1 - 0.4V}{2 \mu\text{A}} = \frac{0.6V}{2 \mu\text{A}} = 0.3 \text{ k}\Omega = 300 \Omega \right]$

2 $\left[R_{DD} = \frac{1 - 0.5V}{2 \mu\text{A}} = \frac{0.5V}{2 \mu\text{A}} = 0.25 \text{ k}\Omega = 250 \Omega \right]$

Part b, 5 points

DC bias



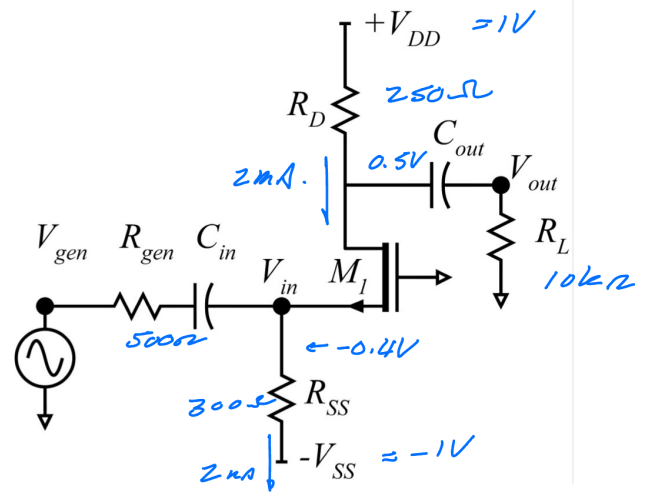
On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**, give **all resistor values**, and give the **FET width**.

*This is mostly for the TAs
to use to check students' work.*

Part c, 5 points

Find the FET small signal parameters

$g_m = \underline{40 \text{ mS}}$ $r_{ds} = \underline{2 \text{ k}\Omega}$



$$M1: K_{\mu} = \mu c_{gs} W_g / 2L_g = 10 \text{ mA/V}^2 \cdot (W_g / 1 \mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 2 \text{ mA/V} \cdot (W_g / 1 \mu\text{m})$$

$$\Delta V = v_{inj} L_g / \mu = 0.1 \text{ V}, V_{th} = 0.3 \text{ V}, 1/\lambda = 4 \text{ V}$$

$$3 \left[g_m = \frac{2 I_D}{V_{gs} - V_{th}} = \frac{2 \cdot 2 \text{ mA}}{0.4 \text{ V} - 0.1 \text{ V}} = \frac{4 \text{ mA}}{0.1 \text{ V}} = 40 \text{ mS} \right]$$

$$2 \left[r_{ds} = \frac{1}{\lambda I_D} = \frac{4 \text{ V}}{2 \text{ mA}} = 2 \text{ k}\Omega \right]$$

40 mS

2 kΩ

Part d, 10 points.

Find the small signal voltage gain V_{out}/V_{in} and the amplifier small-signal input resistance.

$V_{out}/V_{in} =$ 0.866

$R_{in, \text{ amplifier}} =$ 116.8 Ω

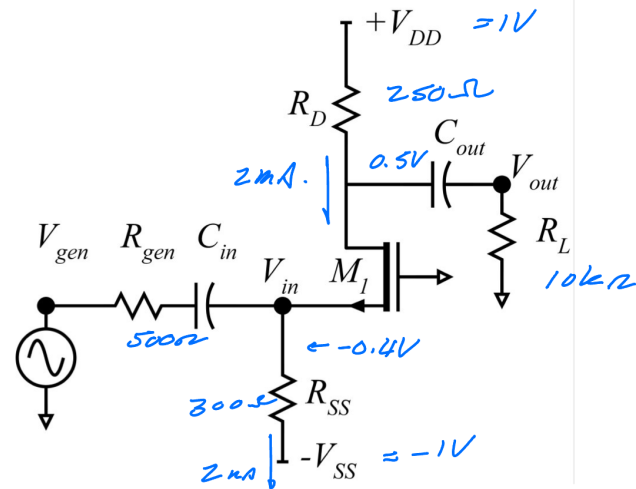
$g_m =$ 40 mS $R_{ds} =$ 2 k Ω

2 $[R_{Leg} = 10 \text{ k}\Omega // 20 \text{ k}\Omega = 243 \Omega$

3 $[R_{inT} = \frac{1}{g_m} \left(\frac{R_{ds} + R_{Leg}}{R_{ds}} \right)$
 $= 250 \Omega \frac{2 \text{ k}\Omega + 243 \Omega}{2 \text{ k}\Omega}$
 $= 280.6 \Omega$

2 $[R_{in} = R_{inT} // R_{ss}$
 $= 280.6 \Omega // 300 \Omega = 116.8 \Omega$

3 $[\frac{v_o}{v_i} = \frac{R_{Leg}}{R_{inT}} = \frac{243 \Omega}{280.6 \Omega}$
 $= 0.866$



$M1: K_{\mu} = \mu c_{gs} W_g / 2L_g = 10 \text{ mA/V}^2 \cdot (W_g / 1 \mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 2 \text{ mA/V} \cdot (W_g / 1 \mu\text{m})$

$\Delta V = v_{inj} L_g / \mu = 0.1 \text{ V}, V_{th} = 0.3 \text{ V}, 1/\lambda = 4 \text{ V}$

Part e, 2 points

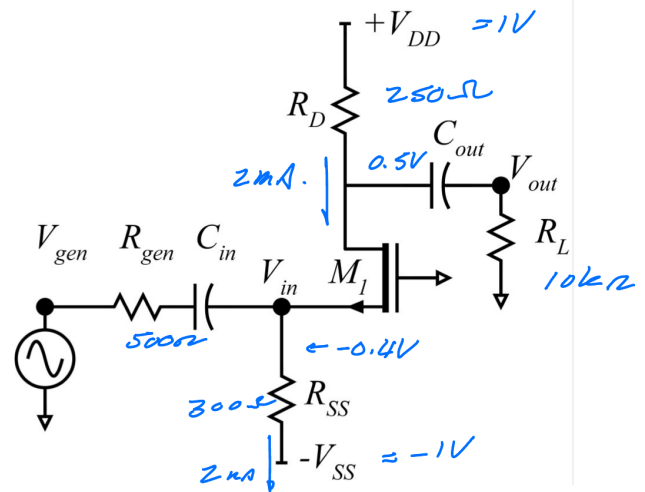
Find (V_{in}/V_{gen}) and (V_{out}/V_{gen})

$(V_{in}/V_{gen}) = \underline{0.189}$

$(V_{out}/V_{gen}) = \underline{0.164}$

$R_{in, \text{ amplifier}} = \underline{116.8 \Omega}$

$$\begin{aligned} \frac{V_{in}}{V_{gen}} &= \frac{R_{in}}{R_{in} + R_{gen}} \\ &= \frac{116.8 \Omega}{116.8 \Omega + 500 \Omega} \\ &= 0.189 \end{aligned}$$



$$M1: K_{\mu} = \mu c_{gs} W_g / 2L_g = 10 \text{mA/V}^2 \cdot (W_g / 1 \mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 2 \text{mA/V} \cdot (W_g / 1 \mu\text{m})$$

$$\Delta V = v_{inj} L_g / \mu = 0.1 \text{V}, V_{th} = 0.3 \text{V}, 1/\lambda = 4 \text{V}$$

$$\begin{aligned} V_{out}/V_{gen} &= V_{out}/V_{in} \cdot V_{in}/V_{gen} \\ &= 0.866 \cdot 0.189 = 0.164 \end{aligned}$$

Part f, 15 points

Now you must find the maximum signal swings. Find the output voltage due to the knee voltage and cutoff in M1. **Give the sign (+ or -) in your answers below.**

Cutoff of M1; Maximum ΔV_{out} resulting = 0.486V $\uparrow$ (+)

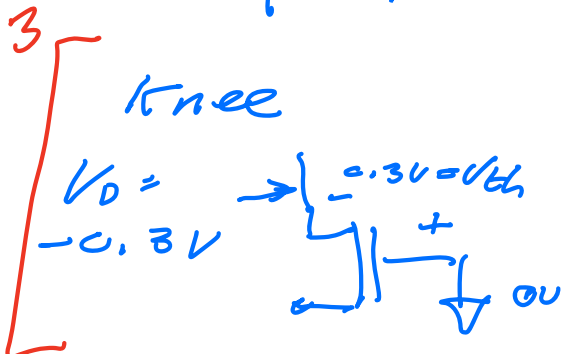
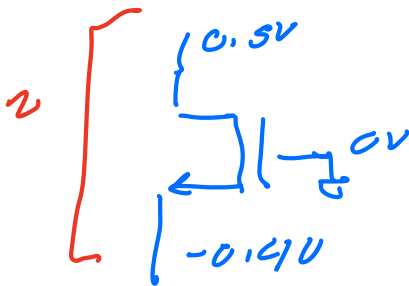
Knee voltage of M1; Maximum ΔV_{out} resulting = 0.8V $\downarrow$ (-)

Cutoff

$$1 \left[R_{eq} = 10k\Omega / 250\Omega = 243\Omega \right]$$

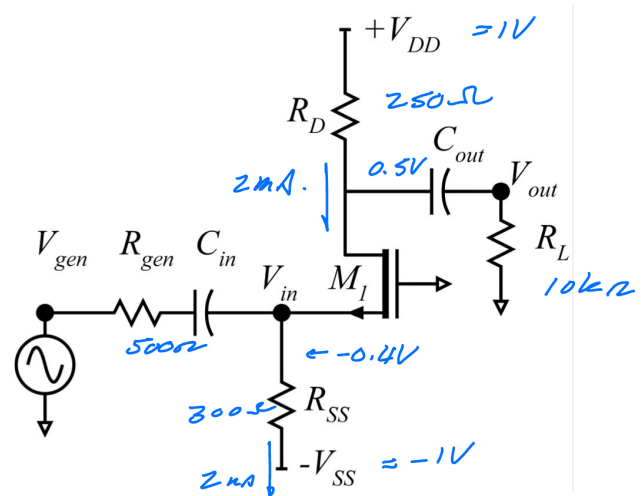
$$3 \left[\begin{aligned} \Delta V_{out} &= 2\mu A \cdot R_{eq} \\ &= 2\mu A \cdot 243\Omega \\ &= 486\text{ mV} \uparrow \end{aligned} \right]$$

BIAS



3

$$\left[\Delta V_{out} = 0.5V + 0.3V = 0.8V \downarrow \right]$$



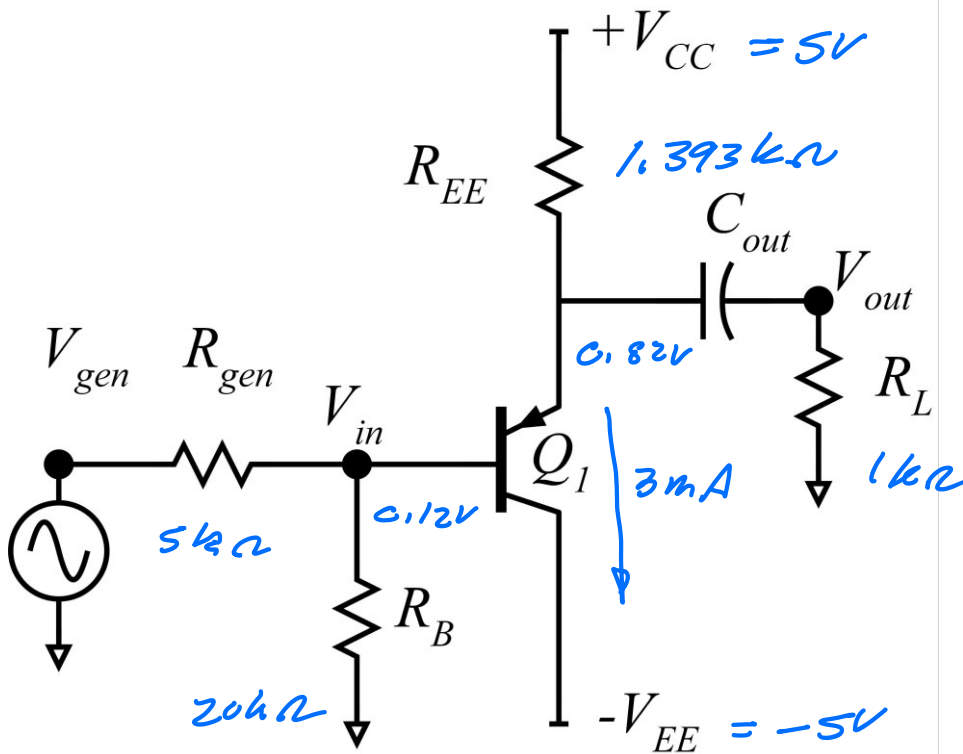
$$M1: K_{\mu} = \mu c_{gs} W_g / 2L_g = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = v_{inj} L_g / \mu = 0.1V, V_{th} = 0.3V, 1/\lambda = 4V$$

Problem 2, 33 points

You will be working on the circuit below:



Q1: $\beta = 100$, $V_A = \text{infinity V}$, $V_{ce,sat} = 0.5V$

The supplies are $+5V$ and $-5V$.

You will bias the transistor with $3mA$ collector current.

R_L is $1k\Omega$, R_{gen} is $5k\Omega$, R_B is $20k\Omega$,

C_{out} is very large (AC short-circuit at the signal frequency)

Part a, 10 points

DC bias.

Find the following:

$$R_{EE} = \underline{1.40 \text{ k}\Omega}$$

$$\text{DC voltage at the base of Q1: } V_{\text{DC_base}} = \underline{+0.12 \text{ V}}$$

Hint: there is no DC blocking capacitor in series with Vgen or Rgen.

$$2 \left[I_B = 3 \text{ mA} / 100 = 30 \mu\text{A} \right]$$

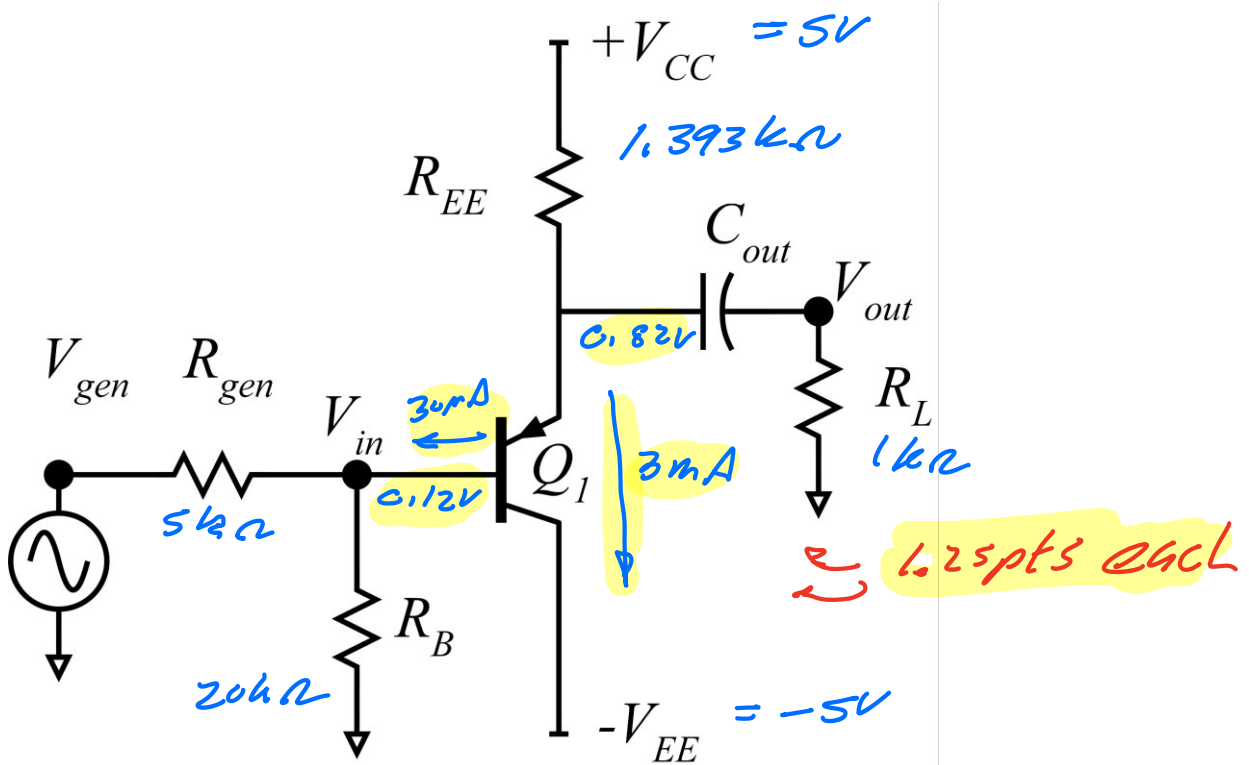
$$3 \left[V_{\text{OC_base}} = I_B \cdot R_B // R_{\text{gen}} = 30 \mu\text{A} \cdot 4 \text{ k}\Omega \right. \\ \left. = +120 \text{ mV} \right]$$

$$2 \left[V_E = V_B + V_{BE} \approx 0.12 \text{ V} + 0.7 \text{ V} = 0.82 \text{ V} \right]$$

$$3 \left[R_{EE} = \frac{5 \text{ V} - 0.82 \text{ V}}{3 \text{ mA}} = 1.40 \text{ k}\Omega \right]$$

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**, and give **all resistor values**,

*This is mostly for the TAs
to use to check students' work.*

Part c, 5 points

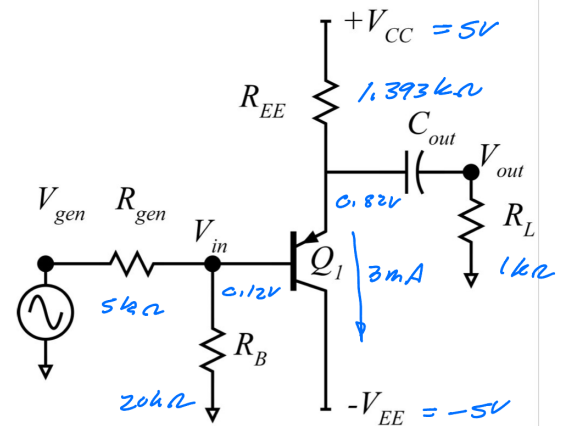
Find the small signal parameters of Q1.

gm = 115 μ S Rce = $\infty \Omega$ Rbe = 866 Ω

$$2 \left[g_m = \frac{3 \text{ mA}}{26 \text{ mV}} = 115 \mu\text{S} = \frac{1}{8.66 \Omega} \right]$$

$$2 \left[R_{ce} \approx \frac{V_D}{I_C} = \frac{\infty \text{ V}}{I_C} = \infty \text{ k}\Omega \right]$$

$$1 \left[V_{be} = \frac{\beta}{\beta + 1} V_{be} = 100 \cdot 8.66 \Omega = 866 \Omega \right]$$



Q1: $\beta = 100$, $V_{be} = 0.7 \text{ V}$, $V_{ce, \text{sat}} = 0.5 \text{ V}$

The supplies are +5V and -5V.

You will bias the transistor with 3mA collector current.

R_L is 1 k Ω , R_{gen} is 5 k Ω , R_B is 20 k Ω ,

C_{out} is very large (AC short-circuit at the signal frequency)

Part d, 10 points.

Find the small signal voltage gain (V_{out}/V_{in}) of Q1 and the amplifier small-signal input resistance.

$V_{out}/V_{in} = 0.985$

$R_{in,amp} = 14.9 k\Omega$

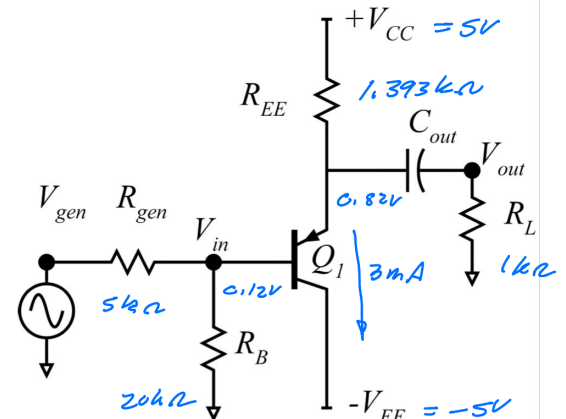
$$g_m = \frac{3 \text{ mA}}{26 \text{ mV}} = 115 \mu\text{S} = \frac{1}{8.66 \Omega}$$

$$\begin{aligned} R_{L_{eq}} &= R_{CC} \parallel R_{EE} \parallel R_L \\ &= \infty \Omega \parallel 1.39 k\Omega \parallel 1 k\Omega = \\ &= 581 \Omega \end{aligned}$$

$$\begin{aligned} A_v &= \frac{V_o}{V_{in}} = \frac{R_{L_{eq}}}{R_{L_{eq}} + 1/g_m} \\ &= \frac{581 \Omega}{581 \Omega + 8.66 \Omega} = 0.985 \end{aligned}$$

$$R_{i,T} = \sqrt{\beta} (581 \Omega + 8.66 \Omega) = 59.0 k\Omega$$

$$R_{i,A} = R_{i,T} \parallel R_B = 59 k\Omega \parallel 20 k\Omega = 14.9 k\Omega$$



Q1: $\beta = 100$, $V_A = \text{infinity V}$, $V_{ce,sat} = 0.5 \text{ V}$
 The supplies are +5V and -5 V.
 You will bias the transistor with 3mA collector current.
 R_L is 1 kΩ, R_{gen} is 5 kΩ, R_B is 20 kΩ,
 C_{out} is very large (AC short-circuit at the signal frequency)

Part e, 3 points

Find (V_{in}/V_{gen}) and (V_{out}/V_{gen})

$(V_{in}/V_{gen}) = \underline{0.750}$

$(V_{out}/V_{gen}) = \underline{0.740}$

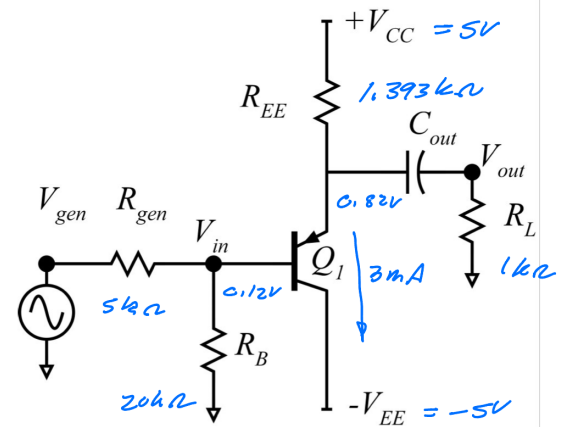
$V_{out}/V_{in} = \underline{0.985}$

$R_{in,amp} = \underline{14.9k\Omega}$

3
$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

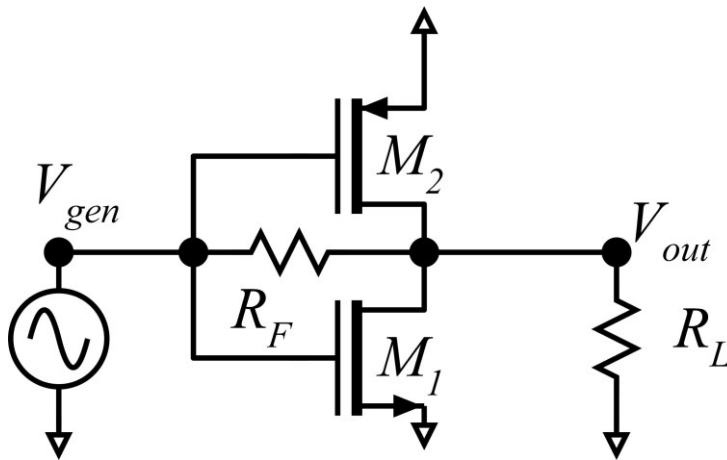
$$= \frac{14.9k\Omega}{14.9k\Omega + 5k\Omega} = 0.750$$

2
$$\frac{V_{out}}{V_{gen}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_{gen}} = 0.986 \cdot 0.750 = 0.740$$



Q1: $\beta = 100$, $V_A = \infty$ V, $V_{ce,sat} = 0.5$ V.
 The supplies are +5V and -5 V.
 You will bias the transistor with 3mA collector current.
 R_L is 1 k Ω , R_{gen} is 5 k Ω , R_B is 20 k Ω ,
 C_{out} is very large (AC short-circuit at the signal frequency)

Problem 3, 20 points
nodal analysis



You will be working on the circuit to the left.

Ignore DC bias analysis. You don't need it.

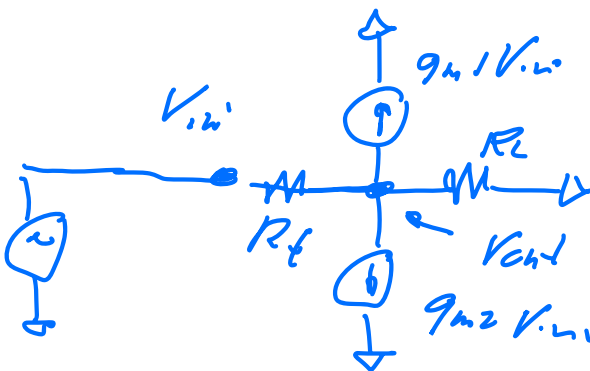
The transistors M1 and M2 have transconductance

$$g_{m1} = g_{m2} = g_m$$

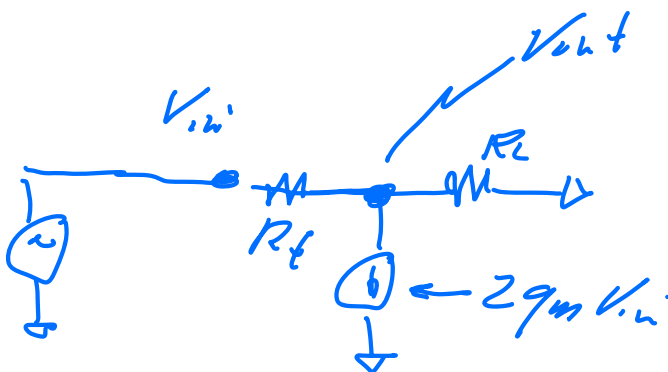
The drain-source resistances are $R_{DS1} = R_{DS2} = R_{DS} = \infty \Omega$ (so you don't need to draw them!)

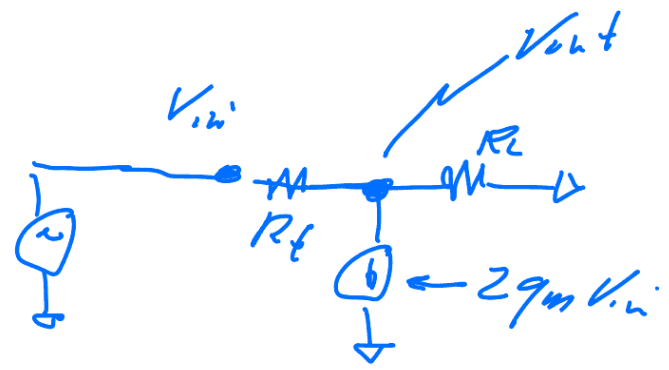
Part a, 8 points

Draw the small-signal equivalent circuit



partial credit breakdown:
Vout & Vin labelled!
correct value of gm knob?
correct topology





Part b, 8 points

Find, by nodal analysis, a small-signal expression for

$$A_v = \underline{- (2g_m - 1/R_f) / (1/R_f + 1/R_L)}$$

16

$$2 \left[\sum I @ V_{out} = 0 \right]$$

$$2 \left[\frac{V_{out} - V_{in}}{R_f} + \frac{V_{out}}{R_L} + 2g_m V_{in} = 0 \right]$$

$$2 \left[V_{out} \left(\frac{1}{R_f} + \frac{1}{R_L} \right) = -2g_m V_{in} + \frac{V_{in}}{R_f} \right]$$

$$2 \left[\frac{V_{out}}{V_{in}} = \frac{- (2g_m - 1/R_f)}{(1/R_f + 1/R_L)} \right]$$

Part c, 4 points

$g_m = 1 \text{ mS}$, $R_F = 1 \text{ k}\Omega$, $R_L = 2 \text{ k}\Omega$.

Give a numerical value for A_v .

$$A_v = \underline{\underline{-1.5}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-(2g_m - 1/R_F)}{(1/R_F + 1/R_L)}$$

$$= -(2g_m - 1/R_F)(R_F || R_L)$$

$$2 \left[= -(2 \text{ mS} - 1 \text{ mS})(1 \text{ k}\Omega || 2 \text{ k}\Omega) \right]$$

$$= -1 \text{ mS} \cdot 666 \Omega$$

$$2 \left[= -2/3 \right]$$