ECE 137 B: Notes Set 13
Feedback loops: the Bode Stability Criterion

Mark Rodwell
Doluca Family Chair
University of California, Santa Barbara

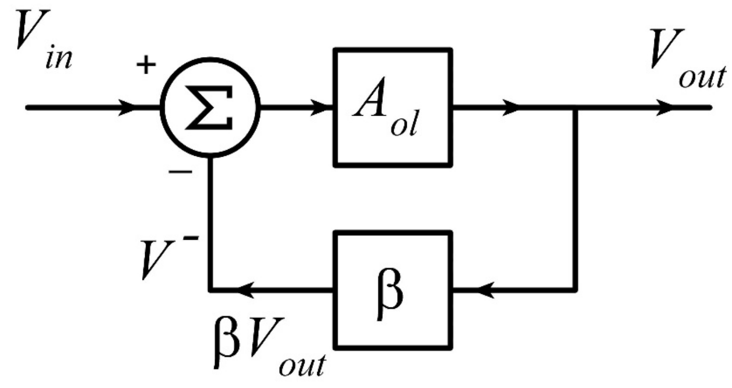
rodwell@ucsb.edu 805-893-3244

Testing for Feedback Loop Stability

$$A_{OL}(s)=\frac{N_{AOL}(s)}{D_{AOL}(s)};\ \beta(s)=\frac{N_\beta(s)}{D_\beta(s)};\ T(s)=A_{OL}\beta=\frac{N_{AOL}N_\beta}{D_{AOL}D_\beta}=\frac{N_T}{D_T}$$

$$A_{CL}(s)=\frac{V_{out}}{V_{in}}=\frac{A_{OL}}{1+A_{OL}\beta}=\frac{N_{AOL}/D_{AOL}}{1+N_{AOL}N_\beta/D_{AOL}D_\beta}$$

$$=\frac{N_{AOL}D_\beta}{D_{AOL}D_\beta+N_{AOL}N_\beta}=\frac{N_{AOL}D_\beta}{N_T+D_T}$$



We have seen that if $T(s)$ has 3 or more poles, the system can be unstable if the magnitude of the loop transmission is sufficiently large.

Algebraically, we can find the pole frequencies s_p of $A_{CL}(s)$ by solving for

$N_T(s_p)+D_T(s_p)=0$

...and then check to see if any of the poles lie in the right half of the s-plane,

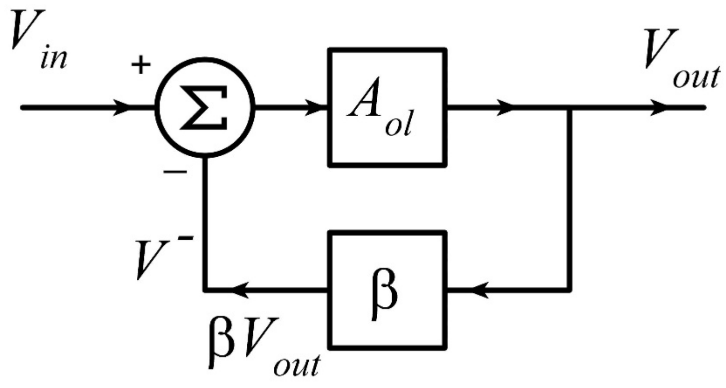
i.e. , $s_p=\sigma_p \pm j\omega_p$; are any of the σ_p positive ? If so, the feedback loop is unstable

This can be very difficult and/or tedious.

The Bode method is an alternative feedback stability test.

Bode Stability Test (1)

We compute $T(j\omega)$ or $T(j2\pi f)$, not $T(s)$,
increasing frequency from DC to $+\infty$.



The feedback loop will be stable if, as frequency increases,
The angle of T does not reach 180^0 until
the magnitude of T decreases below 1.

To restate more mathematically:

$$\text{If both } \begin{cases} \|T(j2\pi f)\| > 1 & \text{for } 0 < f < f_{loop} \\ \|T(j2\pi f)\| < 1 & \text{for } f > f_{loop} \end{cases}$$

and if $\angle T(j2\pi f) < 180^0$ for $0 < f < f_{loop}$

then the feedback loop is stable.

If the feedback loop fails the Bode test,
it might or might not be unstable

Simple (but risky) interpretation of Bode Criterion (1)

$$A_{CL} = \frac{1}{\beta}\frac{T(j2\pi f)}{1+T(j2\pi f)}$$

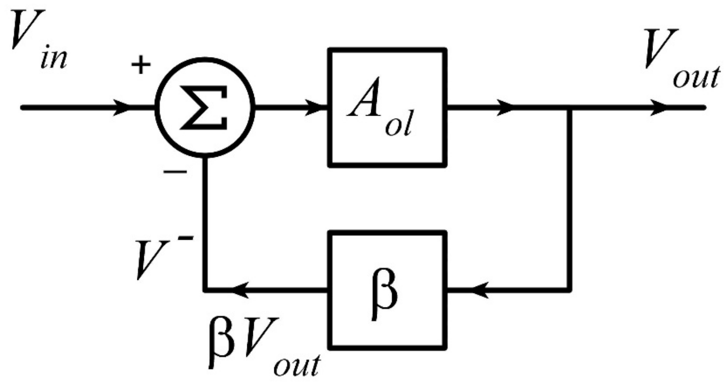
At $f = f_{loop}$, $\|T(j2\pi f)\| = 1$.

If, at $f = f_{loop}$, $\angle T(j2\pi f) = 180^0$ then $T(j2\pi f_{loop}) = -1$

hence

$$A_{CL}(j2\pi f_{loop}) = \frac{1}{\beta}\frac{-1}{1-1} = \infty = \frac{V_{out}(j2\pi f_{loop})}{V_{in}(j2\pi f_{loop})}$$

This implies an nonzero output with zero input = oscillation



This simple interpretation is in fact quite questionable.

Check:

If $\angle T(j2\pi f_{loop}) = 179^0$, then the loop is stable.

If $\angle T(j2\pi f_{loop}) = 180^0$, then the loop is unstable

If $\angle T(j2\pi f_{loop}) = 181^0$, is the loop stable ?

Slightly less risky interpretation of Bode Criterion (1)

Example: feedback loop with 3 poles in $T(s)=\dfrac{T_0}{1+a_1s+a_2s^2+a_3s^3}$,

As we increase T_0 to increase the feedback loop gain, the poles move as shown.

At $T_0=T_{01}$, the poles of $A_{CL}(s)$ lie in the LHP

$$\rightarrow \frac{T(j\omega_{loop})}{1+T(j\omega_{loop})}\neq\infty \text{ and } \angle T(j\omega_{loop})<180^0$$

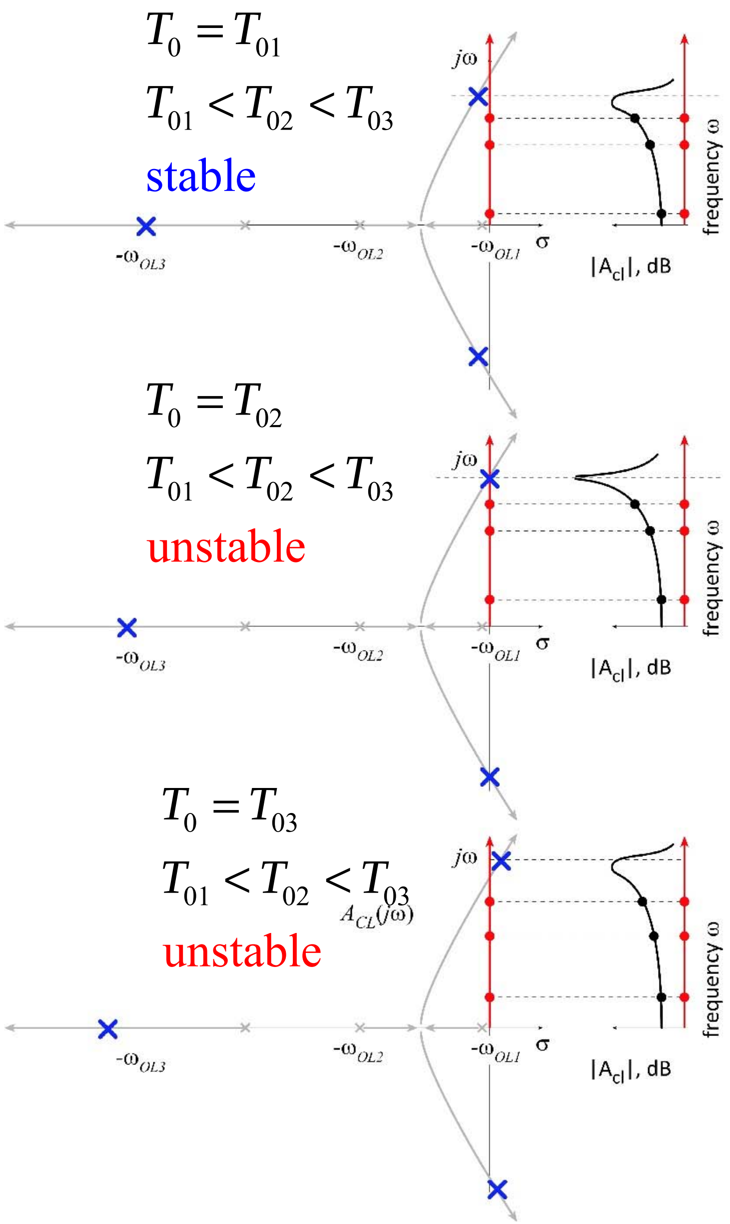
At $T_0=T_{02}$, 2 of 3 poles of $A_{CL}(s)$ lie on the $j\omega$ axis: $s_{pole}=0\pm j\omega_{loop}$

$$\rightarrow \frac{T(j\omega_{loop})}{1+T(j\omega_{loop})}=\infty \;\rightarrow\; \angle T(j\omega_{loop})=180^0$$

At $T_0=T_{03}$, 2 of 3 poles of $A_{CL}(s)$ lie in the RHP

$$\rightarrow \frac{T(j\omega_{loop})}{1+T(j\omega_{loop})}\neq\infty \text{ and } \angle T(j\omega_{loop})>180^0$$

(recollect that, by definition of ω_{loop}, $\|T(j\omega_{loop})\| =1$)



Danger with circuit simulations

With $T_0 = T_{01}$, the poles of $A_{CL}(s)$ lie in the LHP; feedback is stable

$\| A_{CL}(j\omega) \|$ is finite for all frequencies

At $T_0 = T_{02}$, the poles of $A_{CL}(s)$ lie on the $j\omega$ axis: feedback is unstable

$\| A_{CL}(j\omega) \|$ is infinite at $\omega = \omega_{loop}$

At $T_0 = T_{03}$, the poles of $A_{CL}(s)$ lie in the RHP; feedback is unstable

$\| A_{CL}(j\omega) \|$ is finite for all frequencies.

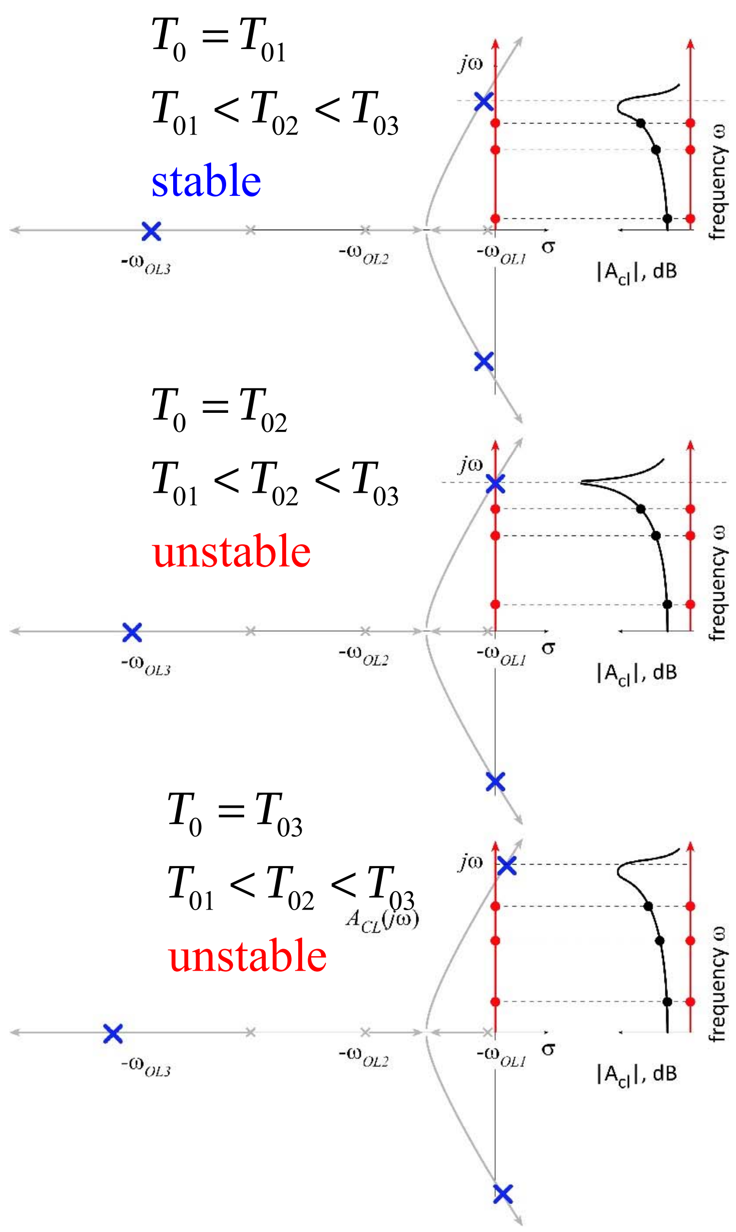
Finite $\|A(j\omega)\|$ for all frequencies does not mean that feedback is stable.

Finite $\|A_V(j\omega)\|$ for all frequencies does not mean that any circuit is stable.

Feedback loops: must use Bode, Nyquist, Root locus etc. stability test.

Other non-explicit-feedback circuits: stability tests can be difficult.

Unambigous test: transient (step or impulse response)



Bode stability test: example 1

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

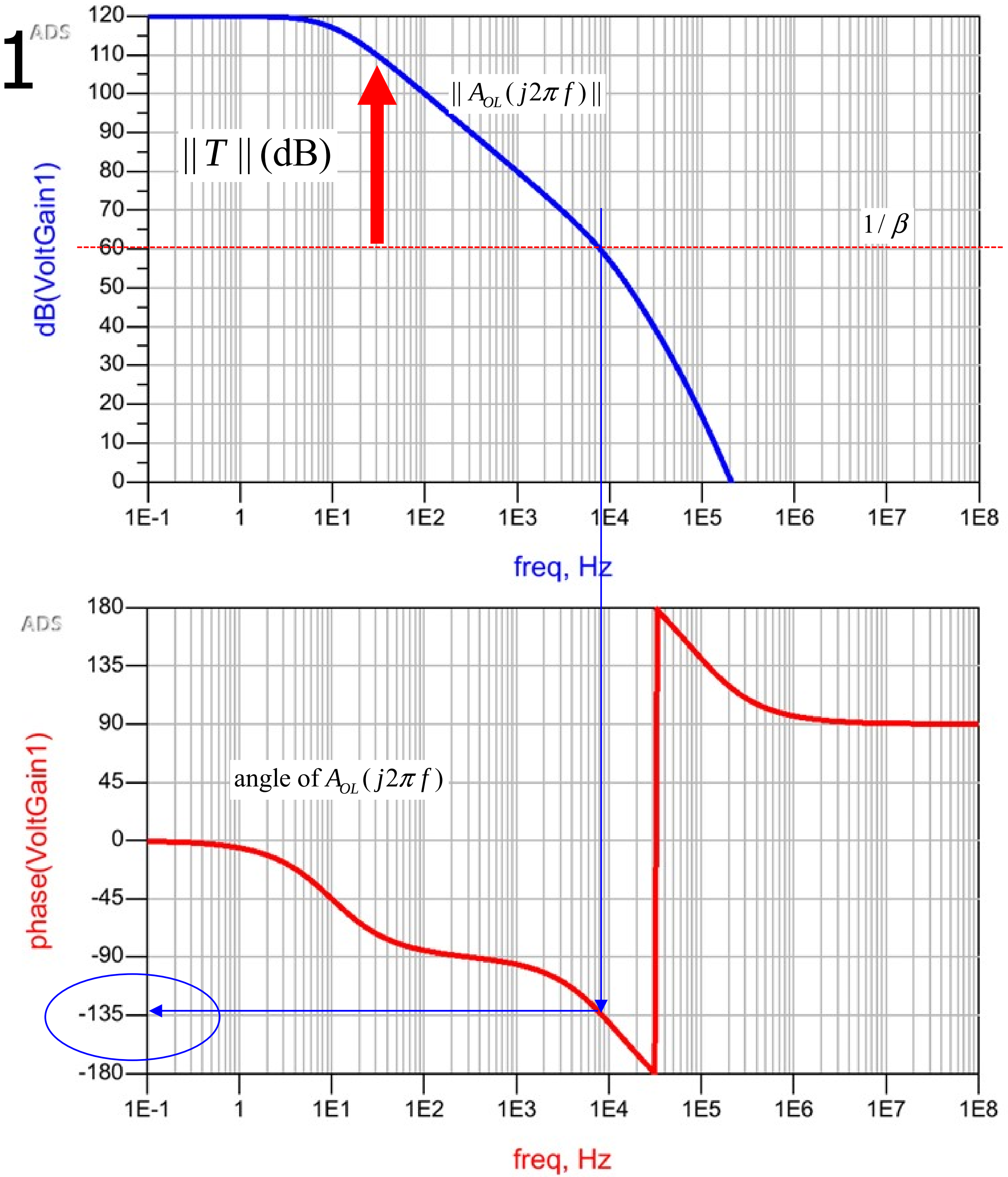
$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$$\beta(j2\pi f) = \beta_0 = 1/1000$$

$f_{loop} \approx 8$ kHz

$\angle T \approx -130^0$ at 8 kHz

phase margin = $180^0 - 130^0 = 50^0$



Bode stability test: example 1

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

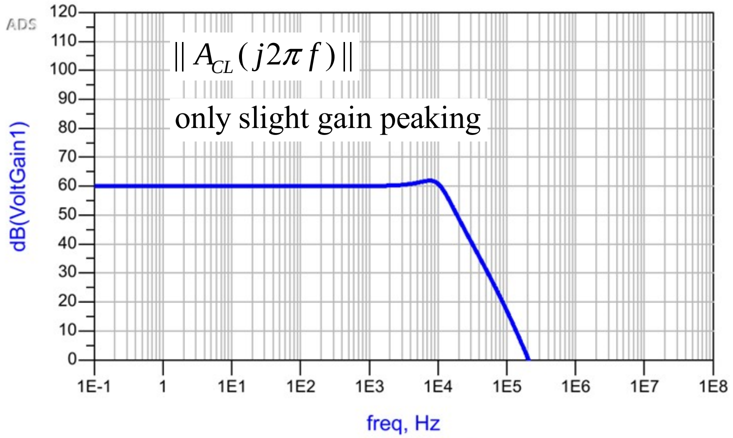
$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

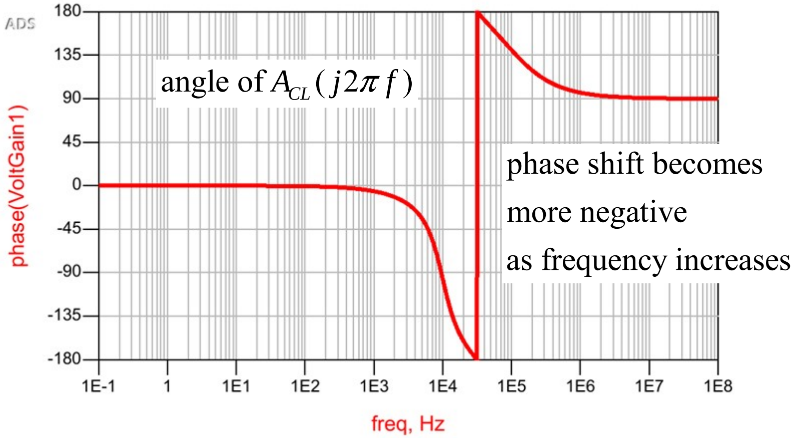
$$\beta(j2\pi f) = \beta_0 = 1/1000$$

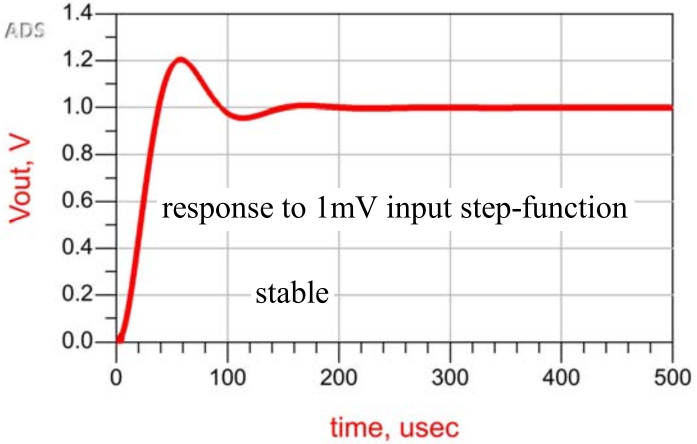
$$f_{loop} \approx 8\text{ kHz}$$

$$\angle T \approx -130^0 \text{ at } 8\text{ kHz}$$

$$\text{phase margin} = 180^0 - 130^0 = 50^0$$







Bode stability test: example 2

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$$\beta(j2\pi f) = \beta_0 = 1/100$$

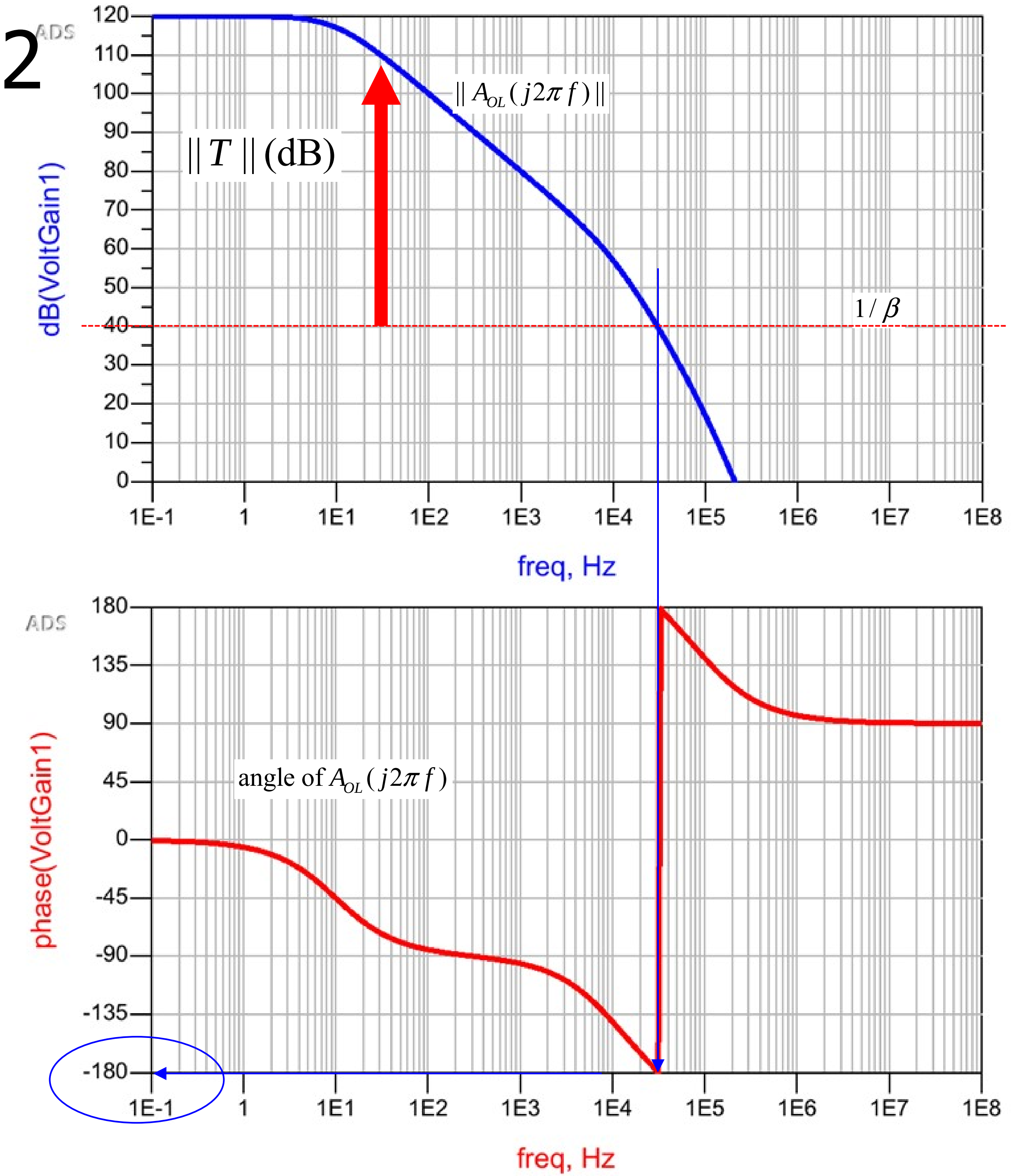
$f_{loop} \approx 25$ kHz

$\angle T \approx -179^0$ at 25 kHz

phase margin = $180^0 - 179^0 = 1^0$

Loop is just barely stable

....had to do transient test to be sure...



Bode stability test: example 2

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$$\beta(j2\pi f) = \beta_0 = 1/100$$

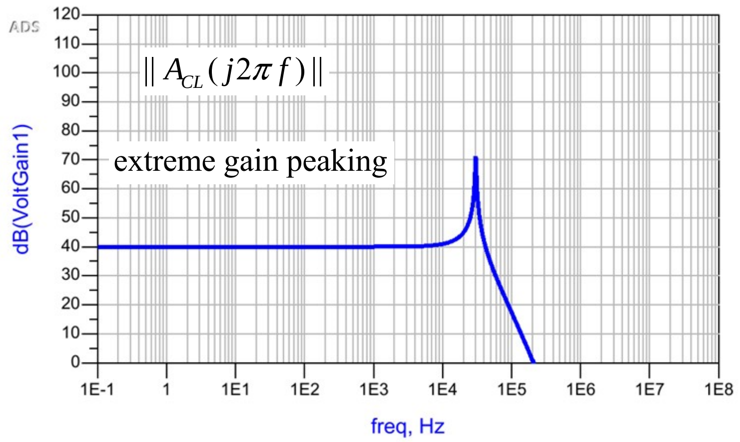
$f_{loop} \approx 25$ kHz

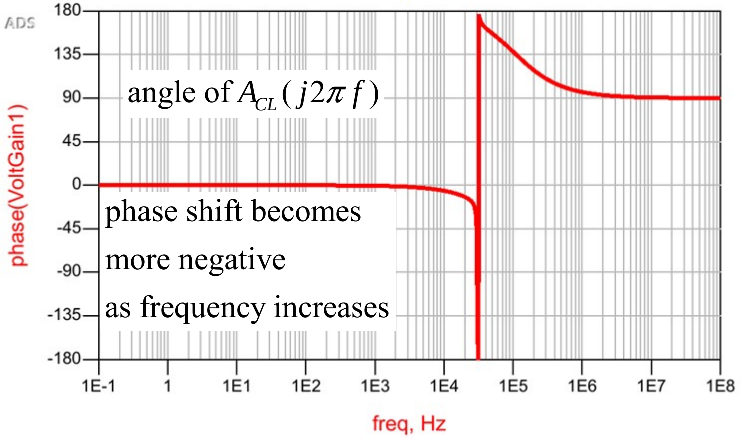
$\angle T \approx -179^0$ at 25 kHz

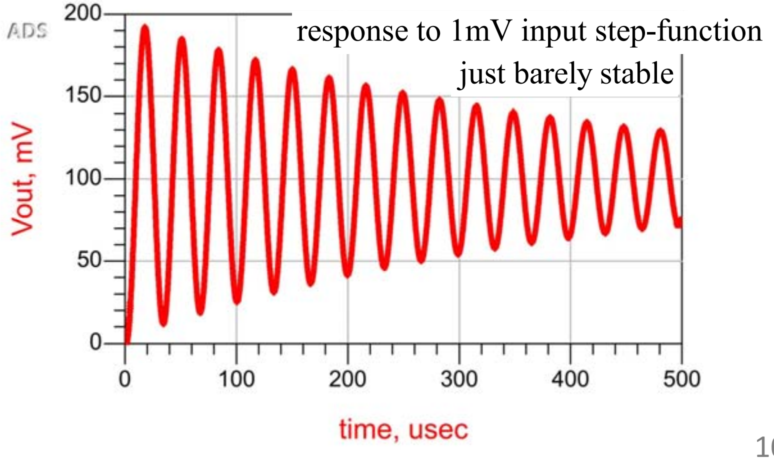
phase margin = $180^0 - 179^0 = 1^0$

Loop is just barely stable

....had to do transient test to be sure...







Bode stability test: example 2

recall: $\| A_{CL}(s)\| \propto (D_{z1}D_{z2}....)/(D_{p1}D_{p2}...)$;
$\angle A_{CL}(s) = \theta_{z1} + \theta_{z2} + ... - \theta_{p1} - \theta_{p2} - ...$

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$\beta(j2\pi f) = \beta_0 = 1/100$

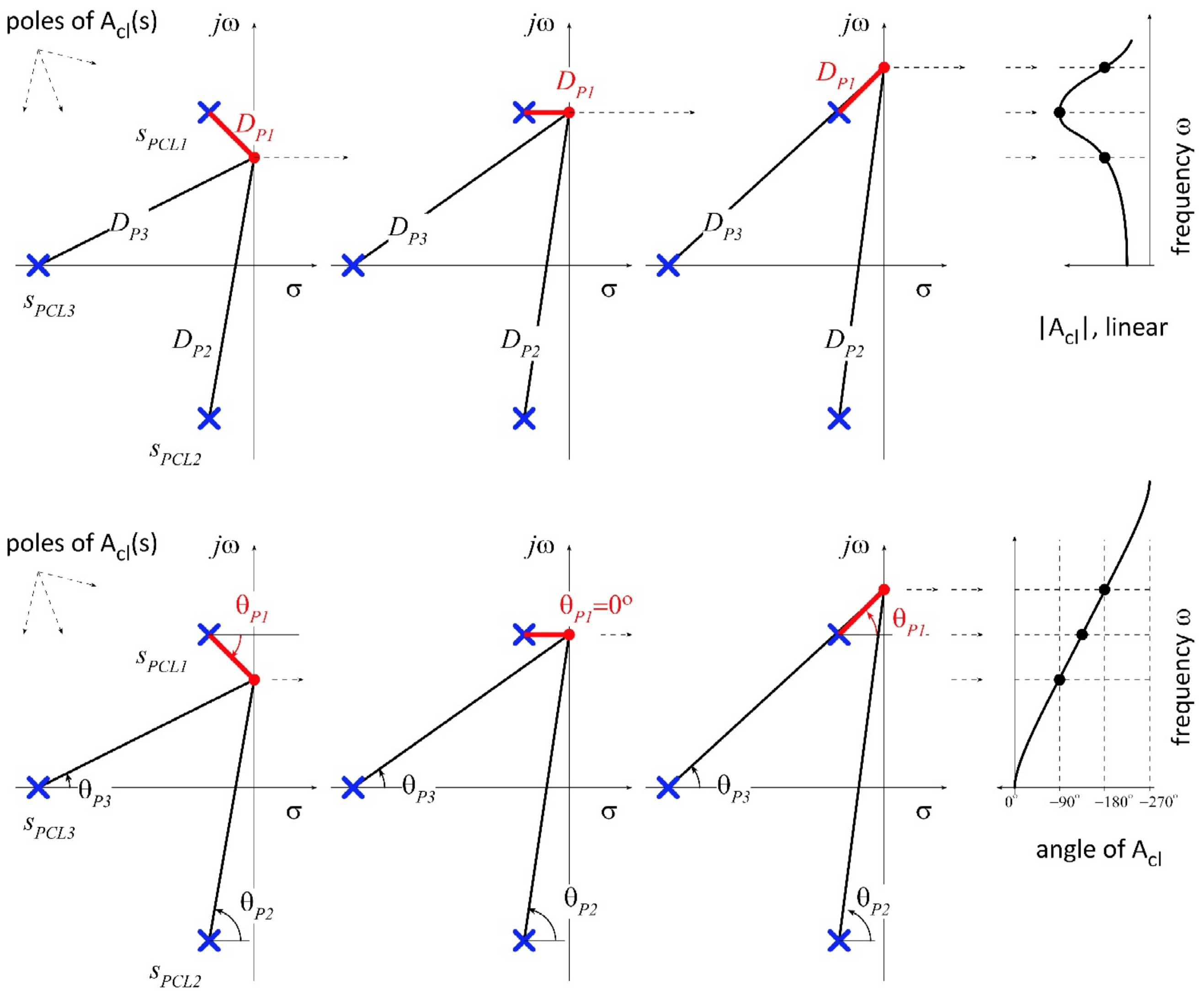
$f_{loop} \approx 25$ kHz

$\angle T \approx -179^0$ at 25 kHz

phase margin $= 180^0 - 179^0 = 1^0$

Loop is just barely stable

Note the variation of the phase of $A_{CL}(j\omega)$ with frequency



Bode stability test: example 3

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$$\beta(j2\pi f) = \beta_0 = 1/81$$

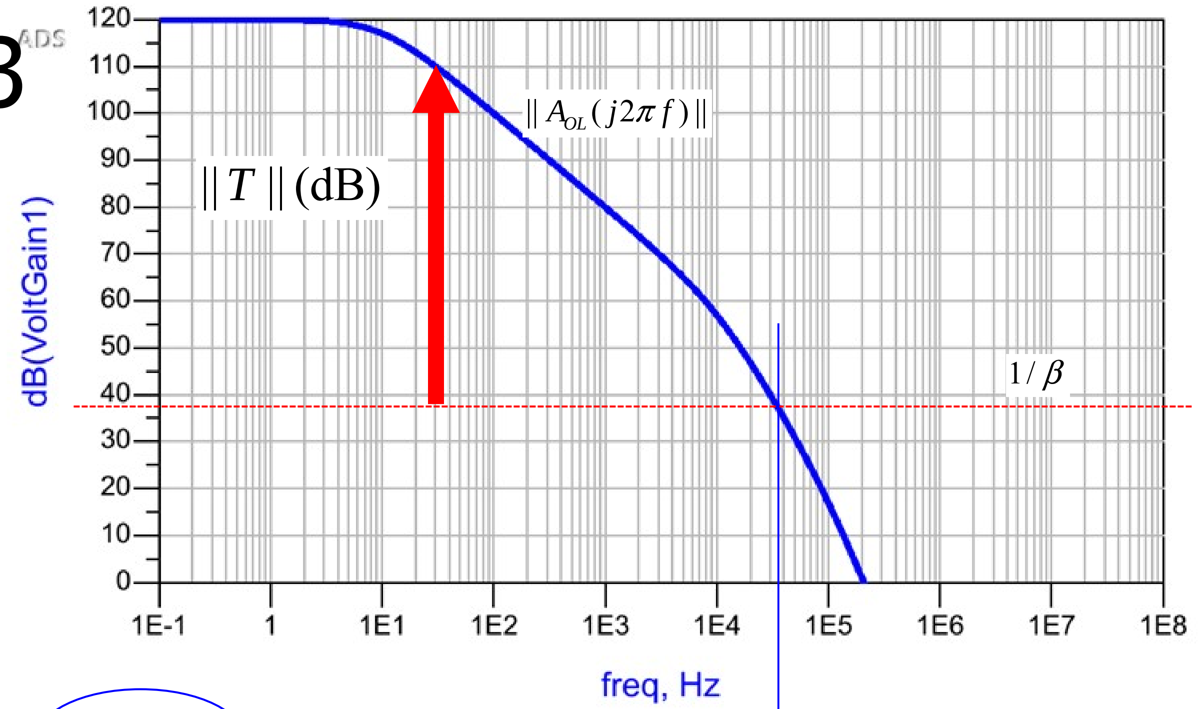
$f_{loop} \approx 35$ kHz

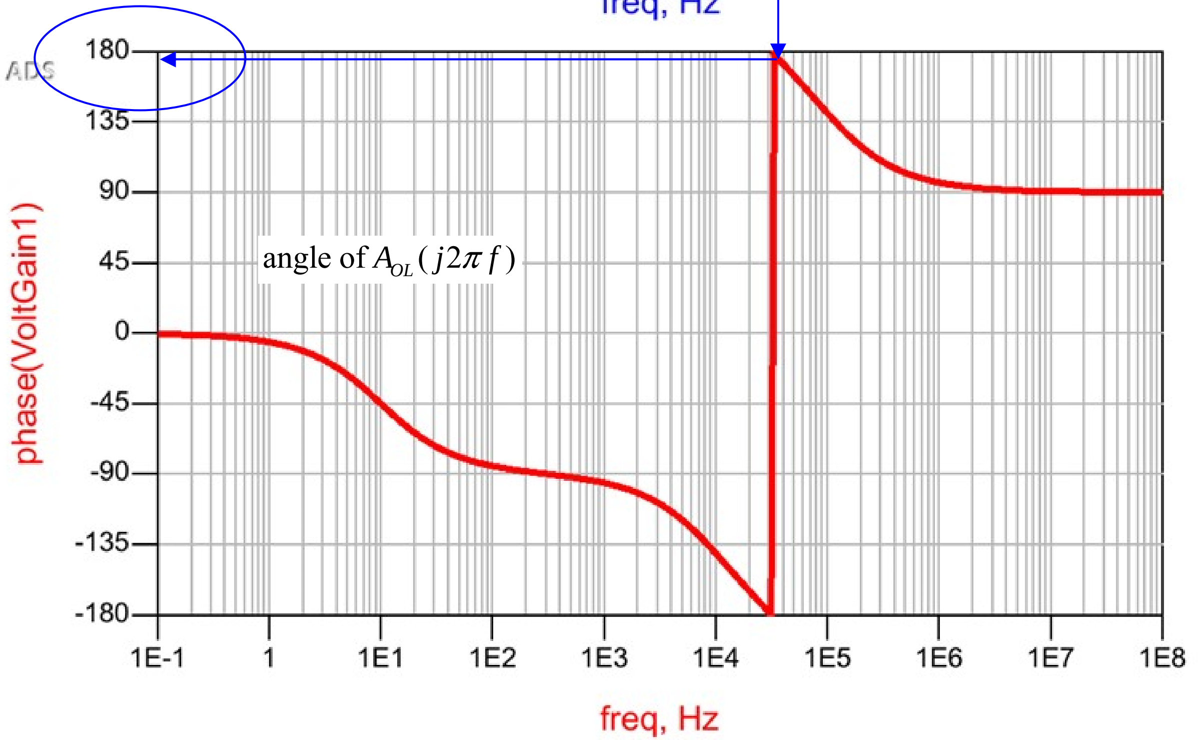
$\angle T \approx +175^0$ at 35 kHz

phase has gone past 180^0

Loop is just unstable

....just barely





Bode stability test: example 3

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$$\beta(j2\pi f) = \beta_0 = 1/81$$

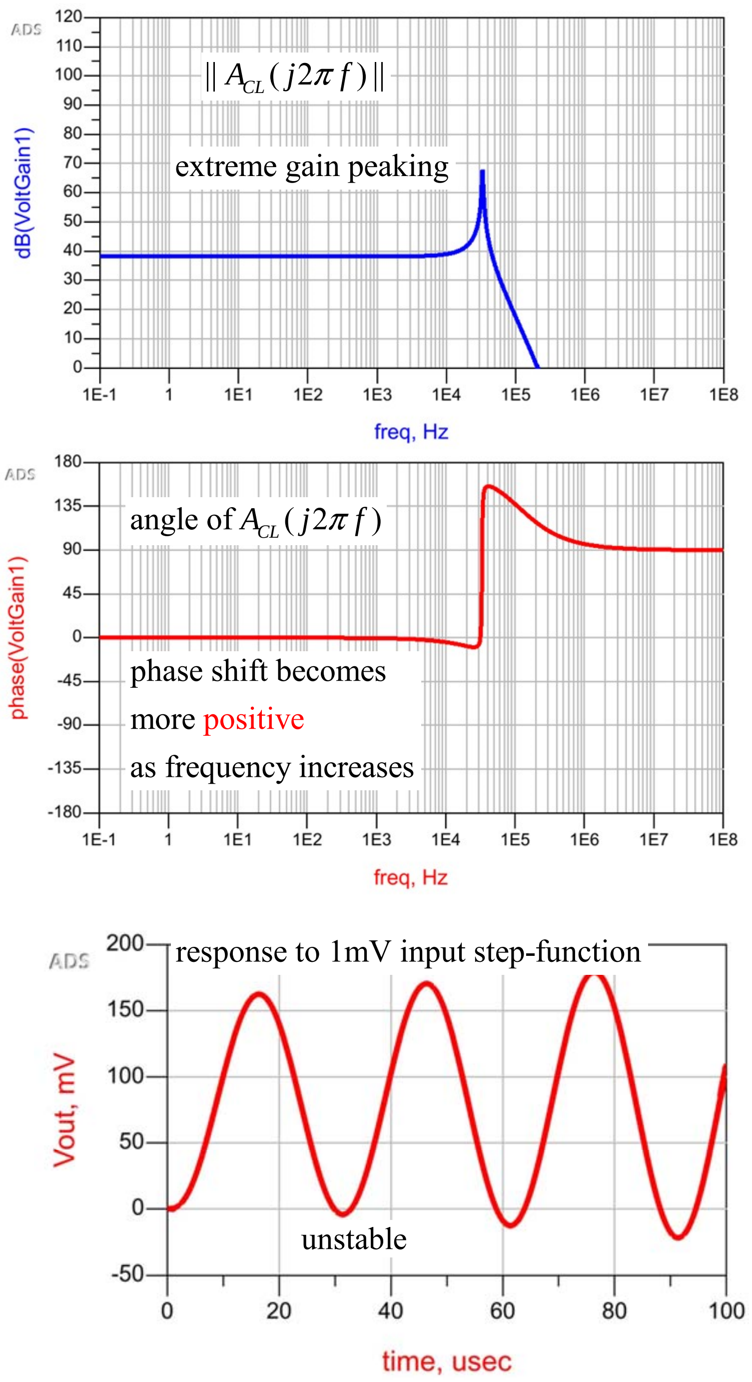
$f_{loop} \approx 35$ kHz

$\angle T \approx +175^0$ at 35 kHz

phase has gone past 180^0

Loop is just unstable

....just barely



Bode stability test: example 3

recall: $\| A_{CL}(s) \| \propto (D_{z1}D_{z2}....)/(D_{p1}D_{p2}...)$;

$\angle A_{CL}(s) = \theta_{z1} + \theta_{z2} + ... - \theta_{p1} - \theta_{p2} - ...$

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

$\beta(j2\pi f) = \beta_0 = 1/81$

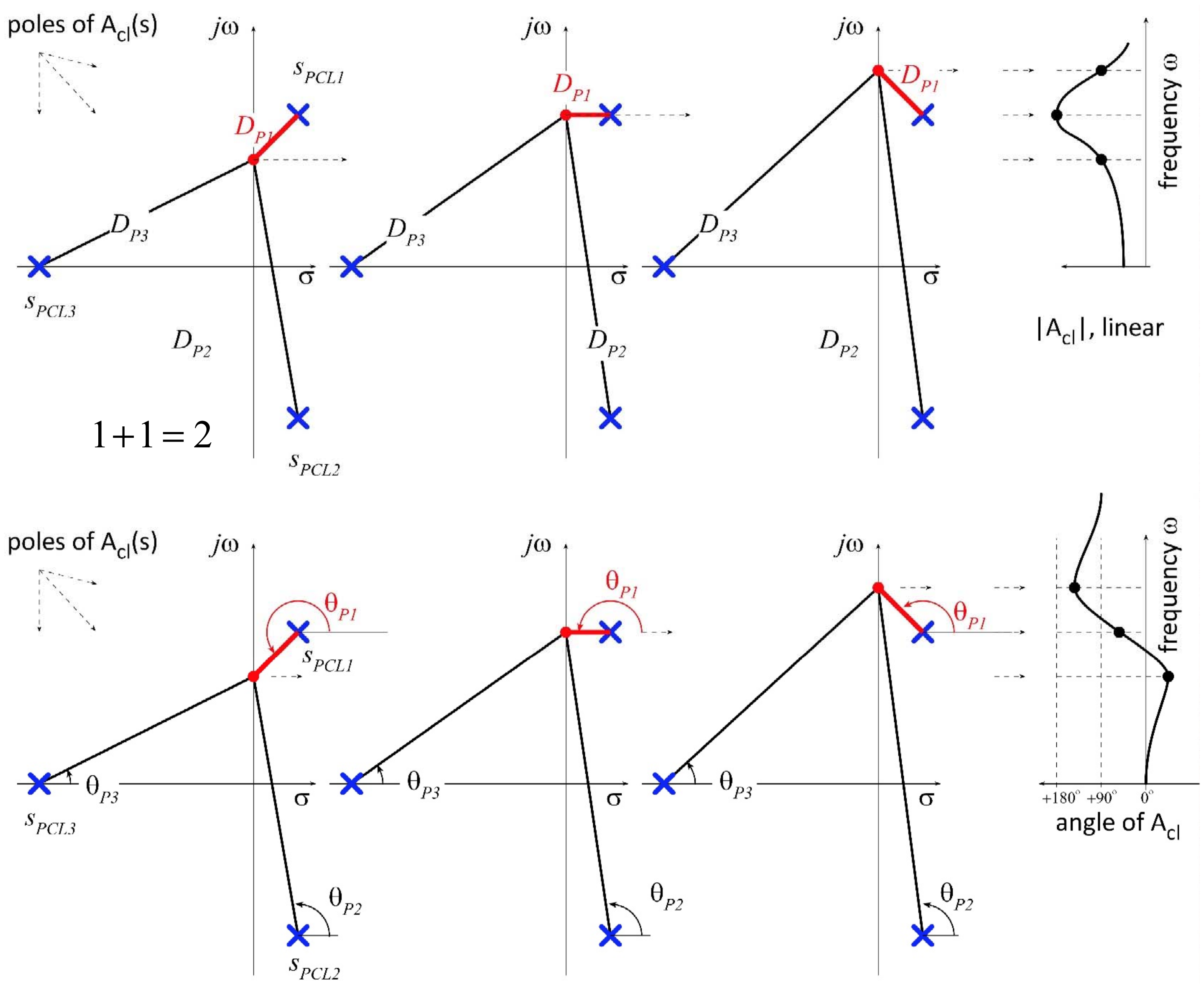
$f_{loop} \approx 35$ kHz

$\angle T \approx +175^0$ at 35 kHz

phase has gone past 180^0

Loop is unstable

Note the variation of the phase of $A_{CL}(j\omega)$ with frequency



Bode stability test: example 4

Example:

$$A_{OL}(j2\pi f) = \frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$= \frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

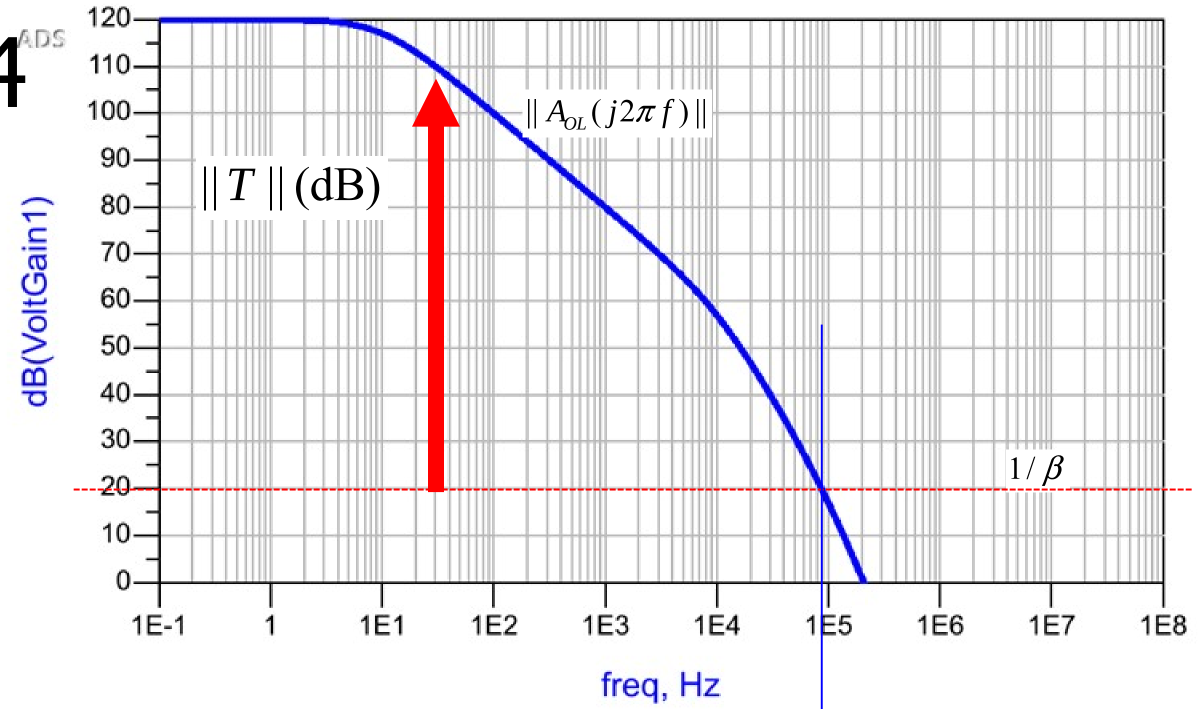
$$\beta(j2\pi f) = \beta_0 = 1/10$$

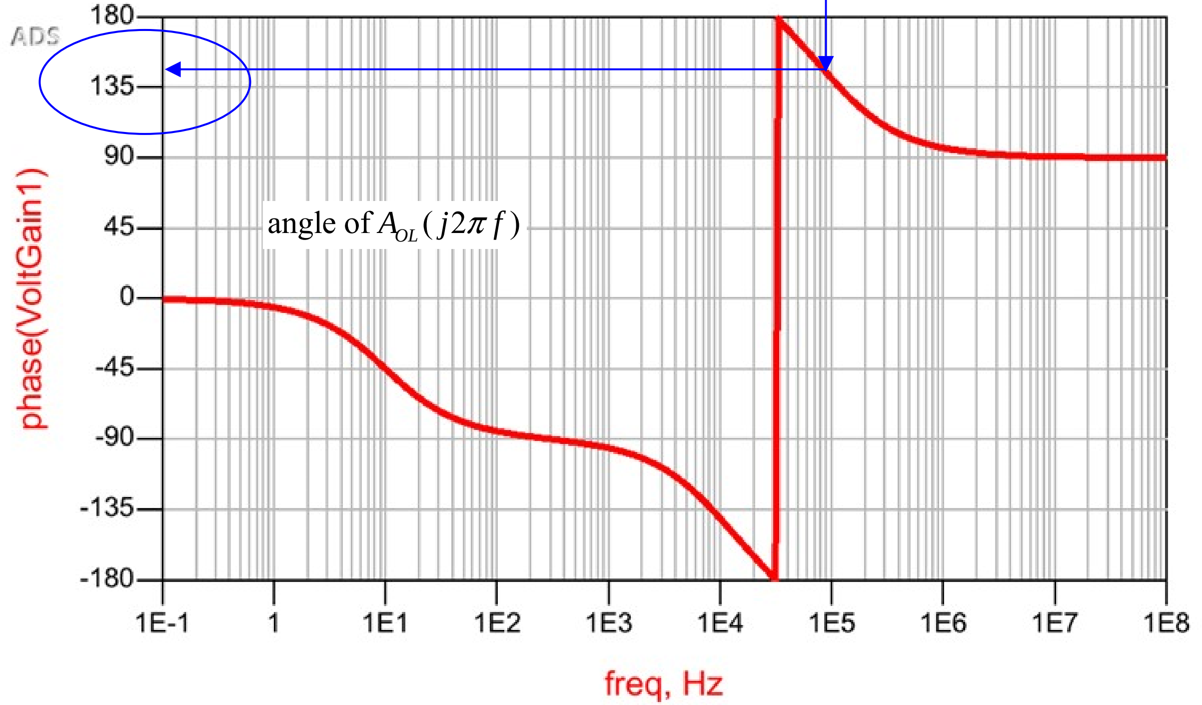
$f_{loop} \approx 900$ kHz

$\angle T \approx +140^0$ at 900 kHz

feedback loop phase has gone well past 180^0

Loop is unstable





Bode stability test: example 4

Example:

$$A_{OL}(j2\pi f)=\frac{A_{OL,DC}}{(1+jf/f_{OL1})(1+jf/f_{OL2})(1+jf/f_{OL3})}$$

$$=\frac{10^6}{(1+jf/10\text{ Hz})(1+jf/10\text{ kHz})(1+jf/100\text{ kHz})}$$

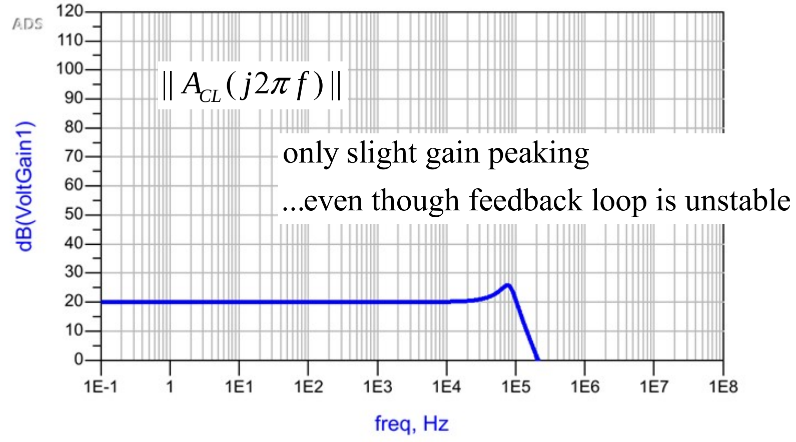
$$\beta(j2\pi f)=\beta_0=1/10$$

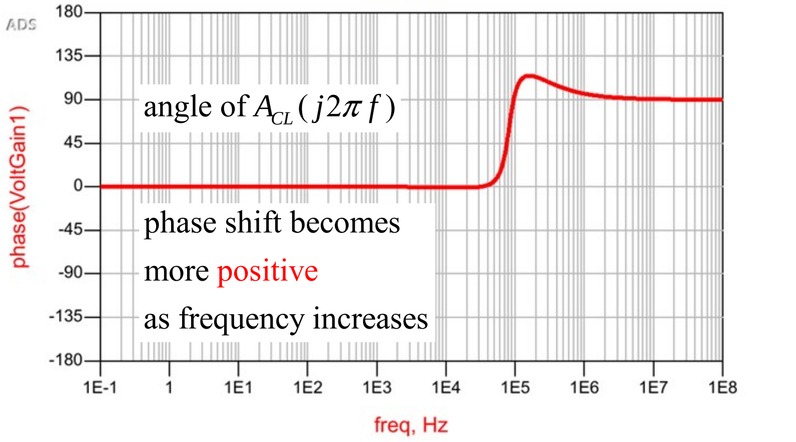
$f_{loop}\approx 900$ kHz

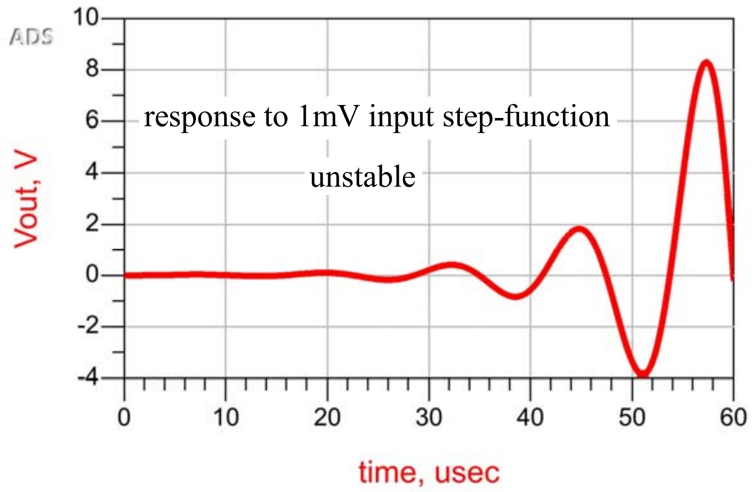
$\angle T\approx +140^0$ at 900 kHz

feedback loop phase has gone well past 180^0

Loop is unstable







Bode stability test by hand

Most of the notes in this set use CAD-generated plots

When working by hand (pencil and paper):

1) Make asymptotic plots of $\mathrm{dB}(\|T\|)$, or of $\mathrm{dB}(\|A_{OL}\|)$ and $\mathrm{dB}(\|1/\beta\|)$

2) If the poles in $T(s)$ are real , calculate the phase of T from

$$\angle T(f) = \arctan(f / f_{z1}) + \arctan(f / f_{z2}) + ... - \arctan(f / f_{p1}) - \arctan(f / f_{p1}) - ...$$

3) Simply evaluate (2) for $f = f_{loop}$ to find the phase margin