:

Final Exam, ECE 137A

Thursday March 20, 2025, Noon- 3 p.m.

Name: Solution

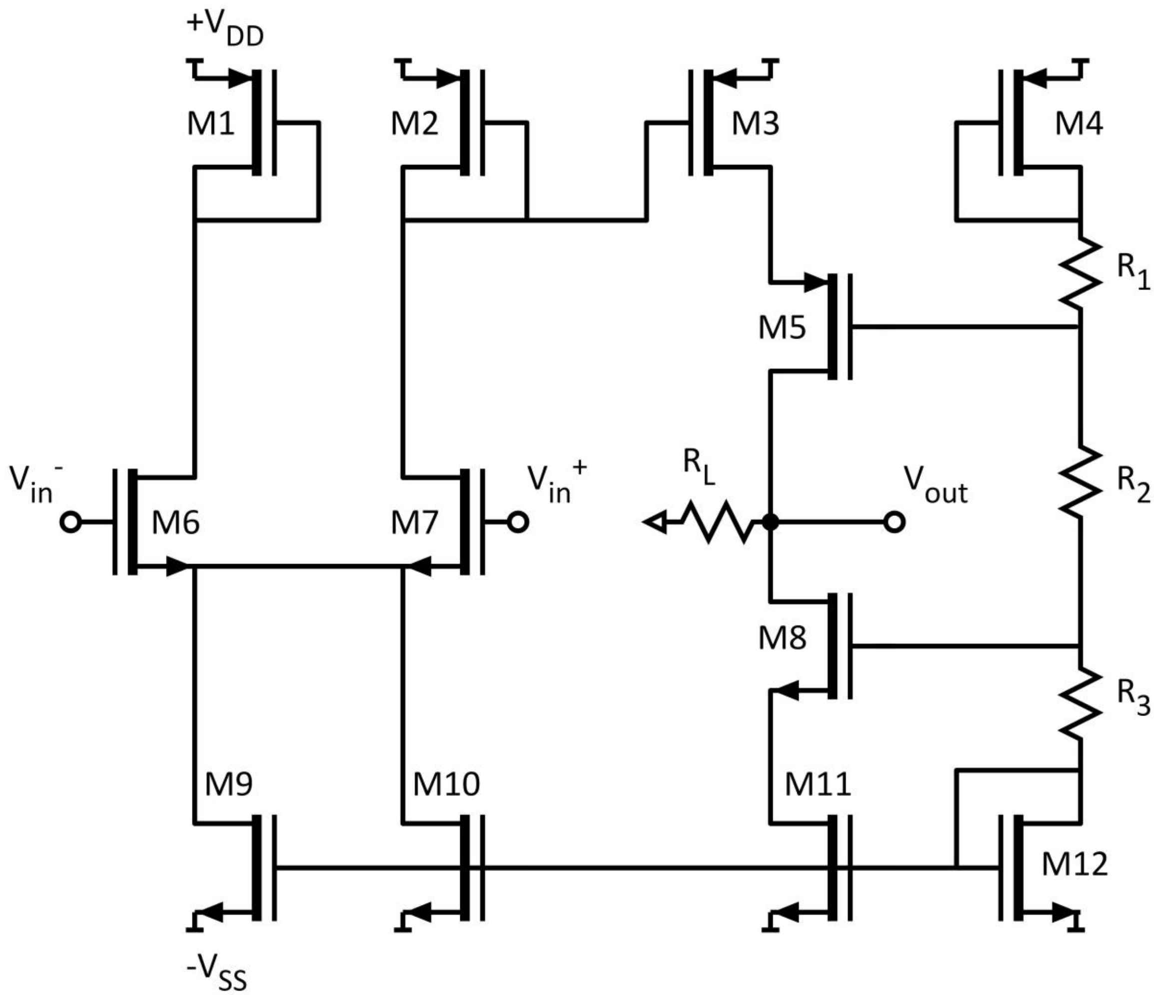
Closed Book Exam:
Class Crib-Sheet and 2 pages (4 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2b		5
1b		5	2c		15
1c		5	2d		10
1d		12	3a		5
1e		8	3b		10
1f		10	3c		5
2a		5			
total		100			

a

Problem 1, 45 points

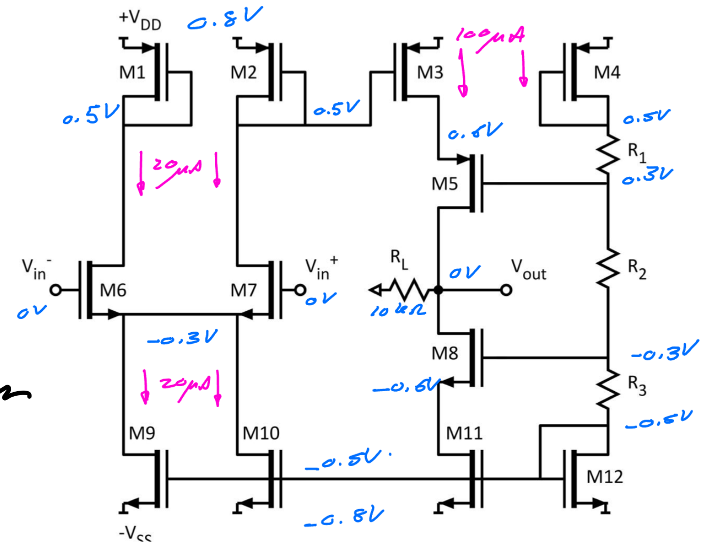
This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts.



The NMOSFETs have $K_{\mu} = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$ $K_v = 2.0\text{mA/V} \cdot (W_g / 1\mu\text{m})$, $\Delta V = 0.10\text{V}$, $V_{th} = 0.2$ V, $1/\lambda = 5$V. The PMOS have identical parameters, except, of course, V_{th} is negative. V_{DD} = +0.8 V, $-V_{SS}$ = -0.8 V

All transistors have |Vgs|=0.3V.
M1,2,6,7,9,10 are biased at I_D=20 µA.
M3,5,8,11,4,12 are biased at I_D =100 µA.
The IR voltage drops across R_1 and R_3 are each 200 mV.
R_L=10 kΩ.

a



Part a, 5 points

DC bias

Find the following:

Gate width of M2 = 0.189 μm

Gate width of M3 = 0.96 μm

Value of R_1 = 2 kΩ

Value of R_2 = 6 kΩ

The NMOSFETs have $K_\mu = 10\text{mA/V}^2 \cdot (W_g/1\mu\text{m})$ $K_v = 2.0\text{mA/V} \cdot (W_g/1\mu\text{m})$, $\Delta V = 0.10\text{V}$, $V_{th} = 0.2$ V, $1/\lambda = 5$V. The PMOS have identical parameters, except, of course, V_{th} is negative. V_{DD} = +0.8 V, $-V_{SS}$ = -0.8 V

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2

M2 $V_{gs} = V_{gs} = 0.3V$

$I_D = K_\mu (V_{gs} - V_{t2})^2 (1 + \lambda V_{DS})$

$20\mu A = K_\mu (0.1V)^2 (1 + 0.3V/5V)$

$20\mu A = K_\mu (0.1V)^2 (1.06)$

$K_\mu = 20\mu A (100)(V^{-2}) / 1.06 = 1.89 \text{ mA/V}^2$

$1.89 \text{ mA/V}^2 = 10 \text{ mA/V}^2 \cdot W_g / 1\mu m \rightarrow W_g = 0.189 \mu m$

a

2

M3 $V_{GS} = 0.2V$, $V_{DS} = 0.2V$

$I_O = K_n (V_{GS} - V_{TL})^2 (1 + \lambda V_{DS})$

$100\,\mu A = K_n (0.1V)^2 (1 + 0.2V/5V)$

$100\,\mu A = K_n (0.1V)^2 (1.04)$

$K_n = 100\,\mu A (100)(V^2)^{-1}/1.04 = 9.6\,mA/V^2$

$K_n = 9.6\,mA/V^2 = 10\,mA/V^2 \cdot W/1\mu m \rightarrow W = 0.96\,\mu m$

$\frac{1}{2}$

$R_1 = \frac{0.2V}{100\,\mu A} = \frac{2V}{1\,mA} = 2\,k\Omega$

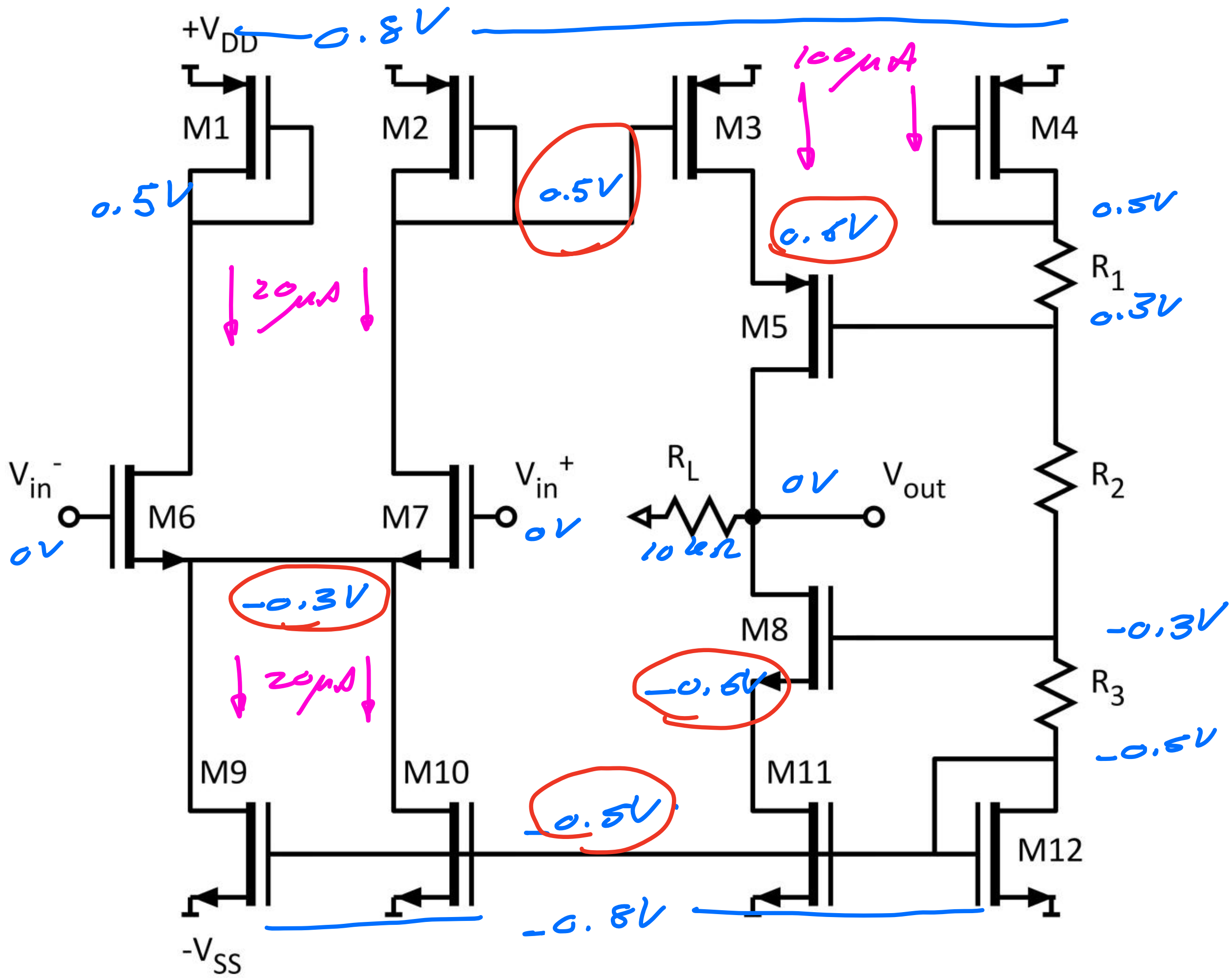
$\frac{1}{2}$

$R_2 = \frac{0.6V}{100\,\mu A} = 6\,k\Omega$

a

Part b, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes**.

1 pt each for the circled #s

a

1.25 pts for each group of answers.

Part c, 5 points

Find the following

MOSFET M6 $g_m =$ 0.4mS $R_{DS} =$ 250 kΩ

MOSFET M7 $g_m =$ 0.4mS $R_{DS} =$ 250 kΩ

MOSFET M2 $g_m =$ 0.4mS $R_{DS} =$ 250 kΩ

MOSFET M3 $g_m =$ 2mS $R_{DS} =$ 50 kΩ

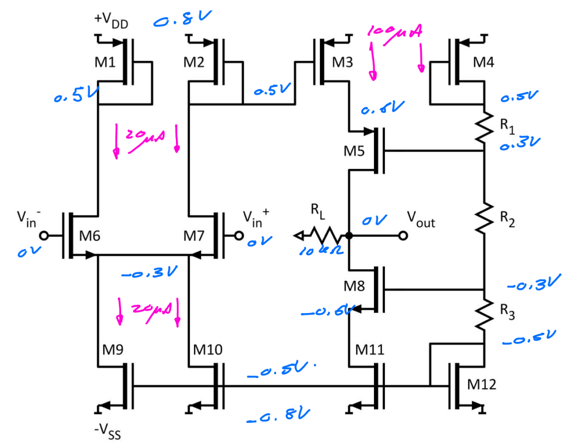
MOSFET M5 $g_m =$ 2mS $R_{DS} =$ 50 kΩ

MOSFET M8 $g_m =$ 2mS $R_{DS} =$ 50 kΩ

MOSFET M11 $g_m =$ 2mS $R_{DS} =$ 50 kΩ

all FETS μ limited.

$$g_m = 2I_D/(V_{gs}-V_{th})$$
$$= 2I_D/0.1V$$



On the circuit diagram above, label the DC voltages at **ALL nodes**.

$$R_{DS} = 1/\lambda I_D$$
$$= 5V/I_D$$

The NMOSFETs have $K_\mu = 10\text{mA/V}^2 \cdot (W_g/1\mu\text{m})$ $K_v = 2.0\text{mA/V} \cdot (W_g/1\mu\text{m})$, $\Delta V = 0.10\text{V}$, $V_{th} = 0.2$ V, $1/\lambda = 5$V. The PMOS have identical parameters, except, of course, V_{th} is negative. V_{DD} = +0.8 V, $-V_{SS}$ = -0.8 V

All transistors have |Vgs|=0.3V.

M1,2,6,7,9,10 are biased at I_D=20 μA.

M3,5,8,11,4,12 are biased at I_D =100 μA.

The IR voltage drops across R_1 and R_3 are each 200 mV.

R =10 kΩ

a

Part d, 12 points.

Find the following

	Voltage Gain	Input impedance
Transistor combination M3,5,8,11	-20	$\infty\ \Omega$

Note: You can analyze M3 and M5 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

1st way of working

step by step:

1 $R_{out11} = R_{DS11} = 50\,k\Omega$

2 $R_{out8} = R_{DS8}(1 + g_{m8} R_{DS11}) = 5.05\,M\Omega$

1 $R_{Leq5} = R_L \| R_{out8} = 10\,k\Omega \| 5.05\,M\Omega = 9.98\,k\Omega \cong 10\,k\Omega$

2 $R_{in5} = 1/g_{m5}\left(\frac{R_{Leq5} + R_{DS5}}{R_{DS5}}\right) = 500\Omega\left[\frac{10k\Omega + 50k\Omega}{50k\Omega}\right]$

$= 600\Omega$

2 $A_{v5} = R_{Leq5}/R_{in5} = 10\,k\Omega/600\Omega = 16.7$

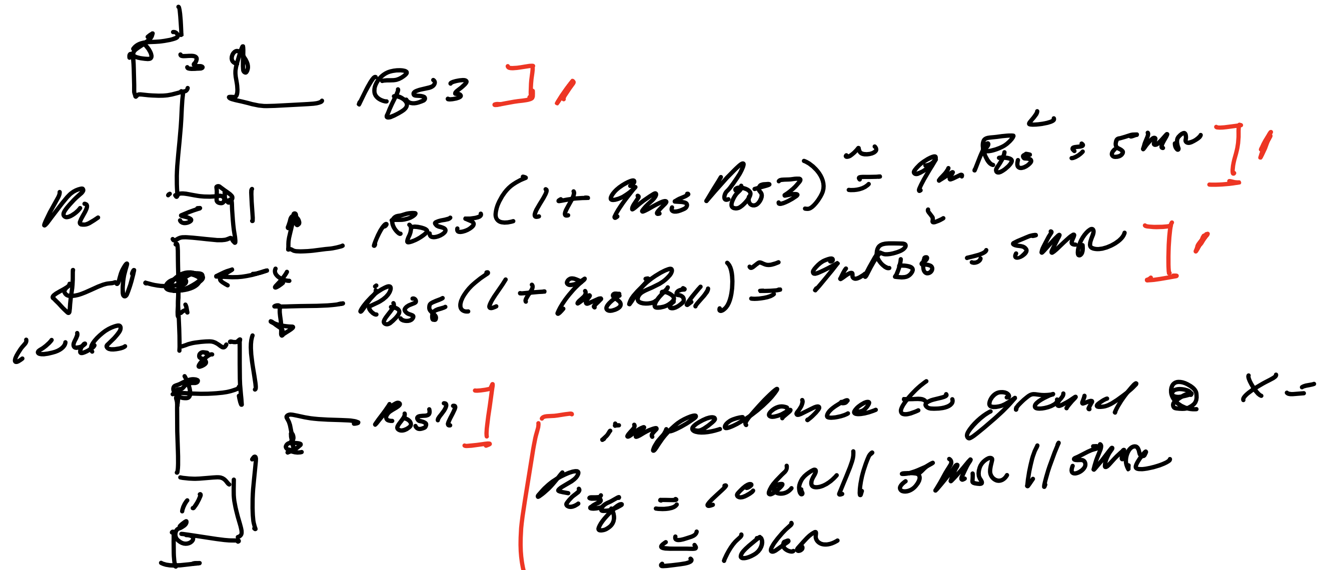
1 $R_{Leq3} = R_{DS3} \| R_{in5} = 50\,k\Omega \| 600\Omega = 593\,\Omega$

2 $A_{v3} = -g_{m3} R_{Leq3} = 2\,ms \cdot 590\,\Omega = -1.19$

1 $A_{v3} A_{v5} = -1.19 \cdot 16.7 = -19.8$

2nd way of working

norton - thevenin:

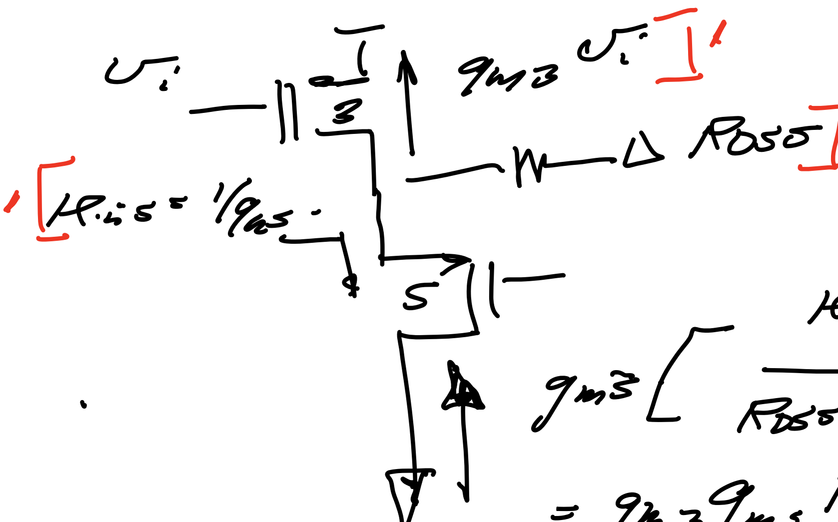


impedance to ground @ X =

$R_{eq} = 10k\Omega \,||\, 5M\Omega \,||\, 5M\Omega$

$\simeq 10k\Omega$

short-circuit current



$g_{m3}\left[\dfrac{R_{DS5}}{R_{DS5} + 1/g_{m5}}\right]v_i$

$= g_{m3} g_{m5} R_{DS5} / (1 + g_{m5} R_{DS5}) v_i$

$\simeq g_{m3} v_i$ 2

1 open-circuit voltage = short-circuit current × thevenin resistance.

$= -g_{m3} \cdot R_{eq}$

$= -2mS \cdot 10k\Omega = -20$

a

1

Part e, 8 points.

Find the following

	Voltage Gain	Input impedance
Transistor combination M1,2,6,7,9,10	-0.49	$\infty\ \Omega$
Overall op-amp $V_{out}/(V_{in}^{+}-V_{in}^{-})$	9.8	$\infty\ \Omega$

Hint: although M2 is a diode-connected FET, the small-signal resistance of this 2-terminal element, although mostly determined by the FET transconductance, also has a slightly contribution from the FET R_{DS}.

3

$$R_{Leq7} = R_{DS7} \,//\, R_{DS2} \,//\, 1/g_{m2} =$$
$$= 250k\Omega \,//\, 250k\Omega \,//\, 2.5k\Omega$$
$$= 2.45\,k\Omega$$

-1 pt if R_{DS2} was omitted.

2

$$A_{VG2} = -\frac{g_m}{2} \cdot R_{Leq}$$
$$= -0.49$$

2

overall

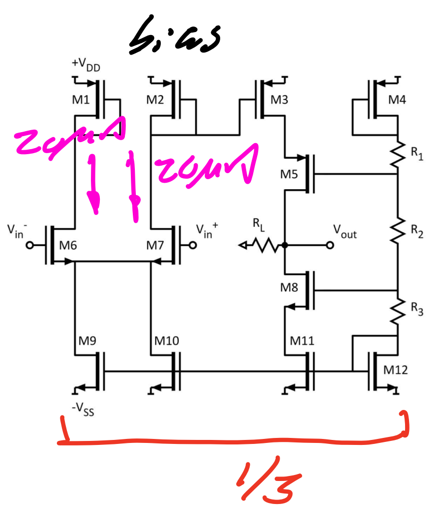
$$A_{V\,overall} = (-0.49) \cdot (-20)$$
$$= \underline{9.8}$$

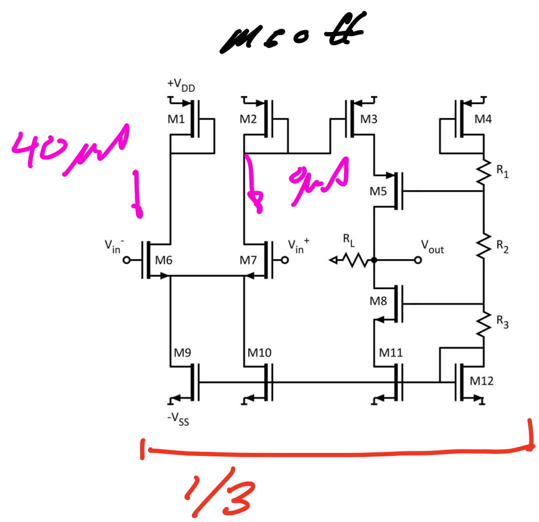
a

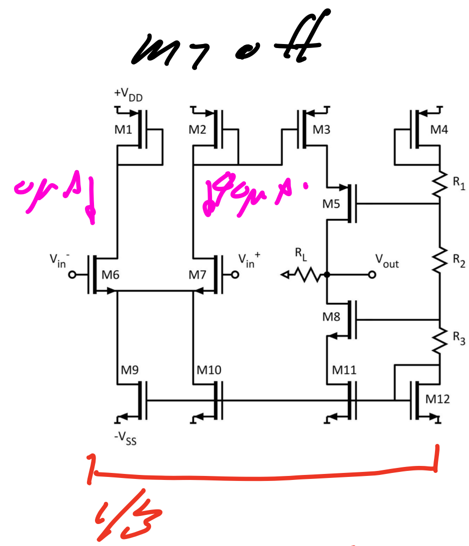
Part f, 10 points

Maximum output voltage swings (***show all your work***).

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***knee voltage***
Transistor M6	+0.98V	Do not calculate
Transistor M7	−0.98V	Do not calculate
Transistor M3	−1.0V	Do not calculate
Transistor M5	Do not calculate	+0.5V
Transistor M8	Do not calculate	−0.5V







→ maximum change in I_{D7} is $\pm 20\mu A$.

1 maximum ΔV_{out} of M7 stage is

$$\Delta V_{max,out,M7} = \pm 20\mu A \cdot R_{eq}$$

$$= \pm 20\mu A \cdot 2.45\,k\Omega = \pm 49\,mV.$$

1 corresponding $\Delta V_{out} = \pm 49\,mV \cdot A_{v3,5}$

$$= \pm 49\,mV \cdot 20$$

$$= \pm 0.98V$$

a

There are 2 ways of calculating M3 cutoff

Method 1

Solution using HS from page 8

M3 Cutoff: $\Delta I_{D3} = 100\mu A$. 1

$\Delta V_{out3} = 100\mu A \cdot R_{Leq3}$
$= 100\mu A \cdot 600\Omega$
$= -60mV$ 1

$\Delta V_{out} = \Delta V_{out3} \cdot A_{v5} =$
$= (-60mV)(+16.7)$
$= -1.0V$ 1

OR

Method 2

Alternate solution: using HS from page 9,

$\Delta I_{D3} = 100\mu A$ 1

$\Delta I_{D5} = \Delta I_{D3}\left[\frac{g_m R_{DS5}}{g_{m3} R_{DS5} + 1}\right] = 10\,0\mu A = 80\mu A$ 1

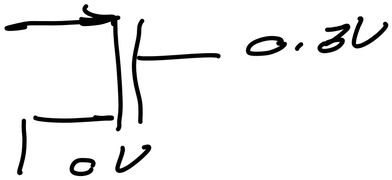
$\Delta V_{out} = \Delta I_{D5} \cdot 10\ k\Omega$
$= -1.0V$ 1

a

Knee voltage M5

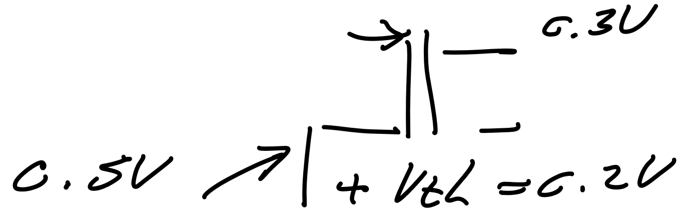
1

Bias



1

Knee



1

$\Delta V_{out} = +0.5V \uparrow$

Knee voltage of M5

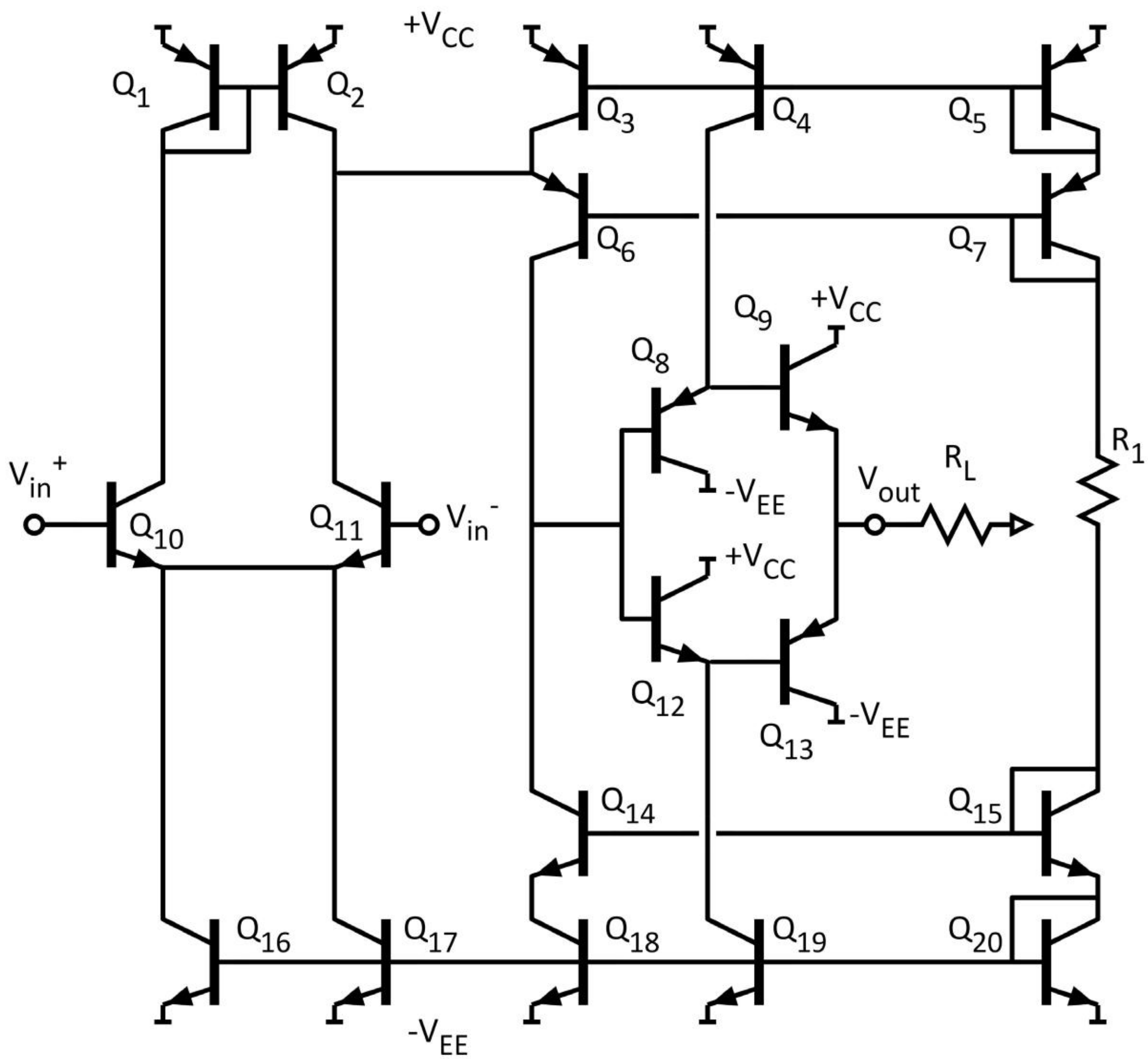
→ same calculation, opposite sign

$\underline{-0.5V \downarrow}$

a

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage V_{out} is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



All the transistors have the same (matched) I_S, have $\beta = \infty$, and $V_A = 100$ Volts . $V_{CE(sat)} = 0.5\text{V}$. V_{be} is *approximately* 0.7 V, but use the *exact relationship* $V_{be} = (kT/q)\ln(I_E / I_S)$ when appropriate. The supplies are +5 Volts and -5 Volts. All transistors have the same I_S, and all are biased at 0.1 mA collector current.
R_L=1 kOhm.

a

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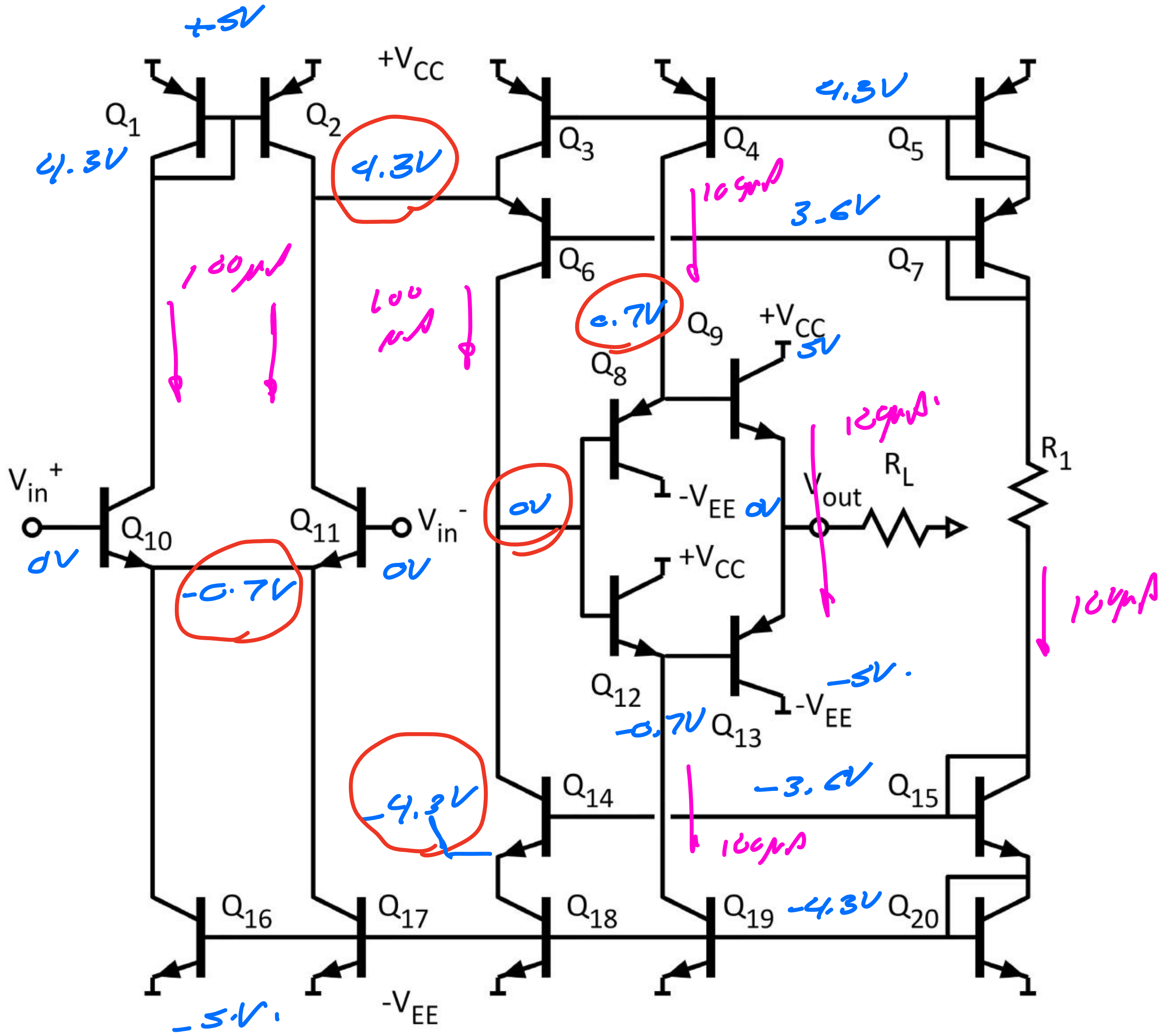
R_L=1 kOhm.

Part a, 5 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes.**

1 pt each for circled values.

a

Part b, 5 points.

Find the following

	Voltage Gain	Input impedance
Transistor combination Q8,9,12,13	0.885 or 0.794	$\infty\,\Omega$

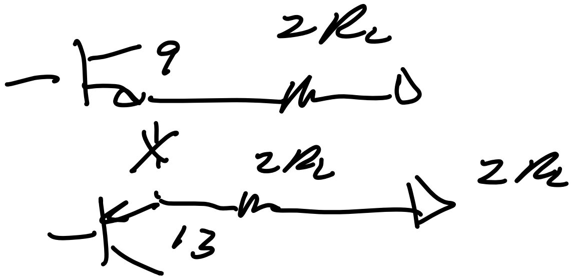
$R_{ce} = V_A/I_C = 100V/0.1mA = 1M\Omega$ $\quad 1/g_m = 260\Omega$

Q9/12 is push/pull: strong + output: Q9 on
strong − output: Q13 on
small signal: both on.

I will work solution for Both on so but all 3 solutions are acceptable.

one way to work

If Q9 & 13 are both on then we can analyze like so



$R_{Leq9} = 2R_L // R_{eq} = 2k\Omega // 1M\Omega \cong 2k\Omega$

$A_{v9} = 2k\Omega/(2k\Omega + 260\Omega) = 0.885$

$R_{in9} = \beta(1/g_m + R_{Leq9}) = \infty\,\Omega.$

$R_{Leq8} = R_{in9} // R_{eq} // R_{ce8} = \infty\Omega // 1M\Omega // 1M\Omega = 500k\Omega$

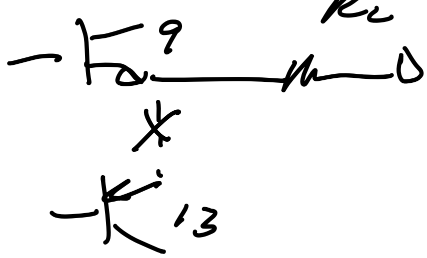
$A_{v8} = R_{Leq8}/(R_{Leq8} + 1/g_{m8}) = 0.999 \cong 1$

$R_{in8} = \infty\Omega$

another way to work.

If Q9 is on but Q13 is off

then we can analyze like so



$R_{Leq9} = R_L \| R_{eq} = 1k\Omega \| 1M\Omega \cong 1k\Omega$

$A_{v9} = 1k\Omega / (1k\Omega + 260\Omega) = 0.794.$

$R_{in9} = \beta(1/g_m + R_{Leq9}) = \infty\,\Omega.$

$R_{Leq8} = R_{in9} \| R_{eq} \| R_{ce8} = \infty\Omega \| 1M\Omega \| 1M\Omega$
$= 500k\Omega$

$A_{v8} = R_{Leq8} / (R_{Leq8} + 1/g_{m8}) = 0.999 \cong 1$

$R_{in8} = \infty\Omega$

You can also work Q13 on, Q9 off
.... same order...

* depends on answer to part 6.

Part c, 15 points.

Find the following

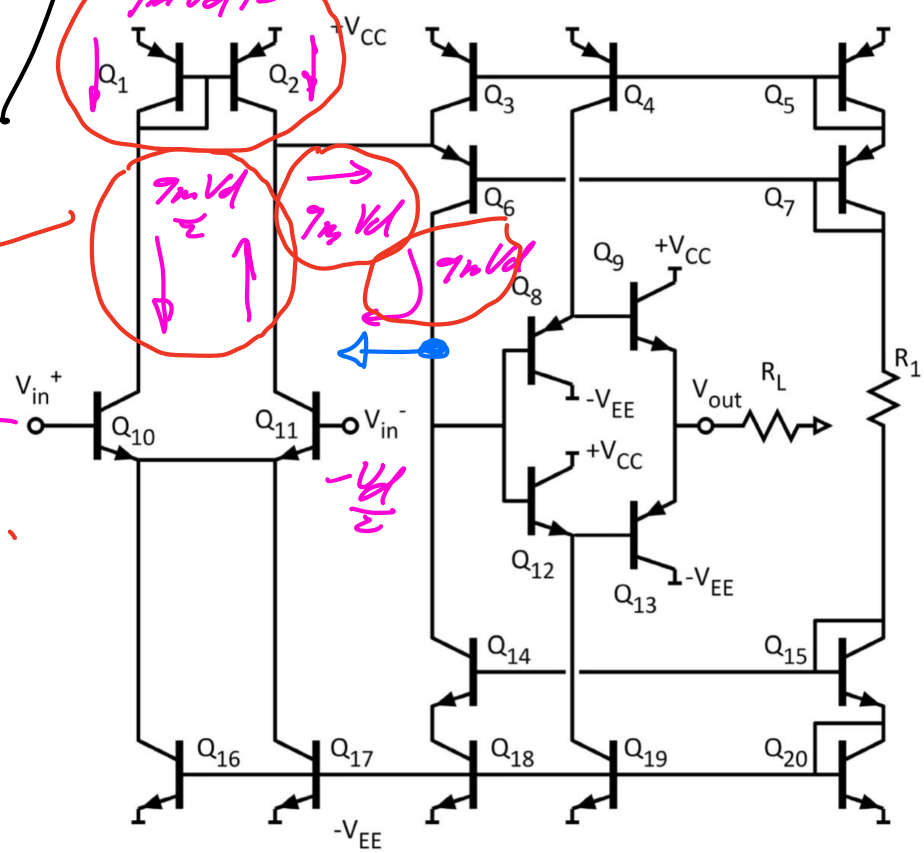
	Voltage Gain	Input impedance
Transistor combination Q1,2,10,11,6	$3.7 \cdot 10^6$	$\infty\ \Omega$
Overall op-amp $V_{out} / (V_{in}^{+} - V_{in}^{-})$	$3.3 \cdot 10^6$ or * $2.9 \cdot 10^6$	$\infty\ \Omega$

Note: You can analyze (Q1,2,10,11) and Q6 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

using Norton/Thevenin methods:

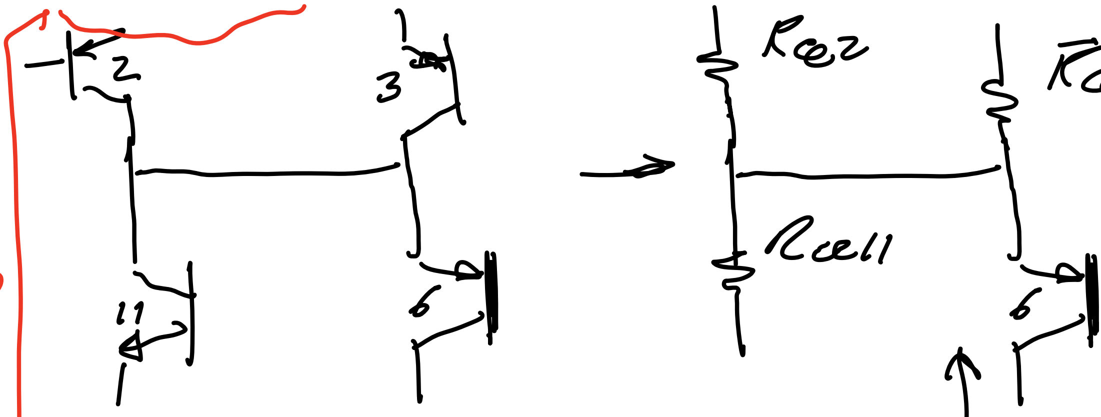
short circuit current:

$= g_m V_d$.



1 pt each.

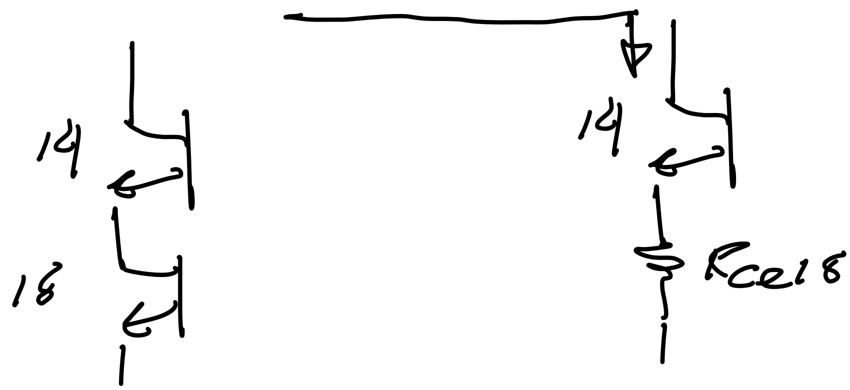
a



3

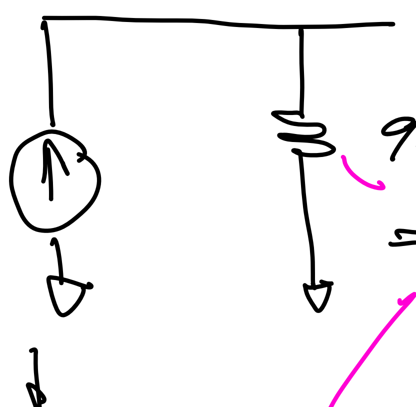
$R_{ce}(1 + g_m R_{CE}/3) \simeq g_m R_{CE}^2/3$

$R_{ce}(1 + g_m R_{ce}) \simeq g_m R_{ce}^2$



3

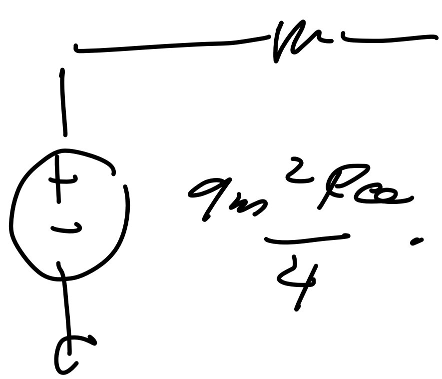
$g_m v_d$



$g_m R_{ce}^2 \,//\, \frac{1}{3} g_m R_{ce}^2$

$= g_m R_{ce}/4.$

3



$\frac{g_m^2 R_{ce}}{4} \cdot v_d = 3.7 \cdot 10^6 \cdot v_d.$

2

One stage of gain.

a

step-by-step method:

3 $R_{out14} = R_{ce14} \cdot (1 + g_{m14} R_{ce18})$

$\simeq g_m R_{ce}^2$

1 $R_{Leq6} = R_{out14}$

3 $R_{in6} = 1/g_{m6} \left(\frac{R_{Leq6} + R_{ce6}}{R_{ce6}} \right)$

$\simeq \frac{1}{g_{m6}} \frac{g_{m6} R_{ce}^2}{R_{ce6}} = R_{ce}$

2 $A_{v6} = R_{Leq6} / R_{in6} = g_m R_{ce}^2 / R_{ce} = g_m R_{ce}$

4 $R_{Leq11} = R_{in6} \| R_{ce3} \| R_{ce2} \| R_{ce11} = R_{ce}/4$

1 $A_{v11} = g_m R_{Leq11} = g_m R_{ce}/4$

1 overall gain $= g_m R_{ce} \cdot g_m R_{ce}/4$

$= (g_m R_{ce})^2/4$

$= 3.70 \cdot 10^6$

another way to work.

a

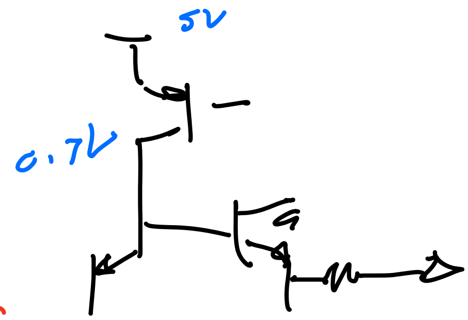
Part d, 10 points

Maximum output voltage swings (***show all your work***).

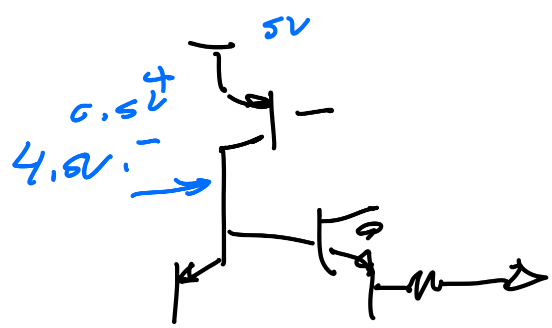
	magnitude and sign of maximum output signal swing due to ***saturation voltage*** (saturation)
Transistor Q4	3.36V
Transistor Q9	4.5V

Note that transistors Q13 and Q19, Because of the circuit symmetry, Will give similar answers but with a change in sign.

Q4 bias:



Q4 saturation



2

2

1 $\Delta V_{b9} = 4.5V - 0.7V = +3.8V$

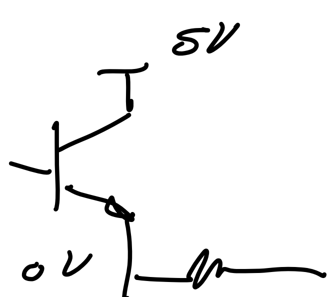
1 $\Delta V_{out} = 3.8V \cdot A_{v9}$

$= 3.8V \cdot 0.885$

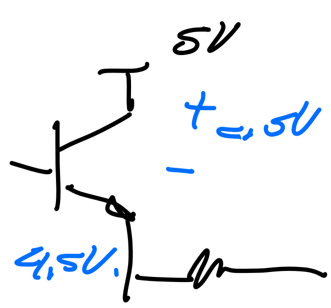
$= 3.36V$

ok to use $A_{v9} =$ either 0.885 or 0.794

Q9 bias



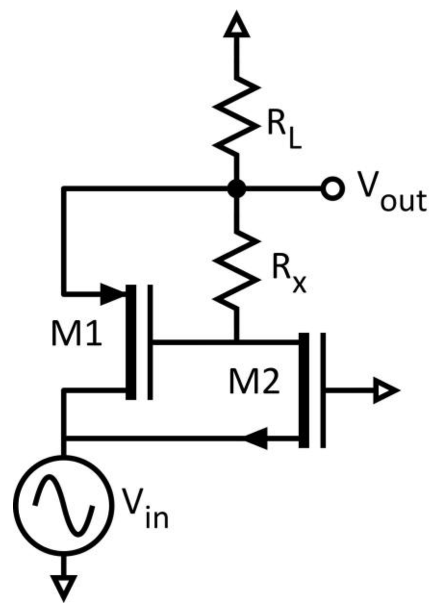
Q9 saturation



2

2

Problem 3, 20 points



You will be working on the circuit to the left

Ignore DC bias analysis. You don't need it.

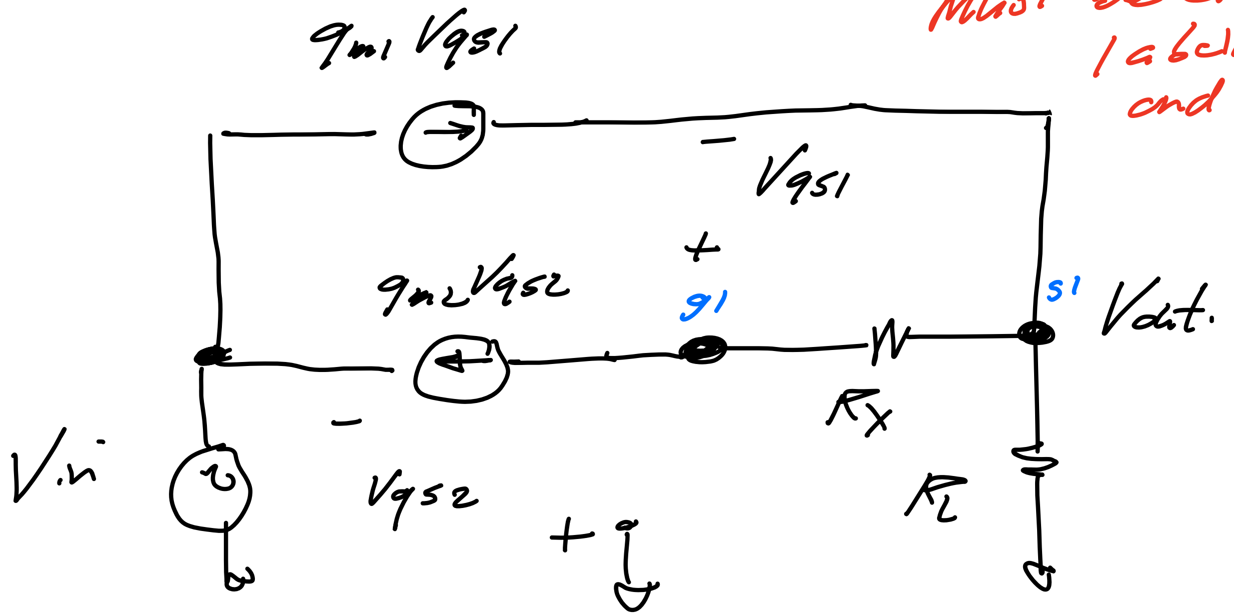
M_1 and M_2 have transconductances g_{m1} and g_{m2}.

Both transistors have **infinite** R_{DS},
...so you don't need to include these elements in the circuit diagram !

Part a, 5 points

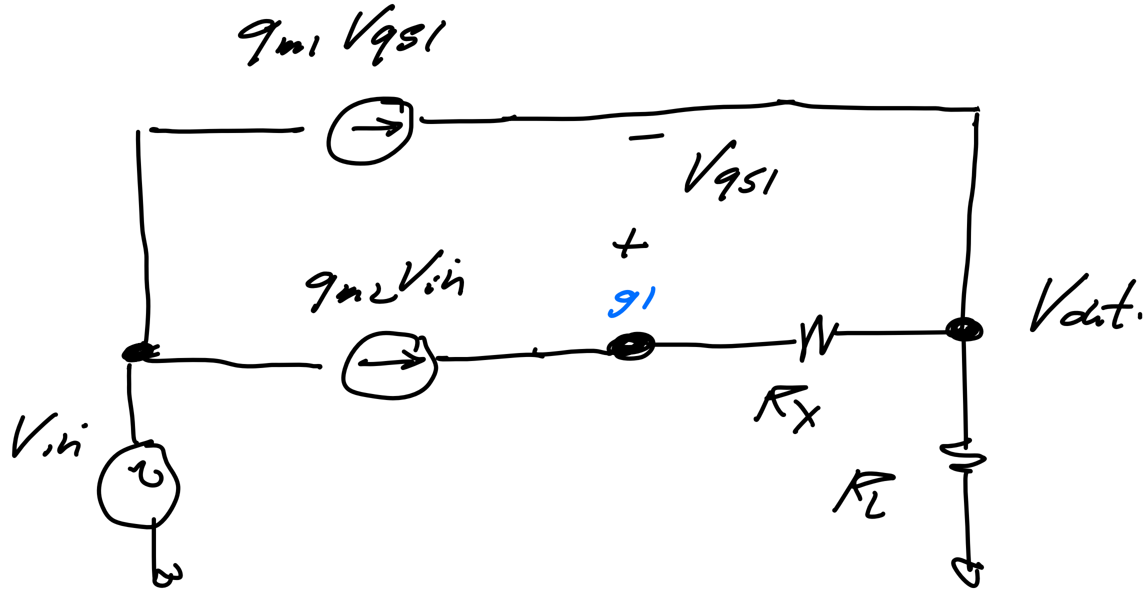
Draw a small-signal equivalent circuit of the circuit.

Voltages controlling gm generators must be clearly labeled and located.



a

alternative answer.

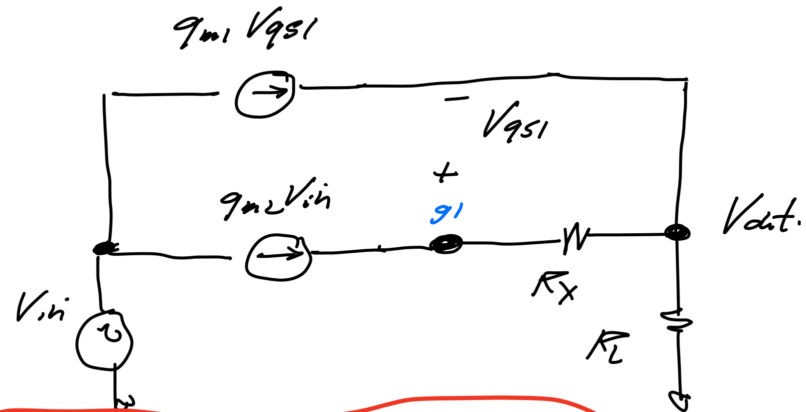


a

Part b, 10 points

Derive an algebraic expression for V_{out} / V_{in} in terms of g_{m1}, g_{m2}, R_L, and R_X

$V_{out} / V_{in} = g_{m2} R_L (1 + g_{m1} R_X)$



$\Sigma I = 0$ @ g_1

A) $-g_{m2} V_{in} + (V_{g1} - V_{out}) / R_X = 0$ 4

$\Sigma I = 0$ @ V_{out}:

$-g_{m1} V_{gs1} - (V_{g1} - V_{out}) / R_X + V_{out} / R_L = 0$ 4

B) $-g_{m1} (V_{g1} - V_{out}) - (V_{g1} - V_{out}) / R_X + V_{out} / R_L = 0$

From A: $(V_{g1} - V_{out}) = (g_{m2} R_X V_{in})$

one solution

a

so from B:

$$-g_{m1}(g_{m2}R_X V_{in}) - (g_{m2}R_X V_{in})/R_X + V_{out}/R_L = 0$$

$$-g_{m1}(g_{m2}R_X V_{in}) - g_{m2}V_{in} + V_{out}/R_L = 0$$

$$V_{out}/R_L = g_{m2}(1+g_{m1}R_X)V_{in}$$

$$V_{out}/V_{in} = g_{m2}R_L(1+g_{m1}R_X)$$

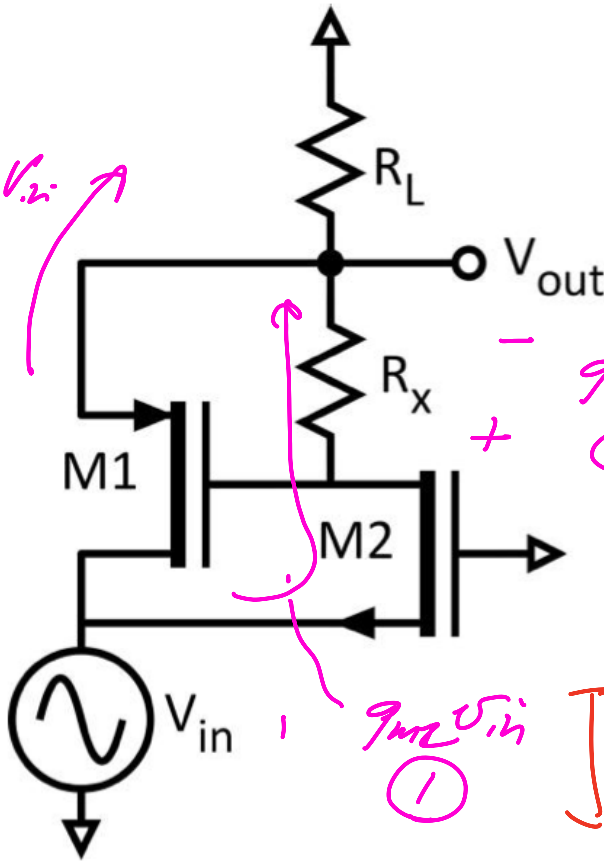
2

a

OK

2pt $g_{m2} \cdot g_{m1} \cdot R_x \cdot V_{in}$

3



$g_{m2} R_x V_{in}$

2

2pt

$g_{m2} v_{in}$

1

2pt

2pt

4 $V_{out} = R_L (g_{m2} V_{in} + g_{m1} g_{m2} R_x V_{in})$

2pt

$\rightarrow V_{out}/V_{in} = g_{m2} R_L (1 + g_{m1} R_x)$

a

Part c, 5 points

Set g_{m1}=1 mS, g_{m2}=2 mS, R_L=3 kΩ, and R_X = 5 kΩ.
Find the numerical value of V_{out}/V_{in}

V_{out}/V_{in} = 36

$A_v = g_{m2} R_L (1 + g_{m1} R_X)$

$= 2mS \cdot 3k\Omega (1 + 1mS \cdot 5k\Omega)$

$= 6[1 + 5]$

$= 36$

5pt

a