ECE137B Final Exam

There are 5 problems on this exam and you have 3 hours
There are pages 1-19 in the exam: please make sure all are there.

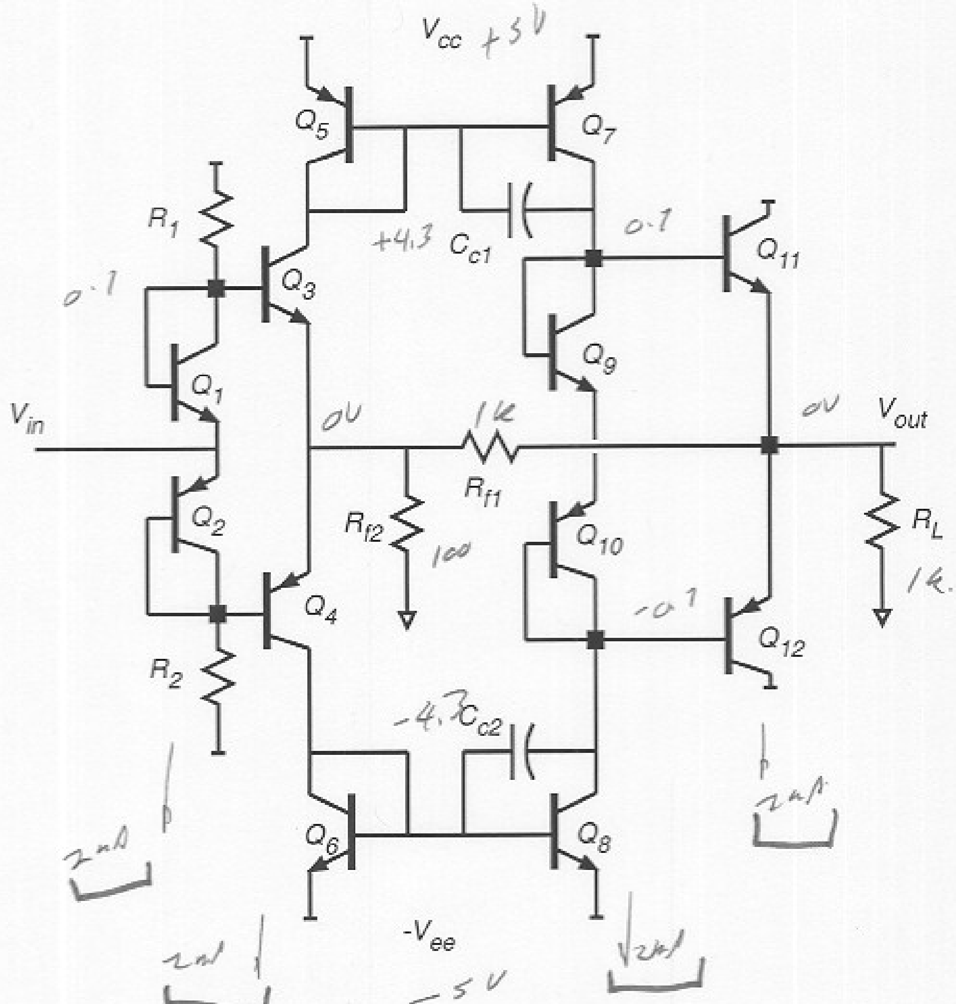
Do not open this exam until told to do so
Show all work:
Credit will not be given for correct answers if supporting work is not shown.
Class Crib sheets and 2 pages (front and back→ 4 surfaces) of your own notes permitted.
Don't panic.

Time function	LaPlace Transform
$\delta(t)$	1
U(t)	1/s
$e^{-\alpha t}\cdot U(t)$	$\frac{1}{s+\alpha}$ or $\frac{1/\alpha}{1+s/\alpha}$
$e^{-\alpha t}\cos(\omega_d t)\cdot U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)\cdot U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Name: solution "B"

Problem	points	possible	Problem	points	possible
1a		3	2		10
1b		2	3a		7
1c		8	3b		13
1d		5	4a		10
1e		12	4b		5
1f		5	5a		5
1g		5	5b		5
			5c		5

Problem 1, 40 points
method of first-order and second-order time constants, some feedback theory



3/4 pts for each current

Part a, 3 points
DC analysis

Find all transistor DC emitter currents, find all node voltages. Make these on the circucit diagrm.

β : infinity, for all transistors.	
Va=200 V for Q7 and Q8	Va=infinity for all other transistors
C_{cb} = zero, for all transistrors.	$C_{c1} = C_{c2}$ =5 fF
τ_f = 1 ps and C_{je} = 5 fF for Q7, Q8, Q11, Q12. τ_f = 0 ps and C_{je} = 0 fF for all other transistors	
All transistors have identical I_s cthe DC component of Vin is zero volts	
The supplies are +/- 5 Volts. Rf1=1 kOhm, Rf2=100 Ohm, RL=1 kOhm	
R1=R2: select their value so that the DC emitter currents in Q1 and Q2 are 2 mA	

$R_1 = R_2 = \frac{5-0.7V}{2mA} = \frac{4.3V}{2mA} = 2.15 k\Omega$

Part b. 2 points

small signal parameters

Find the following:

	$r_e = 1/g_m$	R_{be}	R_{ce}	C_{be}	C_{cb}	f_τ
Q1	13Ω	∞	∞	0		∞
Q3			∞	0		∞
Q5			∞	0		∞
Q7			100kΩ	82fF		149GHz
Q9			∞	~		∞
Q11			∞	82fF		149GHz

1 [$C_{be} = C_{je} + g_m \tau_f = 5fF + 1ps/13\Omega = 82fF$

1 [$f_T = \dfrac{g_m}{2\pi(C_{be} + C_{cb})} = 149\,GHz$

b

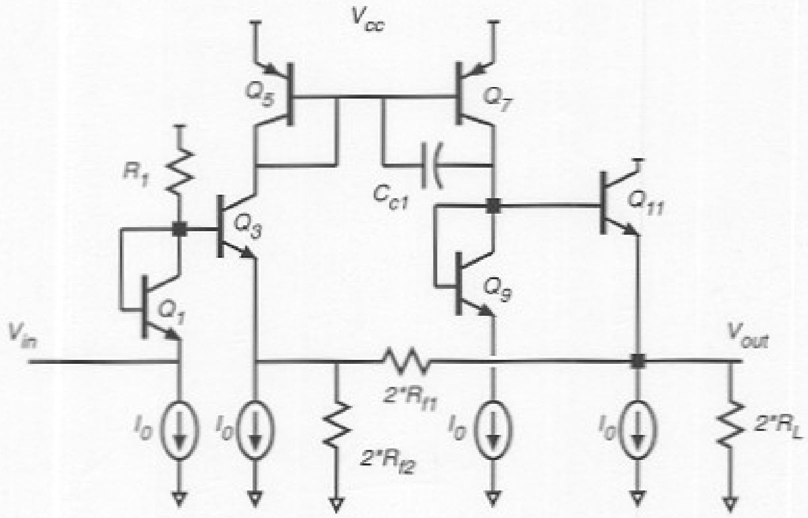
Part c, 8 points

mid-band analysis

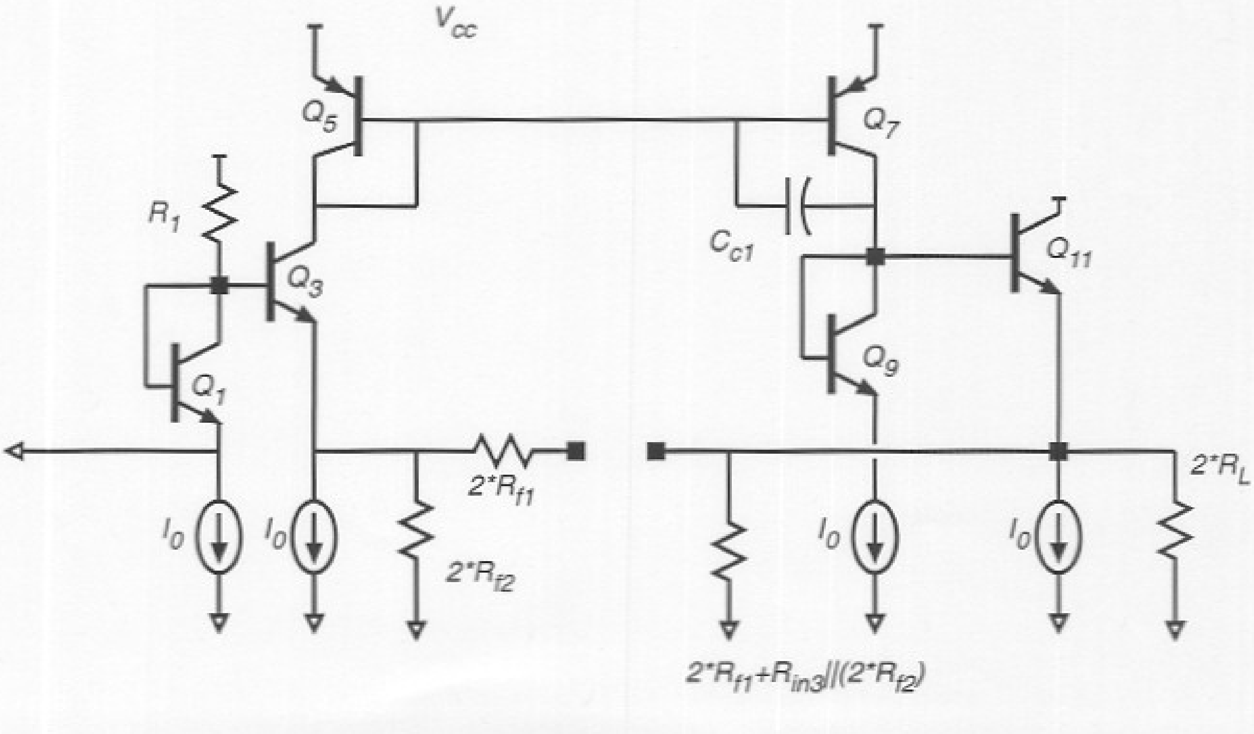
Find the low-frequency loop transmission:

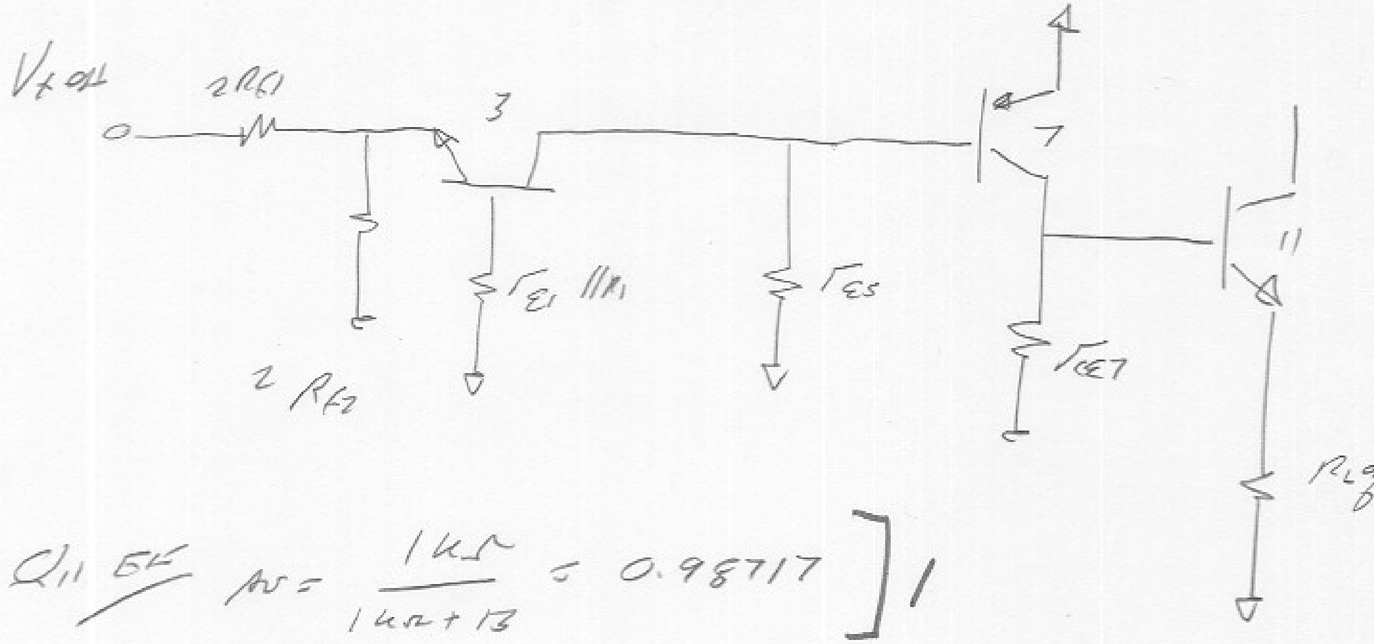
T(f=0 Hz) = 46.11

To do this, you must make 2 changes to the circuit. First, the circuit is symmetric, and can be thus simplified, where I_o is the value of the DC current in R1.



Second, you need to cut the feedback loop, thus, to find the loop transmission





T: V_{Test}

Q11 EF $A_V = \frac{1k\Omega}{1k\Omega + 13} = 0.98717$] 1

Q7 CE

$A_V = r_{CE7} / r_{E7} = \frac{100k\Omega}{13} = 7{,}700$] 2

$R_{Leq} = 2R_{f1} \parallel (2R_{f1} + R_{i3} \parallel 2R_{f2}) = 1k\Omega$

Q3: CS

2 [$A_V = \frac{R_{eq}}{r_E} = \frac{r_{ES}}{r_{E3}} = 1$, $r_{i3} = 13\Omega$

feedback network

2 [$f_{orm} = \frac{r_{i3} \parallel 2R_{f2}}{r_{i3} \parallel 2R_{f2} + 2R_{f1}} = \frac{13 \parallel 200}{13 \parallel 200 + 2000} = 0.0060663$

Loop Transmission $= 0.98717 \cdot 7700 \cdot 1 \cdot 0.0060663$] 1

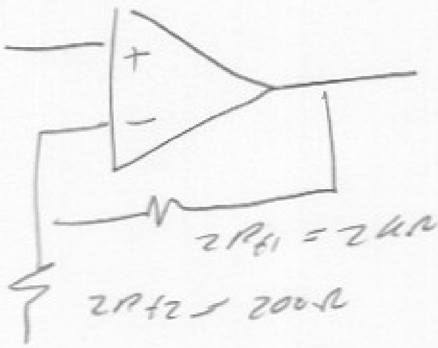
$= 46.11$

Part d, 5 points
feedback theory

At low frequencies, what is the closed-loop gain V_{out}/V_{in} ?

V_{out}/V_{in} = 10.766

2. [$A_{CL} = A_{OL} \frac{T}{1+T}$ $T = 46.1$ at DC



$\Longrightarrow A_{OL} = \frac{2k+200}{200} = 11$] 2

1 [$A_{CL} = 11 \cdot \frac{46.11}{46.11+1} = 10.766$.

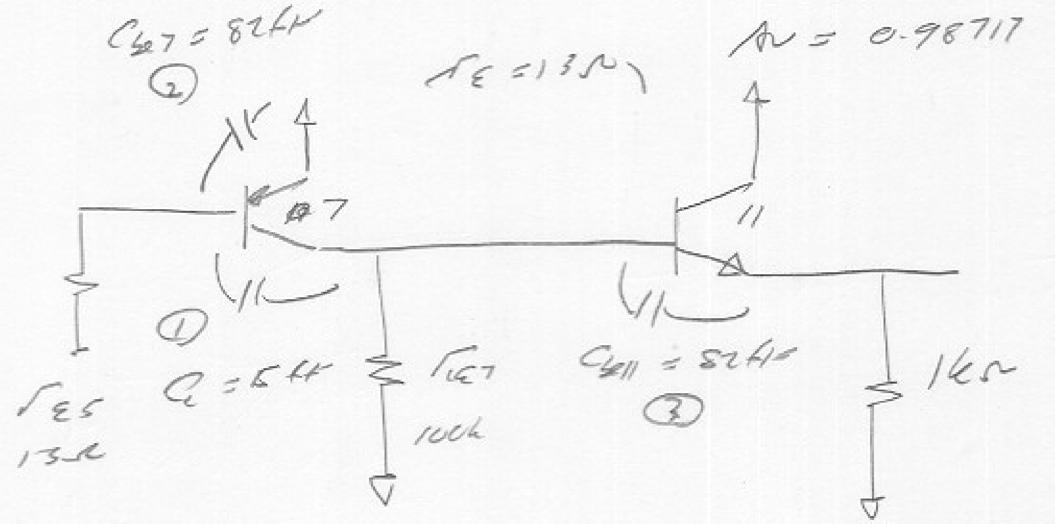
b

Part e, 12 points

motc

Using MOTC, you will find the frequency, in Hz (not rad/sec), of the ***two*** major poles in the transfer function.

capacitor 1:	capacitor 2:	capacitor 3:
$R_{11}^{0}=$ 200kΩ	$R_{22}^{0}=$ 13Ω	$R_{33}^{0}=$ 1296Ω.
$R_{22}^{1}=$ 6.5 Ω	$R_{33}^{1}=$ 13.0Ω.	$R_{33}^{2}=$ 1296Ω.
$f_{p1}=$ 262 MHz	$f_{p2}=$ 825 GHz	
capacitor 1 is the compensation capacitance capacitor 2 is the capacitance between base & emitter of transistor Q7 capacitor 3 is the capacitance between " & " of transistor Q11		



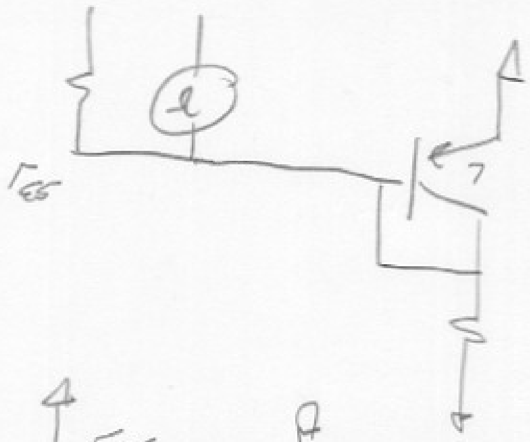
1 [$r_{11}^{0} = 13\Omega(1-A_{v7}) + R_{o7} = 13\Omega(1+7700) + 100k\Omega$
$= 200k\Omega$

1 [$r_{22}^{0} = 13\Omega$

1 [$r_{33}^{0} = 100k\Omega(1-A_{v11}) + r_{E11} \| 1k\Omega = 1.28k\Omega + 13\Omega = 1296\Omega$

1 [$a_1 = r_{11}^{0}C_1 + r_{22}^{0}C_2 + r_{33}^{0}C_3 = 100k\Omega(5fF) + 13\Omega(82fF) + 1296\Omega(82fF)$
$= 500\,ps + 1.06\,ps + 107\,ps = 607\,ps$

2 R_{22}^{1}

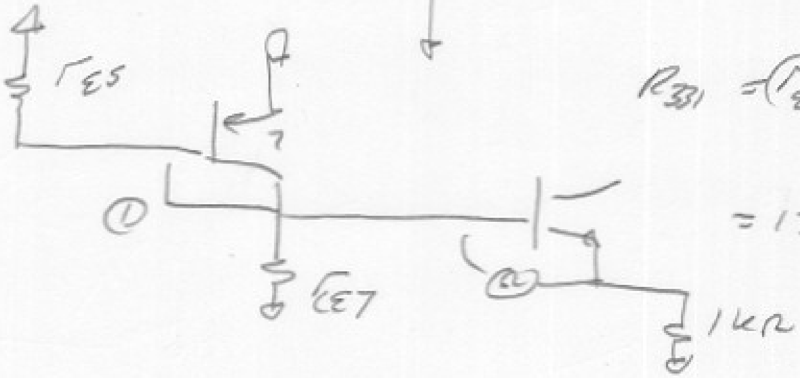


$R_{22}^{1} = r_{E5} \| r_{E7} \| r_{EC7}$

$= 13 \| 13 \| 100k$

$= 6.5\,\Omega$

2 R_{33}^{1}



$R_{33}^{1} = (r_{E5} \| r_{E7} \| r_{E7})(1 - A_{v11}) + r_{E11} \| 1k\Omega$

$= 13\,\Omega(1 - 0.99717) + 13 \| 1k\Omega$

$= 0.166\,\Omega + 12.8$

$= 13.0\,\Omega$

2 $R_{33}^{2} = R_{33}^{0}$ by inspection!

1 $a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{22}^{0} C_2 C_3 R_{33}^{2}$

$= 200k\Omega(5fF)(82fF) \cdot 6.5\,\Omega + 200k\Omega(5fF)\,82fF \cdot 13\,\Omega + 13\,\Omega \cdot 82fF \cdot 82fF \cdot 1276\,\Omega$

$= 5.33 \cdot 10^{-23} + 1.06 \cdot 10^{-21}\,\text{sec}^2 + 1.13(10^{-22})\,\text{sec}^2$

$= 1.17(10^{-21})\,\text{sec}^2$

1 use SPA

$f_{p1} = \frac{1}{2\pi a_1} = 262\,\text{MHz}$

$f_{p2} = \frac{a_1}{2\pi a_2} = 82.5\,\text{GHz}$

SPA ok.

46.11
262 MHz
82.5 GHz

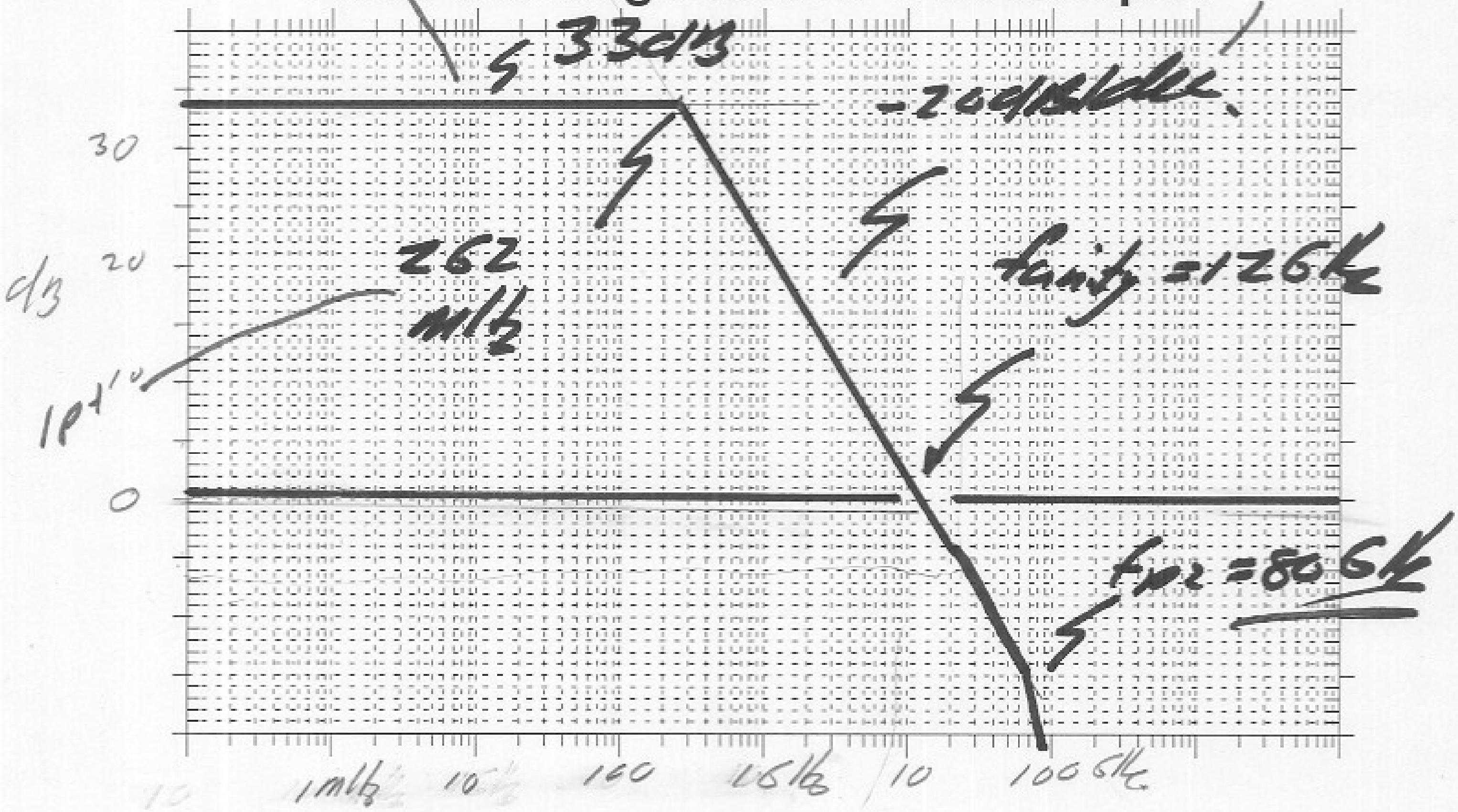
Part f, 5 points

Make accurate asymptotic plots of T. Find the phase margin and the loop bandwidth.

Phase margin = 81.7° Loop bandwidth = 17.1 GHz

1/2 pt 1pt

Draw the magnitude of T on this plot



46.11 → 33.3 dB

1pt [f_{unity} = 46.11 · 262 MHz = 12.1 GHz

1/2 pt [∠T = −90° (pole 1) − arctan(12.1 GHz / 82.5 GHz)
= −90 − 8.33°

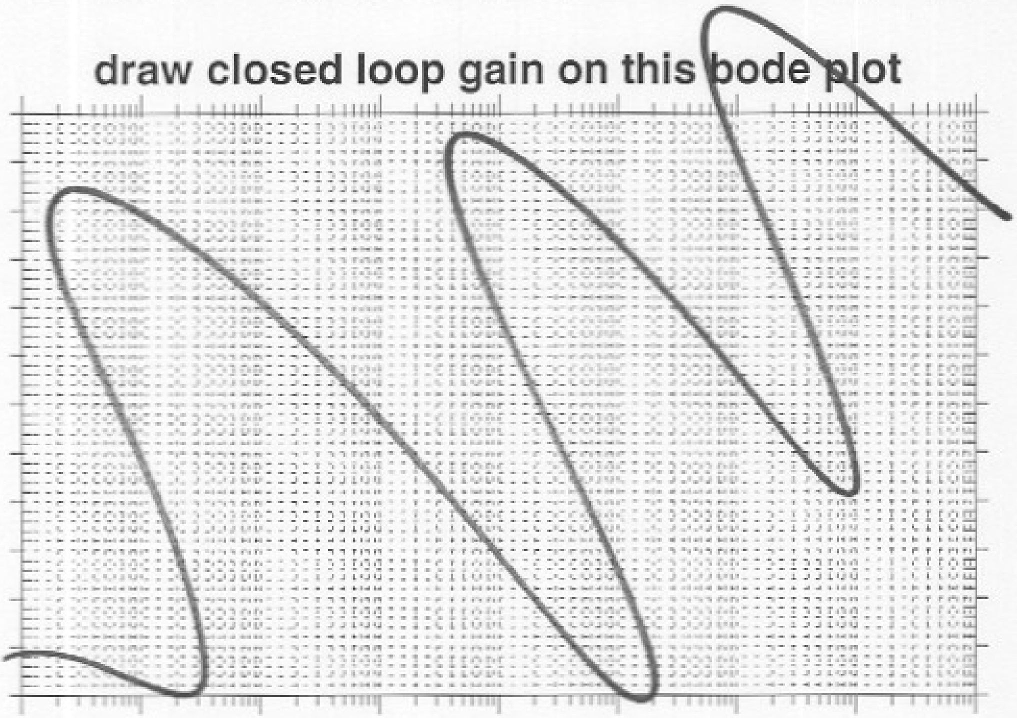
1pt [Phase Margin = 90 − 8.33 = 81.7°

Part g. 5 points

What is the gain and bandwidth of the closed-loop amplifier ?

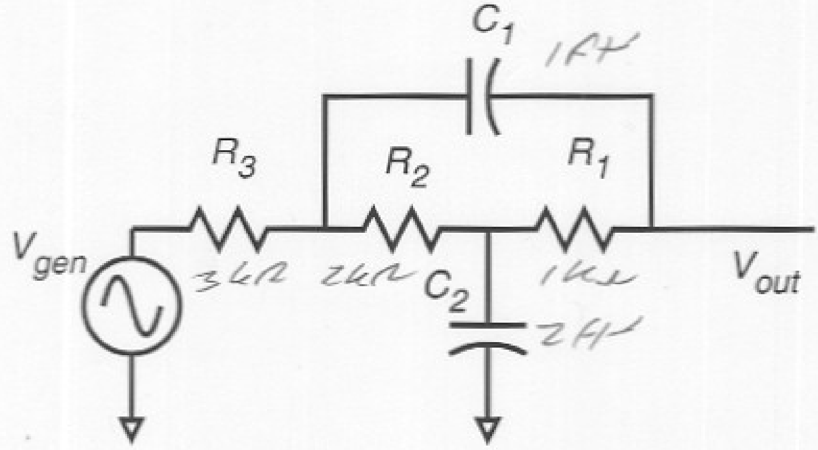
low frequency Vout/Vgen= 10.8 bandwidth of Vout/Vgen= 126k

draw closed loop gain on this bode plot



Frequency, Hz

Problem 2: 10 points
method of time constants analysis



R1=1 KOhm, R2=2kOhm, R3=3kOhm C1= 1 fF C2=2 fF
Using MOTC, find the coefficients a1 and a2 of transfer function Vout(s)/Vgen(s), given a tranfer function in the standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s+b_2 s^2+...}{1+a_1 s+a_2 s^2+...}$

$R_{11}^0 =$ 3kΩ

$\left.\frac{V_{out}}{V_{gen}}\right|_{DC} =$ 1

$R_{22}^0 =$ 5kΩ

$a_1 =$ 13ps

$R_{22}^1 =$ 3.67kΩ

$a_2 =$ $(4.69ps)^2$ $= 2.2(10^{-23})\ sec^2$

2 [$R_{11}^0 = R_1 + R_2 = 3k\Omega$

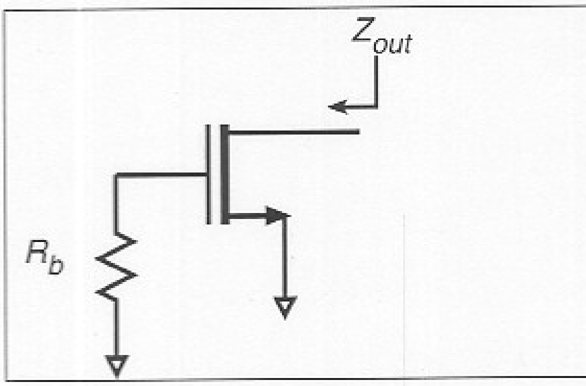
2 [$R_{22}^0 = R_2 + R_3 = 5k\Omega$

2 [$R_{22}^1 = R_3 + R_1 \| R_2 = 3k\Omega + 2k\Omega \| 1k\Omega = 3.67k\Omega$

2 [$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 = 3k\Omega \cdot 1fF + 5k\Omega \cdot 2fF = 13ps$

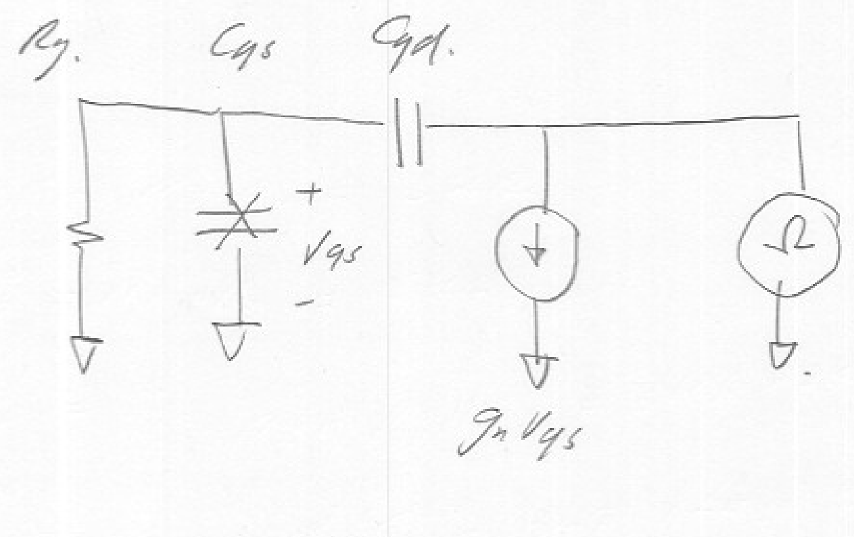
2 [$a_2 =$ ~~$R_{11}^0 ...$~~ $R_{11}^0 C_1 C_2 R_{22}^1 = 3k\Omega \cdot 1fF \cdot 2fF \cdot 3.67k\Omega$
$= 2.2(10^{-23})\ sec^2 = (4.69ps)^2$

Problem 3: 20 points
Nodal analysis and transistor circuit models



Ignore DC bias; you don't need it.
C_{gs} = 10 fF C_{gd} =1 fF. R_{ds} =infinity.
g_m = 100 mS, R_b =1 kOhm

<u>Part a, 7 points</u>
Draw an accurate small-signal equivalent circuit model of the circuit above. Represent the Zout measurement by connecting an Ohm meter. Do not show components whose element values are zero or infinity (!).



partial credit very hard.

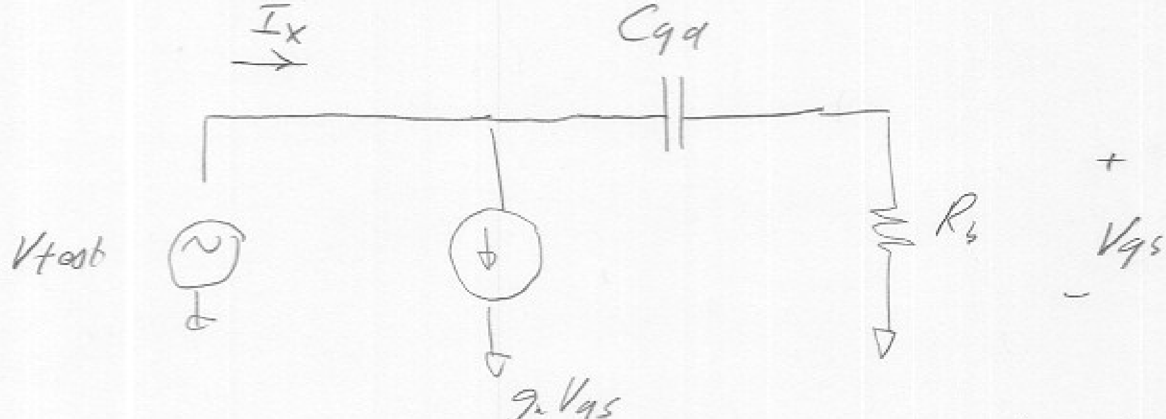
Part b, 13 points

Using NODAL ANALYSIS, find the frequency-dependent output impedance Zout(s)

The answer must be in standard form $Z_{out}(s) = Z_1 * s^n * \frac{1 + b_1 s + b_2 s^2 + ...}{1 + a_1 s + a_2 s^2 + ...}$,

where n might be 0, 1, or 2 , or −1 or −2

Zout(s)=________________________________



by inspection: $V_{gs} = V_{test} \cdot \frac{s\tau}{1+s\tau}$ where $\tau = R_b \cdot C_{gd} = R \cdot C$.

$$\frac{1}{Z_{out}} = \frac{I_x}{V_{test}} = \frac{1}{R + 1/sC_{gd}} + \frac{g_m s\tau}{1+s\tau} = \frac{sC}{1+s\tau} + \frac{g_m s\tau}{1+s\tau}$$

$$\frac{1}{Z_{out}} = \frac{sC + g_m s\tau}{1+s\tau} = \frac{sC + sC g_m R}{1+sCR} = \frac{sC(1+g_m R)}{1+sRC}$$

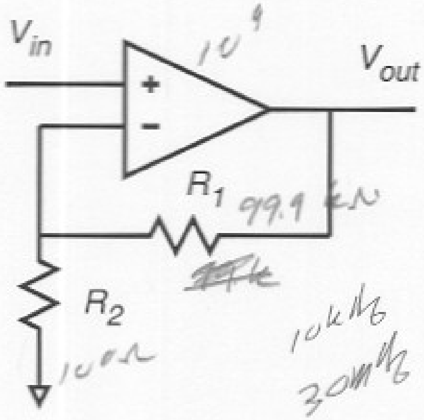
$$Z_{out} = \frac{1+sRC}{sC(1+g_m R)} = \frac{1}{sC(1+g_m R)}(1+sRC)$$

If by NA, 6 pts for $\Sigma I = 0$

7 pts for answer.

Problem 4, 15 points
negative feedback

part a, 10 points



The amplifier has a differential gain of 10^6. R1=99.9 kOhm, R2=100 Ohm. The op-amp has infinite differential input impedance and zero differential output impedance.

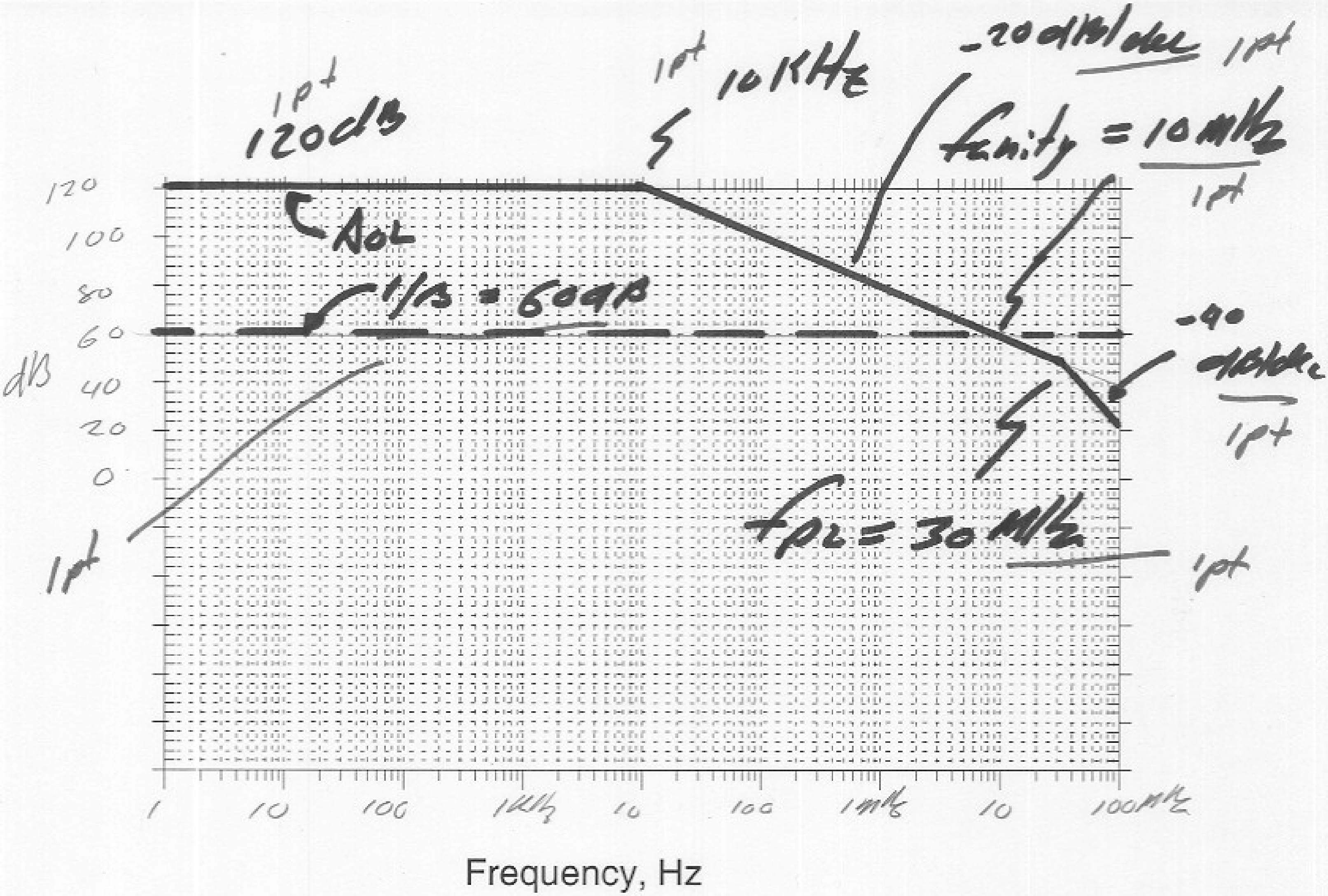
The differential amplifier has 1 pole in its open-loop transfer function at 10 kHz, and one pole at 30 MHz.

Using the Bode plot on the next page, plot the open-loop gain (A_d or A_{ol}), the inverse of the feedback factor ($1/\beta$), closed loop gain (A_{CL}). ***Label all axes, slopes, pole/zero frequencies, etc***. Determine the following:

Loop bandwidth= 10 MHz phase margin= 72°
Vout/Vgen at DC= 1000

1 pt

$\beta = \frac{100\Omega}{100k\Omega} = 10^{-3} \Rightarrow -60dB$



1pt [$f_{unity} = 10\,\text{MHz}$

1pt [$\angle T$ @ $10\,\text{MHz} = -90 - \arctan(10\,\text{MHz}/30\,\text{MHz})$

$= -90 - 18.45°$

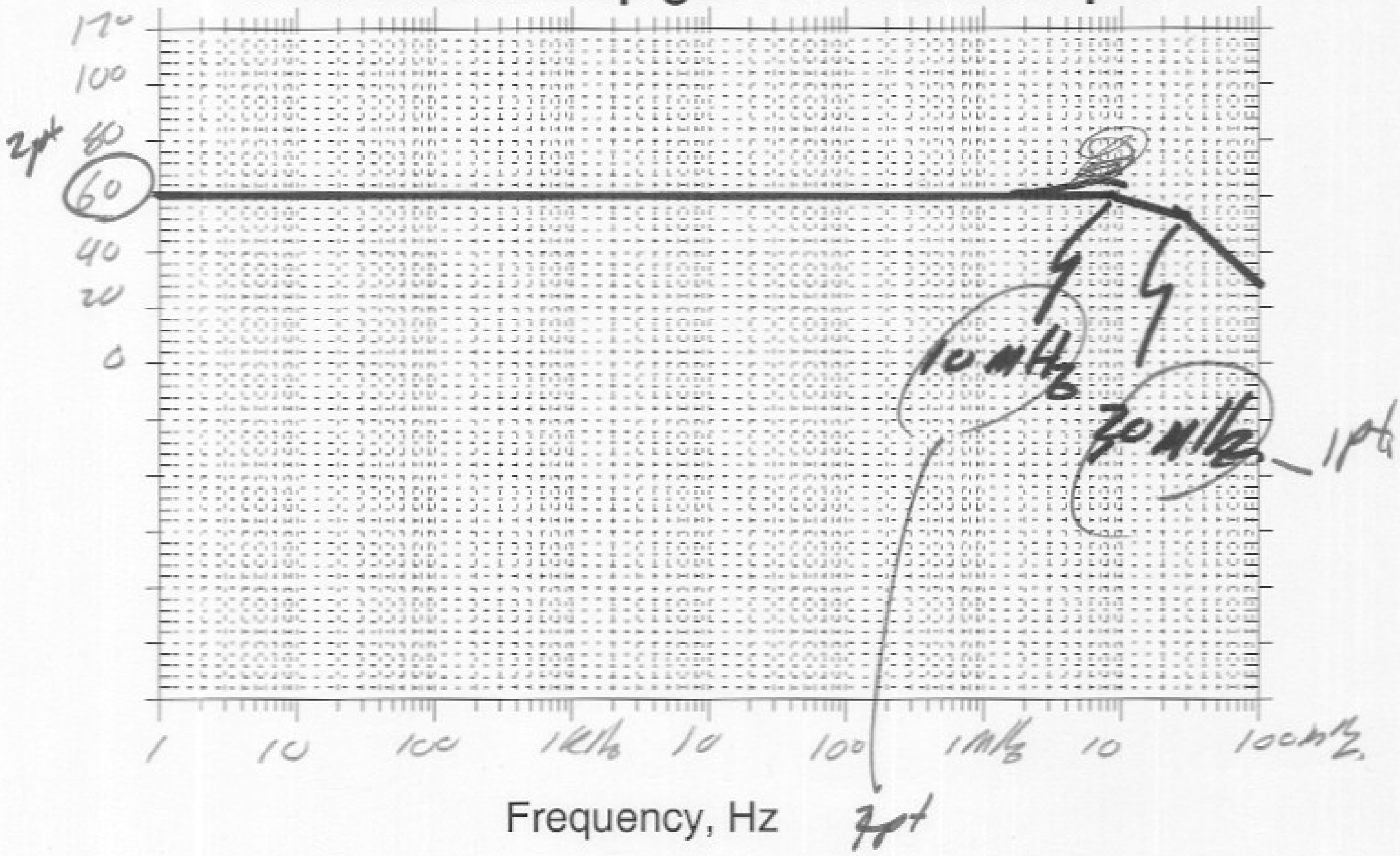
phase margin $= 180 - 90 - 18.45° = 71.6°$

part b. 5 points
What is the gain and bandwidth of the closed-loop amplifier ?

low frequency Vout/Vgen=__1000__ bandwidth of Vout/Vgen=__10MHz__

Draw a plot of the closed loop gain, labeling all axes, slopes, pole/zero frequencies, etc.

draw closed loop gain on this bode plot



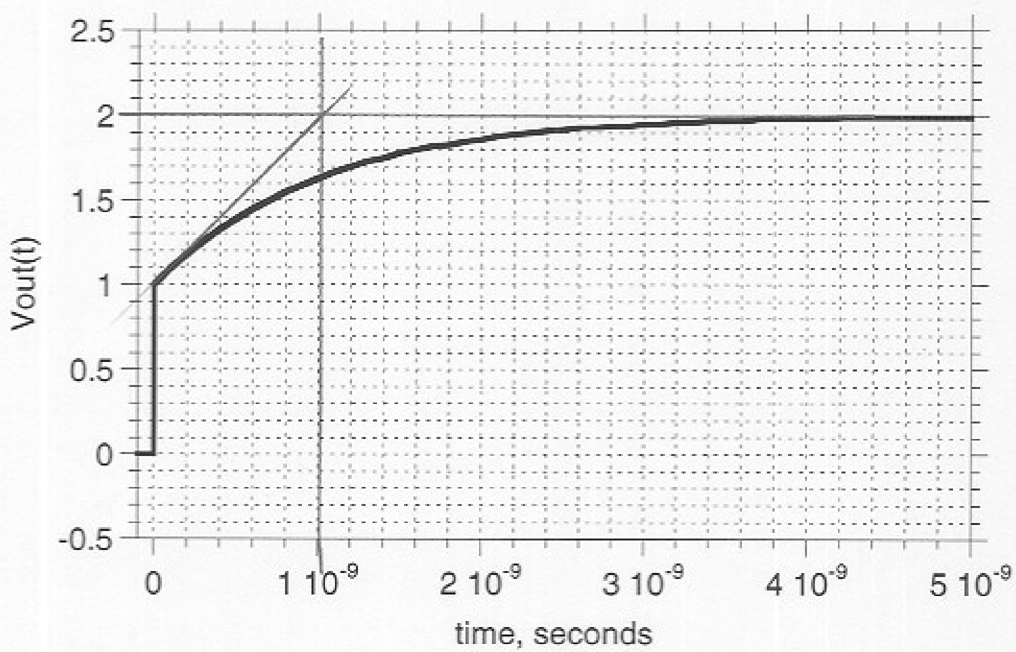
$$A_{cl} = \begin{cases} 1/\beta & T \gg 1 \\ \frac{1}{\beta}\frac{e^{j\theta_T}}{1+e^{j\theta_T}} & \text{for } T = 1 \cdot e^{j\theta_T} \\ A_{ol} & T \ll 1 \end{cases}$$

because pm is large, transfer function shows little or no peaking @ funity.

Problem 5: 15 points
transfer functions

Part a, 5 points

A transistor circuit has a step response (input is a 1-V step function) as shown.



identify all pole and zero frequencies in the transfer function

pole frequencies: 160MHz, X, X Hz

zero frequencies: 320MHz, X, X Hz

3) [function IS $2 \cdot u(t) - u(t)\, e^{-t/1ns}$

Part b. 5 points

Give the transfer function

Vout(s)/Vgen(s). Give the answer in standard form $\frac{V_{out}(s)}{V_{gen}(s)} = \left.\frac{V_{out}}{V_{gen}}\right|_{DC} \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$

Vout(s)/Vgen(s)= ______________________

1

2 $V_o(s) = 2V/s - \frac{1V \cdot T}{1+sT}$ / $V_i(s) = 1V/s$

2 $H(s) = \frac{\frac{2V}{s} - \frac{1V \cdot T}{1+sT}}{1V/s} = 2 - \frac{sT}{1+sT} = \frac{2+2sT-sT}{1+sT}$

$= \frac{2+sT}{1+sT} = 2\,\frac{1+sT/2}{1+sT}$ $T = 1\,ns.$

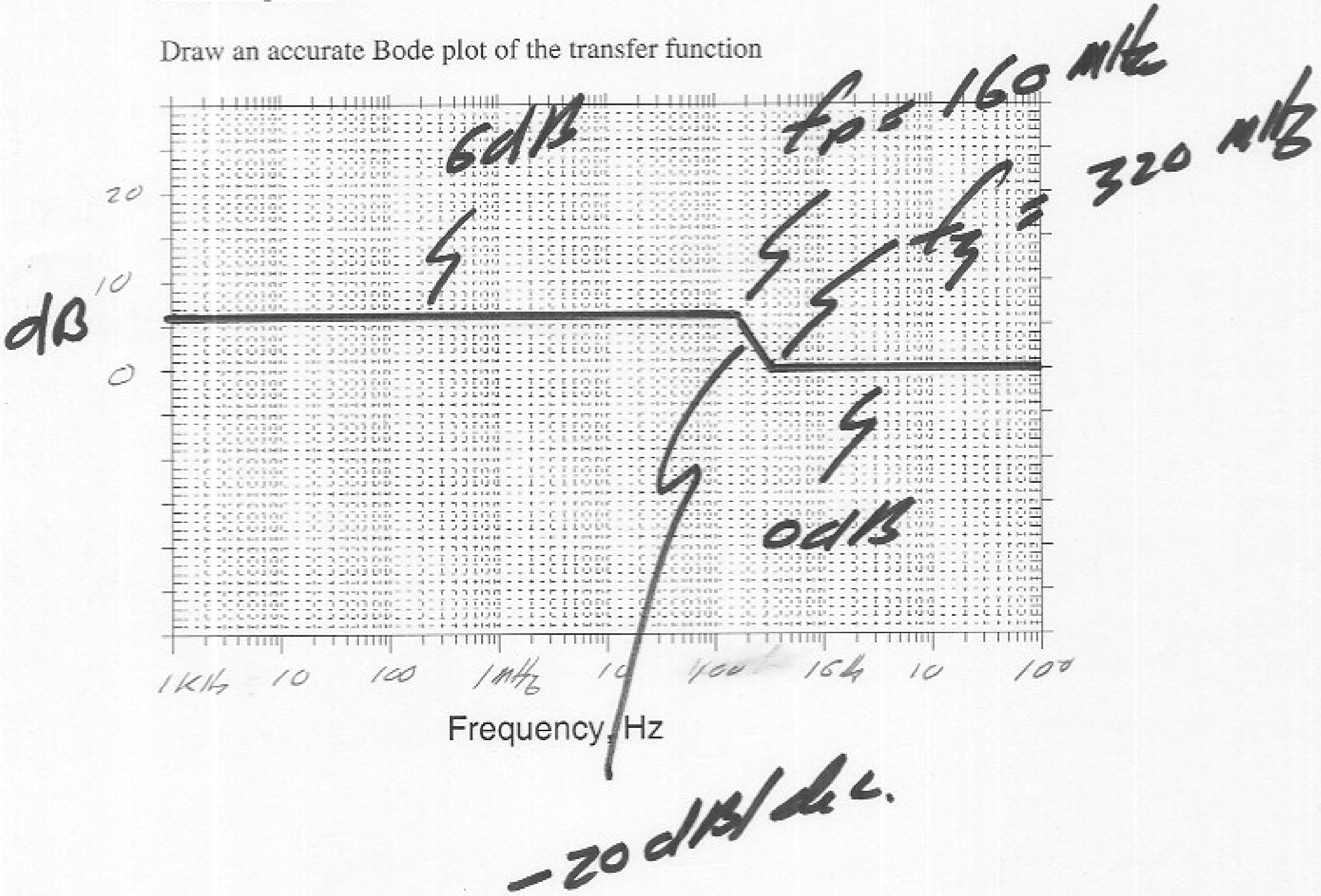
ad

1) pole @ $f_{p1} = \frac{1}{2\pi(1ns)} = 160\,MHz$

1) zero @ $f_{z1} = \frac{1}{2\pi(1/2\,ns)} = 320\,MHz.$

Part c, 5 points

Draw an accurate Bode plot of the transfer function



1pt each for two