# *ECE145a / 218a*
# *Signal Flow Graphs*

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# Signal Flow Graphs

Mason : control system theory

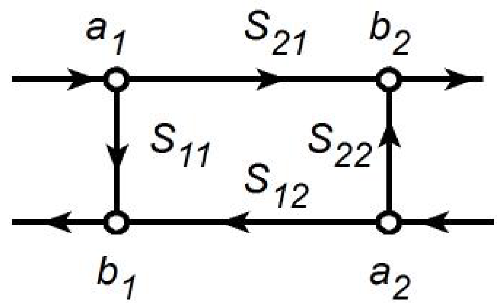
System of equations

(example : S - parameters)

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

Represent as below :




Variables represented as nodes :


Value of variable = sum of entering branches

= sum of values of connecting nodes

times weight of branches.

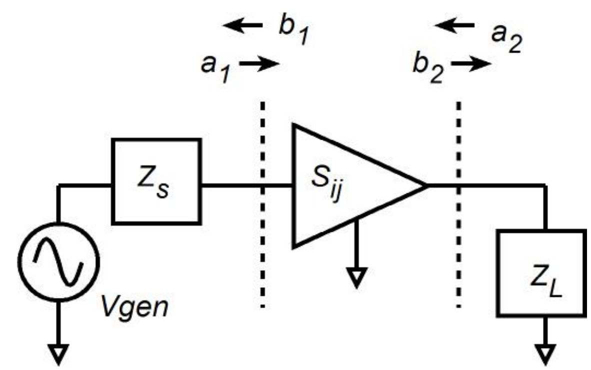
# Representation of Generator & Load

$$V^+ = T_s V_{gen} + \Gamma_s V^-$$

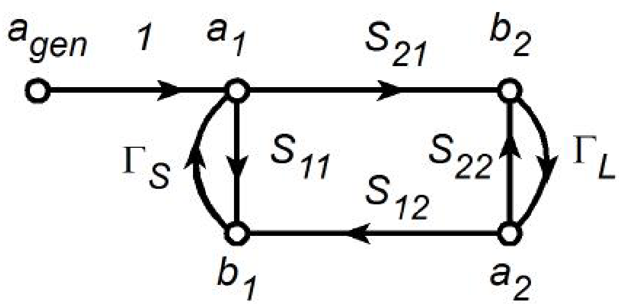
$$\rightarrow V^+ \Big/ \sqrt{Z_0} = T_s V_{gen} \Big/ \sqrt{Z_0} + \Gamma_s V^- \Big/ \sqrt{Z_0}$$

$$\rightarrow a_1 = a_{gen} + \Gamma_s b_1$$

$$\text{further}: a_2 = \Gamma_L b_2$$

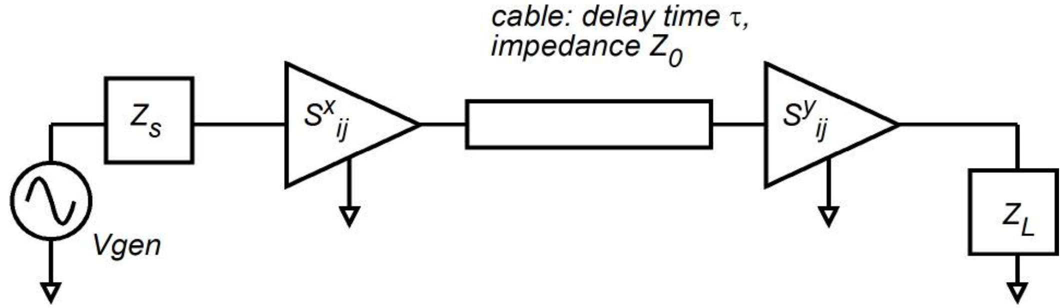



Representation:

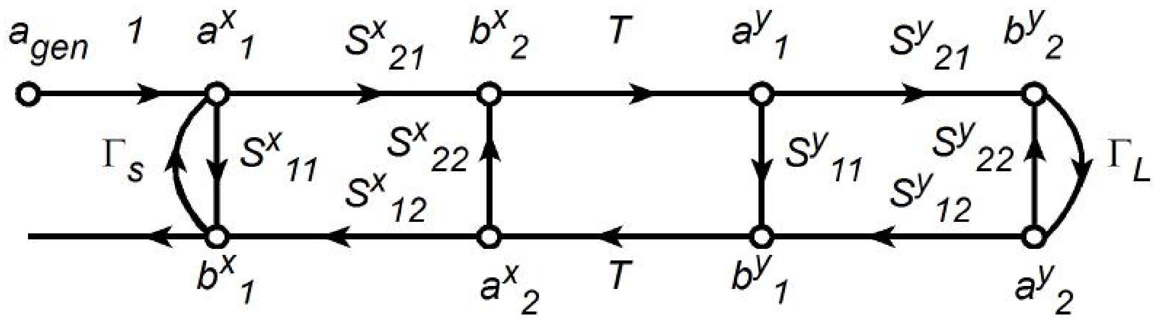


# 2nd Example: Cascaded Amplifiers

Circuit




Representation




$$T = e^{-j\omega\tau}$$

The signal flow graph compactly and visually represents the many equations describing the system.

# Why Use Signal Flow Graphs ?

Signal flow graphs most heavily used in control system theory :

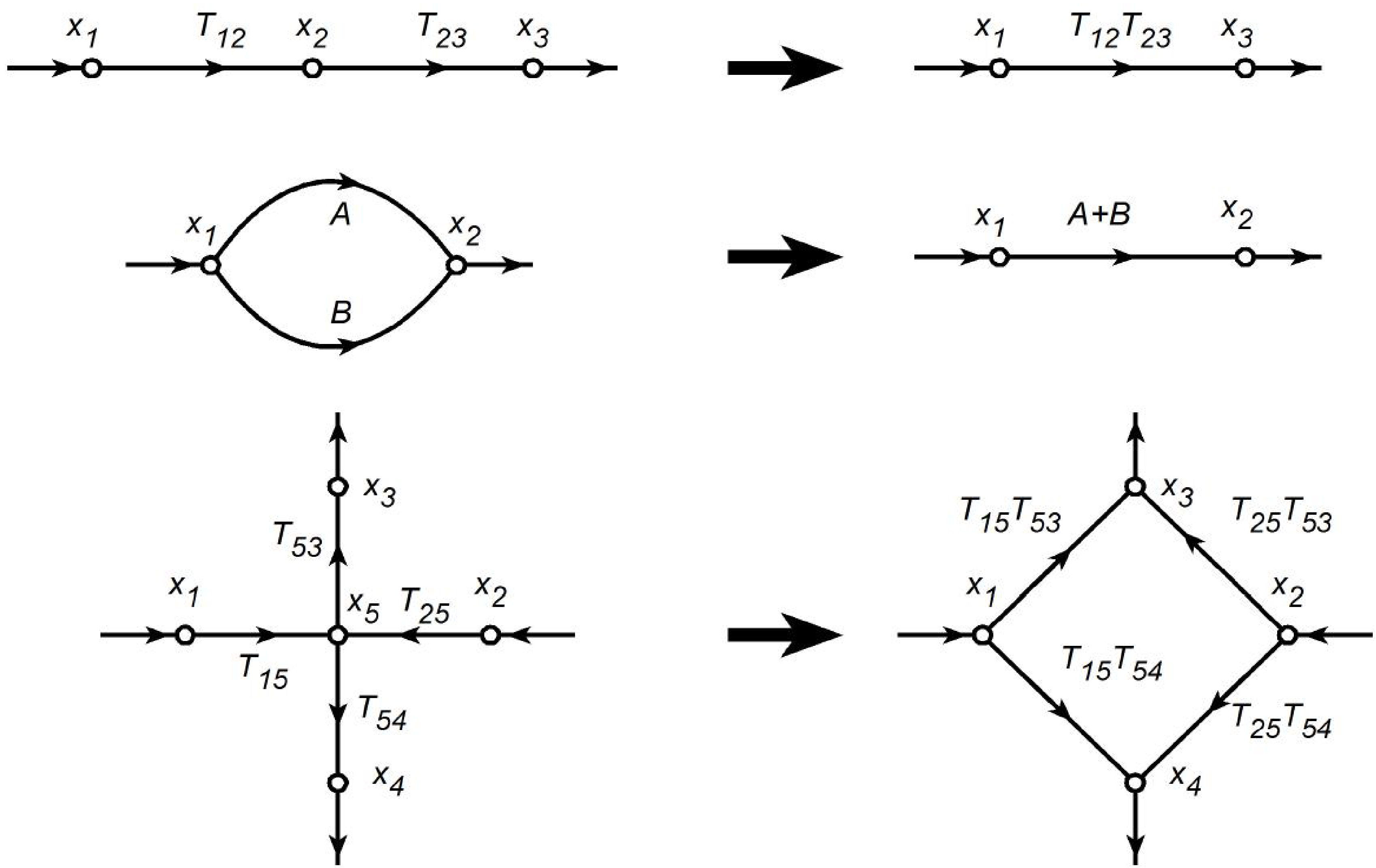
- Organizes the representation of a set of linear equations
- Lends visual intuition in analysis.
- Provides efficient solution through * Mason's Gain Rules *

S.J. Mason : "Feedback theory – Some Properties of Signal Flow Graphs"
Proc. IRE, 41, p. 1141, Sept. 1953.

or : Many texts on control system theory.

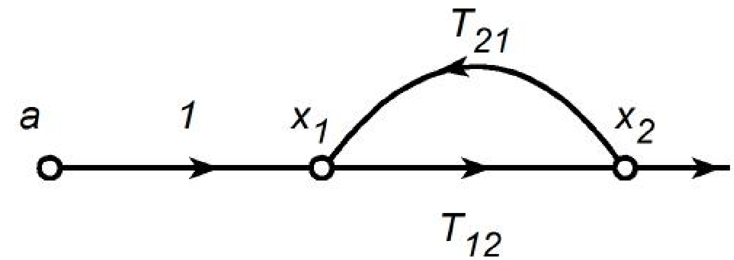
# Manipulating Signal Flow Graphs

Elementary manipulations:

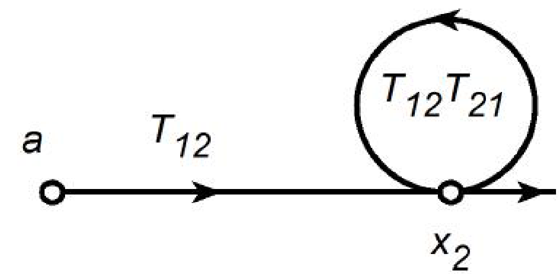


# Reducing a Feedback Loop

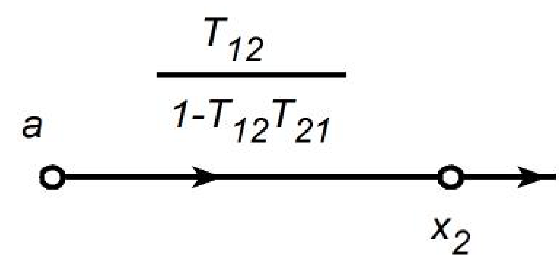
System with feedback :




$$x_2 = T_{12}x_1$$
$$x_1 = a + T_{12}x_2$$
$$\rightarrow x_2 = T_{12}a + T_{12}T_{21}x_2$$



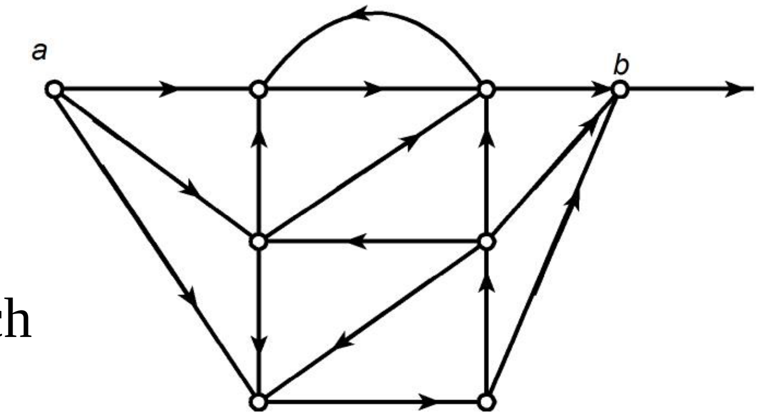

$$x_2 = T_{12}a + T_{12}T_{21}x_2$$
$$\rightarrow x_2 = \frac{T_{12}}{1 - T_{12}T_{21}}a$$



# Mason's Gain Rule

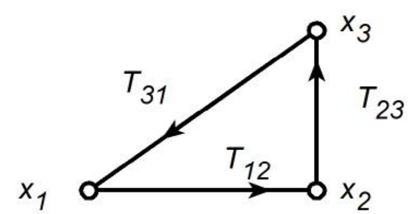
Define $T = b/a =$ "transmission"

How do we find $T$?



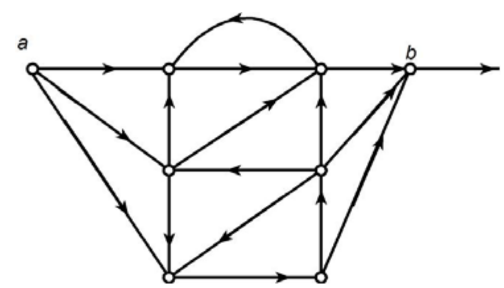

Define a path $P_i$ as any route from $a$ to $b$ which does not go through any node twice.

Define a loop coefficient $L_i$ as the product $(T_{12}T_{23}T_{31})$ of the transmission coefficients around any closed loop.



# Mason's Gain Rule

$$T=\frac{b}{a}=\frac{P_1\left[1-\sum L(1)^{(1)}+\sum L(2)^{(1)}-\cdots\right]+P_2\left[1-\sum L(1)^{(2)}+\sum L(2)^{(2)}-\cdots\right]+\cdots}{1-\sum L(1)\;+\sum L(2)\;-\sum L(3)\;+\cdots}$$




Where:

$\sum L(1)$ = sum of all loop coefficients

$\sum L(1)^{(1)}$ = sum of all loop coefficients for loops which do not touch path $P_1$

$\sum L(2)$ = sum of all second-order loops

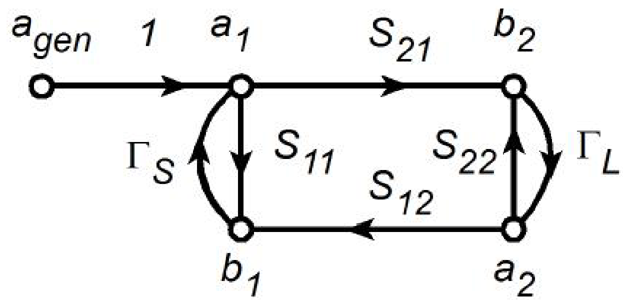
A second-order loop is the product of the coefficients of any pair of non-touching loops.

$\sum L(2)^{(1)}$ = sum of all second-order loops which do not touch path $P_1$.

etc.

# Analysis of Simple Amplifier

Find $T = b_2 / a_{gen}$



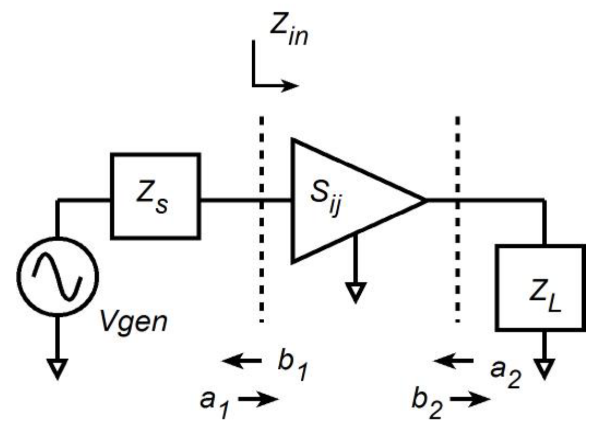

$$T = \frac{P_1\left[1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \cdots\right]}{1 - \sum L(1) + \sum L(2)}$$

$$= \frac{S_{21}}{1 - \underbrace{\Gamma_s S_{11} - \Gamma_l S_{22} - \Gamma_s \Gamma_l S_{21} S_{12}}_{\sum L(1)} + \underbrace{\Gamma_s S_{11} \Gamma_l S_{22}}_{\sum L(2)}}$$

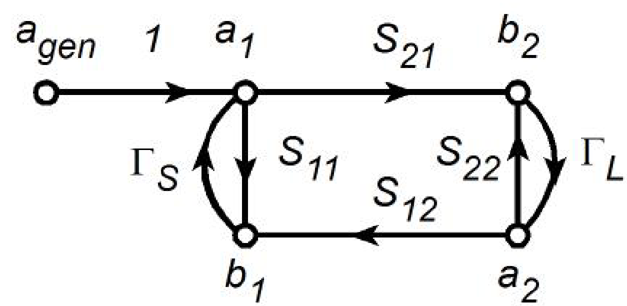
$= \text{easy}\,!$

# Input Reflection Coefficient

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

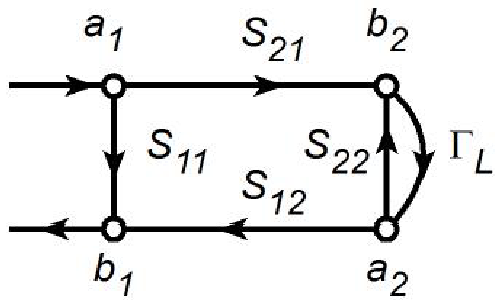



Overall Representation



# Input Reflection Coefficient

Relationship between incident and reflected waves at input :




$$T = \frac{b_1}{a_1} = \Gamma_{in} = \frac{S_{11}\left[1 - S_{22}\Gamma_L\right] + S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}$$

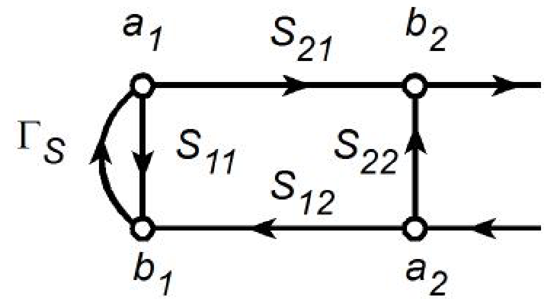
$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$

$$\text{Input}\left\{\begin{array}{c}\text{impedance}\\\text{reflection coefficient}\end{array}\right\}\text{depends upon}$$

$$\text{load}\left\{\begin{array}{c}\text{impedance}\\\text{reflection coefficient}\end{array}\right\}\text{unless } S_{12}S_{21} = 0.$$

# Output Reflection Coefficient

Relationship between incident and reflected waves at output :




$$T = \frac{b_2}{a_2} = \Gamma_{out} = \frac{S_{22}\left[1 - S_{11}\Gamma_S\right] + S_{21}\Gamma_S S_{12}}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

$$\text{Output} \left\{ \begin{matrix} \text{impedance} \\ \text{reflection coefficient} \end{matrix} \right\} \text{depends upon}$$

$$\text{source} \left\{ \begin{matrix} \text{impedance} \\ \text{reflection coefficient} \end{matrix} \right\} \text{unless } S_{12}S_{21} = 0.$$

# Implication for Impedance Matching

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

If $S_{12}S_{21} = 0$, then either $S_{12} = 0$ or $S_{21} = 0$.

In either case, the amplifier cannot pass signals in both directions.

If $S_{12}S_{21} = 0$, the amplifier is *unilateral*.

Unilateral amplifiers have $\Gamma_{\text{in}} = S_{11}$ and $\Gamma_{\text{out}} = S_{22}$.

In this case, tuning the input match does not disturb the output tuning,

nor does tuning the output match disturb the input tuning.

In bilateral amplifiers ($S_{12}S_{21} \neq 0$), input and output tuning are interactive.

Interactive tuning $\rightarrow$ at a minimum: design is more difficult.

If $S_{12}S_{21}$ is sufficiently large, we will find that matching is not possible.

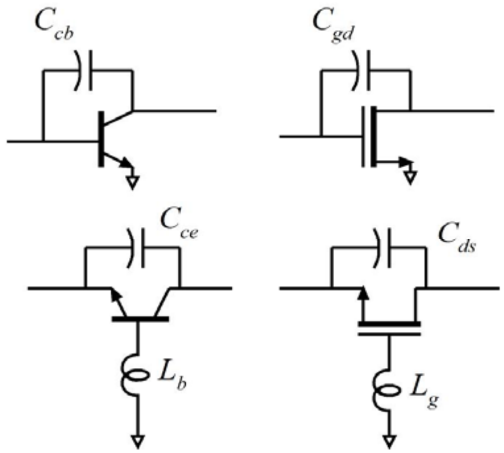
# Origin of Nonzero $S_{12}S_{21}$

Reverse coupling in common-source FETs: $C_{gd}$

Reverse coupling in common-emitter BJTs: $C_{cb}$

Reverse coupling in common-gate FETs: $L_g, C_{ds}$

Reverse coupling in common-base BJTs: $L_b, C_{ce}$

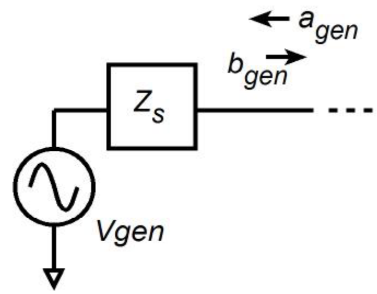



Some of these are device parasitics, some arise only from poor interconnect design near the device terminals.

High reverse isolation (low $S_{12}$) increases amplifier stability and (usually) increases device maximum stable gain.

# Available Source Power

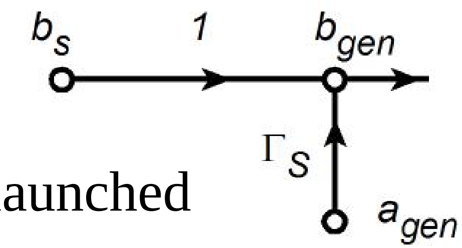
$$P_{AV,G} = \left\| V_{gen} \right\|^2 \Big/ 4 \cdot \mathrm{Re}\{Z_{gen}\}$$




$$b_{gen} = T_s V_{gen} / \sqrt{Z_0} + \Gamma_s a_{gen}$$

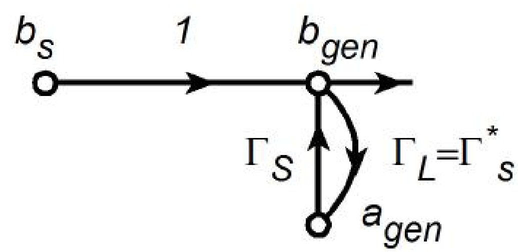
$$b_{gen} = b_s + \Gamma_s a_{gen}$$

note that $b_s$ is the wave amplitude launched into a load $Z_L = Z_0$

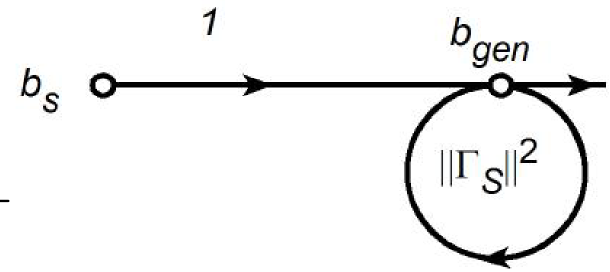



Now: connect conjugate-matched load

$Z_L = Z_S^*$ i.e. $\Gamma_L = \Gamma_S^*$

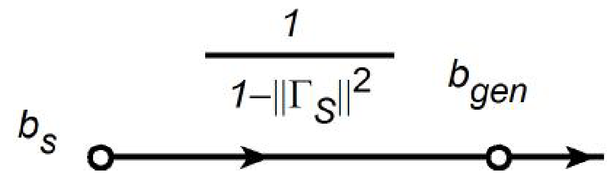


# Available Source Power




$$\text{Reverse Power} = \|b_s\|^2 \frac{\|\Gamma_L\|^2}{\left[1-\|\Gamma_S\|^2\right]^2} = \|b_s\|^2 \frac{\|\Gamma_s\|^2}{\left[1-\|\Gamma_S\|^2\right]^2}$$

$$\text{Forward Power} = \|b_s\|^2 \frac{1}{\left[1-\|\Gamma_S\|^2\right]^2}$$




$$\text{Load Power} = \text{Available Power} = \|b_s\|^2 \frac{1}{1-\|\Gamma_S\|^2}$$

$$P_{AVG} = \frac{\|b_s\|^2}{1-\|\Gamma_S\|^2} \quad \text{where } \|b_s\|^2 \text{ is the power delivered to } Z_L = Z_0$$