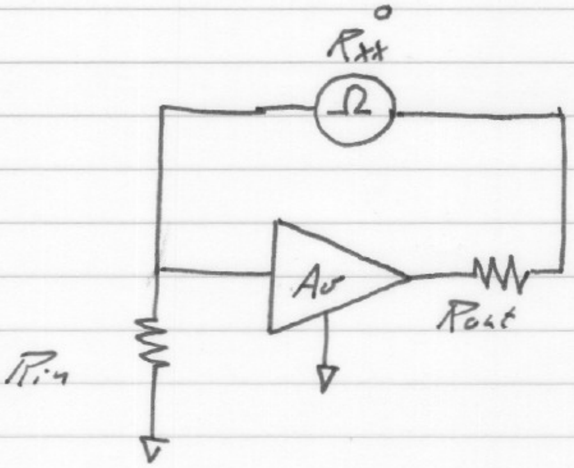
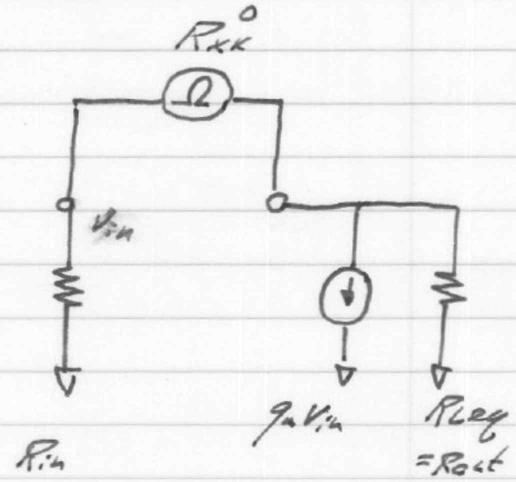
ECE 137B Notes Set 7:

1) MORE Regarding motc,

2) Emitter and source follower by motc.

We had earlier found:

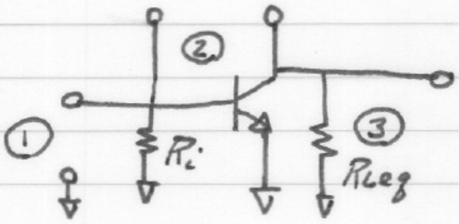




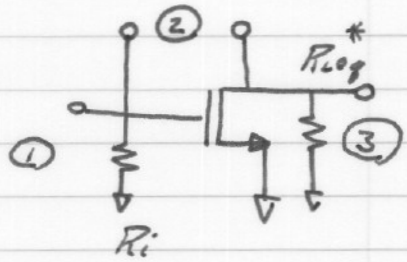
$$R_{xx}^0 = R_i\left[1 - A_v\right] + R_{out} \qquad R_{xx}^0 = R_i\left[1 + g_m R_{Leq}\right] + R_{Leq}$$

this will be useful for source and emitter follower

There are some other useful relation-ships:



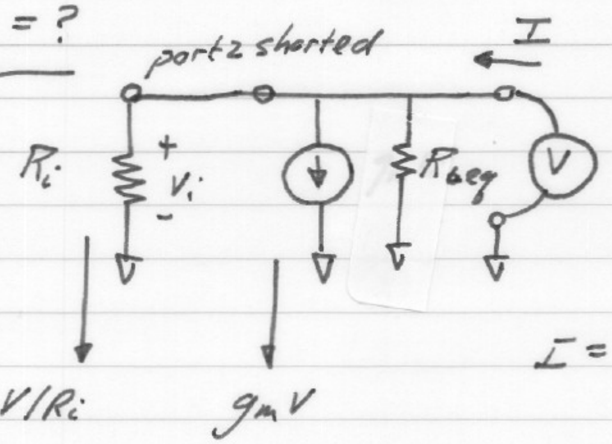
or



*R_i includes R_{be}
*R_{Leq} includes R_{CE}

*R_{Leq} includes R_{ds}

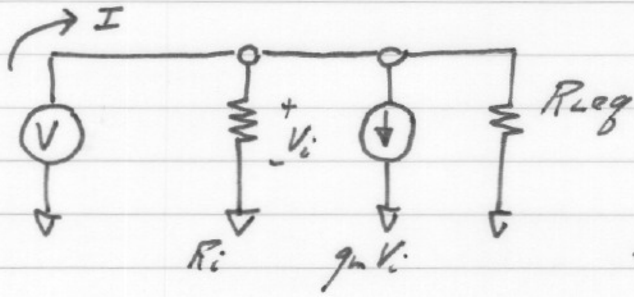
$R_{33}^{2} = ?$



$I = V(g_m + 1/R_i) + V_0/R_{Leq}$

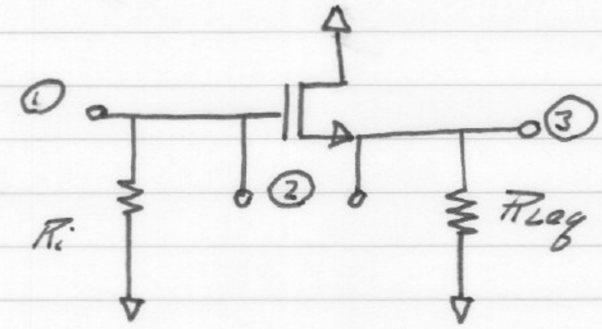
$\Rightarrow V/I = R_{33}^{2} = R_i \,\|\, \frac{1}{g_m} \,\|\, R_{Leq}$

$R_{11}^{2} = ?$



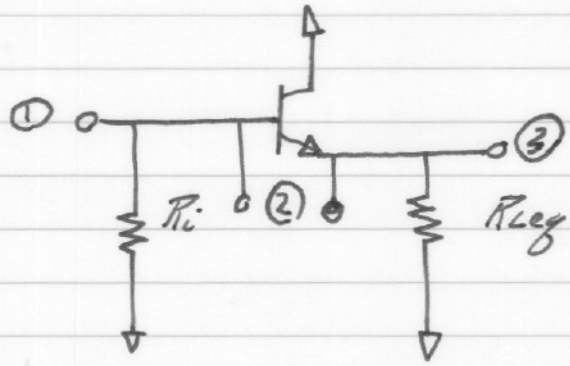
$I = V(g_m + 1/R_i + 1/R_{Leq})$

$\Rightarrow R_{11}^{2} = R_i \,\|\, \frac{1}{g_m} \,\|\, R_{Leq}$



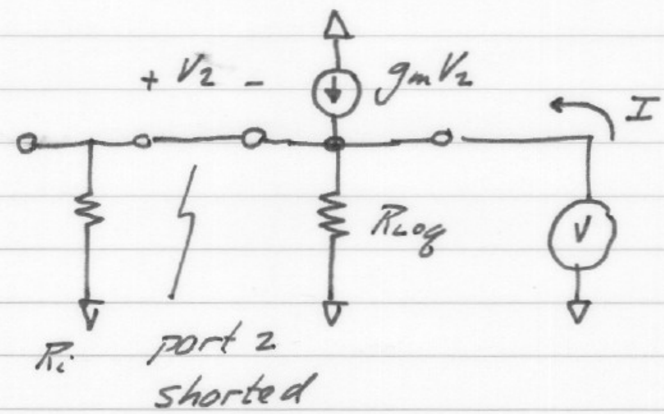
* R_{Leq} includes R_{ds}

or



* R_{Leq} includes R_{CE}

$\underline{R_{33}^{2} = ?}$

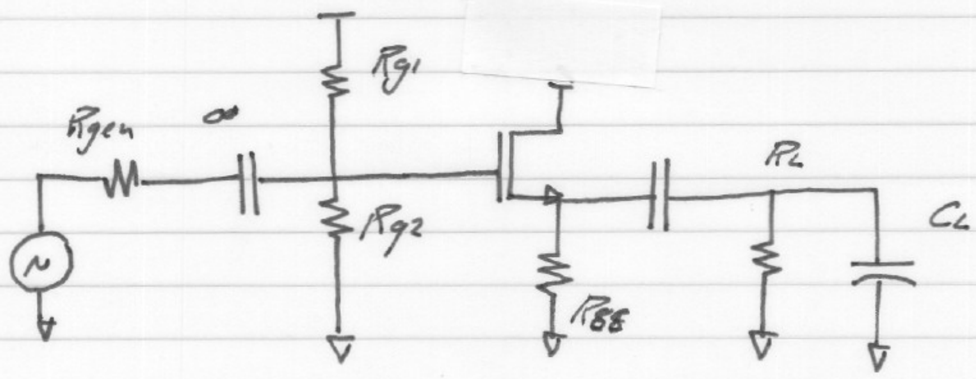


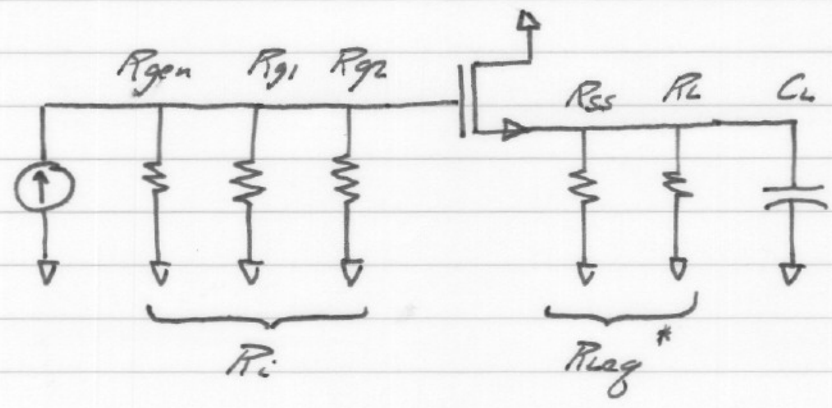
shorting port 2 makes $V_2 = 0 \Rightarrow g_m V_2 = 0$

$\underline{R_{33}^{2} = R_{Leq} \parallel R_i}$

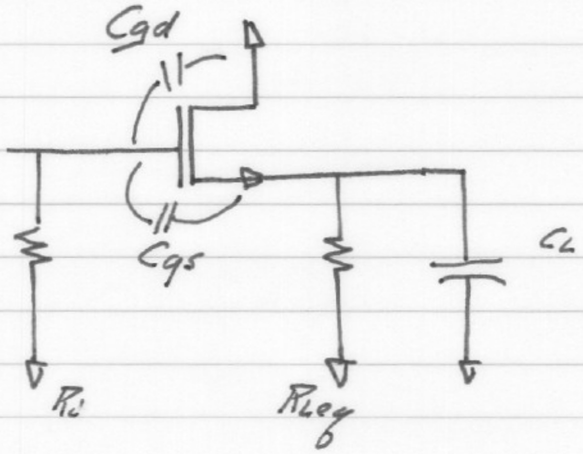
By a very similar construction, $R_{11}^{2} = R_{Leq} \parallel R_i$

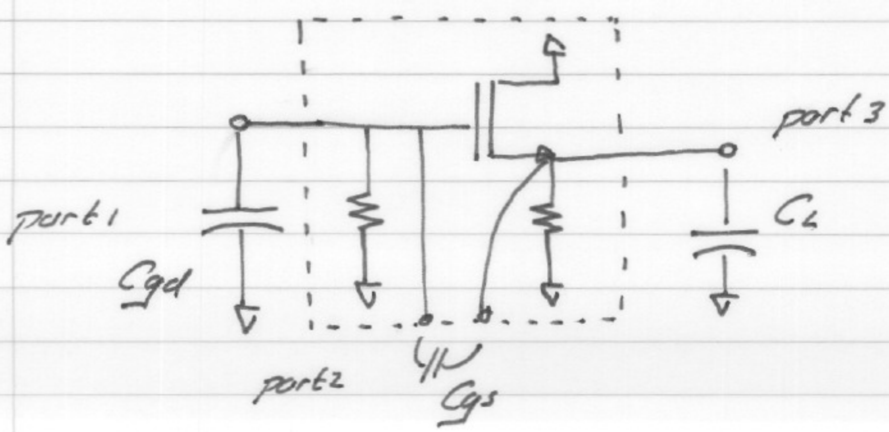
We now can solve the source follower easily



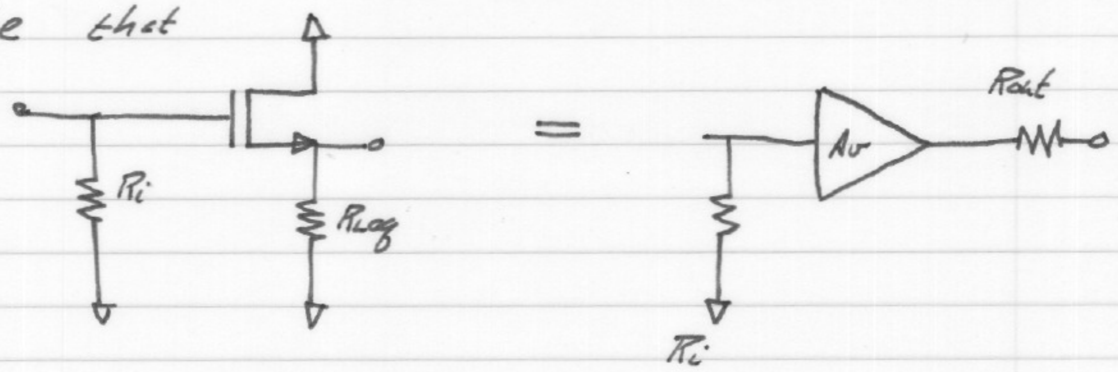


*Rleq includes Rds





note that

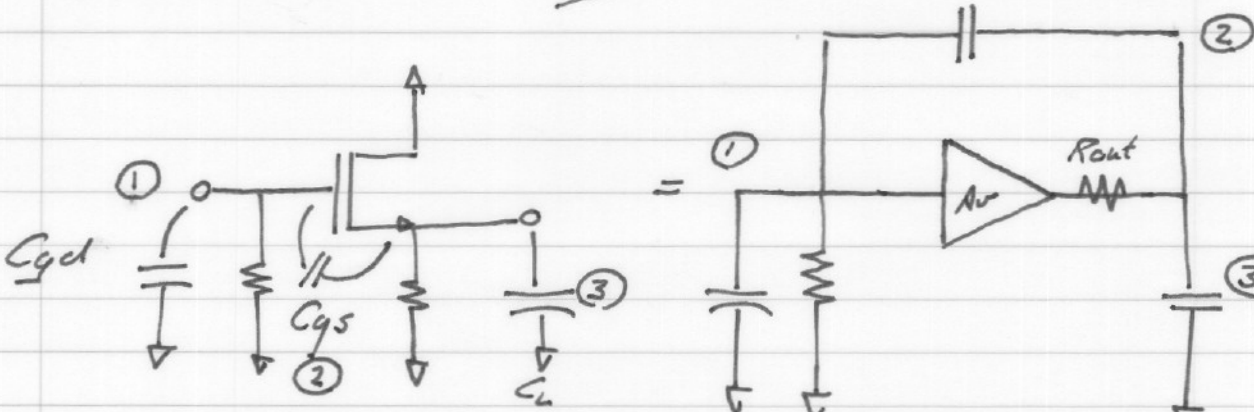


$$R_i = R_{Leq}$$

$$A_v = R_{Leq} / (R_{Leq} + 1/g_m)$$

$$R_{out} = R_{Leq} \,||\, \frac{1}{g_m}$$

So we can immediately write the time constants:



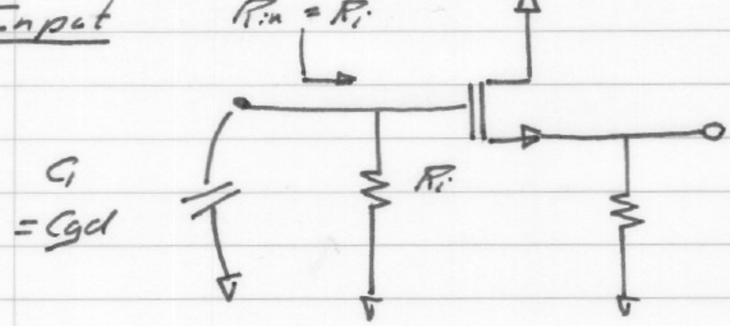
$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3$$

$$= C_{gd} \cdot R_i + C_{gs}\left[\left(1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right) R_i + R_{Leq} \,||\, \frac{1}{g_m}\right]$$

$$+ C_L\left[R_{Leq} \,||\, \frac{1}{g_m}\right]$$

Lets look at those terms again:

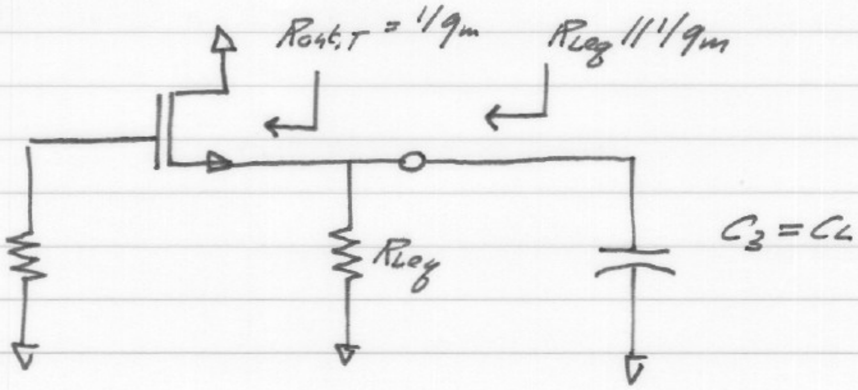
Input



the capacitance C_{gd} sees resistance R_i

$\Rightarrow$ time constant of $C_{gd} \cdot R_i$

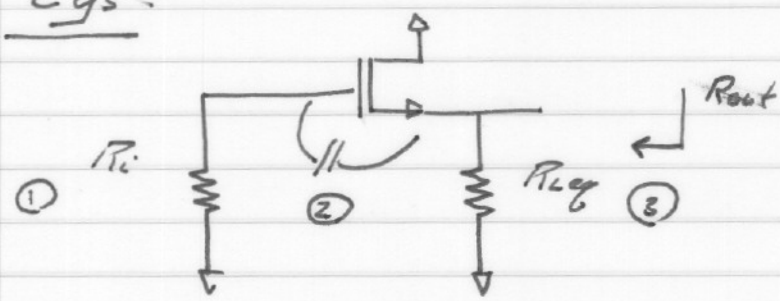
Output



the capacitance C_L sees resistance $R_{Leq} \,\|\, 1/g_m$

$\Rightarrow$ time constant of $C_L (R_{Leq} \,\|\, 1/g_m)$

C_{gs}:



$$C_{gs} \cdot R_{22}^{\circ} = C_{gs}\Big[R_i(1 - A_v) + \overbrace{(1/g_m) \,||\, R_{Leq}}^{R_{out}} \Big]$$

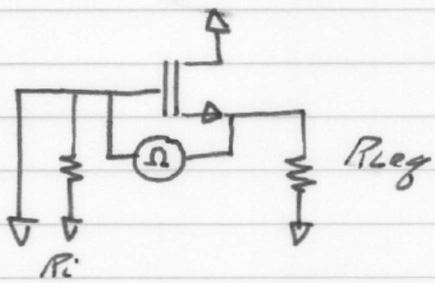
$$= C_{gs}\left[R_i\left[1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right] + (1/g_m) \,||\, R_{Leq}) \right]$$

Now find a_2

$$a_2 = R_{11}^{0} C_1 C_2 R_{22}^{1} + R_{11}^{0} C_1 C_3 R_{33}^{1} + R_{22}^{3} C_2 C_3 R_{33}^{0}$$

$R_{11}^{0} = R_i$, $R_{33}^{0} = (1/g_m) \,||\, R_{Leq}$

$\underline{R_{22}^{1}:}$



$$R_{22}^{0} = R_i\left(1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right) + \frac{1}{g_m} \,||\, R_{Leq}$$

$$R_{22}^{1} = 0\left(1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right) + \frac{1}{g_m} \,||\, R_{Leq}$$

$$= (1/g_m) \,||\, R_{Leq}$$

R_{33}^{1} = expression for R_{33}^{0} except with $R_i \to 0$

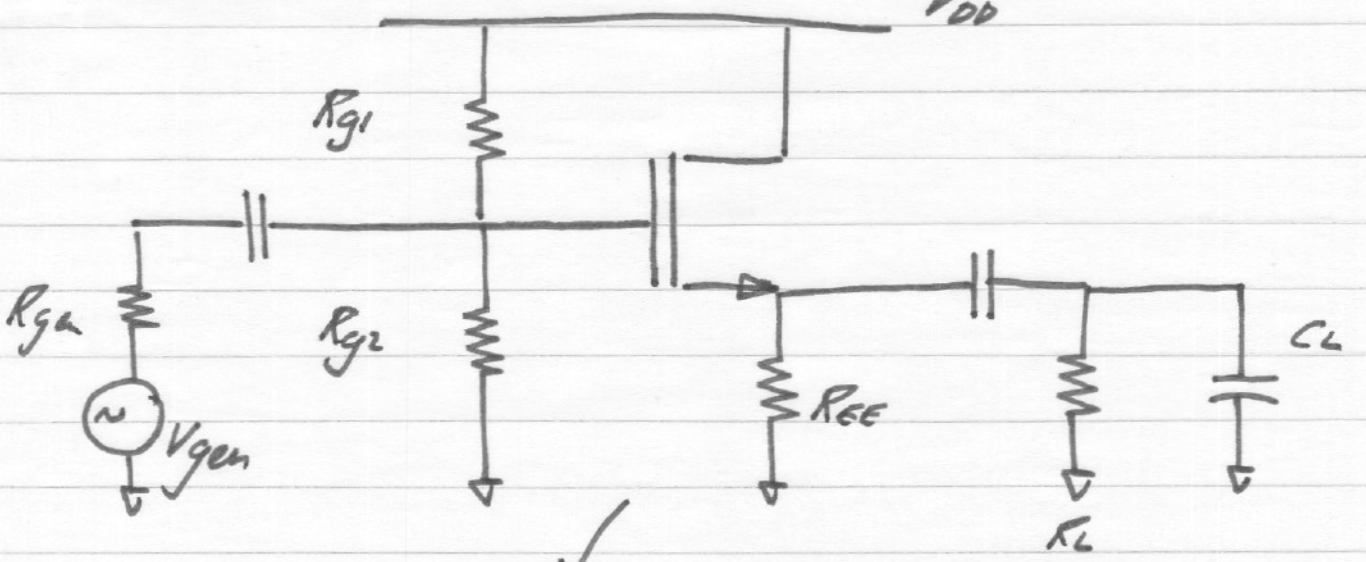
$= (1/g_m) \,||\, R_{Leq}$

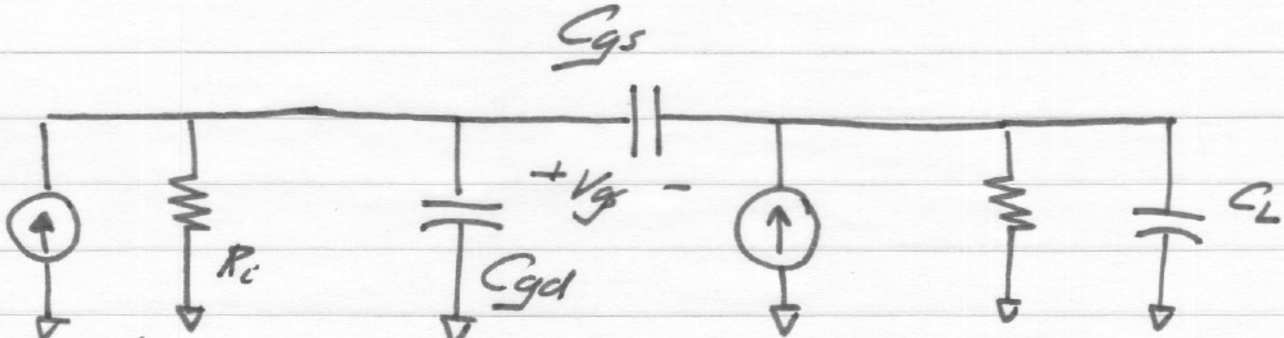
R_{22}^{3} = expression for R_{22}^{0} except with $R_{Leq} = 0$

$= R_i$

$$a_2 = R_i \cdot \left[\frac{1}{g_m} \,||\, R_{Leq}\right]\left[C_{gs} C_{gd} + C_{gs} C_L + C_{gd} C_L\right]$$

MOSFET Source Follower





$I_i = V_{gen}/R_{gen}$ $\qquad$ $g_m V_{gs}$ $\qquad$ $R_{eq} = R_L \| R_{EE}$

$R_i = R_{g1} \| R_{g2} \| R_{gen}$

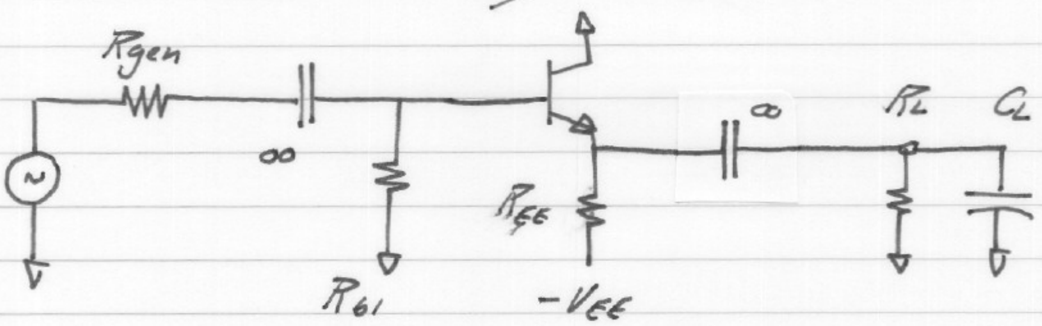
we can, once again, apply exactly the same methods as for the bjt case.

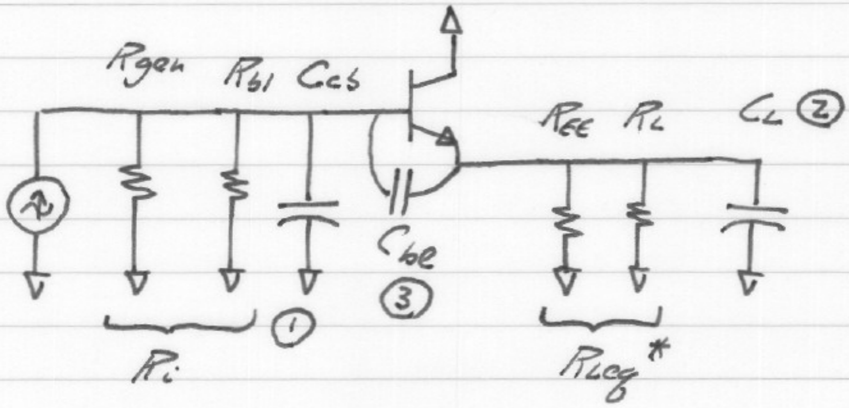
$$v_o / v_{gen} = \left.\frac{v_o}{v_{gen}}\right|_{MB} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

b_1 = by other methods = C_{gs} / g_m

$$a_1 = C_{gd} \cdot R_i + C_L (R_L \| 1/g_m)$$
$$+ C_{gs} \left[(1/g_m) \| R_{Leq} + R_i (1 - A_v) \right]$$

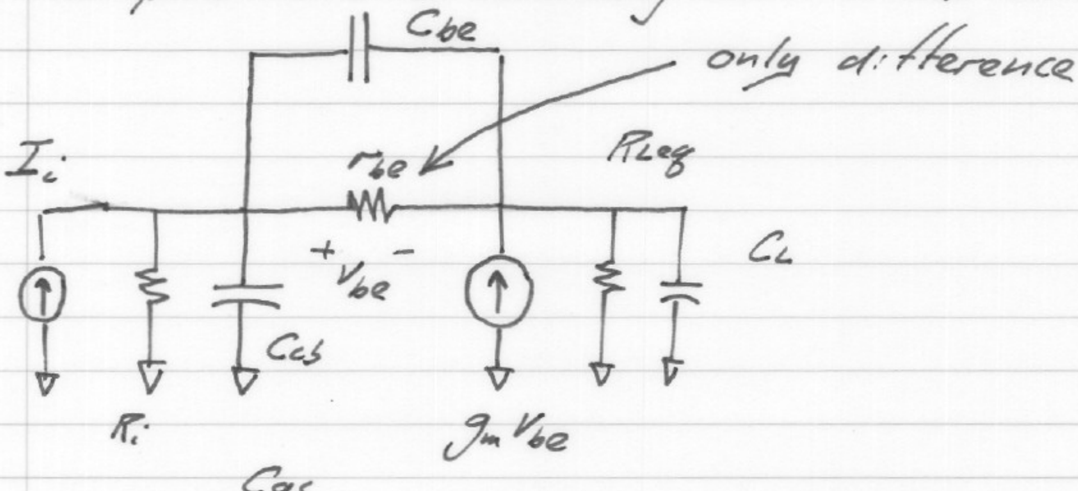
Emitter follower by MOTC

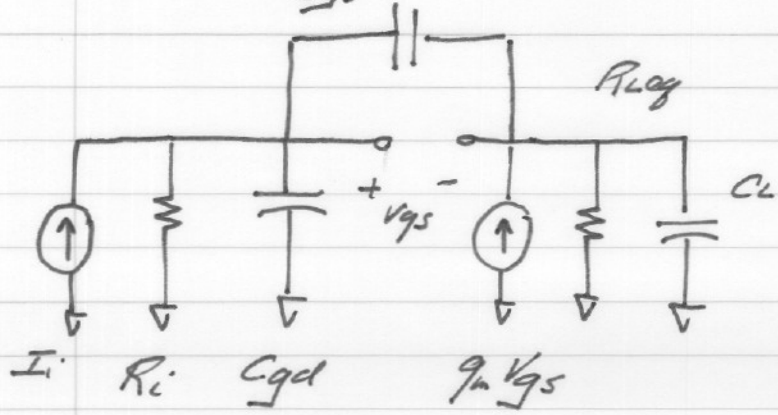


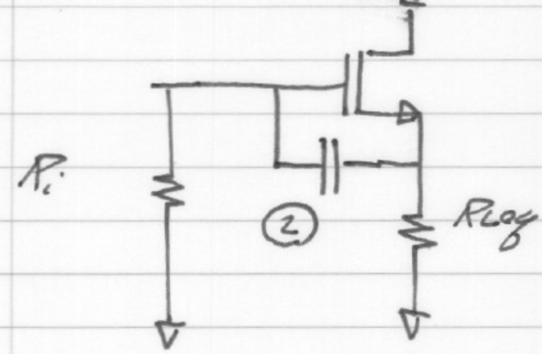


*includes REE

Compare the circuit diagrams of & followers

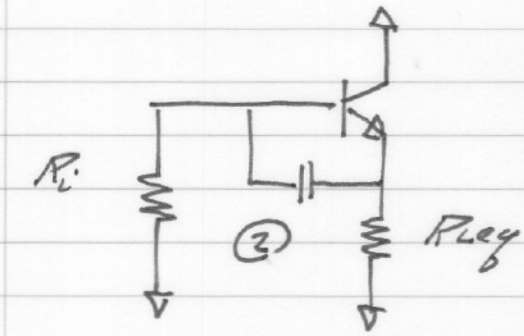






Given that for the MOSFET

$$R_{22}^{o} C_{gs} = C_{gs}\left[\left[1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right] R_i + \frac{1}{g_m} \,\|\, R_{Leq}\right]$$

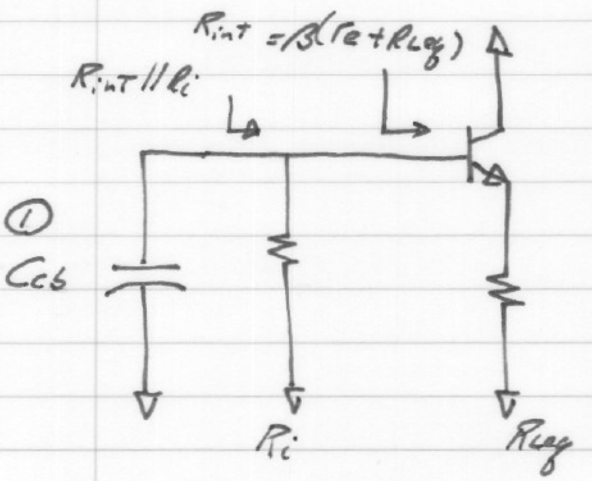


We have simply added $R_\pi = \beta / g_m$

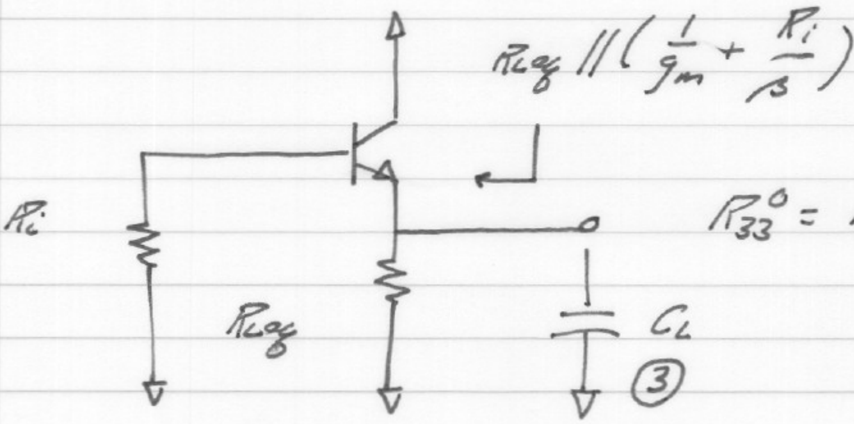
in parallel:

$$R_{22}^{o} = \left[R_i\left(1 - \frac{R_{Leq}}{R_{Leq} + 1/g_m}\right) + \frac{1}{g_m} \,\|\, R_{Leq}\right] \,\Big\|\, \frac{\beta}{g_m}$$

The other time constants for the E.F. are simple:



$$R_{11}^{0} = R_i \| \beta(r_e + R_{Leq})$$



$$R_{33}^{0} = R_{Leq} \| \left(\frac{1}{g_m} + \frac{R_i}{\beta}\right)$$

So the overall EF transfer function is:

$$\frac{v_o}{v_{gen}} = \left.\frac{v_o}{v_{gen}}\right|_{MB} \quad \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

$$a_1 = C_{cb} \cdot R_i \,||\, R_{inT} \qquad R_{inT} = \beta(r_e + R_{Leq})$$

$$+ C_{be} \cdot \left\{ R_{be} \,\middle|\middle|\, \left\{ R_i(1 - A_v) + R_{Leq} \,||\, 1/g_m \right\} \right\}$$

$$+ C_L \cdot \quad R_{Leq} \,||\, (1/g_m + R_i/\beta)$$

$$a_2 = (R_i \,||\, R_{inT})(1/g_m \,||\, R_{Leq})$$

$$\times \left[C_{cb} C_{be} + C_{cb} C_L + C_{be} C_L \right]$$

Source follower example:

MOSFET : $C_{ox} v_{sat} = 1\,mS/\mu m$ $\quad V_{th} = 0.25V$

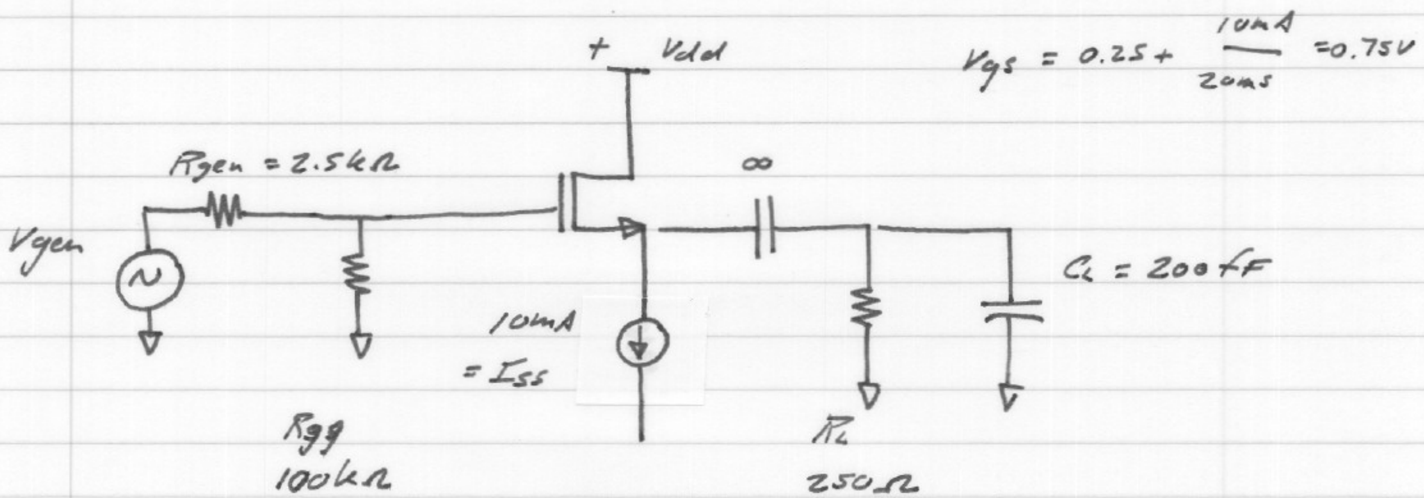
$\lambda = 0$ (simplify) $\quad W_g = 20\mu m \quad L_g = 200nm \quad v_{sat} = 10^7 cm/s$

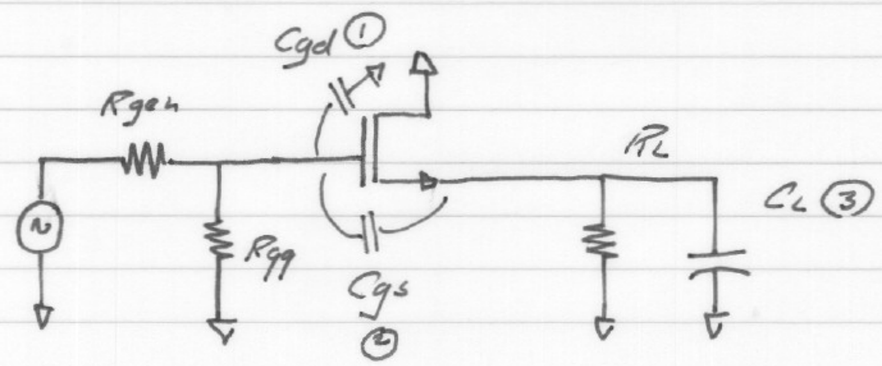
$$\Rightarrow C_{gs} = C_{ox} \cdot L_g \cdot W_g = \frac{1mS/\mu m}{10^5 m/s} \cdot 200nm \cdot 20\mu m = 40fF$$

$$g_m = 1mS/\mu m \times 20\mu m = 20mS \qquad (f_T = 79.5\,GHz)$$

lets assume $C_{gd} = 0.1\,C_{gs} = 4fF$

$$V_{gs} = 0.25 + \frac{10mA}{20mS} = 0.75V$$





Mid-band analysis

$R_{Leq} = 250\Omega \qquad A_{vMB} = \dfrac{250}{250\Omega + \underbrace{1/20\,mS}_{50\Omega}} = 0.8333$

$v_{in}/v_{gen} = 0.976 \qquad v_o/v_{gen} = 0.813$

$R_i = R_{gen} \| R_{gg} = 2.44\ k\Omega$

$R_{Leq} \| 1/g_m = 49\Omega$

High Frequency Analysis

$R_{11}^o\, C_{gd} = 2.44 K\Omega \cdot 4 fF = 9.8 ps$

$R_{33}^o\, C_L = 49\Omega \cdot 200\,fF = 9.8\,ps$

$C_{gs} \cdot R_{22}^o = 40fF\left[2.44k\Omega \cdot (1 - 0.8333) + 49\Omega\right]$

$= 40fF\left[456\Omega\right] = 18.2\ ps.$

$a_1 = 37.8\ ps$

$a_2 = 2.44k\Omega \cdot 49\Omega \cdot \left[4fF \cdot 200fF + 4fF \cdot 40fF + 40fF \cdot 200fF\right]$

$= 1.07 (10^{-21})\ sec^2 = (32.7 ps)^2$

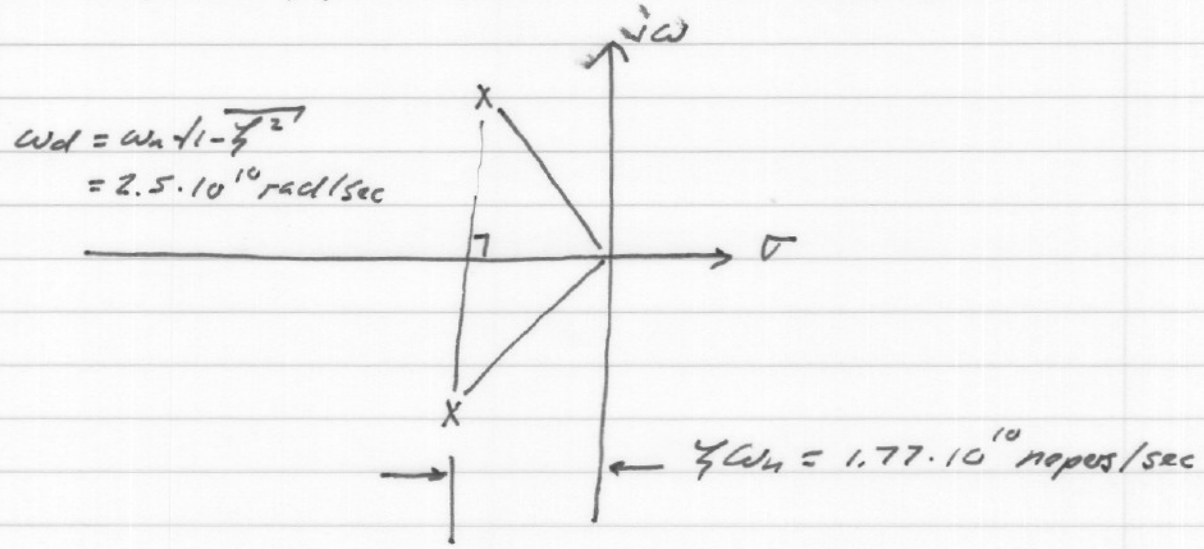
The ratio of a_2/a_1^2 is such that the SPA is not valid. See if the poles are complex

$$1 + a_1 s + a_2 s^2 = 1 + (2\zeta/\omega_n) s + s^2/\omega_n^2$$

$$\omega_n = 1/\sqrt{a_2} = 3.06 \cdot 10^{10} \text{ rad/sec} \; ; \; \underline{f_n = 4.9\text{ GHz}}$$

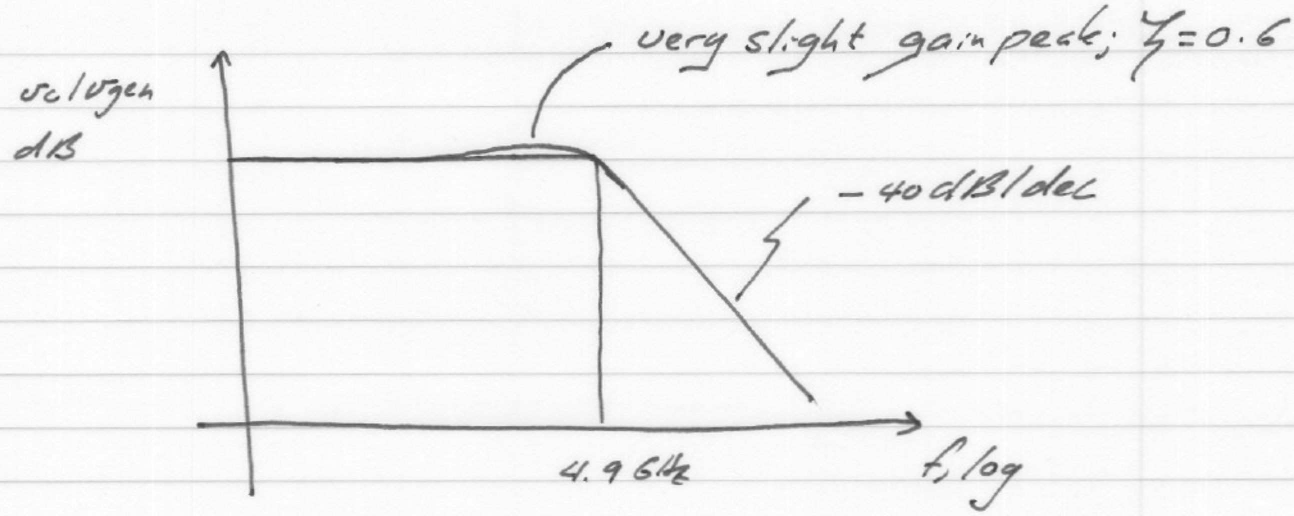
$$\zeta = a_1/2\sqrt{a_2} = 0.58$$

Since $\zeta < 1$, poles are indeed complex.



the transfer function also has a zero at

$$f_z = (1/2\pi)\, g_m/C_{gs} \cong f_T = 80\text{ GHz}.$$


very slight gain peak; ζ=0.6
vc/vgen
dB
-40dB/dec
4.9GHz
f, log