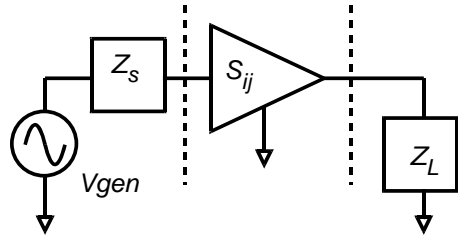

ECE145a / 218a
Power Gain Definitions

Mark Rodwell

University of California, Santa Barbara

Power Gain Definitions: Summary

Transducer Gain

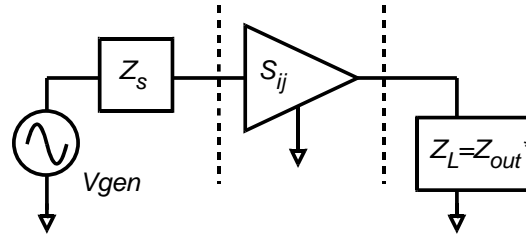


$$G_T = P_{load} / P_{av,gen}$$

$$= \frac{\text{load power}}{\text{poweravailable from generator}}$$

= general - case gain

Available Gain

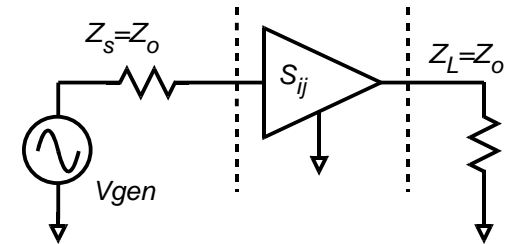


$$G_A = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{poweravailable from amplifier}}{\text{poweravailable from generator}}$$

= gain without output matched

Insertion Gain

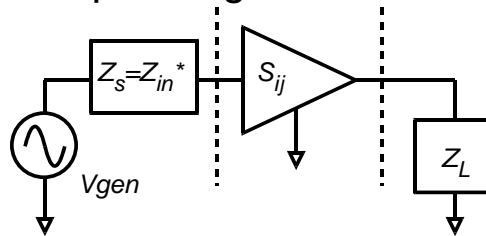


$$\|S_{21}\|^2 = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{powerdelivered to } Z_o \text{ load}}{\text{poweravailable from } Z_o \text{ generator}}$$

= gain in a 50 Ohm environment

Operating Gain

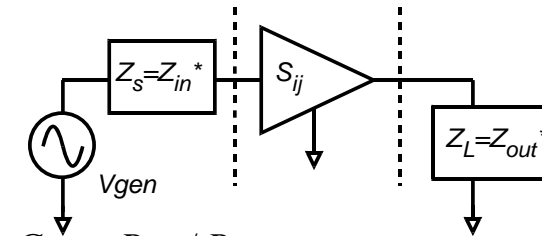


$$G_P = P_{load} / P_{gen,delivered}$$

$$= \frac{\text{load power}}{\text{powerdelivered from generator}}$$

= gain with input matched

Maximum Available Gain

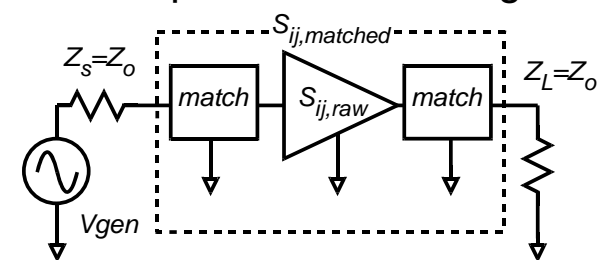


$$G_{MA} = P_{av,a} / P_{gen,delivered}$$

$$= \frac{\text{poweravailable from amplifier}}{\text{powerdelivered from generator}}$$

= gain with both ports matched
...MAG may not exist...

After impedance-matching:



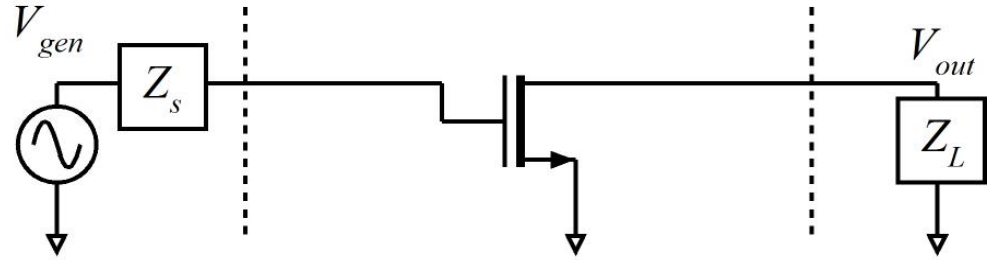
$$\|S_{21,matched}\|^2 = G_{max,raw}$$

$$S_{11,matched} = S_{22,matched} = 0$$

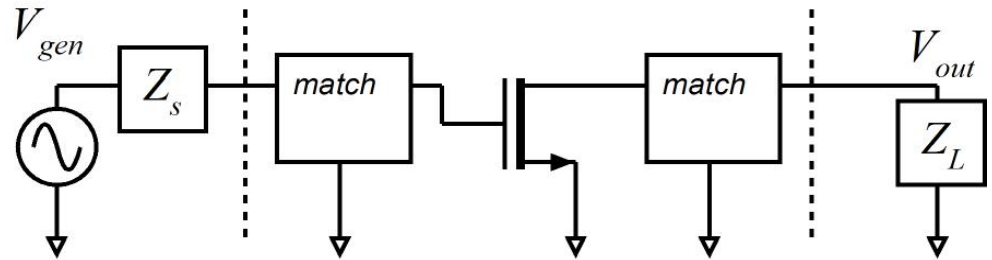
...but only if unconditionally stable...

Types of 2-Ports To Consider

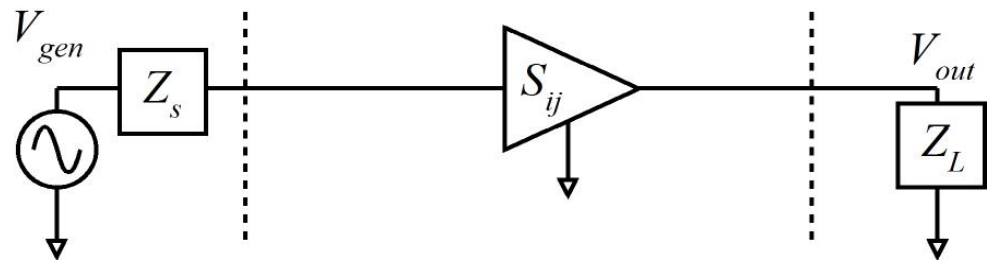
The 2 - port might be a transistor
neither S_{11} nor $S_{22} = 0$



The 2 - port might be a
 transistor with matching built - in
 S_{11} & S_{22} close or equal to zero



The 2 - port might be a general
 amplifier with arbitrary S_{ij}



For all of these, we could choose to add * additional * matching networks
 at the generator and at the load.

Insertion Power Gain

From S - parameter properties

$$S_{21} = 2 \frac{V_{out}}{V_{gen}} \Big|_{Z_{gen}=Z_L=Z_0}$$

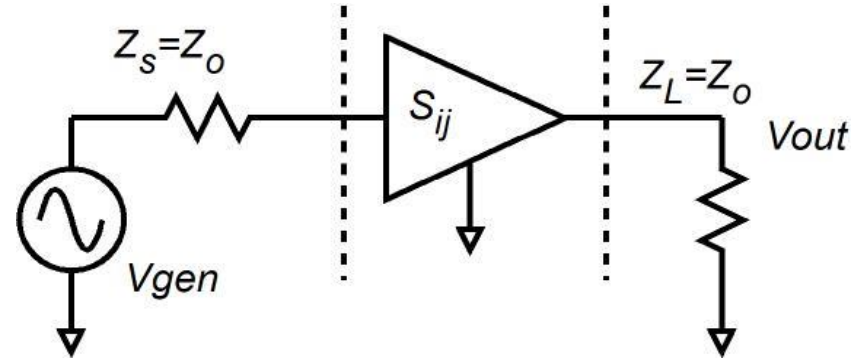
But if $Z_{gen} = Z_0 \rightarrow P_{av,gen} = |V_{gen}|^2 / 4Z_0$

And if $Z_L = Z_0 \rightarrow P_{load} = |V_{out}|^2 / Z_0$

$\rightarrow P_{load} / P_{av,gen} = |V_{out}|^2 / 4|V_{gen}|^2 = \|S_{21}\|^2$ but only if $Z_{gen} = Z_L = Z_0$

hence

$$\|S_{21}\|^2 = \frac{\text{power delivered to } Z_0 \text{ load}}{\text{power available from } Z_0 \text{ generator}} = \text{gain in a } Z_0 \text{ (50 Ohm?) environment}$$

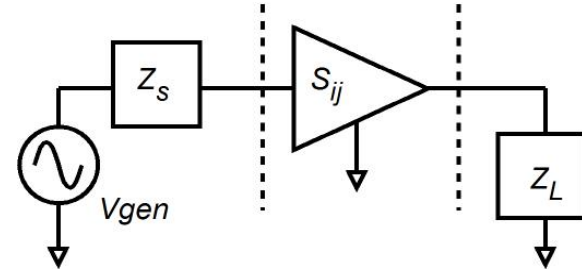


Transducer Power Gain

$$G_T = P_{load} / P_{av,gen}$$

$$= \frac{\text{power delivered to load}}{\text{power available from generator}}$$

$$= \text{general - case gain}$$



Amplifier input impedance might or might not be matched to Z_{source} (Z_{gen}).

Amplifier output impedance might or might not be matched to Z_{load} .

Generator impedance might or might not be Z_o

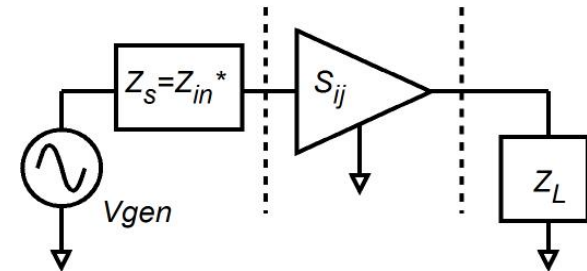
Load impedance might or might not be Z_o

G_T depends upon both Z_s and Z_L .

Operating Power Gain

$$G_P = P_{load} / P_{gen,delivered}$$

$$= \frac{\text{load power}}{\text{power delivered from generator}}$$



Note: $P_{load} / P_{gen,delivered} = P_{load} / P_{avg} = G_T$ iff $Z_{gen} = Z_{in}^*$

hence $G_p =$ gain with input matched

Amplifier input impedance *is* matched to Z_{source} (Z_{gen}).

Amplifier output impedance might or might not be matched to Z_{load} .

Generator impedance might or might not be Z_o

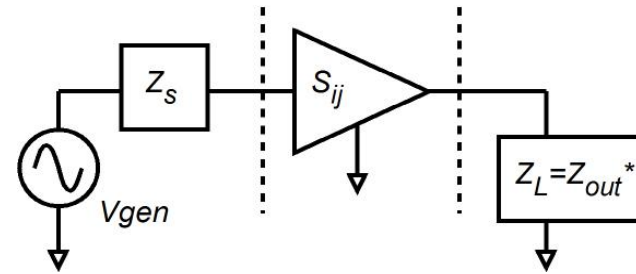
Load impedance might or might not be Z_o

G_P depends upon Z_L but not upon Z_s .

Available Power Gain

$$G_A = P_{av,a} / P_{av,gen}$$

$$= \frac{\text{power available from amplifier}}{\text{power available from generator}}$$



Note: $P_{ava} / P_{avg} = P_{load} / P_{avg} = G_T$ iff $Z_{load} = Z_{out}^*$

hence $G_a = \text{gain with output matched}$

Amplifier input impedance might or might not be matched to Z_{source} (Z_{gen}).

Amplifier output impedance * is * matched to Z_{load} .

Generator impedance might or might not be Z_o

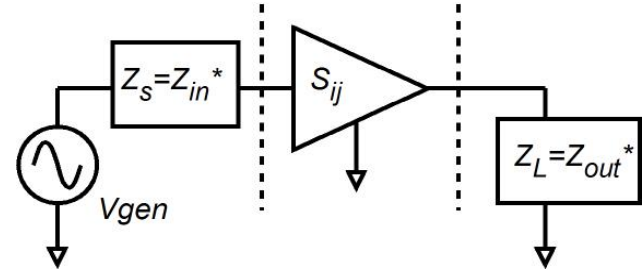
Load impedance might or might not be Z_o

G_A depends upon Z_s but not upon Z_L .

Maximum Available Power Gain

$$G_{\max} = P_{av,a} / P_{gen,delivered}$$

$$= \frac{\text{power available from amplifier}}{\text{power delivered from generator}}$$



Note: $P_{ava} / P_{gen,delivered} = P_{load} / P_{avg} = G_T$ iff ($Z_{gen} = Z_{in}^*$ and $Z_{load} = Z_{out}^*$)

hence G_{\max} = gain with both input and output matched

Simultaneous input/output matching may not be possible \rightarrow MAG may not exist.

Amplifier input impedance *is* matched to Z_{source} (Z_{gen}).

Amplifier output impedance *is* matched to Z_{load} .

Generator impedance might or might not be Z_o

Load impedance might or might not be Z_o

G_{\max} depends upon neither Z_s nor Z_L .

MAG Does Not Always Exist

S_{21} and S_{12} represent interaction terms between input and output.

If $S_{21}S_{12} \neq 0$, then input & output tuning become mutually interactive.

With sufficiently large $S_{21}S_{12}$, simultaneous matching of input & output is not possible.

In this case, G_{\max} no longer exists.

This condition corresponds to potential amplifier instability.

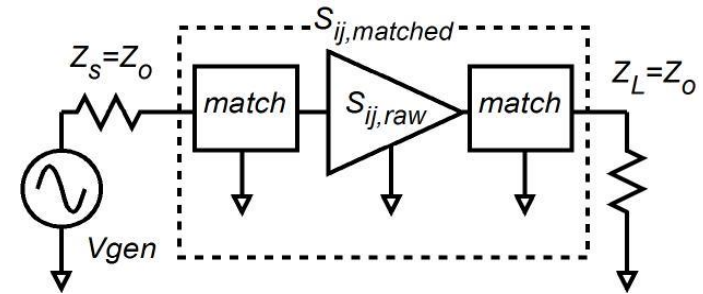
Will be covered later.

$\|S_{21}\|^2$ After Matching to $Z_0 = \text{MAG}$ Before Matching

$$\begin{aligned} \|S_{21,matched}\|^2 &= \left\| \frac{2V_{out}}{V_{gen}} \right\|^2 = \frac{\|V_{out}\|^2 / Z_o}{\|V_{gen}\|^2 / 4Z_o} \\ &= P_{load} / P_{avg} \end{aligned}$$

but $P_{load} / P_{avg} = G_{\max,raw}$

So : $\|S_{21,matched}\|^2 = G_{\max,raw}$



$$S_{11,matched} = S_{22,matched} = 0$$

The dB insertion gain of the amplifier is the dB MAG of the transistor.

Implication : examining the transistor MAG before we design a matching network tells us the gain we should expect to obtain after matching.

Mason's Unilateral Power Gain (1)

1) Cancel device feedback with external lossless feedback

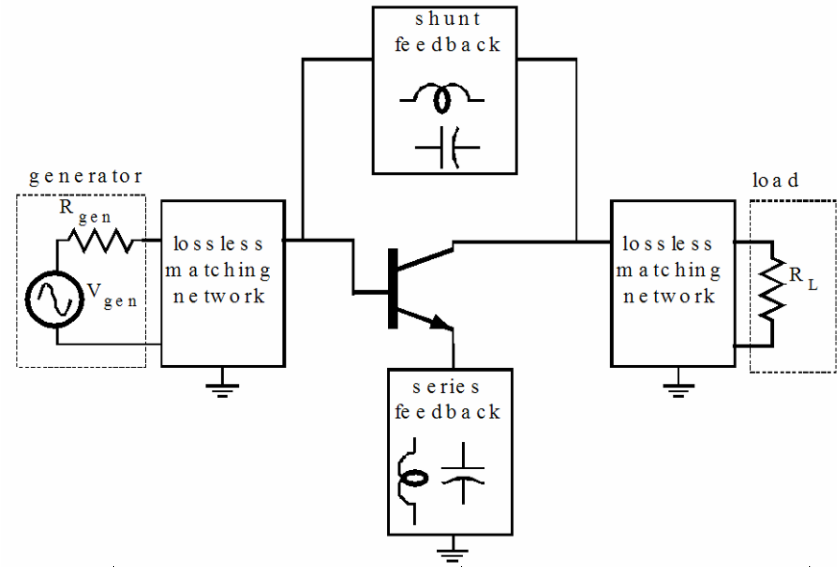
$$\rightarrow Y_{12}^{\text{overall}} = S_{12}^{\text{overall}} = 0$$

2) Match input and output

Resulting power gain is Mason's Unilateral Gain

$$U = \frac{|Y_{21}^{\text{FET}} - Y_{12}^{\text{FET}}|^2}{4(G_{11}^{\text{FET}} G_{22}^{\text{FET}} - G_{21}^{\text{FET}} G_{12}^{\text{FET}})}$$

Note carefully the difference between Y_{ij}^{FET} and Y_{ij}^{overall} .



Monolithic amplifiers are not easily made unilateral

$\rightarrow U$ mostly of historical relevance to IC design

For simple BJT model, U rolls off at -20 dB/decade

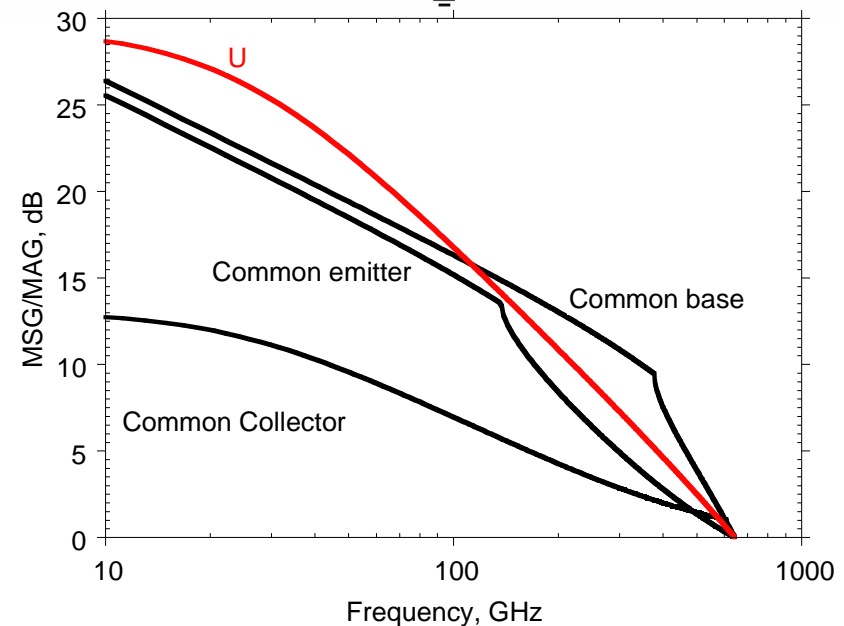
$\rightarrow U$ useful for extrapolation to find f_{max}

In III - V FETs, U shows peak from $C_{ds} - R_s - R_d$ interaction

$\rightarrow U$ hard to use for f_{max} extrapolation

For bulk CMOS, C_{ds} is shielded by substrate

$\rightarrow U$ should be OK for f_{max} extrapolation



Mason's Unilateral Power Gain (2): Additional Points

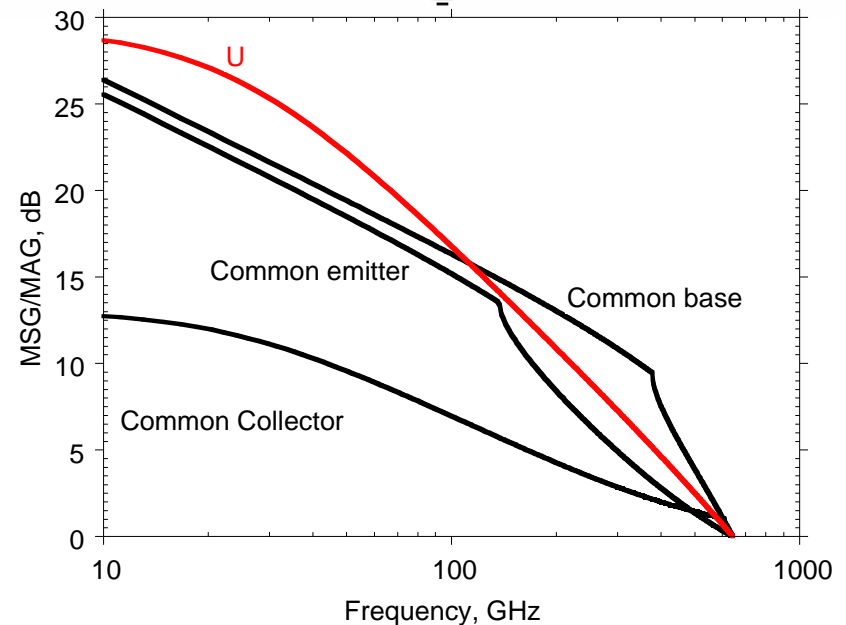
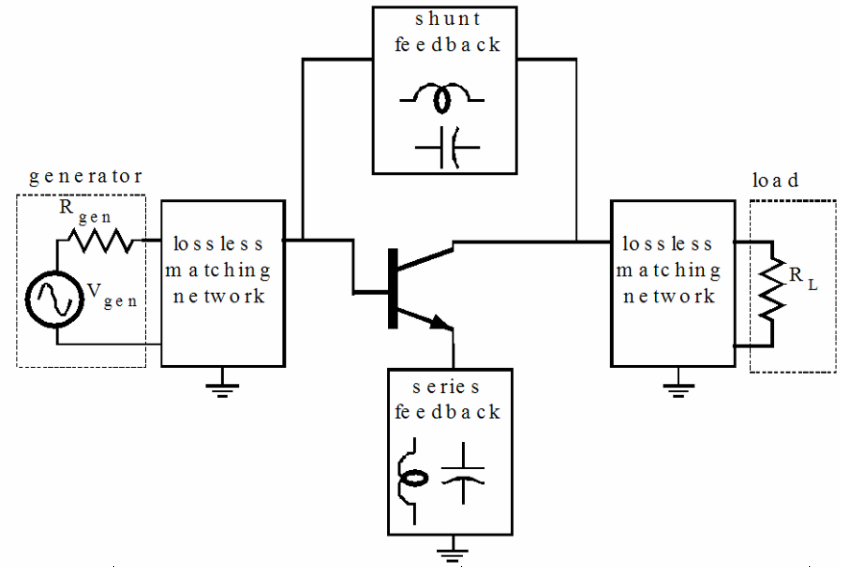
Makes input-output coupling = 0

→ No interaction of input, output tuning networks mostly of historical relevance.

Mason's gain is invariant with respect to embedding the device in a lossless reciprocal network.

It is the only* function of the DUT 2-port parameters which is invariant with respect to such embedding

*except, of course, any other function which has a 1:1 mapping with U



S. Mason, "Power Gain in Feedback Amplifier," in *Transactions of the IRE Professional Group on Circuit Theory*, vol. CT-1, no. 2, pp. 20-25, June 1954. doi: 10.1109/TCT.1954.1083579

<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=1083579&isnumber=23422>

M. S. Gupta, "Power gain in feedback amplifiers, a classic revisited," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 40, no. 5, pp. 864-879, May 1992.

doi: 10.1109/22.137392

<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=137392&isnumber=3740>

Singhakowinta's gain

- 1) Add external lossless feedback to maximize the MAG/MSG

$$\rightarrow Y_{12}^{\text{overall}} \neq 0, S_{12}^{\text{overall}} \neq 0$$

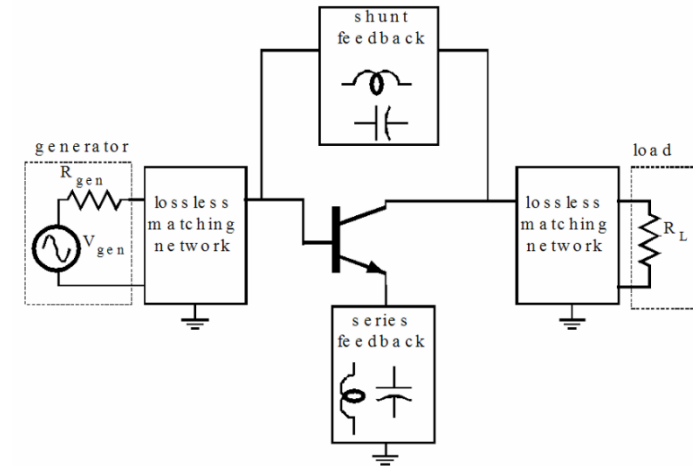
- 2) Stabilize if necessary
- 3) Match input and output

Resulting power gain is Singhakowinta's Gain

$$G_{\text{max},S} = (2U - 1) + 2U^{1/2}(U - 1)^{1/2}$$

This is the highest gain obtainable under conditions of unconditional stability.

$G_{\text{max},S}$ has a 1:1 relationship with U .

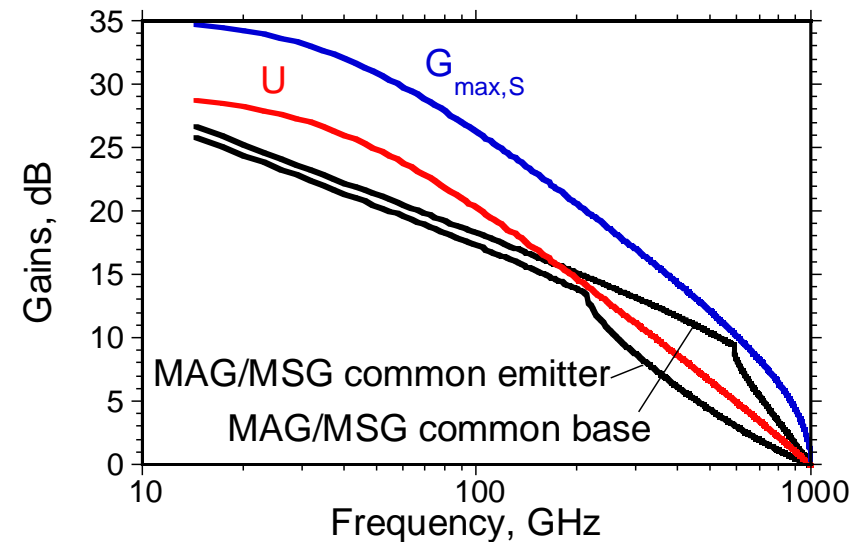


Gain Capability of Two-port Amplifiers
A. Singhakowinta & A. R. Boothroyd
International Journal of Electronics
Volume 21, Issue 6, 1966

<http://dx.doi.org/10.1080/00207216608937931>

A. Singhakowinta and A. Boothroyd, "On Linear Two-Port Amplifiers," in IEEE Transactions on Circuit Theory, vol. 11, no. 1, pp. 169-169, March 1964. doi: 10.1109/TCT.1964.1082263

<http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=1082263&isnumber=23381>

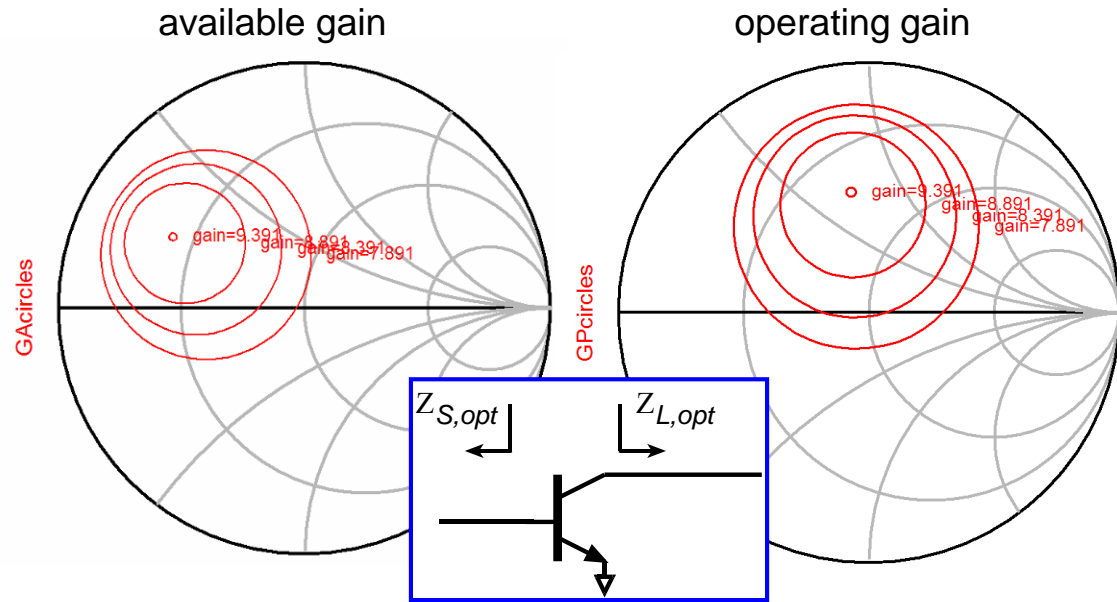


Why Ga and Gp Matter: Matching Network Design

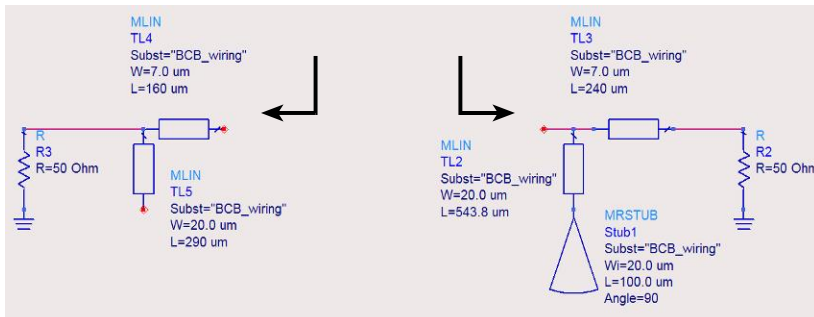
Circuit simulators (ADS, etc) provide contour plots of G_a vs source impedance and G_p vs load impedance.

These, the G_a & G_p circles, show the variation in transistor gain as generator and load impedance are tuned.

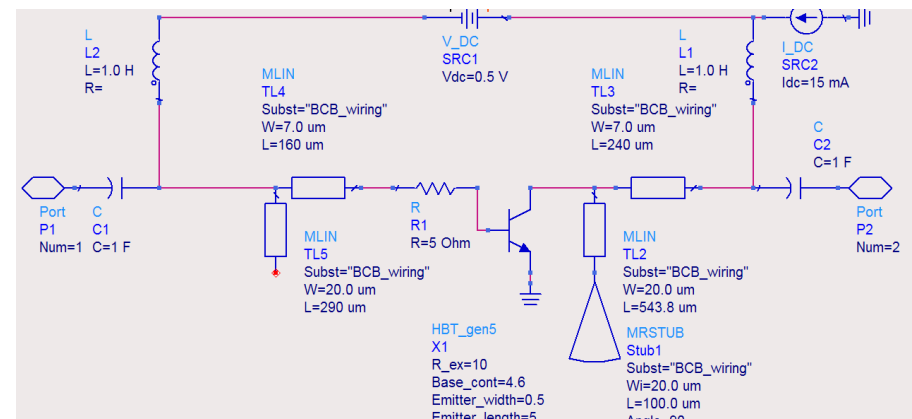
The center of these circles are the (generator, load) impedances required for maximum gain



We can then separately design Input & Output Tuning Networks to provide these impedances...



...added to device, the amplifier is realized



Caution: the above assumes that MAG exists; we must examine this critical point in the next lecture

Simple Matching Example: Unilateral Device Model

Simple FET model; $C_{gd} = 0 \rightarrow S_{12} = 0$

\rightarrow unilateral device.

$G_{\max} = P_{ava} / P_{in} = P_{load} / P_{in}$ because $R_L = R_{out}$

$P_{in} = I_{in}^2 R_{in}$ and $V_{gs} = I_{in} / \omega C_{gs}$

$$P_{out} = I_{load}^2 R_{load} = (g_m V_{gs} / 2)^2 R_{ds} = \frac{g_m^2 \|I_{in}\|^2}{4\omega^2 C_{gs}^2} R_{ds}$$

$$\frac{P_{out}}{P_{in}} = \frac{g_m^2 R_{ds}}{4\omega^2 C_{gs}^2 R_{in}} = \frac{f_{\max}^2}{f^2} \text{ where } f_{\max} = \frac{g_m}{2\pi C_{gs}} \sqrt{\frac{R_{ds}}{4R_{in}}}$$

If we impedance-match, then $\|S_{21}\|^2 = \frac{f_{\max}^2}{f^2}$ at $f = f_{match}$

while, from inspection, $S_{21,FET} = -\frac{2g_m (R_{DS} \parallel Z_0)}{1 + j\omega(R_{in} + Z_0)C_{gs}}$

