ECE137A, Notes Set 16:
First-Order Circuits

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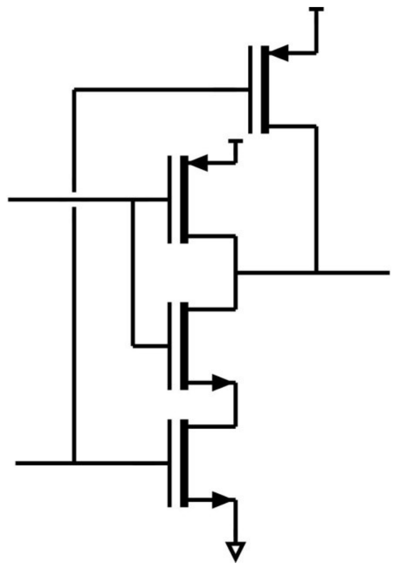
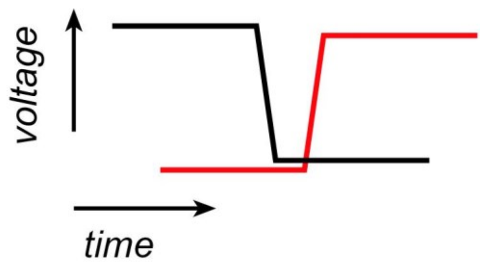
Pulse and Frequency Response: Transistor Circuits





Transistor and wirinig capacitances determine the gate propagation delays in digital logic circuits

Pulse and Frequency Response: Transistor Circuits

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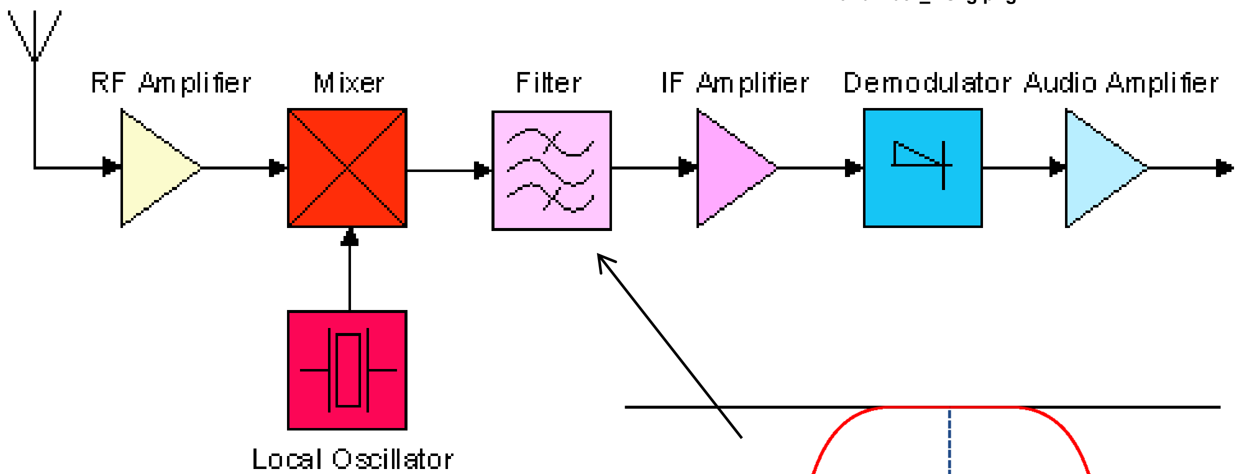
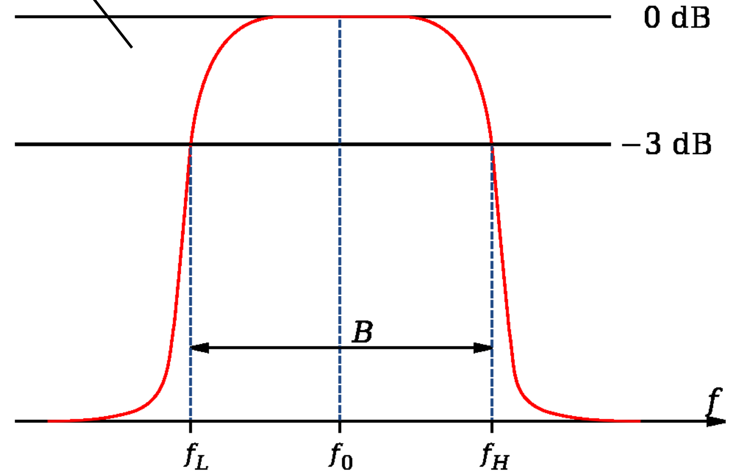
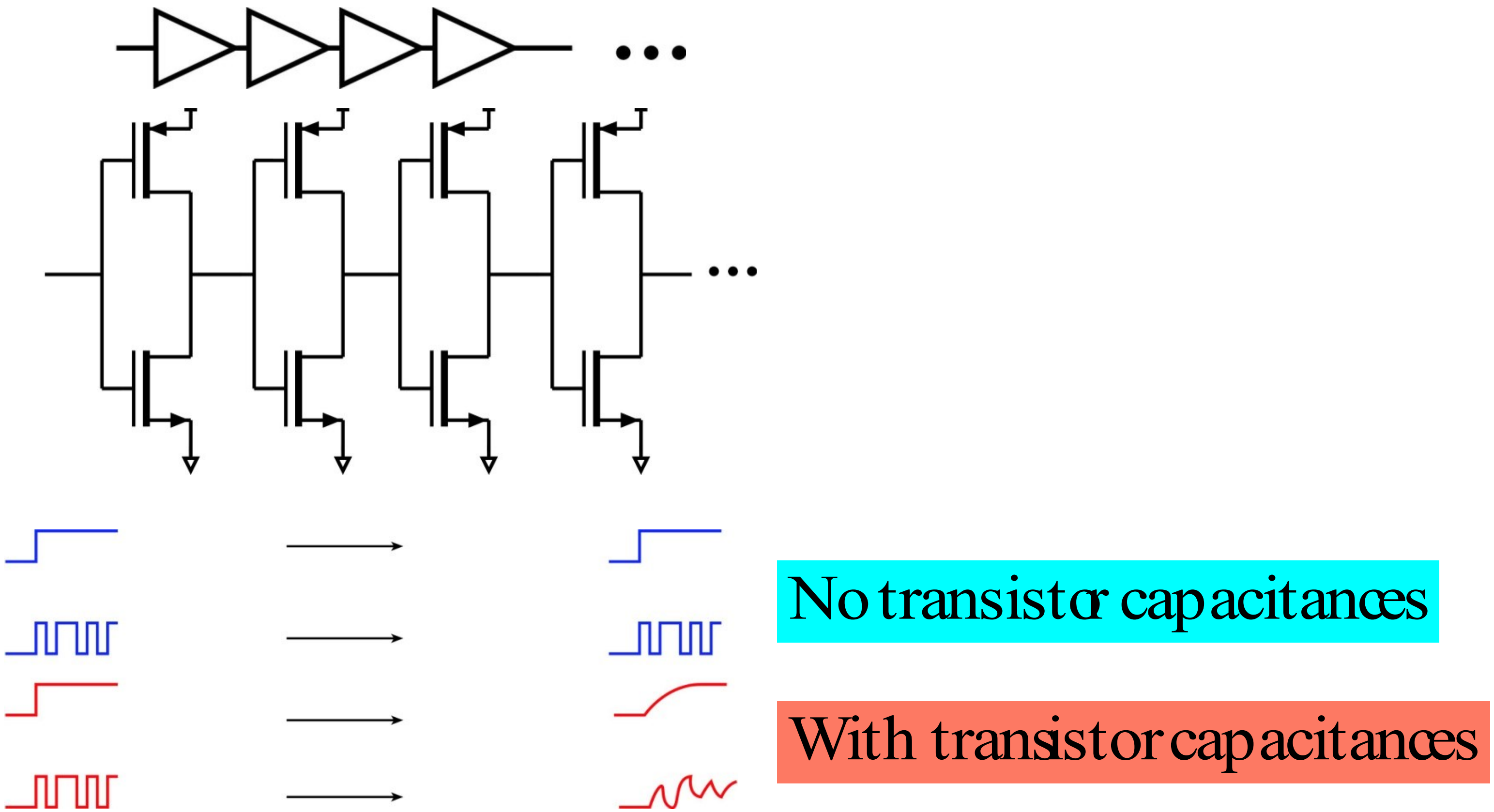




Frequency-selective circuits are used in radio receivers to select the desired signal.

Pulse and Frequency Response: Transistor Circuits



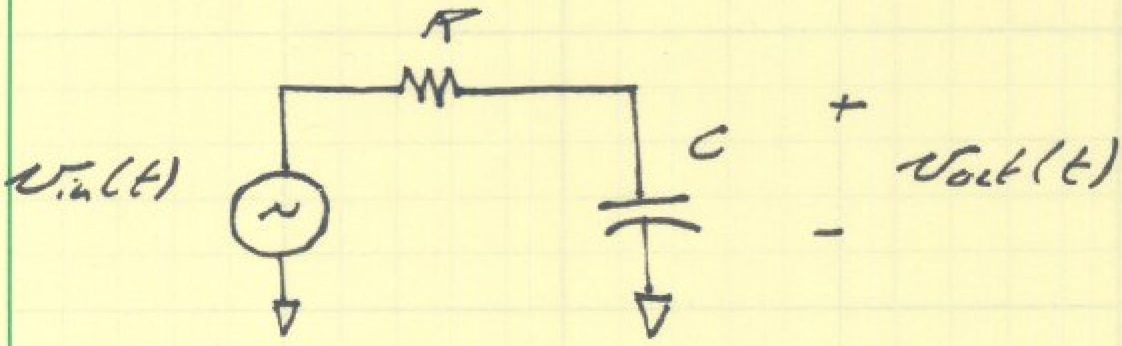
Transistor capacitances control the pulse-response rise time and hence maximum transmission bit-rate in optical fiber and similar pulse-code data transmission systems

Frequency and Pulse Response.

We will shortly be analyzing transistor circuits for pulse and frequency response.

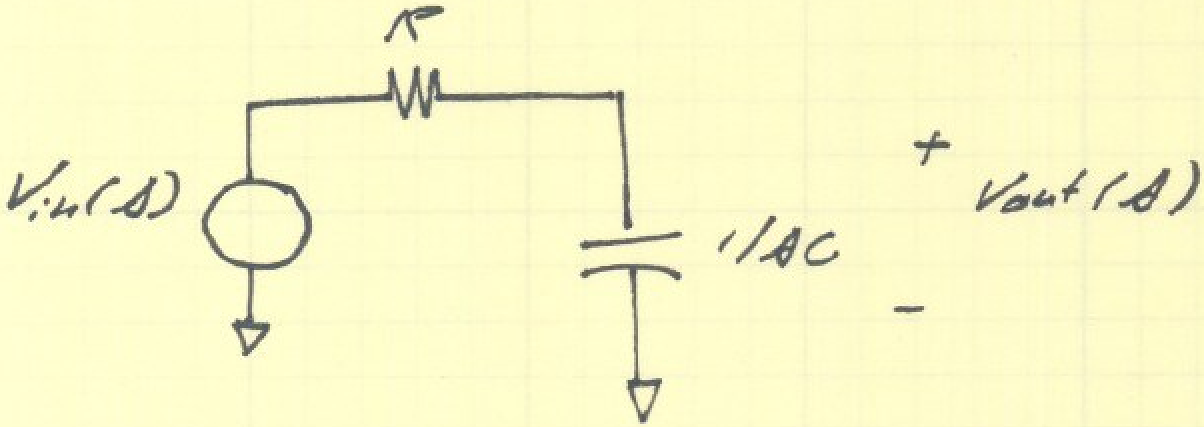
Let us first review our methods of analysis and of presenting data.

First-Order Circuits: Review



let us restrict ourselves now to problems with zero initial conditions

First-Order Circuits: Laplace Domain



$$V_{out}(s) = V_{in}(s)\,\frac{1}{1 + s\tau}$$

where $\tau = RC$

$H(s)$ = transfer function

$= V_{out}(s)/V_{in}(s)$

Sinusoidal Response→ phasors

we can work with complex exponentials,
e.g. exp{jωt} as stimulus and (implicitly) take the real part of both stimulus and response.

Input voltage $= v_{in}(t) = v_{in} \cdot e^{j\omega t}$

output voltage $= v_{out}(t) = v_{out} \cdot e^{j\omega t}$

where v_{in} & v_{out} are complex numbers.

Sinusoidal Response→ phasors

In our example: $\tilde{v}_{out} = \dfrac{\tilde{v}_{in}}{1 + j\omega\tau}$

Specifically, the amplitude of the output is $\| H(j\omega) \|$ times the input amplitude.

and

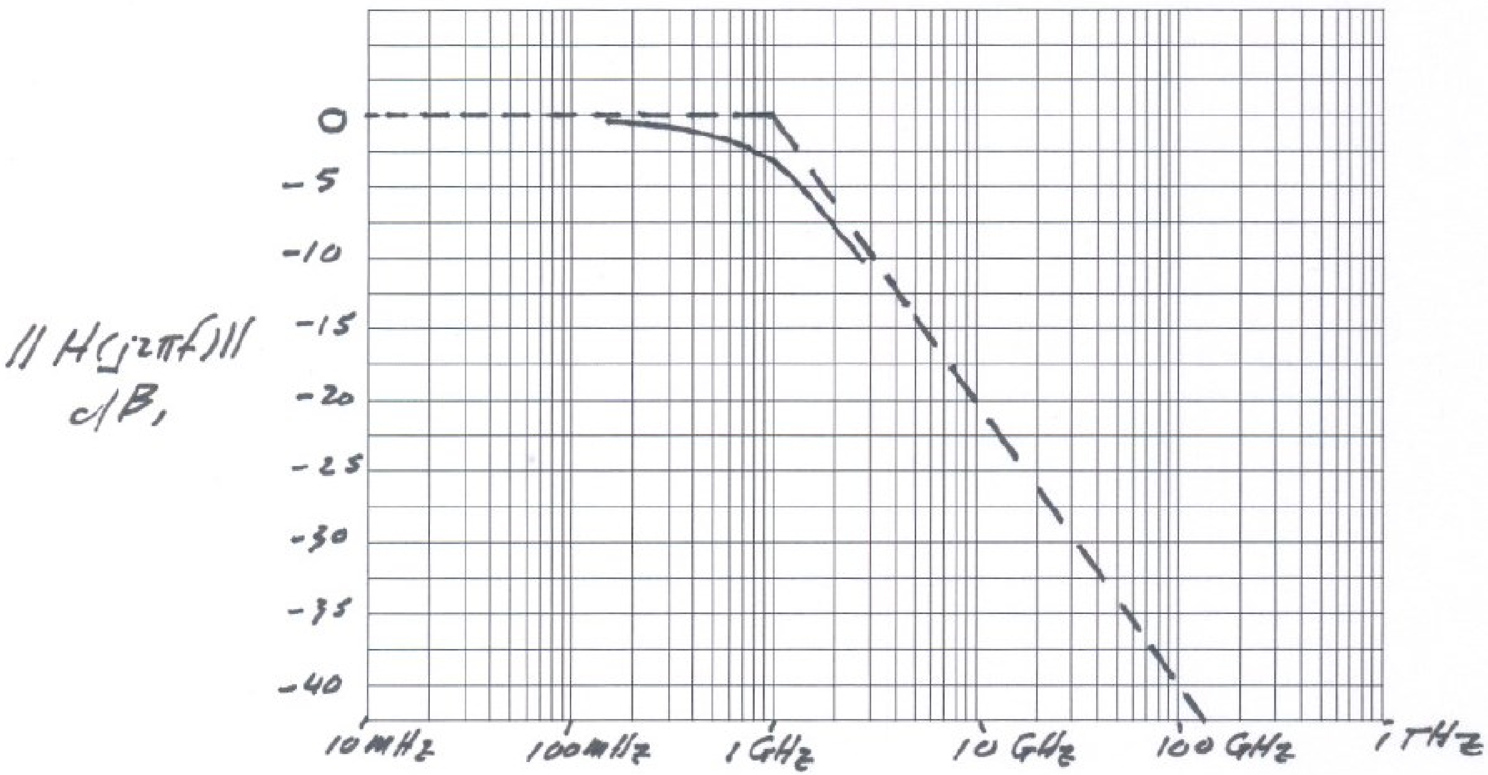
The phase of the output leads the phase angle of the input by an angle $= \angle H(j\omega)$

$$H(j\omega) = \frac{1}{1 + j\omega\tau} \rightarrow \| H(j\omega) \| = \frac{1}{\sqrt{1+\omega^2\tau^2}} \text{ and } \angle H(j\omega) = -1 \cdot \arctan(\omega\tau)$$

Bode Plots To Represent Frequency Responsee.

Represent the amplitude and phase of $H(j\omega) = H(j2\pi f)$ vs. frequency.

Vertical axis in dB: $10 \cdot \log_{10}(\text{power ratio}) = 20 \cdot \log_{10}(\text{voltage ratio})$

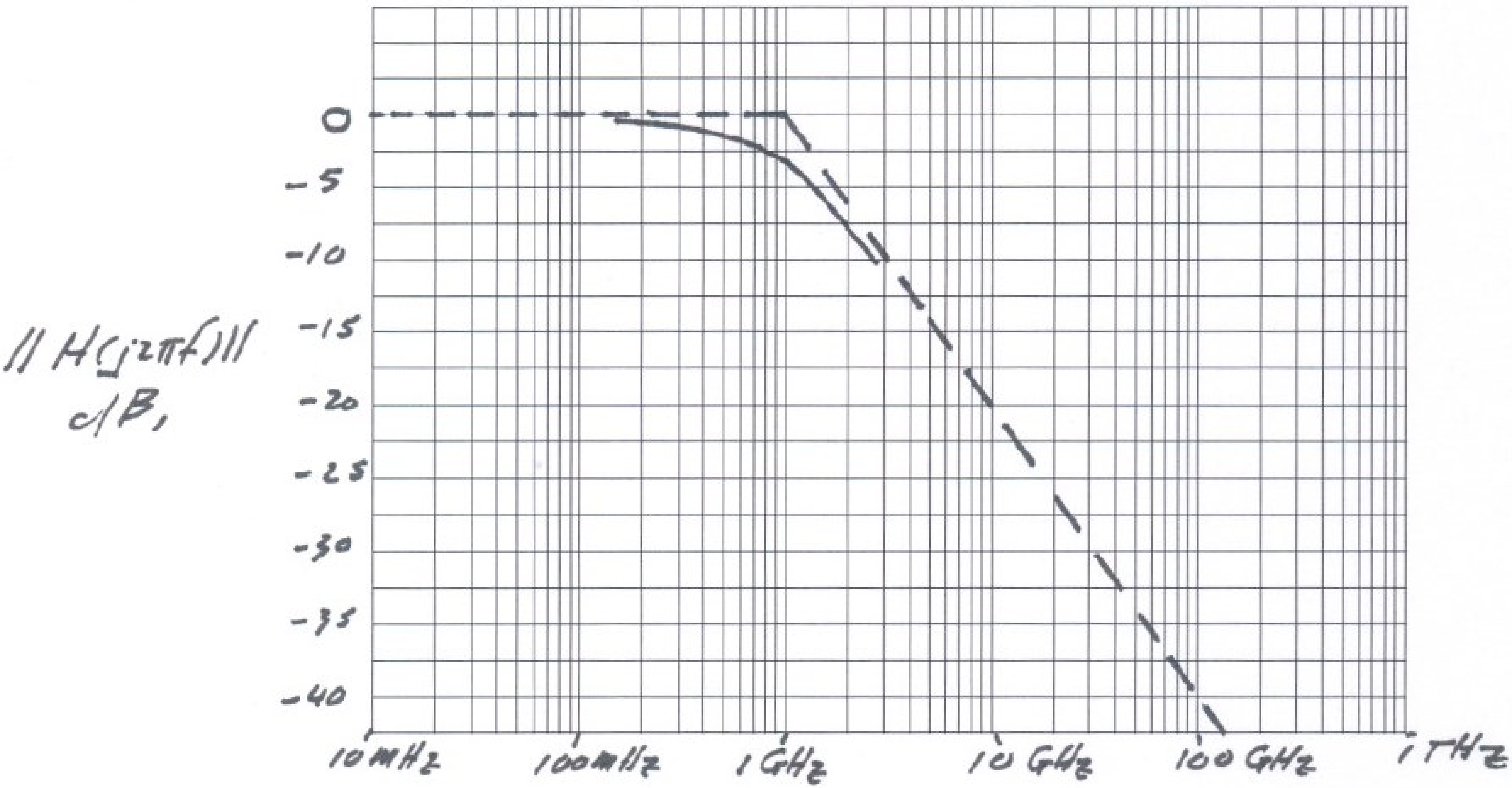


Example: $H(j\omega) = (1 + j\omega T)^{-1}$; $T = 159\text{ ps}$

↓

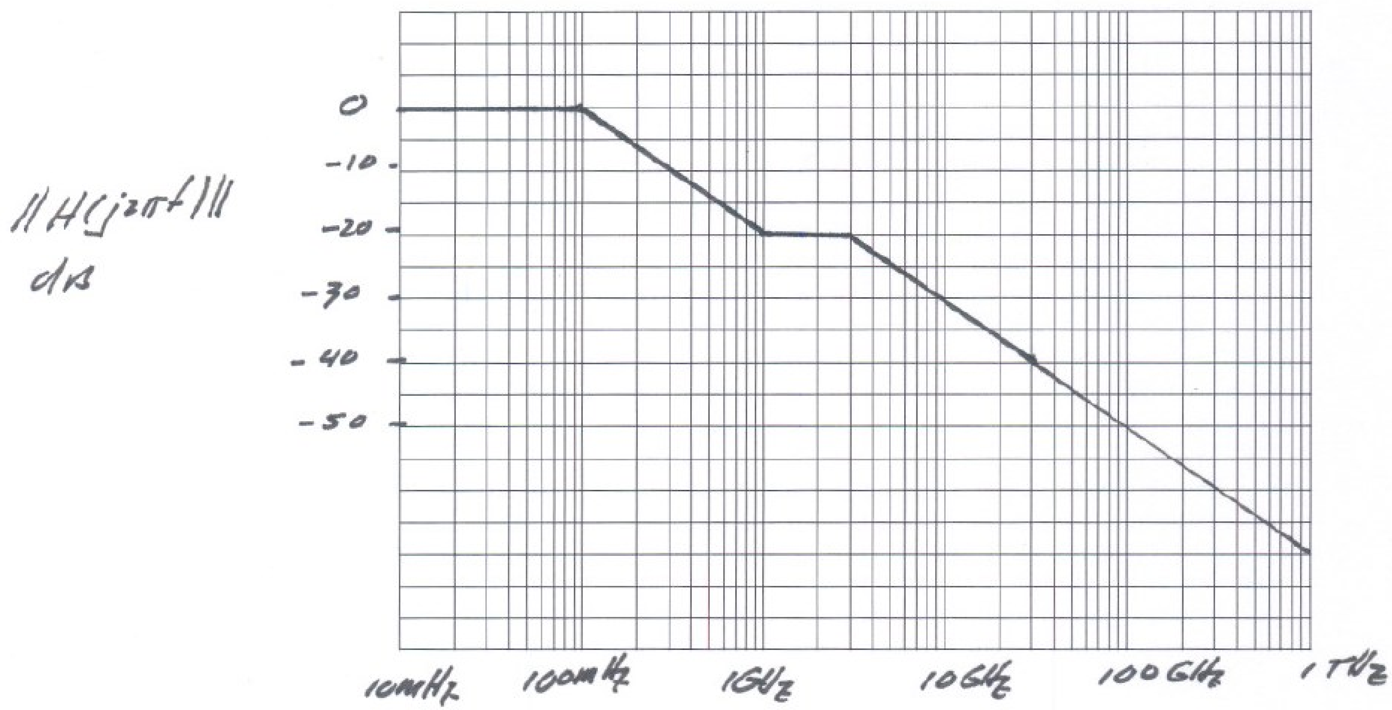
$H(j2\pi f) = (1 + jf/f_{pole})^{-1}$ where $f_{pole} = 1\text{ GHz}$

Asymptotic (Straight-Line) vs. Actual Bode Plots



Asymptotic plot is often more informative than actual curve.

Example of More Complex Asymptotic Plot:



Example:

$$H(j2\pi f) = \frac{(1 + jf/f_{z1})}{(1 + jf/f_{p1})(1 + jf/f_{p2})}$$

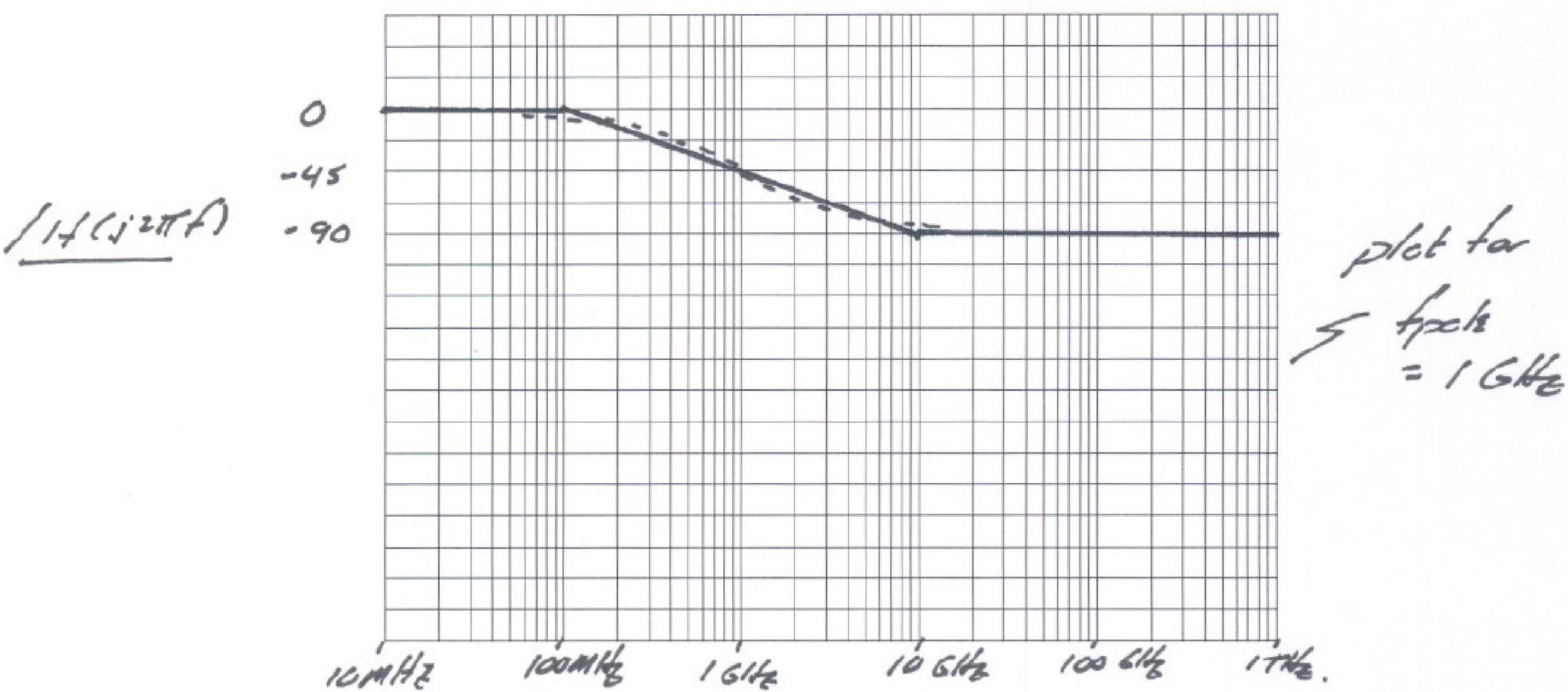
$f_{p1} = 100\text{MHz}$ $\quad f_{p2} = 3\text{ GHz}.$

$f_{z1} = 1\text{ GHz}$

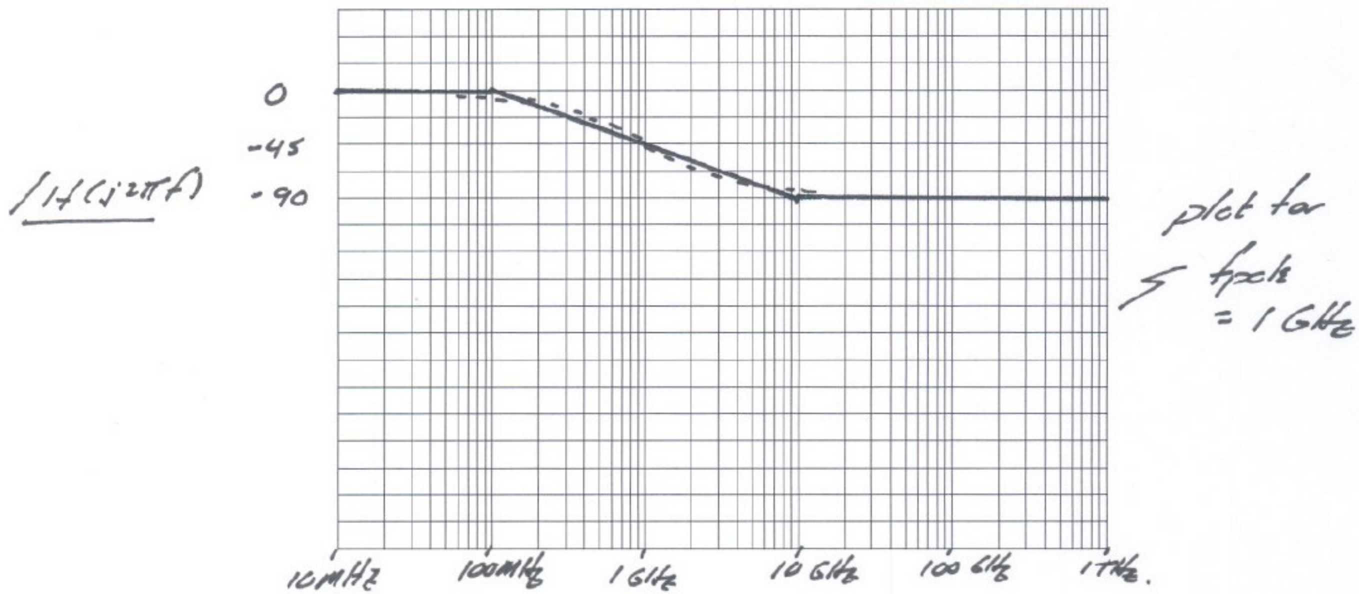
It is much easier to recognize the poles and zeros in the asymptotic plot.

Bode Phase Plot

This is $\angle H(j2\pi f)$ plotted vs. frequency, using a logarithmic frequency axis.



Bode Phase Plot



= For a single-pole transfer function:

$$H(j2\pi f) = \frac{1}{1 + jf/f_p} \qquad f_p = 1/2\pi\tau$$

$$\angle H(j2\pi f) = -\arctan(\omega\tau) = -\arctan(f/f_{pole})$$

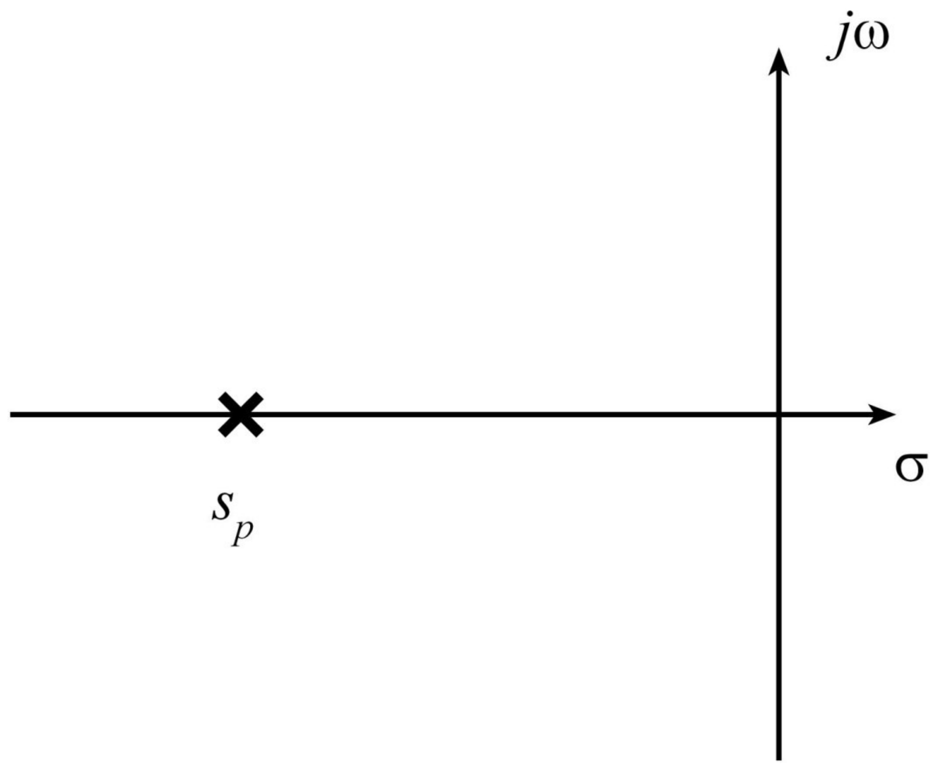
= exact plot is well-approximated by 3 asymptotes as indicated.

Root (Pole-Zero) Constellation

This is a graphical tool to represent and calculate frequency response.

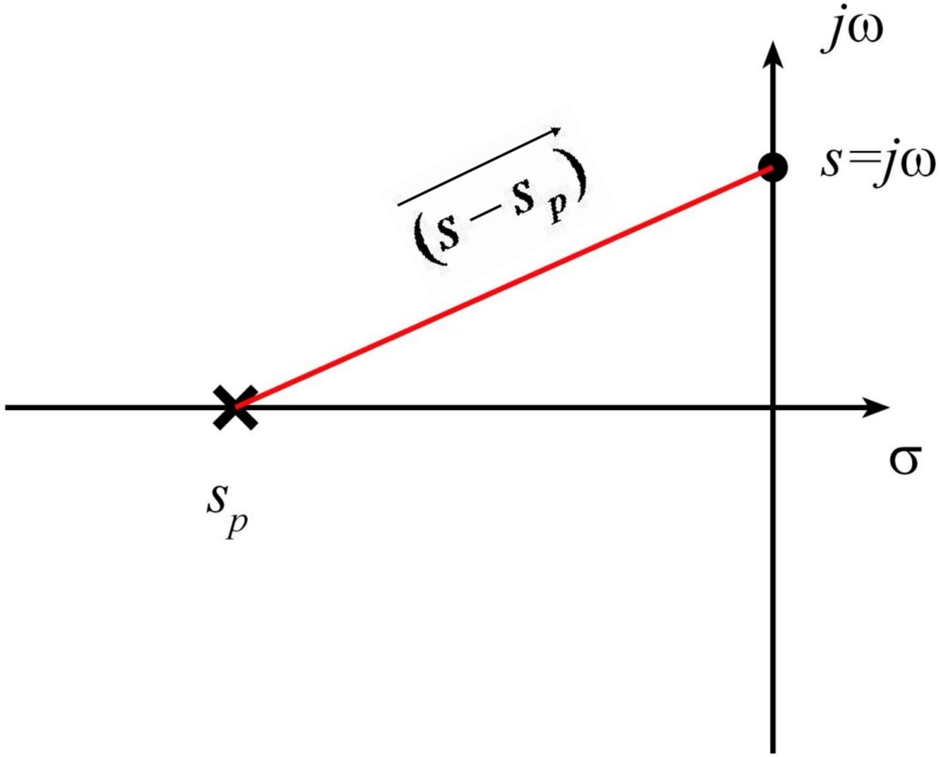
Given a transfer function $H(s)=\dfrac{1}{1+s\tau}=\dfrac{1}{\tau}\dfrac{1}{s-s_p}$ where $s_p=-1/\tau$,

the root locus is represented like so:



Root Constellation: Magnitude

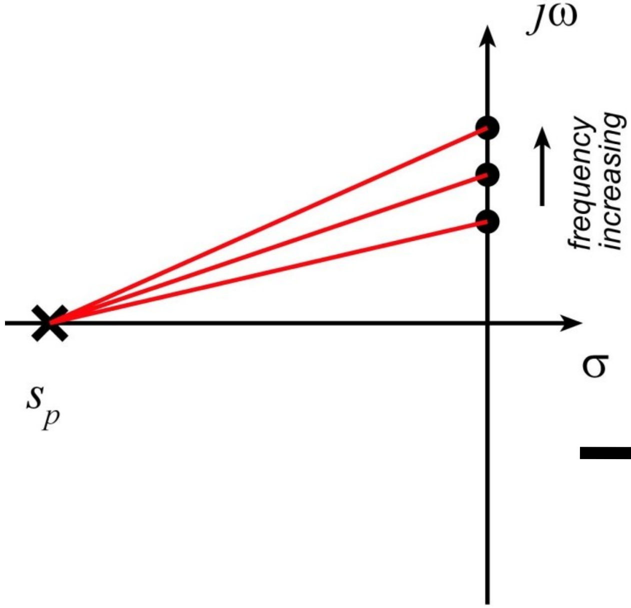
$$\| H(s) \| = \frac{1}{\| 1 + s\tau \|} = \frac{1}{\| \tau \|} \frac{1}{\| s - s_p \|} = \frac{1}{\| \tau \|} \frac{1}{D_z}$$

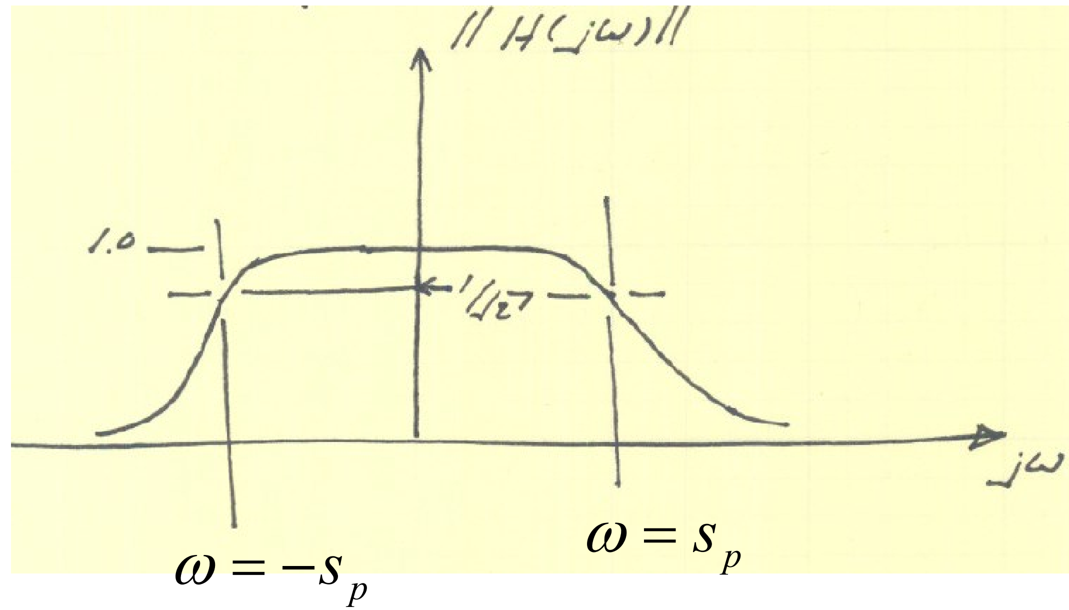


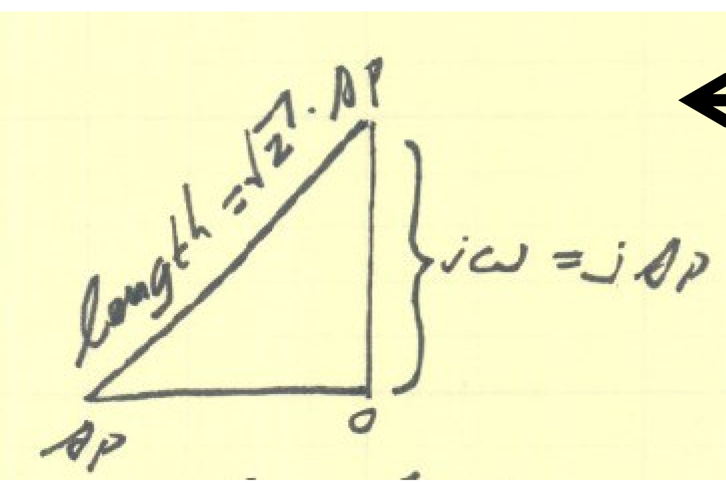
$(s - s_p)$ is a vector, and $H(s)$ varies as the inverse of its length:

$H(s) \propto 1 / \| (s - s_p) \|$

Root Constellation: Plotting Frequency Response



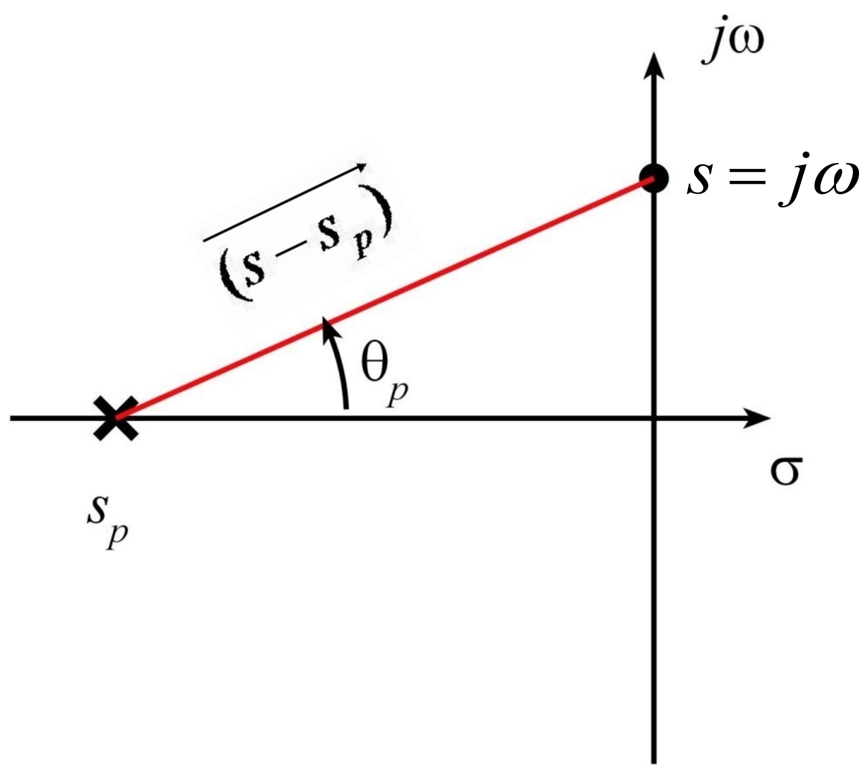




From this diagram, it is clear that the transfer function must be reduced to $1/\sqrt{2} = -3$ dB when $\omega = s_p$

Root Constellation: Plotting Phase Response

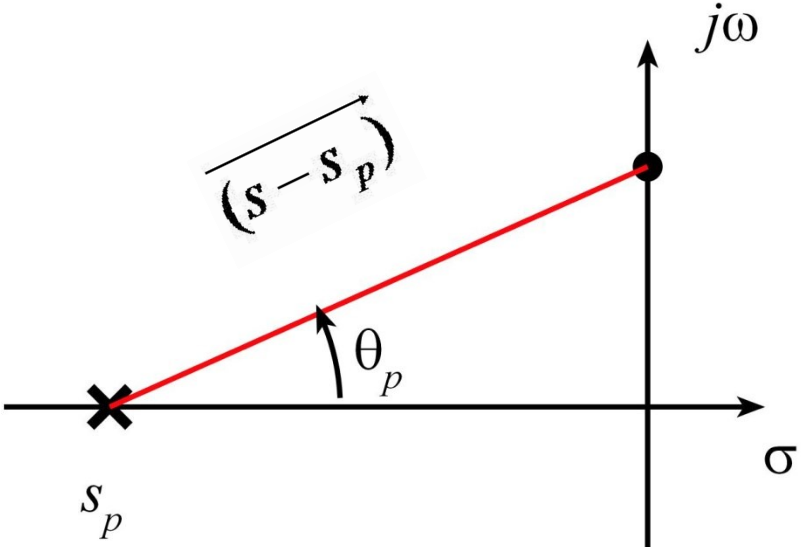
$$\angle H(s) = -1 \cdot \angle(1 + s\tau) = -1 \cdot \angle(s - s_p) = -\theta_p$$

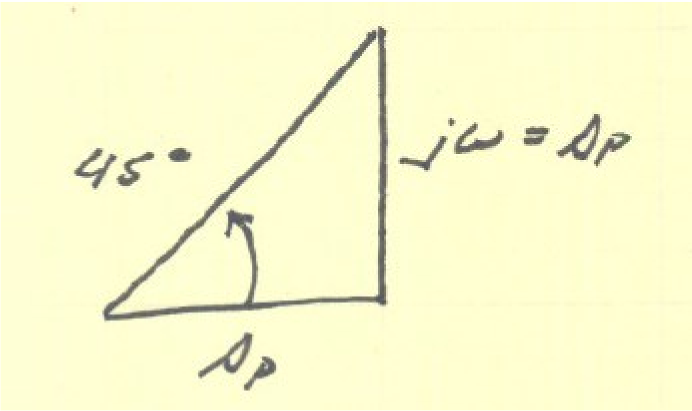


θ_p is the angle of the vector $(s - s_p)$

Root Constellation: Plotting Phase Response

$$\angle H(s) = -1 \cdot \angle(1 + s\tau) = -1 \cdot \angle(s - s_p) = -\theta_p$$





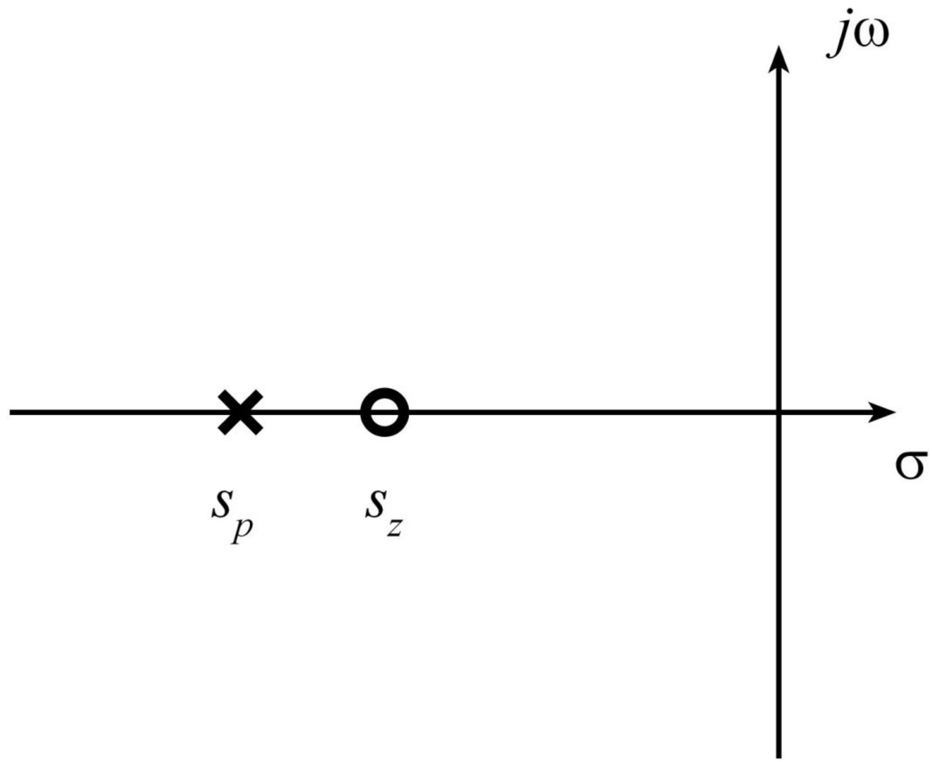
It is clear that $\angle H(j\omega) = -45^o$ when $\omega = s_p$.

Further, $\angle H(j\omega)$ clearly varies from 0^o to -45^o as the frequency varies from DC to infinity.

Root Constellation: A pole and a zero

$$\text{If } H(s)=\frac{1+s\tau_{zero}}{1+s\tau_{pole}}=\frac{\tau_z}{\tau_p}\frac{s-s_z}{s-s_p} \text{ where } s_p=-1/\tau_p \;, s_p=-1/\tau_z$$

then the root locus is :

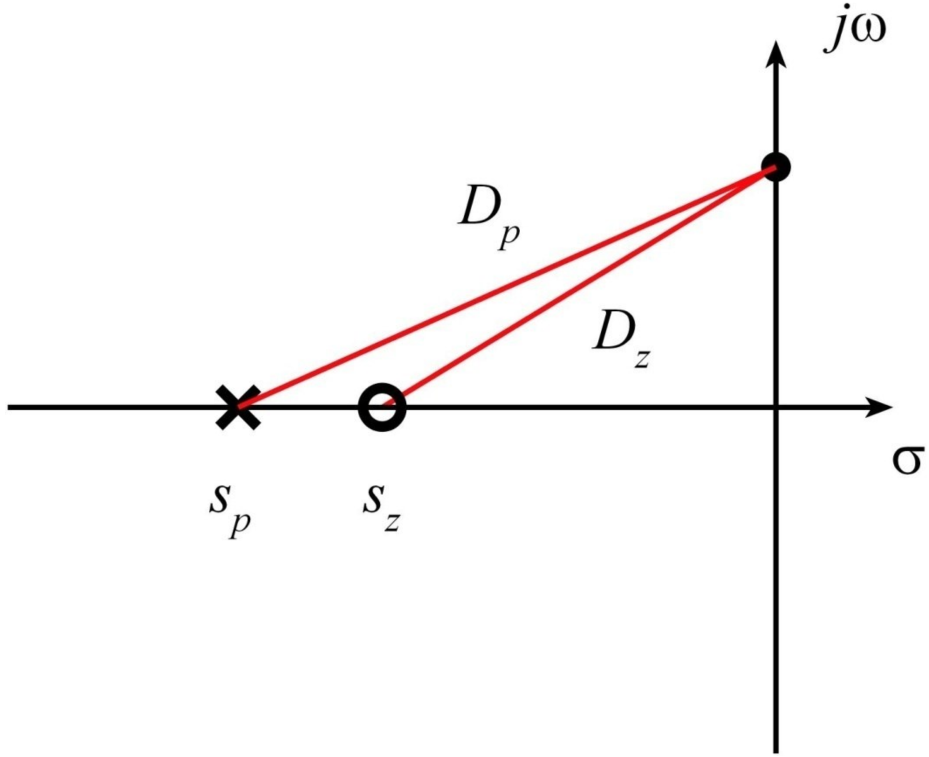


Transfer Function Magnitudes: Poles and Zeros

$$\| H(s) \| = \frac{\| 1 + s\tau_{zero} \|}{1 + s\tau_{pole} \|} = \frac{\tau_z}{\tau_p} \frac{\| s - s_z \|}{\| s - s_p \|} = \frac{\tau_z}{\tau_p} \frac{D_z}{D_p}$$

where D_z is the distance to the zero and D_p is the distance to the pole.

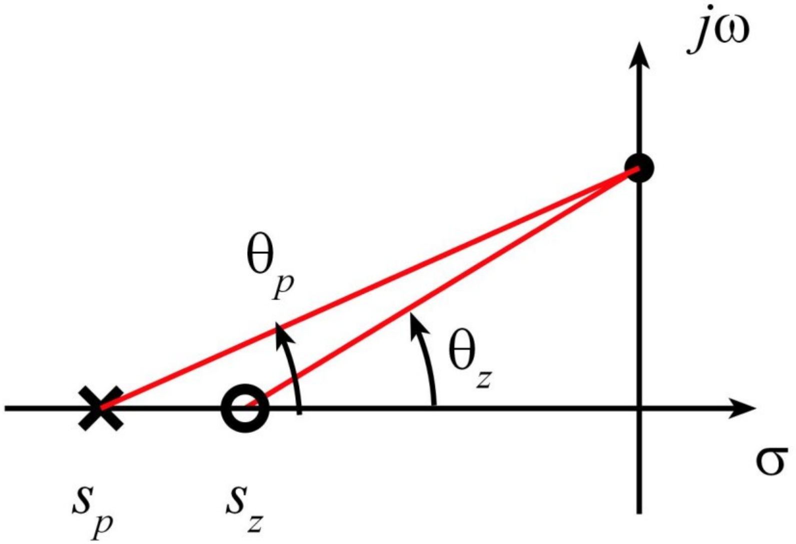
What would the answer be if there were 4 poles and 3 zeros?



Transfer Function Phase: Poles and Zeros

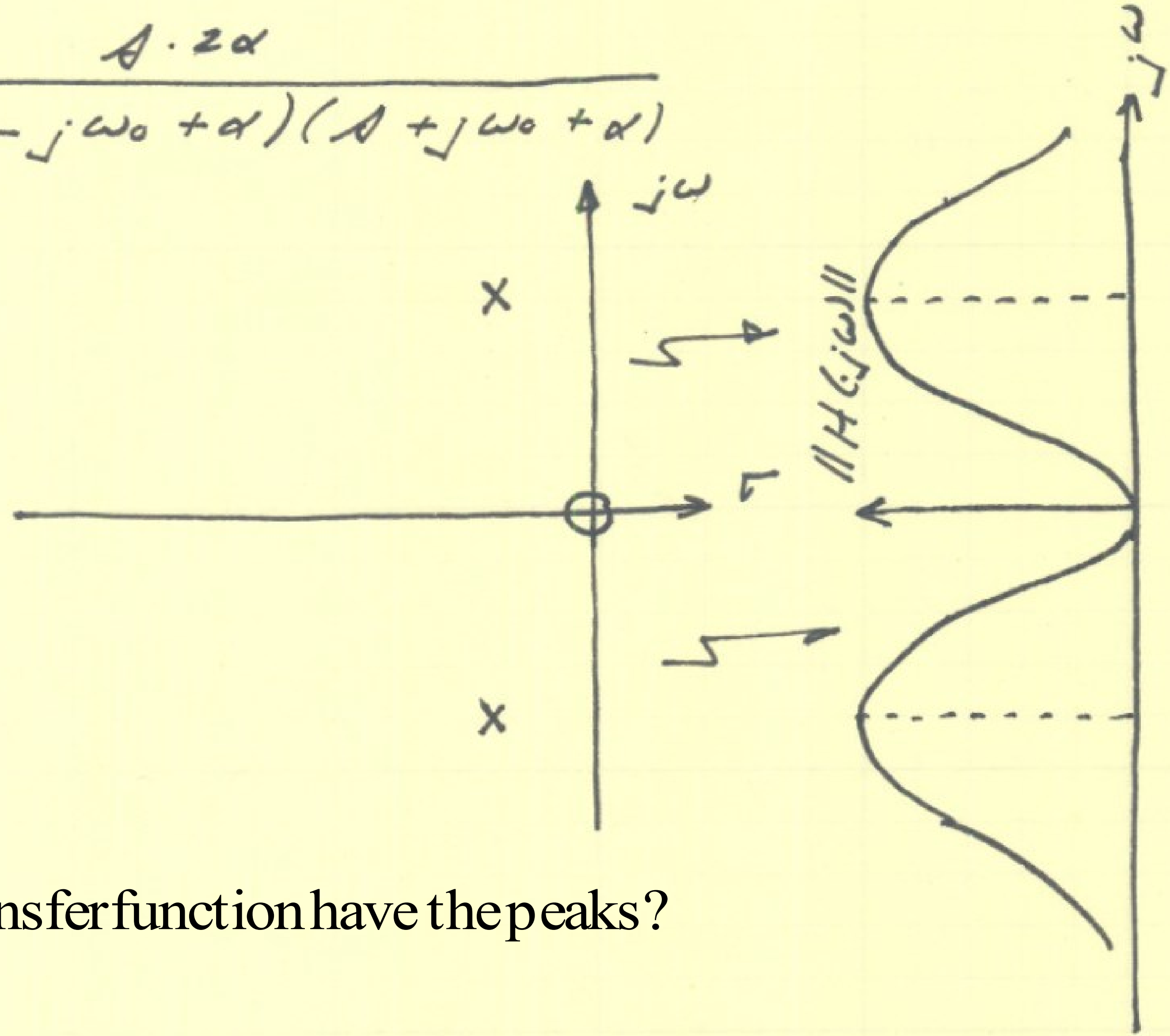
$$\angle H(s) = \frac{\angle(1+s\tau_{zero})}{\angle(1+s\tau_{pole})} = \angle(s-s_z) - \angle(s-s_p) = \theta_z - \theta_p$$

What would the answer be with 3 poles and 2 zeros?



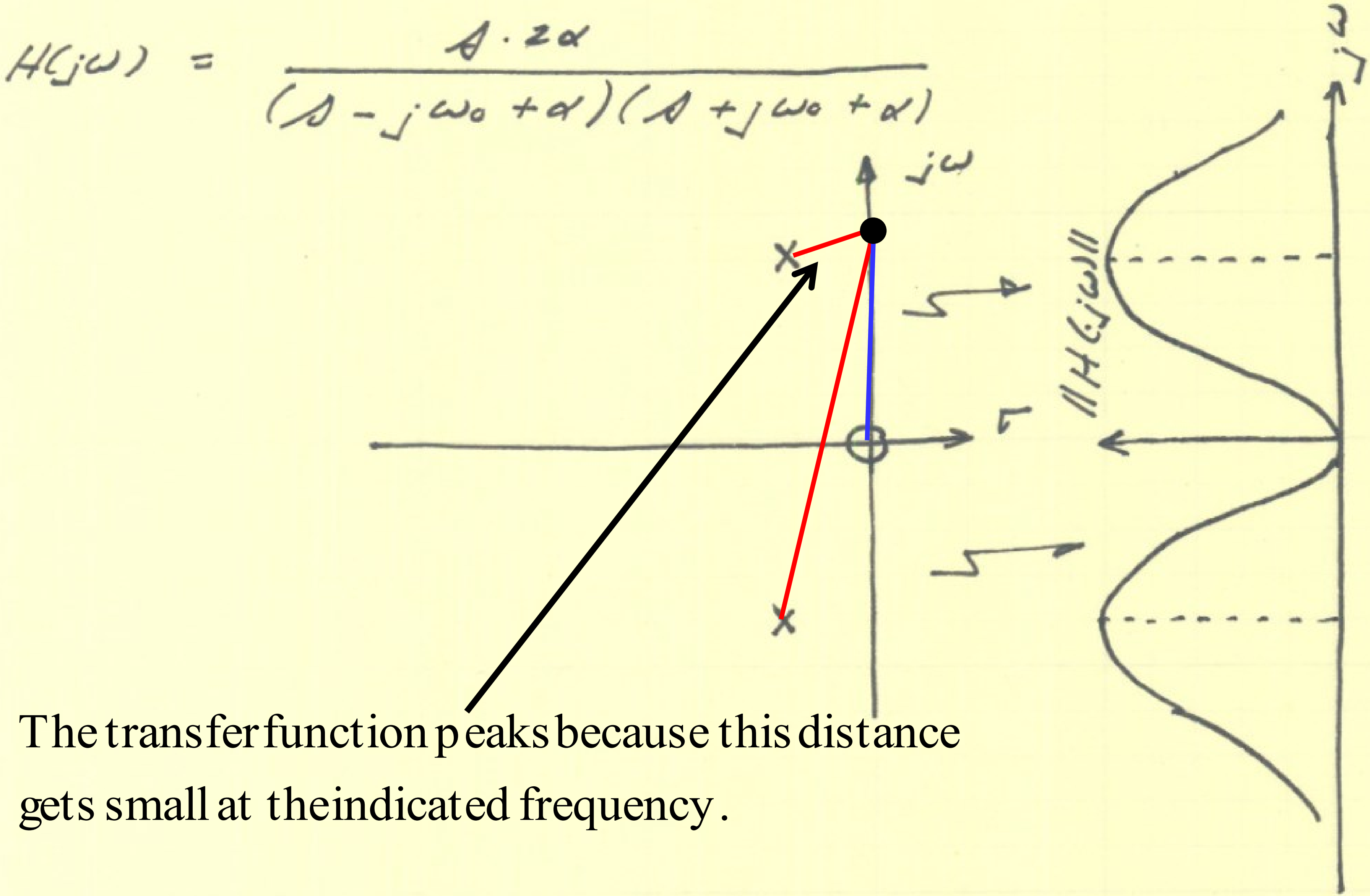
Root Constellation and Frequency Response: Complex Poles

$$H(j\omega) = \frac{s \cdot 2\alpha}{(s - j\omega_0 + \alpha)(s + j\omega_0 + \alpha)}$$



Why does the transfer function have the peaks?

Root Constellation and Frequency Response: Complex Poles



The transfer function peaks because this distance gets small at the indicated frequency.

Impulse Response: First we should check units.

Recall that $\int_{-\infty}^{+\infty} \delta(t)\, dt = 1.$

But "1" has no units, while t has units of time,

so $\delta(t)$ has units of 1/(time).

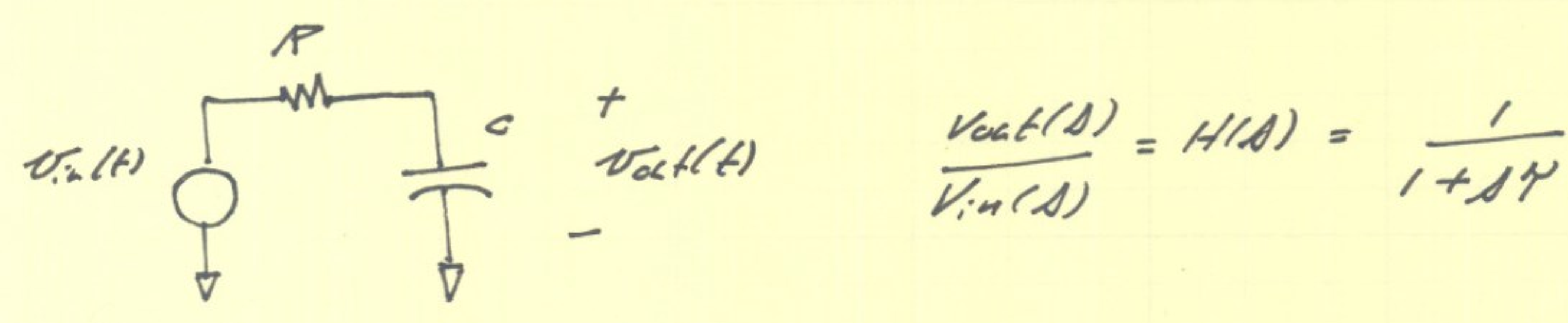
Now consider $V(s) = \int_{0}^{+\infty} v(t) e^{-st}\, dt.$

$v(t)$ has units of volts, t has units of time, so

$V(s)$ has units of (volts)$\cdot$(time)

Checking units with each line of a set of calculations is an excellent way to help catch mistakes.

Impulse Response



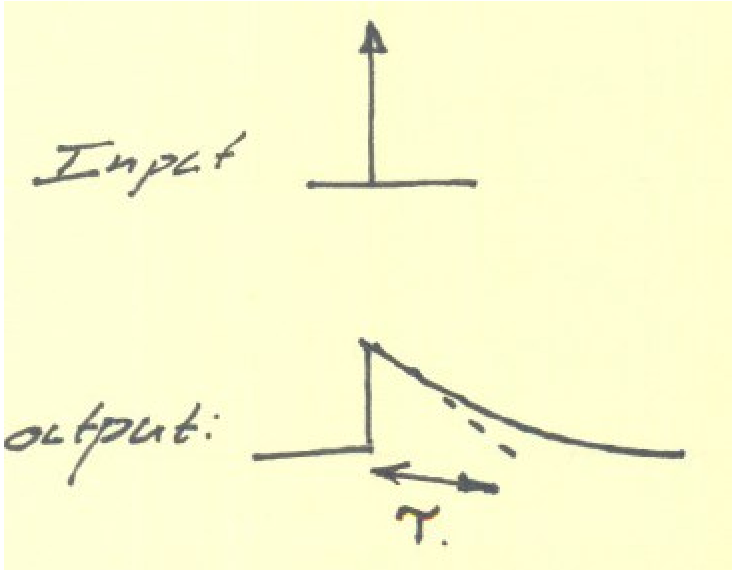
$v_{in}(t) = k \cdot \delta(t)$ where $\delta(t)$ has units of 1/(time), k has units of volts·time.

$V_{in}(s) = k$ consistent units, as $V(s)$ has units of volts * time.

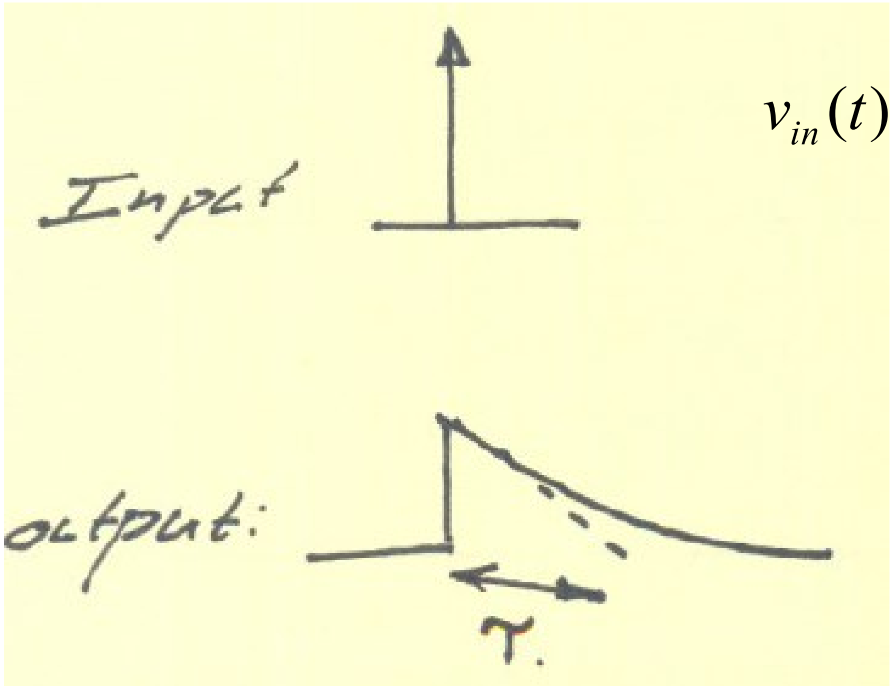
$$V_{out}(s) = \frac{k}{1+s\tau}$$

$$v_{out}(t) = \frac{k}{\tau} e^{-t/\tau} \cdot u(t)$$

units again check correctly.

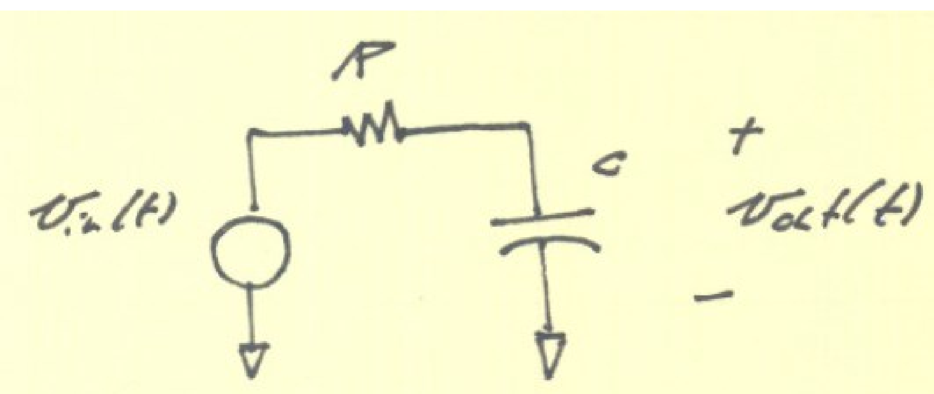


Impulse Response



$$v_{in}(t) = k \cdot \delta(t)$$

$$v_{out}(t) = \frac{k}{\tau} e^{-t/\tau} \cdot u(t)$$



Output has decayed to 50% of its original value when

$$t_{50\%} = \tau \cdot \ln(2) = 0.693 \cdot \tau$$

Step Response

$$v_{in}(t) = v_0 \cdot u(t)$$ } units: volts

$$V_{in}(s) = v_0 / s$$

$$V_{out}(s) = \frac{v_0}{s} \frac{1}{1+s\tau}$$

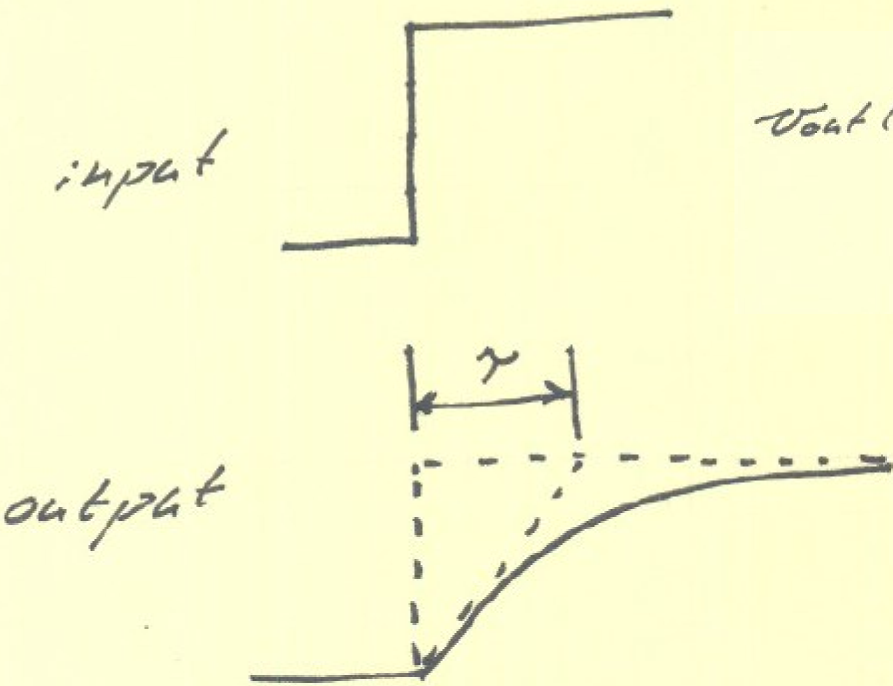
$$= \frac{v_0}{s} - \frac{v_0 \tau}{1+s\tau}$$

} units: volts x time

$$v_{out}(t) = v_0 \cdot u(t) - v_0 \cdot u(t) \cdot e^{-t/\tau}$$

$$= v_0 \cdot \left(1 - e^{-t/\tau}\right) \cdot u(t)$$

Step Response: 10%-90% Risetime



$$v_{out}(t) = v_0 \cdot u(t) - v_0 \cdot u(t) \cdot e^{-t/\tau}$$

$$= v_0 \cdot (1 - e^{-t/\tau}) \cdot u(t)$$

$$\text{10\%-90\% risetime} = T_{10-90} = \tau[\ln(0.9) - \ln(0.1)]$$

$$= 2.2 \cdot \tau$$

Risetime and 3 dB bandwidth

So for a single-pole system

$$\|H(j\omega)\| = \sqrt{\frac{1}{1+\omega^2/\omega_p^2}} \qquad \omega_p = 1/\tau$$

-3 dB when $\omega = 1/\tau$ $(f = f_p)$

this is the 3-dB-bandwidth f_{3dB}

step response risetime $= 2.2 \cdot \tau$

$$T_{10\%-90\%} = 2.2\,\tau$$

$$= 2.2/\omega_{3dB}$$

$$= 2.2/(f_{3dB}/2\pi)$$

$$\boxed{T_{10-90} = 0.35/f_{3dB}}$$

Exact for single pole system. Rough approximation in other cases.