# ECE137B Final Exam

There are 5 problems on this exam and you have 3 hours
There are pages 1-19 in the exam: please make sure all are there.

Do not open this exam until told to do so
Show all work:
Credit will not be given for correct answers if supporting work is not shown.
Class Crib sheets and 2 pages (front and back→ 4 surfaces) of your own notes permitted.
Don't panic.

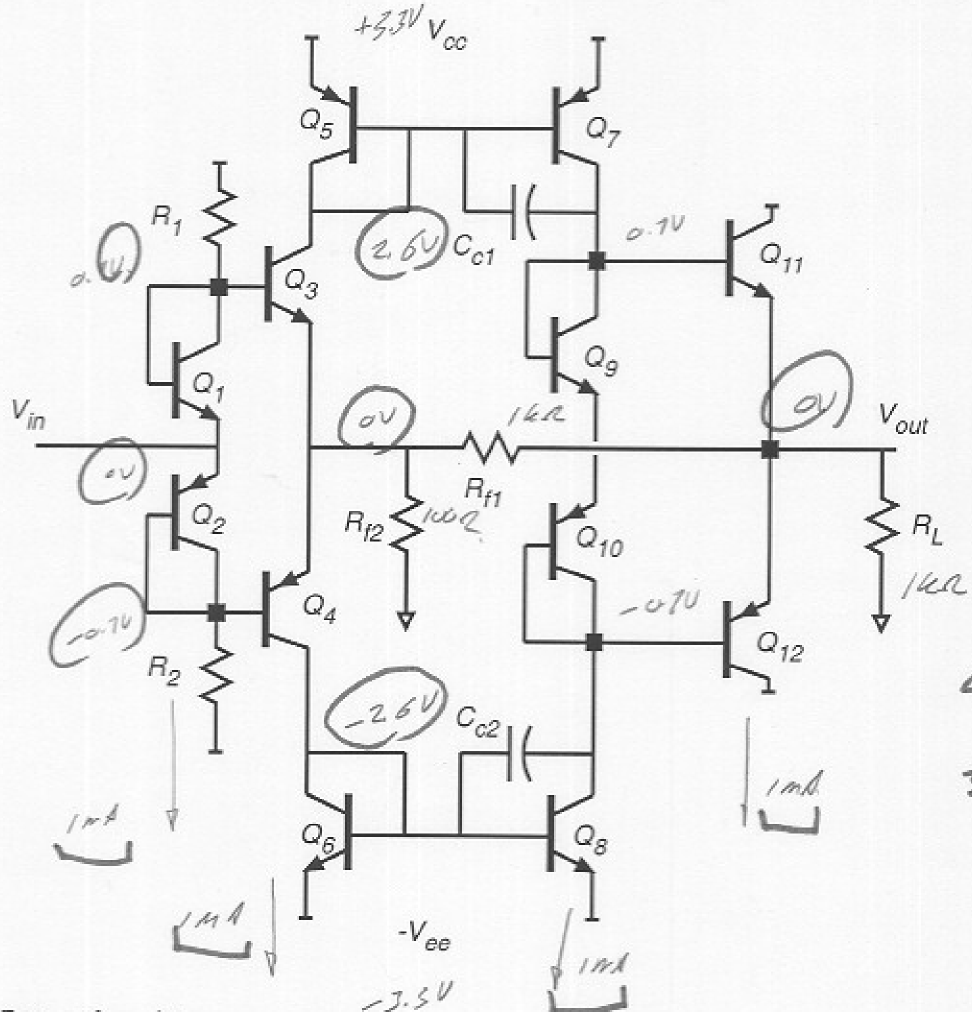
| Time function | LaPlace Transform |
|---|---|
| $\delta(t)$ | 1 |
| U(t) | 1/s |
| $e^{-\alpha t}\cdot U(t)$ | $\frac{1}{s+\alpha}$ or $\frac{1/\alpha}{1+s/\alpha}$ |
| $e^{-\alpha t}\cos(\omega_d t)\cdot U(t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$ |
| $e^{-\alpha t}\sin(\omega_d t)\cdot U(t)$ | $\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ |

Name: solution "A"

| Problem | points | possible | Problem | points | possible |
|---|---|---|---|---|---|
| 1a | | 3 | 2 | | 10 |
| 1b | | 2 | 3a | | 7 |
| 1c | | 8 | 3b | | 13 |
| 1d | | 5 | 4a | | 10 |
| 1e | | 12 | 4b | | 5 |
| 1f | | 5 | 5a | | 5 |
| 1g | | 5 | 5b | | 5 |
| | | | 5c | | 5 |

**Problem 1, 40 points**

*method of first-order and second-order time constants, some feedback theory*




Currents are important

3/4 pts each

Part a, 3 points

*DC analysis*

Find all transistor DC emitter currents, find all node voltages. Make these on the circucit diagrm.

| | |
|---|---|
| $\beta$ : infinity, for all transistors. | |
| Va=200 V for Q7 and Q8 | Va=infinity for all other transistors |
| $C_{cb}$ = zero, for all transistrors. | $C_{c1} = C_{c2}$ =10 fF |
| $\tau_f$ = 2 ps and $C_{je}$ = 10 fF for Q7, Q8, Q11, Q12.<br>$\tau_f$ = 0 ps and $C_{je}$ = 0 fF for all other transistors | |
| All transistors have identical $I_s$ cthe DC component of Vin is zero volts | |
| The supplies are +/- 3.3 Volts. | Rf1=1 kOhm, Rf2=100 Ohm, RL=1 kOhm |
| R1=R2: select their value so that the DC emitter currents in Q1 and Q2 are 1 mA | |

$R_1 = R_2 = \frac{3.3V - 0.7V}{1mA} = 2.6k\Omega$

Part b, 2 points
*small signal parameters*

Find the following:

| | $r_e = 1/g_m$ | $R_{be}$ | $R_{ce}$ | $C_{be}$ | $C_{cb}$ | $f_\tau$ |
|---|---|---|---|---|---|---|
| Q1 | 26 Ω | ∞ | ∞ | 0 | 0 | ∞ |
| Q3 | | | ∞ | 0 | | ∞ |
| Q5 | | | ∞ | 0 | | ∞ |
| Q7 | | | 200kΩ | 87 fF | | 70 GHz |
| Q9 | | | ∞ | 0 | | ∞ |
| Q11 | | | | 87 fF | | 70 GHz. |

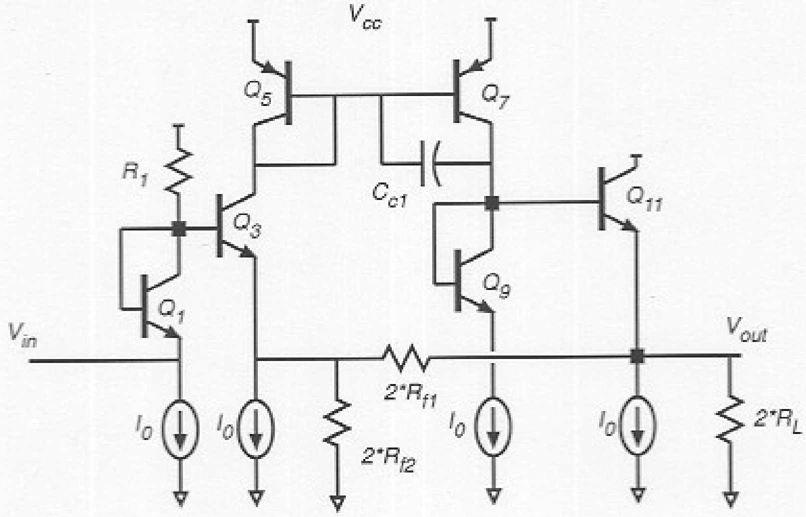
1 $C_{be} = C_{je} + g_m \tau_f = 10\,fF + \frac{2\,ps}{26\,\Omega} = 87\,fF$

1 $f_T = \frac{g_m}{2\pi (C_{be} + C_{cb})} = 70\,GHz$

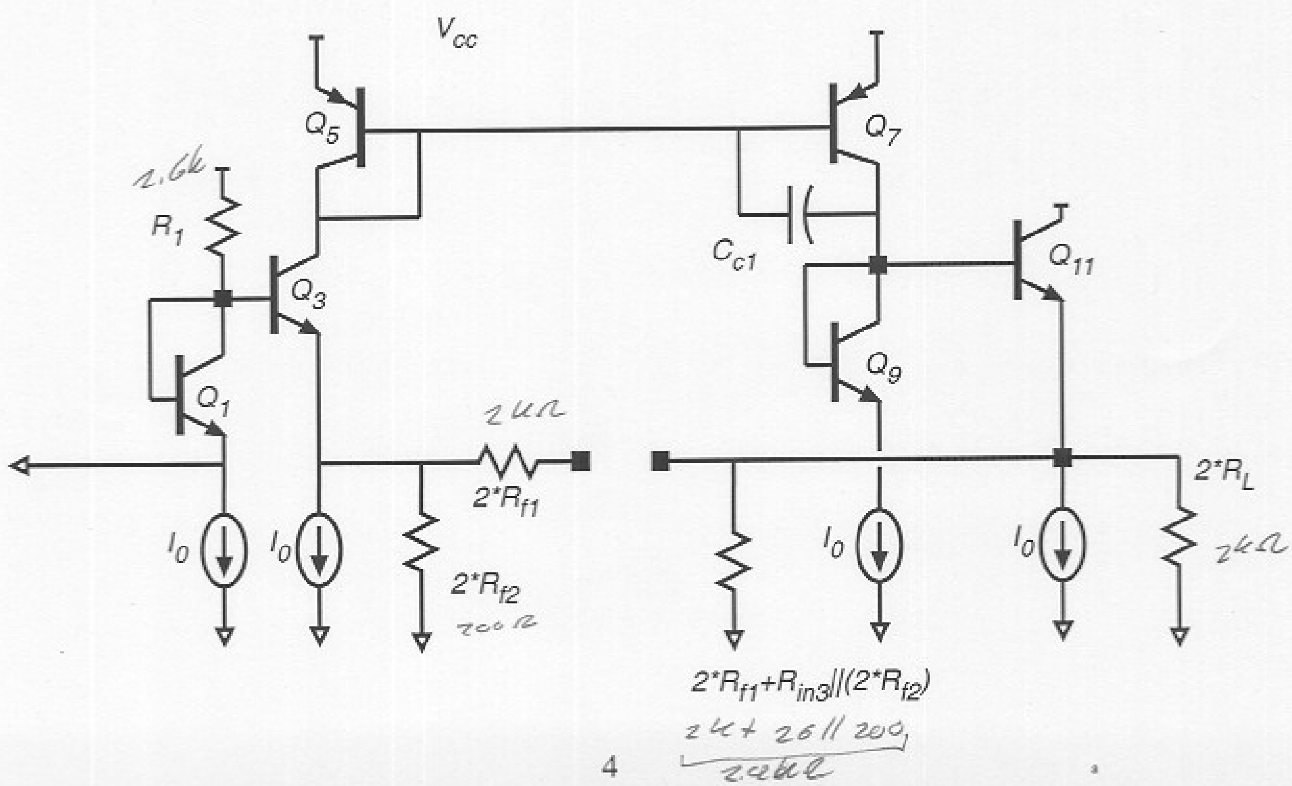
Part c. 8 points
*mid-band analysis*

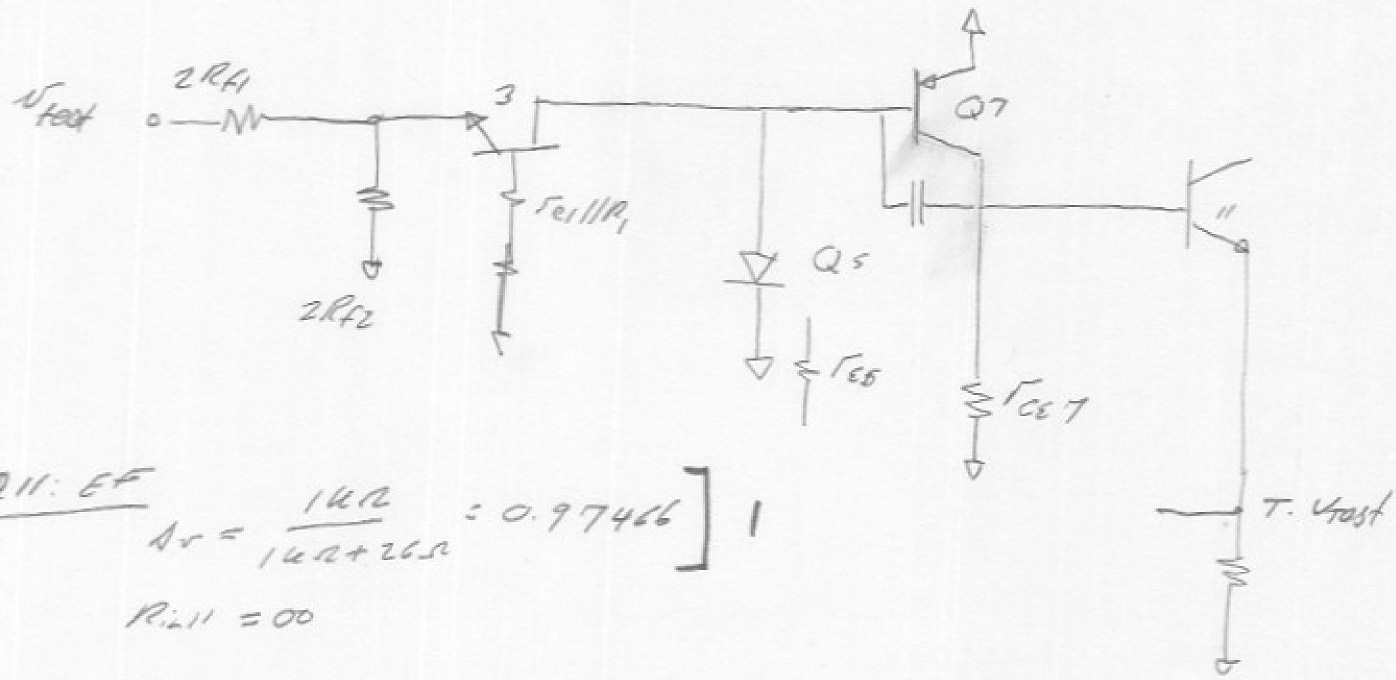
Find the low-frequency loop transmission:
T(f=0 Hz) = 85

To do this, you must make 2 changes to the circuit. First, the circuit is symmetric, and can be thus simplified, where $I_o$ is the value of the DC current in R1.




Second, you need to cut the feedback loop, thus, to find the loop transmission







**Q11: EF**

$$A_v = \frac{1k\Omega}{1k\Omega + 26\Omega} = 0.97466 \quad \Big] \; 1$$

$$R_{i11} = \infty$$

**Q7: CE**

$$A_v = r_{CE7}/r_{E7} = \frac{200k\Omega}{26\Omega} = 7,700 \quad \Big] \; 2$$

$$\underbrace{2R_L \| (2R_{f1} + R_{i3}\|2R_{f2})}_{1k\Omega}$$

**Q3 CB**

$$2 \; \Big[ \; A_v = \frac{R_{load}}{r_E} = \frac{r_{E5}}{r_{E3}} = 1 \quad , \; r_{i3} = 26\Omega$$

**feedback network**

$$\text{term} = \frac{r_{in3} \| 2R_{f2}}{r_{in3}\|2R_{f2} + 2R_{f1}} = \frac{26\Omega \| 200\Omega}{26\Omega\|200\Omega + 2k\Omega}$$

$$= \frac{23\Omega}{23\Omega + 2k\Omega} = \underline{0.0114} \quad \Big] \; 2$$

Loop transmission = product of these terms

$$= 0.0114 \cdot 1 \cdot 7,700 \cdot 0.97466 = \underline{85.36} \quad \Big] \; 1$$

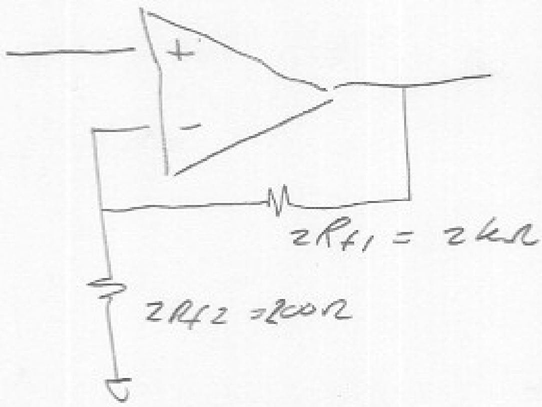
Part d. 5 points
*feedback theory*

At low frequencies, what is the closed-loop gain $V_{out}/V_{in}$ ?

$V_{out}/V_{in} =$ 10.87

2 $\left[ A_{CL} = A_{\infty} \dfrac{T}{1+T} \right.$

√ [ T = 85.4 at low frequencies.




If $T \to \infty$, $V^+ = V^- = V_i$

but $V^- = \dfrac{2R_{f2}}{2R_{f2}+2R_{f1}} \cdot V^-$

so:

$\left. A_{\infty} = \dfrac{R_{f1}+R_{f2}}{R_{f1}} = 11 \right]$ 2

$\left. A_{CL} = 11 \cdot \dfrac{85.4}{86.4} = 10.87 \right]$ 1

Part e, 12 points

*motc*

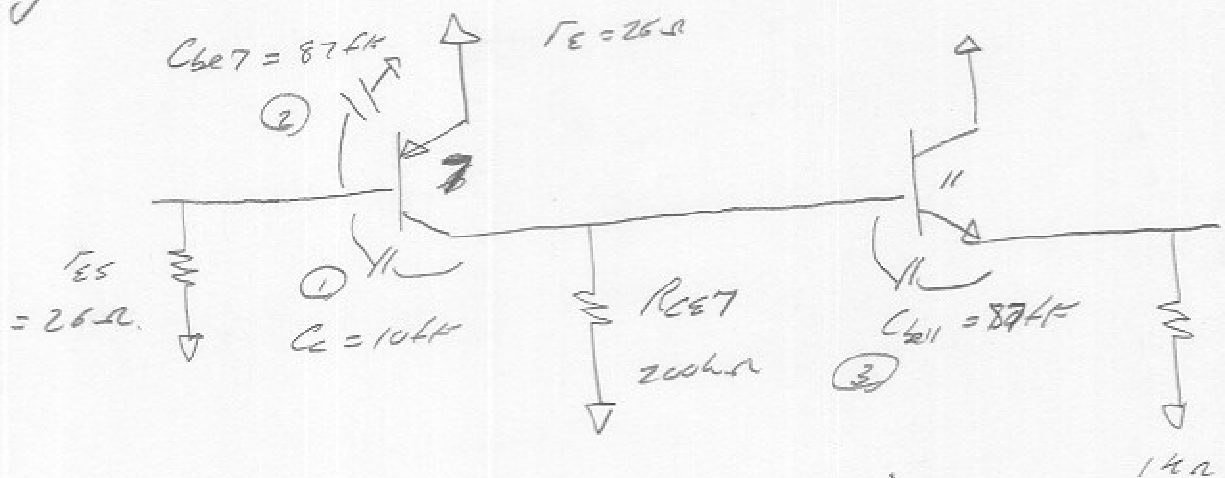
Using MOTC, you will find the frequency, in Hz (not rad/sec), of the ***two*** major poles in the transfer function.

| capacitor 1: | capacitor 2: | capacitor 3: |
|---|---|---|
| $R_{11}^0 =$ 400kΩ | $R_{22}^0 =$ 26Ω | $R_{33}^0 =$ 5093Ω |
| $R_{22}^1 =$ 13Ω | $R_{33}^1 =$ 26.6Ω (26 is ok) | $R_{33}^2 =$ 5093Ω |
| $f_{p1} =$ 35.8 MHz | $f_{p2} =$ 47.1 GHz | |

capacitor 1 is the compensation capacitance
capacitor 2 is the capacitance between base & Emitter of transistor Q 7
capacitor 3 is the capacitance between base & Emitter. of transistor Q 11

$A_v = 0.97466$

only Q7 & Q11 have $\frac{1}{T}$



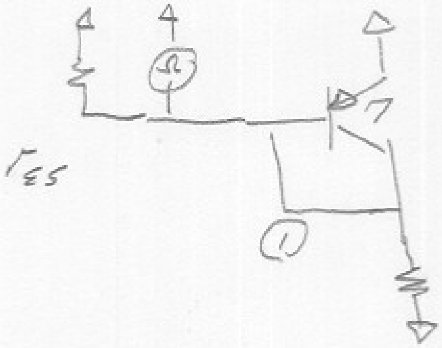

1 $\left[ r_{11}^0 = 26\Omega(1 - A_{v7}) + R_{CE7} = 26\Omega(1 + 7700) + 200k\Omega \right.$
$= 400k\Omega$

1 $[ r_{22}^0 = 26\Omega$

1 $[ r_{33}^0 = 200k\Omega(1 - A_{v11}) + r_{E11} \| 1k\Omega = 5068\Omega + 25.3\Omega = 5093\Omega$

1 $[ a_1 = r_{11}^0 C_1 + r_{22}^0 C_2 + r_{33}^0 C_3 = 400k\Omega(10fF) + 26\Omega(87fF) + 5.1k\Omega(87fF)$
$= 4ns + 2.26ps + 0.444ns = \underline{4.44ns}$

2 $R_{22}^1$:

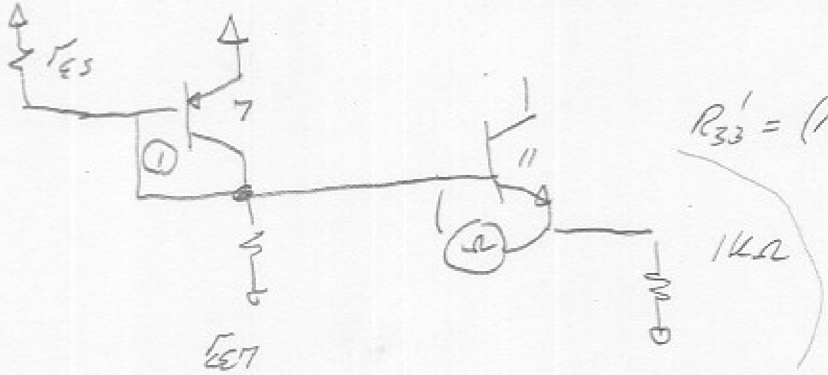


$R_{22}^1 = r_{E5} \| r_{E7} \| r_{CE7}$

$= 26\Omega \| 26\Omega \| 200k\Omega$

$\cong 13\Omega$

2 $R_{33}^1$:



$R_{33}^1 = (r_{E5} \| r_{E7} \| r_{CE7})(1 - A_{v11})$

$+ r_{E11} \| 1k\Omega$

$= 26\Omega(1 - 0.97466)$

$+ 26\Omega$

$= 26.6\Omega.$

2 $R_{33}^2$: by inspection, $R_{33}^2 = R_{33}^0 = 5093\Omega$

1

$a_2 = R_{11}^0 C_1 C_2 R_{22}^1 + R_{11}^0 C_1 C_3 R_{33}^1 + R_{22}^0 C_2 C_3 R_{33}^2$

$= 400k\Omega(10fF)(87fF)(13\Omega) + 400k\Omega(10fF)(87fF)\,26.6\Omega$

$+ 26\Omega(87fF)(87fF)\,5093\Omega$

$= 4.52 \cdot 10^{-21} + 9.25(10^{-21}) + 1.00(10^{-21})\ sec^2$

$= 1.5 \cdot 10^{-20}\ sec^2 = (0.122\ ns)^2$

1 use SPA:

$f_{p1} \overset{?}{=} \dfrac{0.159}{a_1} = 35.8\ MHz$

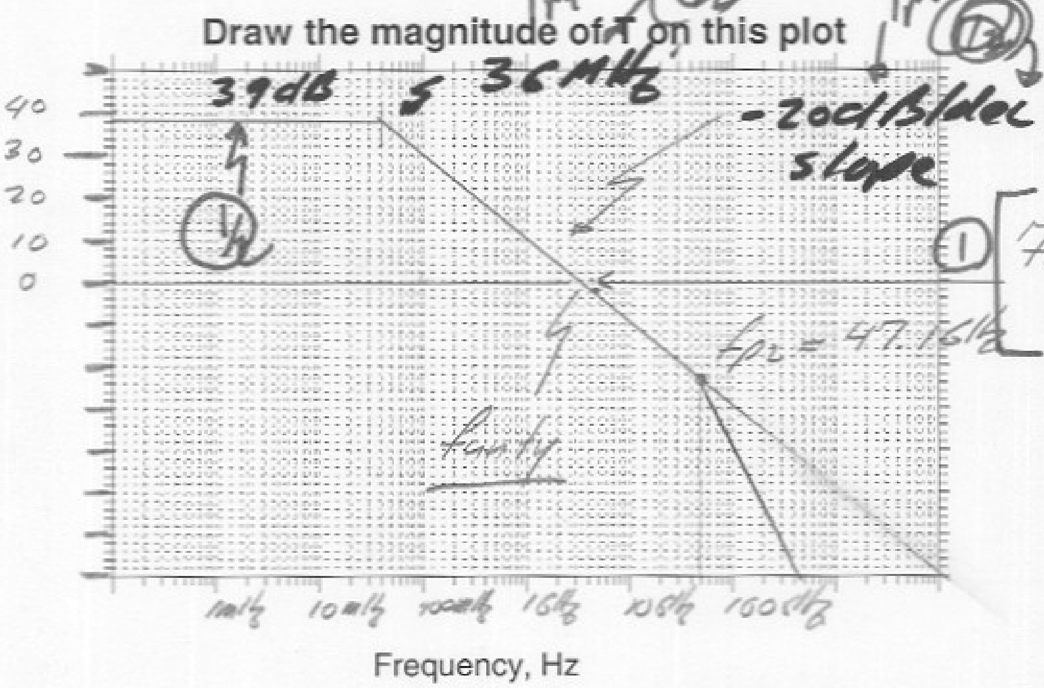
$f_{p2} \overset{?}{=} \dfrac{0.159\, a_1}{a_2} = \underline{47.1\ GHz}$

SPA checks!

Part f. 5 points

Make accurate asymptotic plots of T. Find the phase margin and the loop bandwidth.

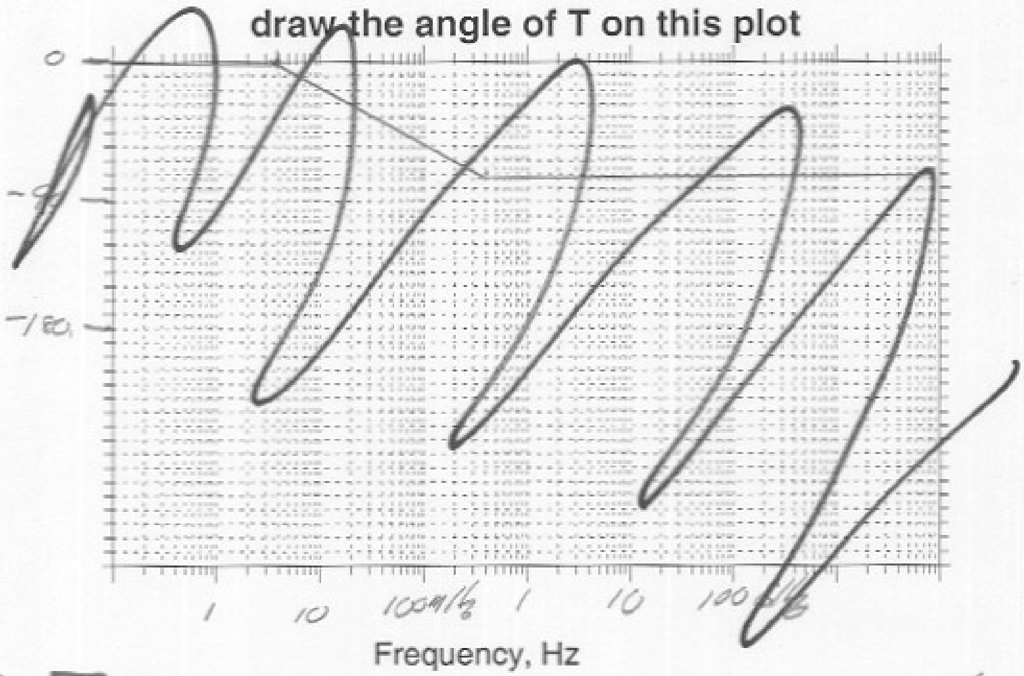
Phase margin = 86.4° Loop bandwidth = 3.04 GHz




T: 85 @ DC

85 => 38.6 dB

$f_{unity} = 85 \cdot 35.8\text{MHz} = 3.04\text{GHz}$




½ [ $\angle T$ @ 3.04 GHz $= -90°$ (pole 1) $- \arctan(3.04\text{GHz} / 47.1\text{GHz})$
$= -90 - 3.64°$

1 [ Phase margin $= 180 - (90 + 3.64) = 86.4°$

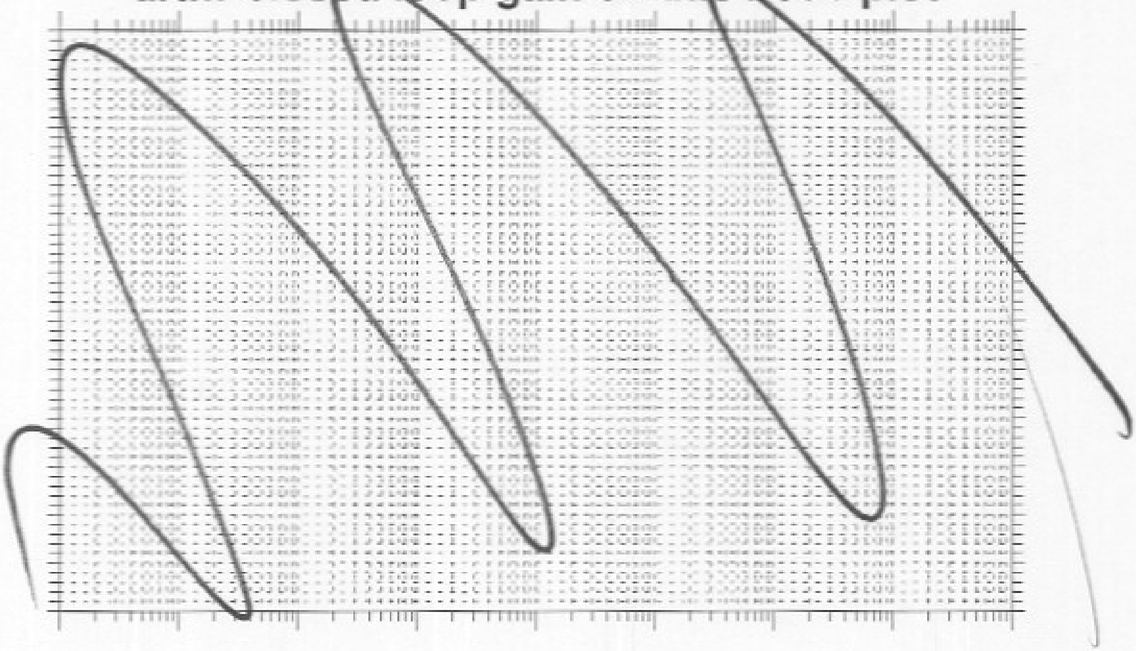
Part g. 5 points

What is the gain and bandwidth of the closed-loop amplifier ?

2 3

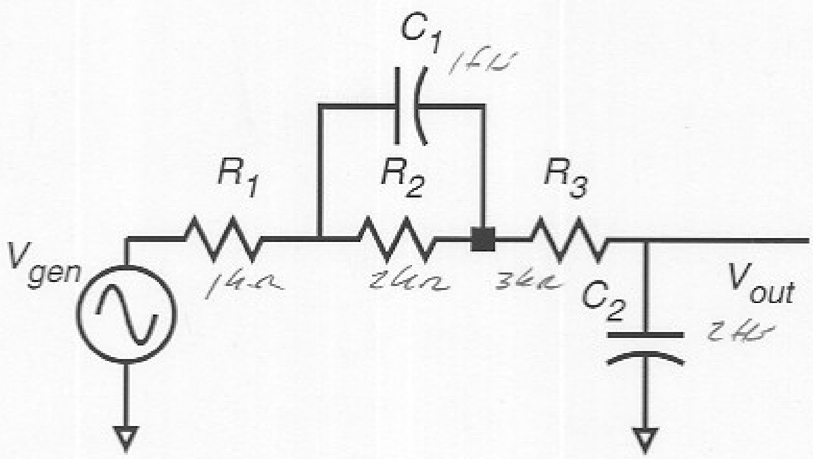
low frequency Vout/Vgen= 10.87 bandwidth of Vout/Vgen= 3.046 Hz

**draw closed loop gain on this bode plot**



Frequency, Hz

**Problem 2: 10 points**
*method of time constants analysis*




R1=1 KOhm, R2=2kOhm, R3=3kOhm  C1= 1 fF C2=2 fF
Using MOTC, find the coefficients a1 and a2 of transfer function Vout(s)/Vgen(s), given a tranfer function in the standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1s+b_2s^2+\ldots}{1+a_1s+a_2s^2+\ldots}$

$R_{11}^0 =$ 1.33 kΩ  $R_{22}^0 =$ 6 kΩ  $R_{22}^1 =$ 4 kΩ

$\left.\frac{V_{out}}{V_{gen}}\right|_{DC} =$ 1.0  $a_1 =$ 13.33 ps  $a_2 =$ 1.06 $(10^{-23})$ sec$^2$

2 [ $R_{11}^0 = 2k\Omega \,\|\, (1k\Omega + 3k\Omega) = 2k\Omega \,\|\, 4k\Omega = 1.33k\Omega$

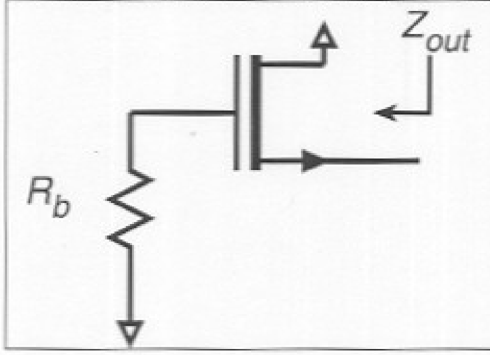
2 [ $R_{22}^0 = 1k\Omega + 2k\Omega + 3k\Omega = 6k\Omega$

2 [ $R_{22}^1 = 1k\Omega + 3k\Omega = 4k\Omega$

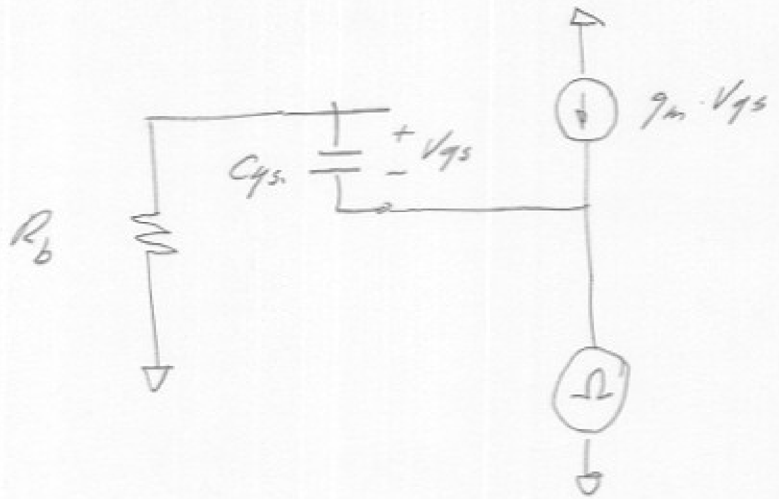
2 [ $a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 = 1.33k\Omega \cdot 1fF + 6k\Omega \cdot 2fF = 13.33\,ps$

2 [ $a_2 = R_{11}^0 C_1 C_2 R_{22}^1 = (1.33k\Omega \cdot 1fF)(2fF)(4k\Omega) = 1.06(10^{-23})\,sec^2$
$= (3.26\,ps)^2$

**Problem 3: 20 points**
*Nodal analysis and transistor circuit models*

| | |
|---|---|
|   | Ignore DC bias; you don't need it.<br>$C_{gs}$ = 10 fF $C_{gd}$ =0. $R_{ds}$ =infinity.<br>$g_m$ = 10 mS, $R_b$=10 kOhm |

Part a, 7 points
Draw an accurate small-signal equivalent circuit model of the circuit above. Represent the Zout measurement by connecting an Ohm meter. Do not show components whose element values are zero or infinity (!).




partial
credit
very
hard.

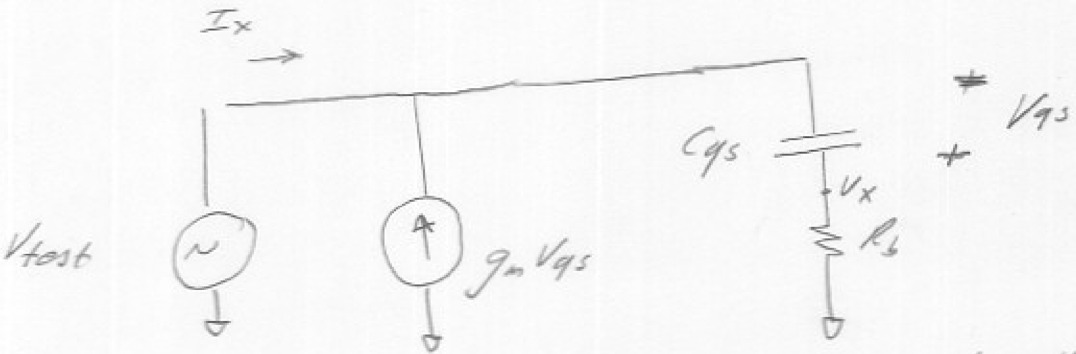
Part b, 13 points

**Using NODAL ANALYSIS**, find the frequency-dependent output impedance Zout(s)

The answer must be in standard form $Z_{out}(s) = Z_1 * s^n * \frac{1+b_1 s + b_2 s^2 + ...}{1+a_1 s + a_2 s^2 + ...}$,

where $n$ might be 0, 1, or 2 , or -1 or -2.

Zout(s)=____________________________




by inspection: $V_x = V_{test} \cdot \frac{sT}{1+sT}$ where $T = C_{gs} \cdot R_b$

$\Rightarrow V_{gs} = -V_{test}/(1+sT)$

so $I_x/V_{test} = \frac{1}{R+1/sC} + \frac{g_m}{1+sT}$

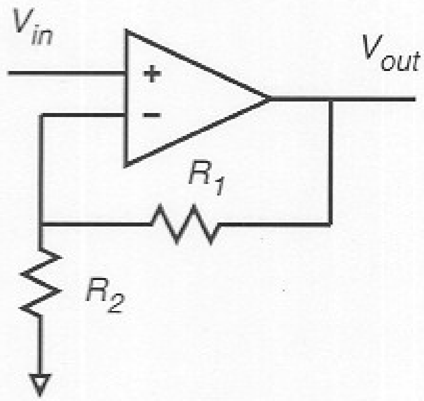
$= \frac{sC}{1+sT} + \frac{g_m}{1+sT} = \frac{g_m + sC}{1+sR_b C_{gs}}$

$$\boxed{Z_{out} = \frac{1+sR_b C_{gs}}{g_m + sC} = \frac{1}{g_m}\frac{1+sR_b C_{gs}}{1+sC/g_m}}$$

if by N.A: 6 pts for $\Sigma I = 0$ eqn
7 pts for answer.

**Problem 4, 15 points**
*negative feedback*

part a, 10 points




The amplifier has a differential gain of $10^5$. R1=99 kOhm, R2=1 kOhm. The op-amp has infinite differential input impedance and zero differential output impedance.

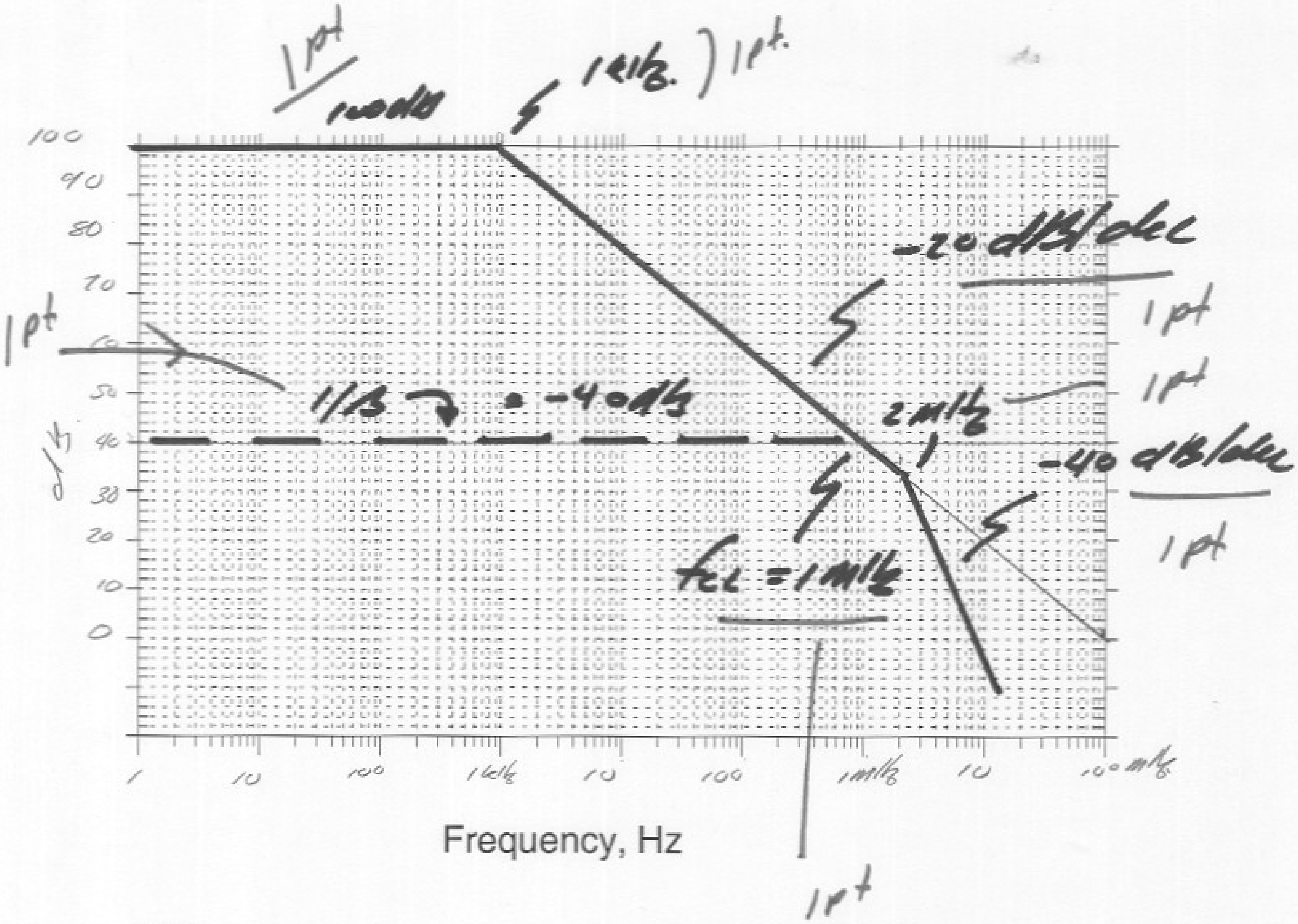
The differential amplifier has 1 pole in its open-loop transfer function at 1 kHz, and one pole at 2 MHz.

Using the Bode plot on the next page, plot the open-loop gain ( $A_d$ or $A_{ol}$ ), the inverse of the feedback factor ($1/\beta$) , closed loop gain ( $A_{CL}$). ***Label all axes, slopes, pole/zero frequencies, etc***. Determine the following:

Loop bandwidth=____1 MHz____ phase margin=____64°____
Vout/Vgen at DC=____100____

1pt.

$$\beta = \frac{1}{1+99} = \frac{1}{100} \Rightarrow -40\,dB$$




1pt [ $F_{unity}$ = 1MHz.

$\angle T$ @ 1MHz = $-90° - \arctan(1\text{MHz}/2\text{MHz})$

= $-90 - 26.6°$

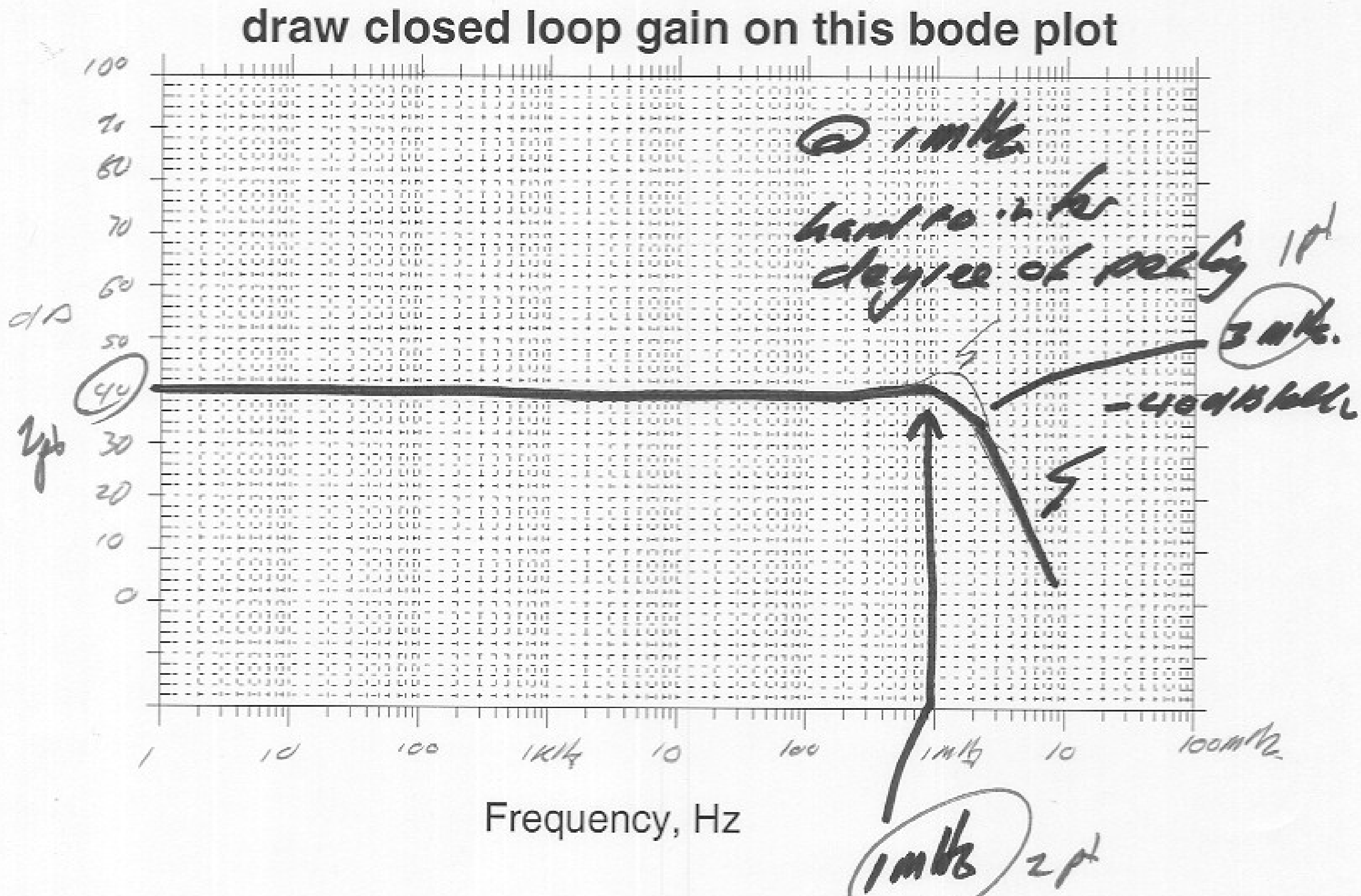
1pt [ $\phi m$ = 180 − 90 − 26.6 = 63.4°.

part b, 5 points

What is the gain and bandwidth of the closed-loop amplifier ?

low frequency Vout/Vgen= 100 bandwidth of Vout/Vgen= 1 MHz

***Draw a plot of the closed loop gain, labeling all axes, slopes, pole/zero frequencies, etc.***

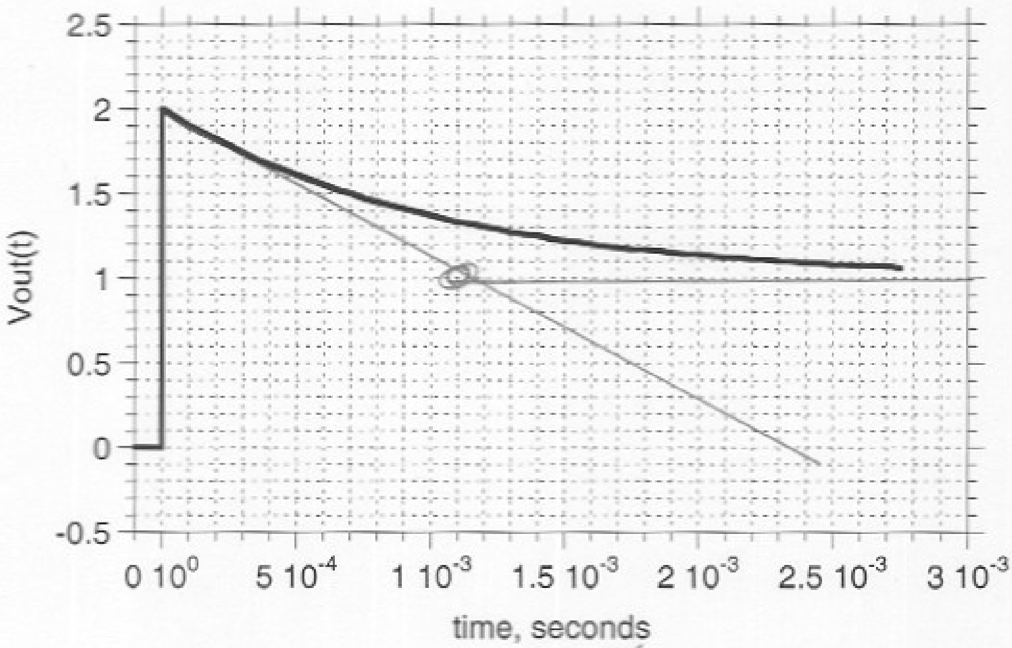



$$A_{cL} = \begin{cases} 1/\beta & T>>1 \quad \Rightarrow \text{constant at low freq.} \\ \dfrac{1}{\beta}\dfrac{e^{j\theta_T}}{1+e^{j\theta_T}} & \text{when } T = 1 e^{j\theta_T} \quad \Rightarrow \\ A_{ol} & T<<1 \quad \Rightarrow -40\text{dB/dec @ high freq.} \end{cases}$$

**Problem 5: 15 points**
*transfer functions*

Part a, 5 points

A transistor circuit has a step response (input is a 1-V step function) as shown.




3 [ function is $u(t) + u(t)\, e^{-t/1\,ms}$

identify all pole and zero frequencies in the transfer function

pole frequencies: 159, X, X Hz

zero frequencies: 80, X, X Hz

Part b, 5 points

Give the transfer function

Vout(s)/Vgen(s). Give the answer in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$

Vout(s)/Vgen(s)= ______________________

2 $\left[\; V_o(s) = 1V/s + \frac{1V\cdot\tau}{1+s\tau}\right.$

1 $\left[\; V_i(s) = 1V/s\right.$

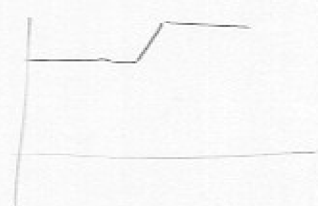
2

$$H(s) = \frac{1/s + \frac{\tau}{1+s\tau}}{1/s} = 1 + \frac{s\tau}{1+s\tau} = \frac{1+s\tau+s\tau}{1+s\tau}$$

$$= \frac{1+s\tau\cdot 2}{1+s\tau} = \underbrace{\left[\frac{1+s\,2\tau}{1+s\tau}\right] = H(s)}_{\text{where } \tau = 10^{-3}\text{ sec} = 1\text{ms.}}$$

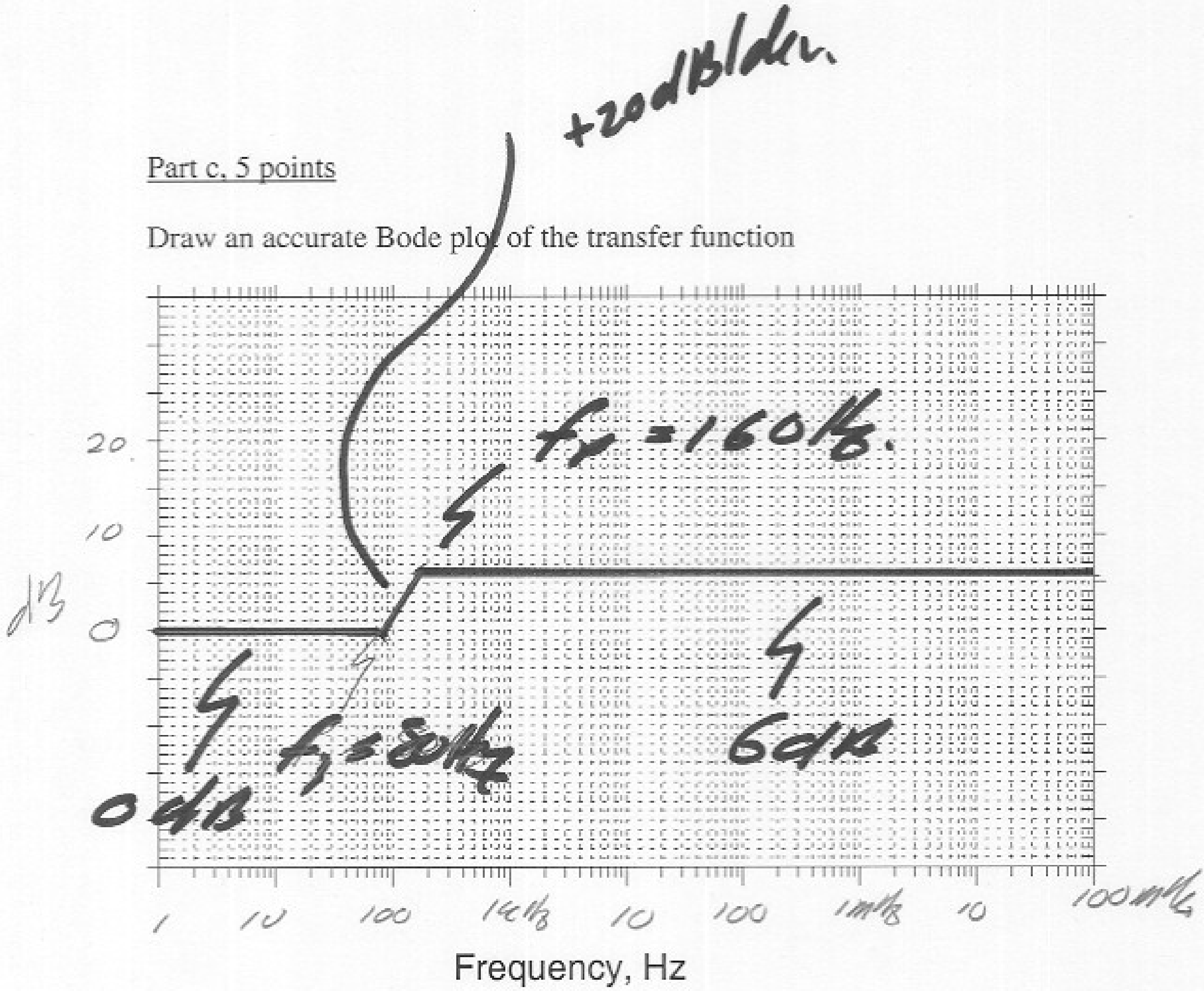
9d 1) pole @ $f_{p1} = \frac{0.159}{1ms} = 159 Hz$.

1) zero @ $f_{z1} = \frac{0.159}{2ms} = 80 Hz$.



Part c, 5 points

Draw an accurate Bode plot of the transfer function




($f_p$, $f_z$)  1 pt each feature