ECE145a / 218a
Bilateral Tuned Amplifier Design: Stability

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Stability

Instability : non - zero output with zero input.

Stability theory : many equivalent versions :

- Classical control system theory
  - Bode & Nyquist methods : find phase margin.
  - Find real part of closed - loop poles.
  - Do any lie in right half of s - plane ?
- Network theory :
  - Analyze circuit by nodal analysis.
  - Find poles in $V_{out}(s)/V_{gen}(s)$.
  - Do any lie in right half of s - plane ?
- Impedance viewpoint
  - Does $Z_{in}(j\omega)$ have a negative real part ?
- Reflection (S) viewpoint
  - Is the magnitude of Γ_{in} greater than 1 ?

Stability: LaPlace Transform / Eigenvalue Method

Physical system (circuits, etc.) in small-signal limit $\rightarrow$ transfer function.

$$\frac{V_{out}(s)}{V_{gen}(s)} \text{ or } \frac{b(s)}{a(s)} \text{ or } \frac{V_{in}(s)}{I_{in}(s)} \text{ etc} = H(s)$$

$$H(s) = c_1 \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} = c_2 \frac{(s-s_{z1})(s-s_{z2})(s-s_{z3})\ldots}{(s-s_{p1})(s-s_{p2})(s-s_{p3})\ldots}$$

Impulse response:

$$h(t) = k_1 \exp(s_{p1}t) + k_2 \exp(s_{p2}t) + k_3 \exp(s_{p3}t) + \ldots$$

Poles are (generally) complex:

$$s_{pi} = \sigma_{pi} + j\omega_{pi}$$

If any poles lie in right half of s-plane (σ_{pi} positive)

then $k_i \exp(s_{pi}t)$ will grow without limit.

$\rightarrow$ unstable system.

Unstable system if any pole has positive real part

$s_{pi} = \sigma_{pi} + j\omega_{pi}$; negative σ_{pi}

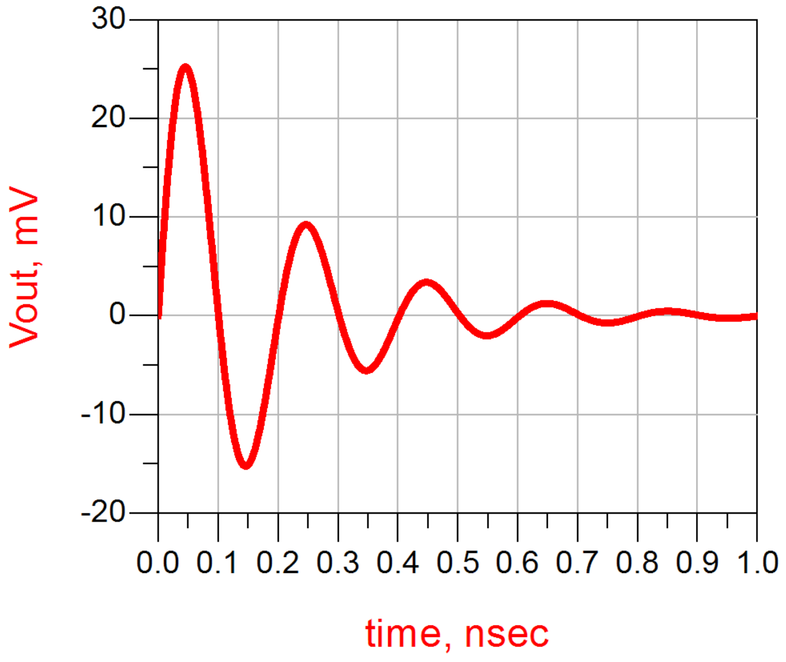
$\rightarrow \exp(s_{pi}t)$ decays.

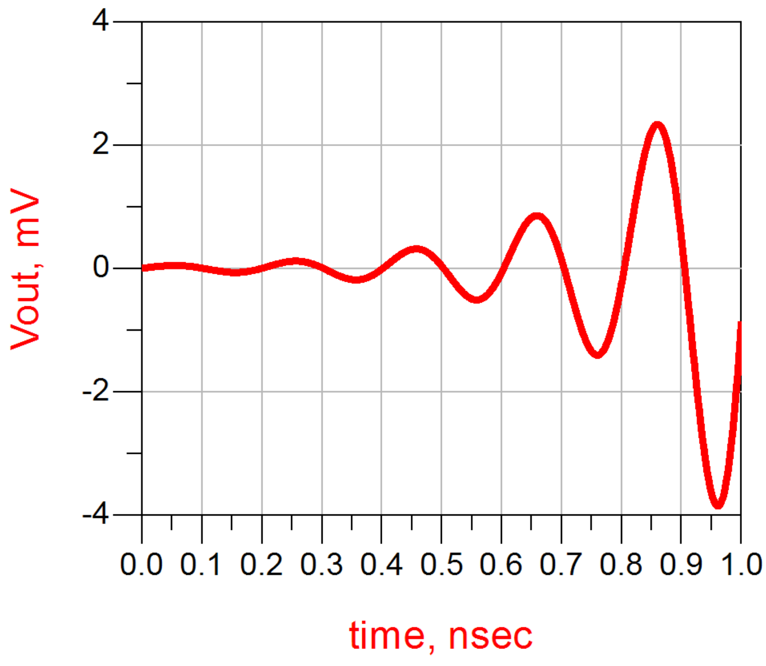
$\rightarrow$ stable system.

$s_{pi} = \sigma_{pi} + j\omega_{pi}$; positive σ_{pi}

$\rightarrow \exp(s_{pi}t)$ grows.

$\rightarrow$ unstable system.





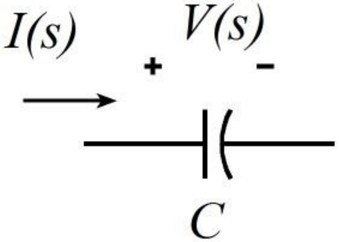
Stability from Network Viewpoint

Impedances:

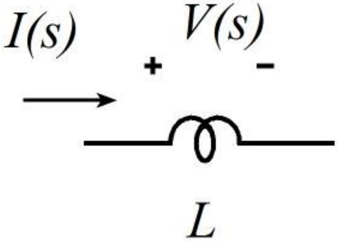
$$I(t) = Ie^{st}$$

$$V(t) = Ve^{st}$$

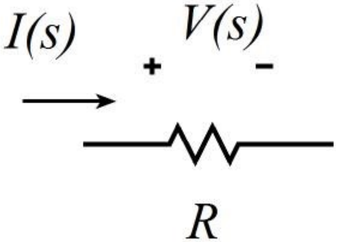
$$Z = 1/sC$$



$$Z = sL$$



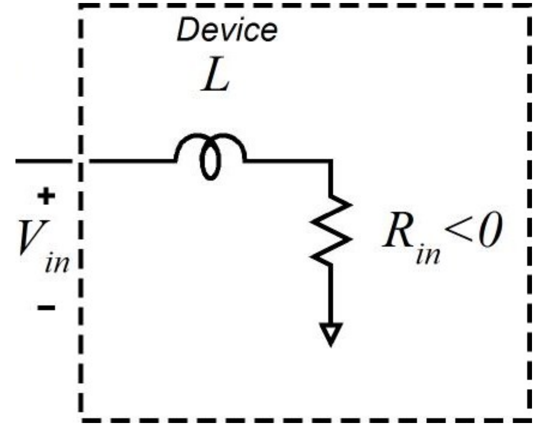
$$Z = R$$



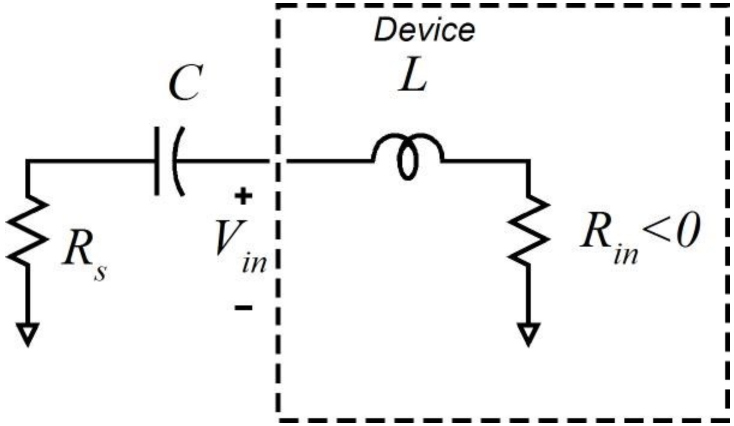
This assumes zero initial conditions

Network Theory: One-Port Potential Instability.

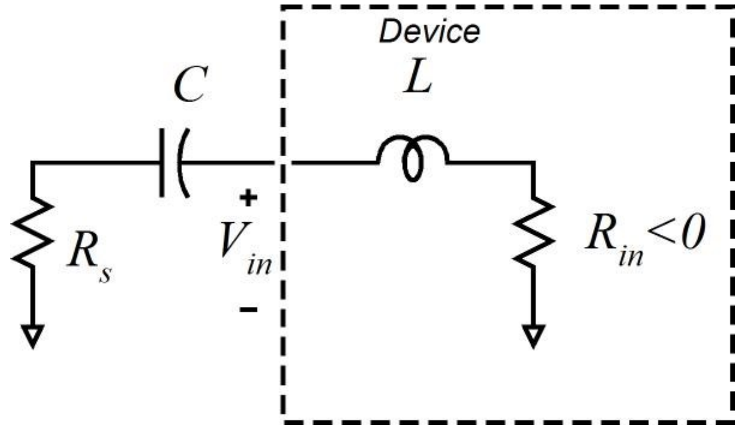
Consider a 1 - port device having a negative real part to $Z_{in}(j\omega)$



We connect an external load to consider stability of the combined system.



Network Theory: One-Port Potential Instability.



$$\text{Nodal analysis}: \frac{V_{in}}{R_s + 1/sC} + \frac{V_{in}}{R_{in} + sL} = 0$$

$$\text{Hence } V_{in} = 0 \text{ (stable) or } \frac{1}{R_s + 1/s_pC} + \frac{1}{R_{in} + s_pL} = 0$$

$$R_s + 1/s_pC + R_{in} + s_pL = 0 \quad \rightarrow \; s_p^2 LC + s(R_{in} + R_s)C + 1 = 0$$

$$s_{p1,2} = -\left(\frac{R_{in} + R_s}{2L}\right) \pm \sqrt{\left(\frac{R_{in} + R_s}{2L}\right)^2 - \frac{1}{LC}}$$

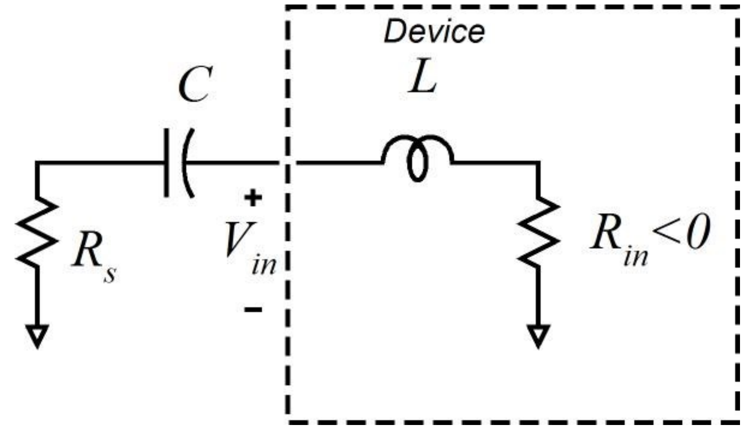
$$(R_{in} + R_s) \text{ positive} \rightarrow \text{Re}\{s_{p1,2}\} < 0 \rightarrow \text{stable}$$

$$(R_{in} + R_s) \text{ negative} \rightarrow \text{Re}\{s_{p1,2}\} > 0 \rightarrow \text{unstable}$$

Ideas: Unconditional stability, Potential instability

$(R_{in}+R_s)$ positive $\rightarrow$ $\mathrm{Re}\{s_{p1,2}\}<0 \rightarrow$ stable

$(R_{in}+R_s)$ negative $\rightarrow$ $\mathrm{Re}\{s_{p1,2}\}>0 \rightarrow$ unstable



If R_{in} is positive, no (positive) value of R_s produces an unstable system

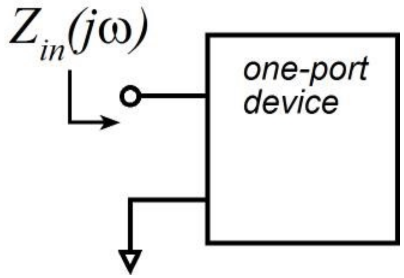
$\rightarrow$ device is unconditionally stable.

If R_{in} is negative, some (positive) values of R_s produces an unstable system

$\rightarrow$ device is potentially unstable

Unconditional stability, Potential instability

From impedance viewpoint:



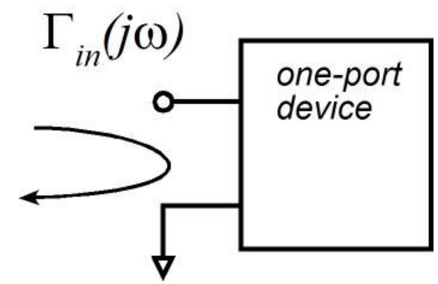
A one-port is unconditionally stable if:
$\mathrm{Re}\{Z_{in}(j\omega)\} > 0$ for all frequencies.

Alternatively, a one-port is unconditionally stable if:
$\mathrm{Re}\{Y_{in}(j\omega)\} > 0$ for all frequencies.

If $\mathrm{Re}\{Z_{in}(j\omega)\} < 0$ for some frequencies,
then the device is potentially unstable.

Potential instability: Relfection Viewpoint

From reflection viewpoint:



$$\Gamma_{in}(j\omega) = \frac{Z_{in}(j\omega) - Z_0}{Z_{in}(j\omega) + Z_0}$$

A one-port is unconditionally stable if:

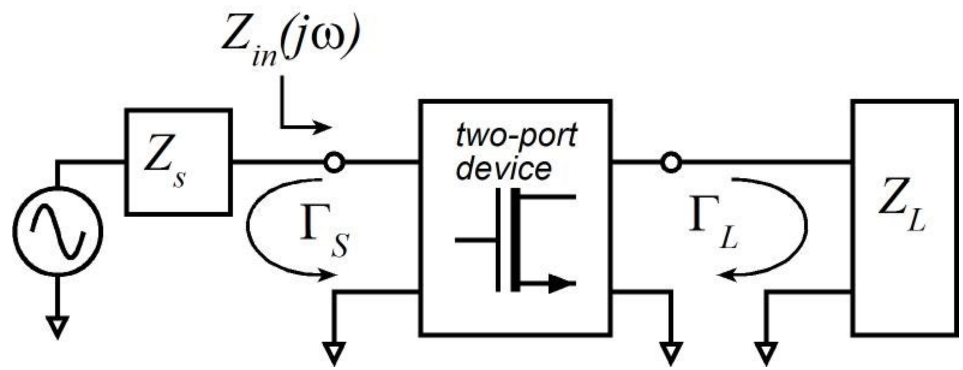
$\|\Gamma_{in}(j\omega)\| < 1$ for all frequencies.

If $\|\Gamma_{in}(j\omega)\| > 1$ for some frequencies,

then the device is potentially unstable.

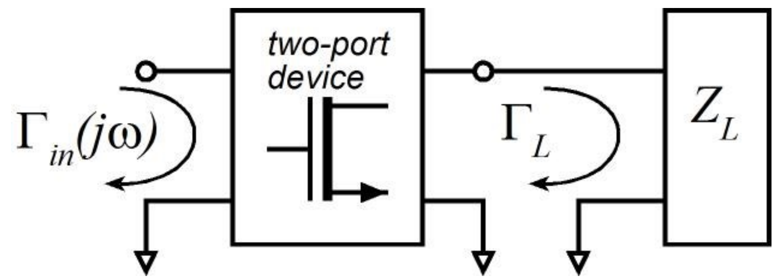
Stability of a Two-Port: Look at Γ_{in}

There are two degrees of freedom : Γ_S and Γ_L .



Think of it as a 1-port:

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



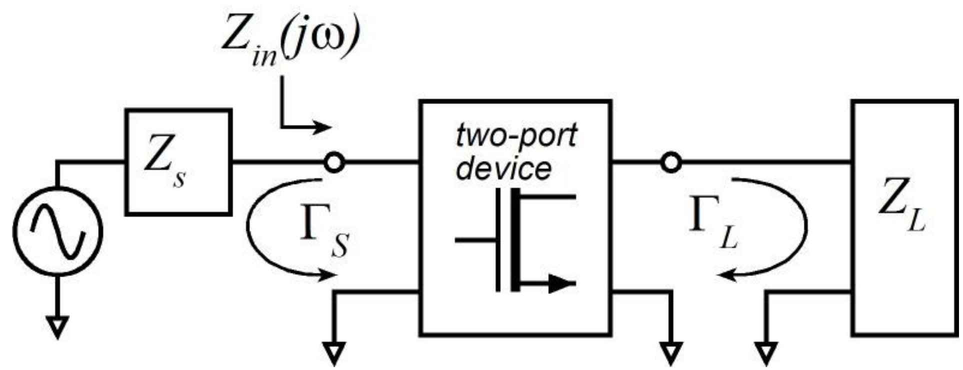
The device is unconditionally stable if $\|\Gamma_{in}(j\omega)\| < 1$

for all load reflection coefficients Γ_L such that $\|\Gamma_L\| \leq 1$

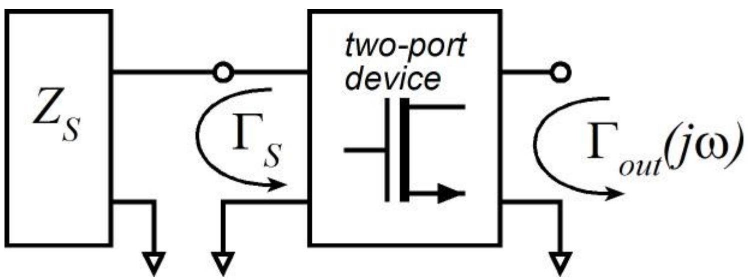
Why do we not consider $\|\Gamma_L\| > 1$?

Stability of a Two-Port: Look at Γ_{out} Instead.

There are two degrees of freedom : Γ_S and Γ_L .



Think of it as a 1-port:

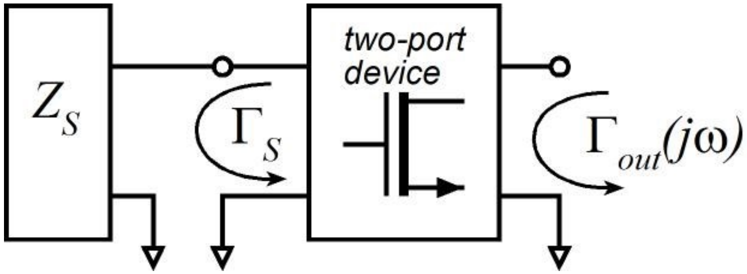


$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

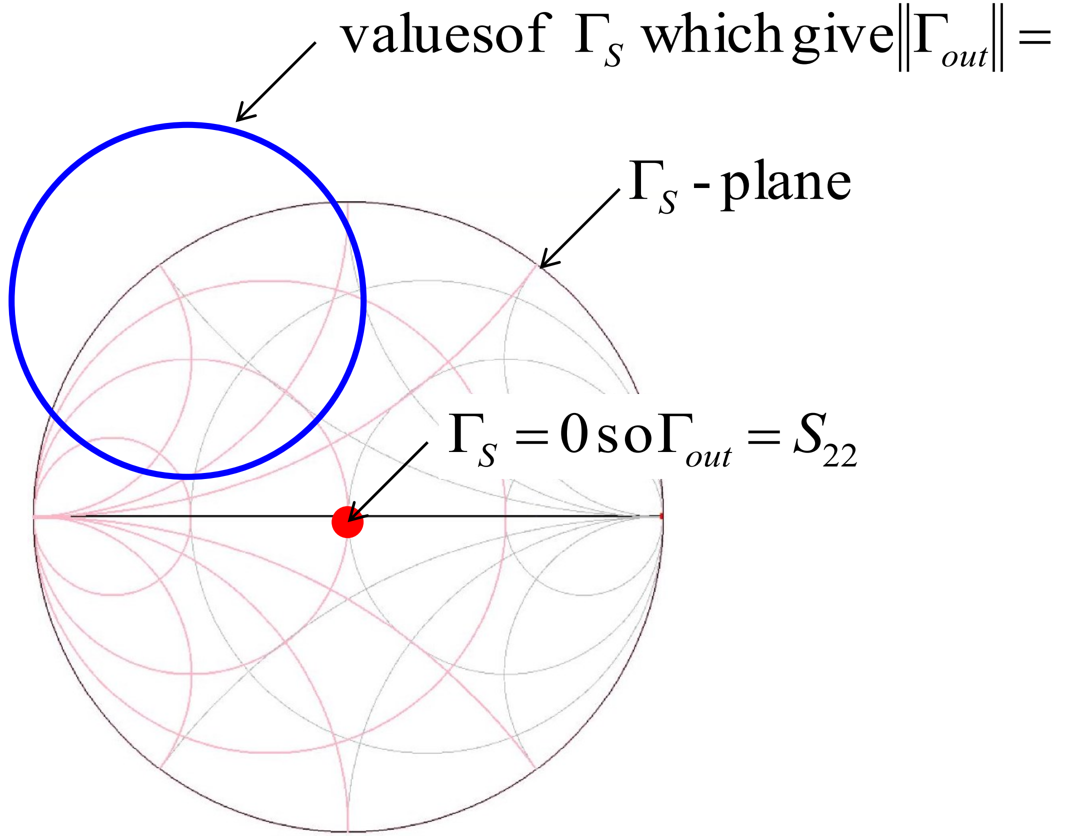
The device is unconditionally stable if $\|\Gamma_{out}(j\omega)\| < 1$ for all load reflection coefficients Γ_S such that $\|\Gamma_S\| \le 1$

Why do we not consider $\|\Gamma_S\| > 1$?

Input Stability Circle

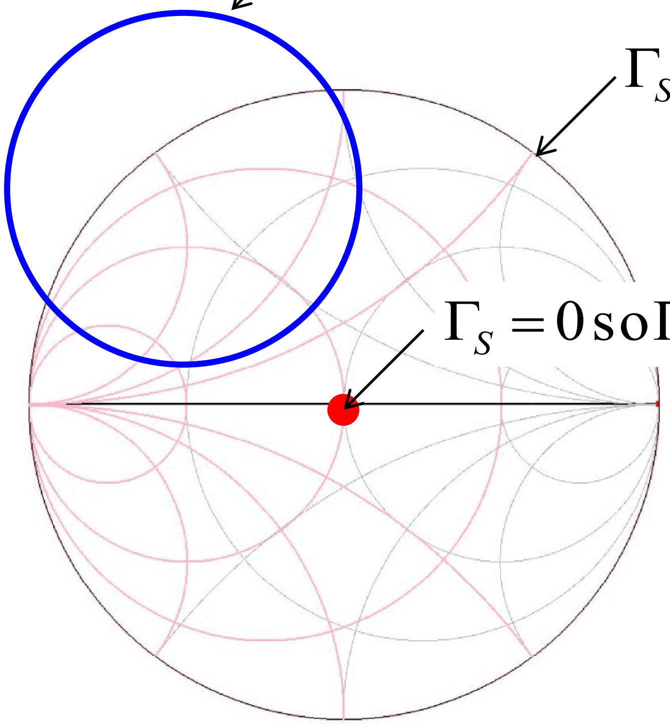


$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$



Is the inside or the outside of the circle stable ?

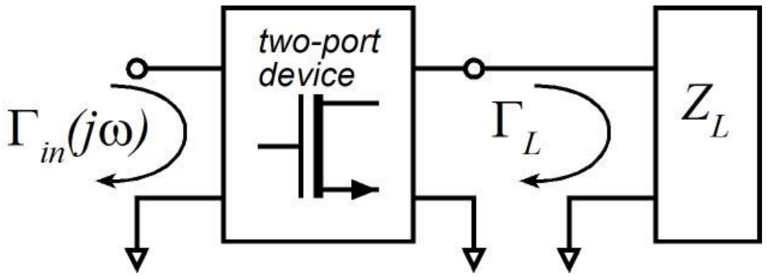
$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$



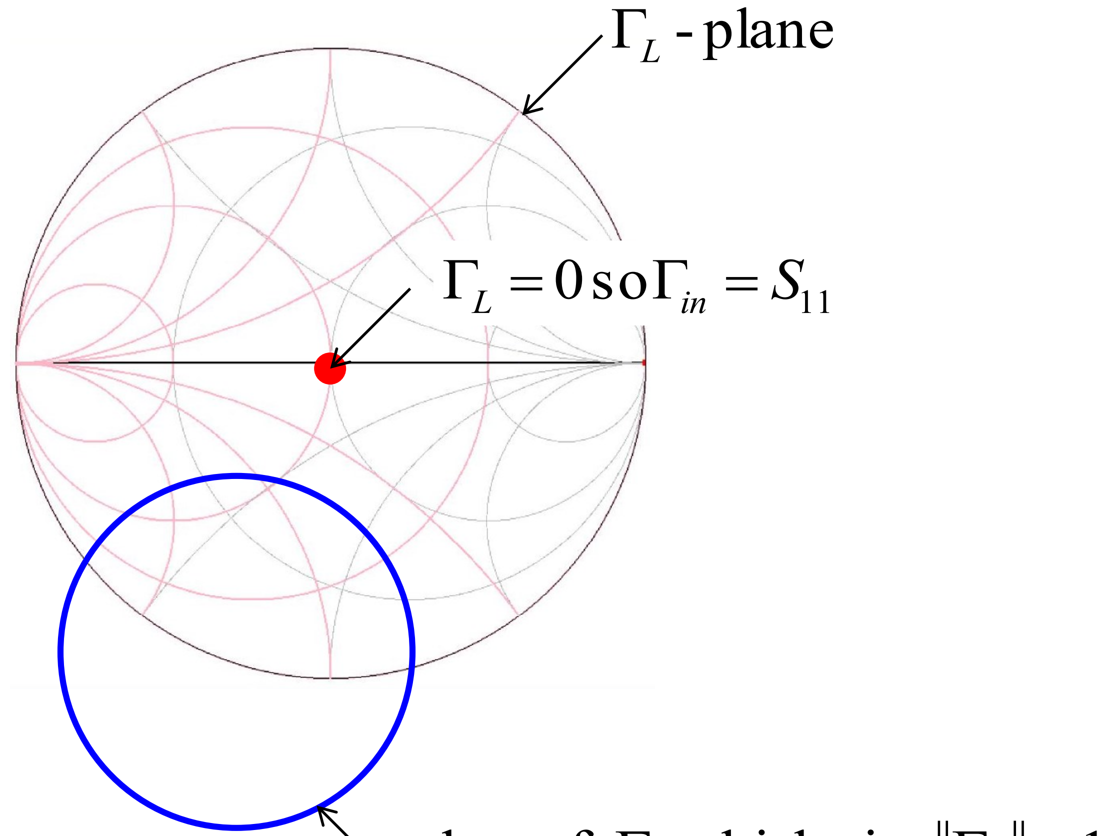
If $\|S_{22}\| < 1$, then the center of the Smith chart is stable.

If $\|S_{22}\| > 1$, then the center of the Smith chart is unstable

Output Stability Circle

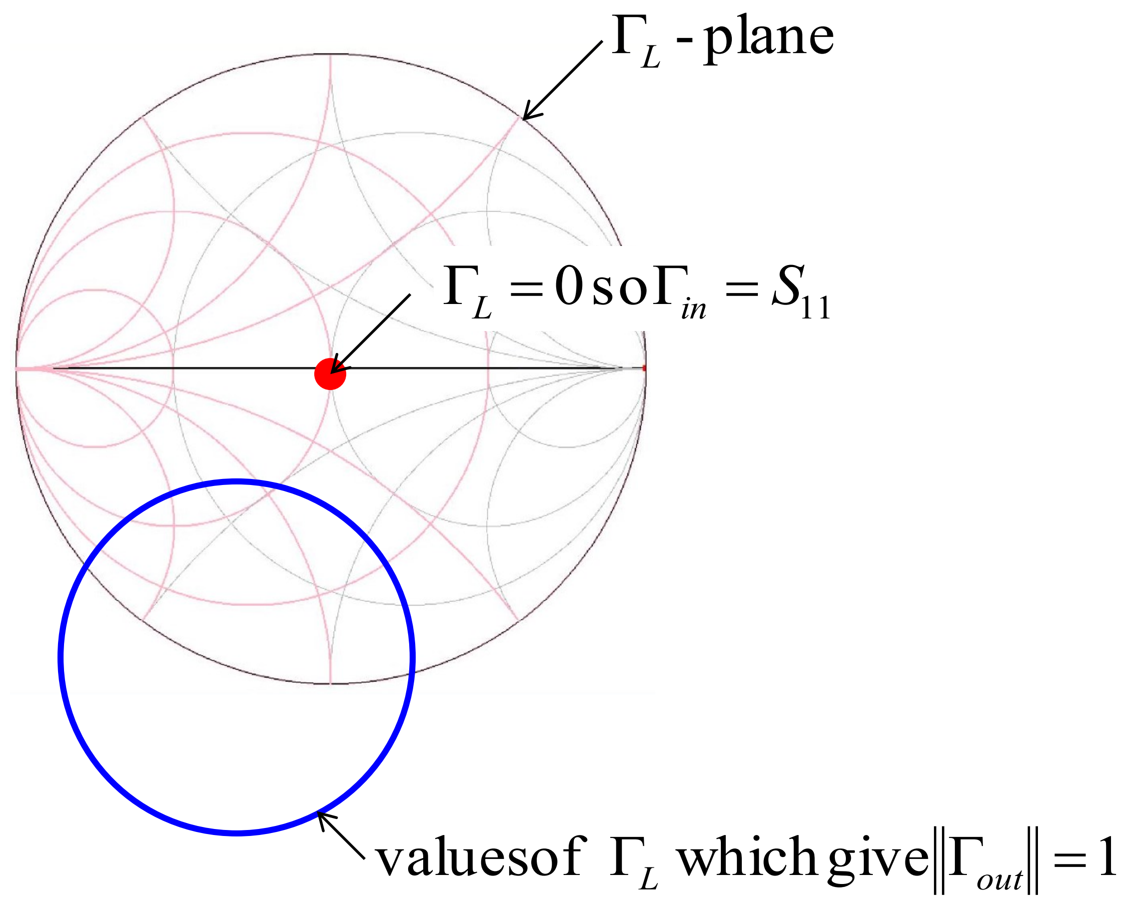


$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



Is the inside or the outside of the circle stable ?

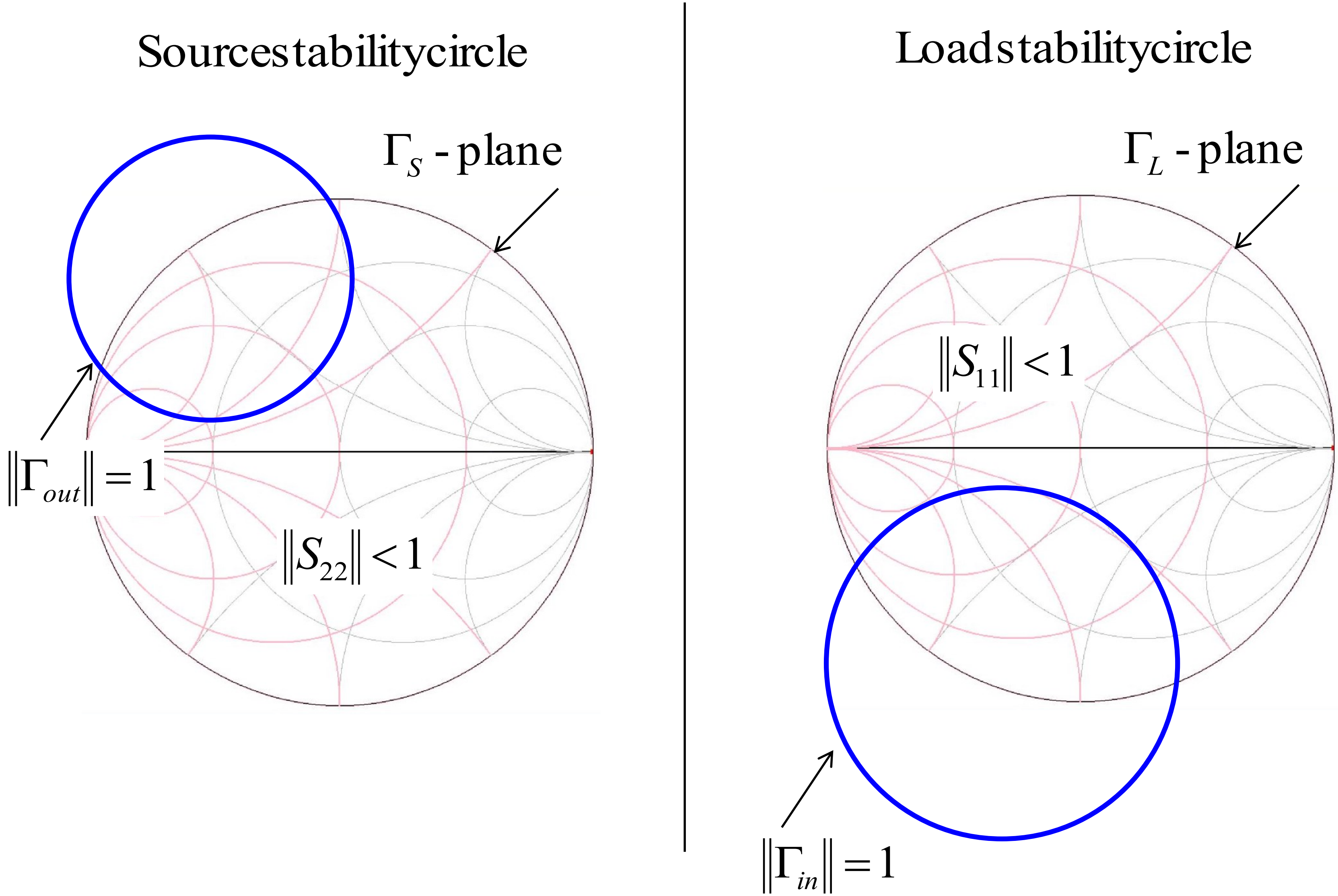
$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



If $\|S_{11}\| < 1$, then the center of the Smith chart is stable.

If $\|S_{11}\| > 1$, then the center of the Smith chart is unstable

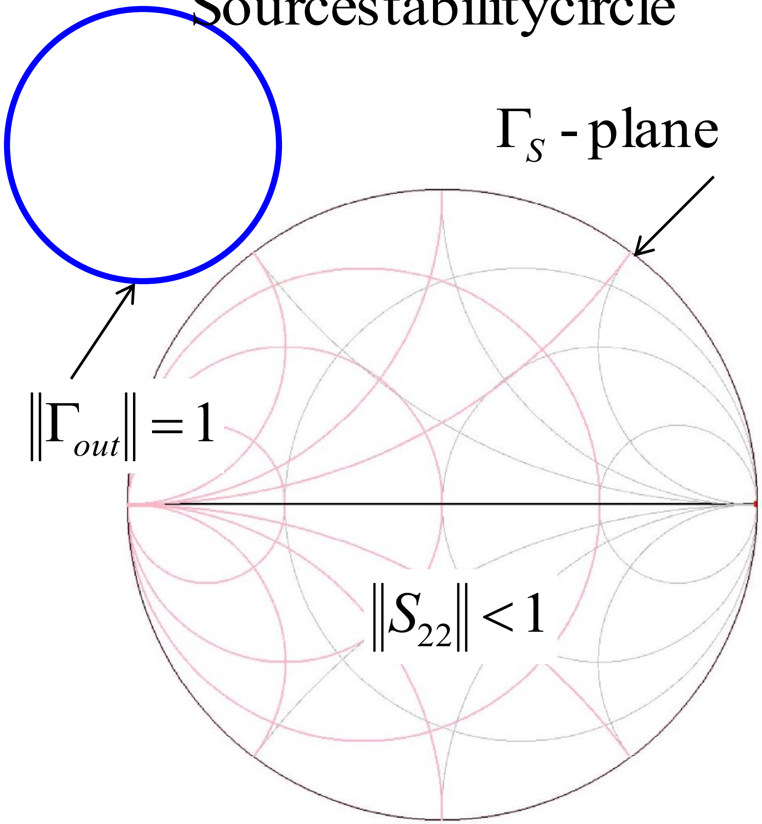
Potentially Unstable Amplifier



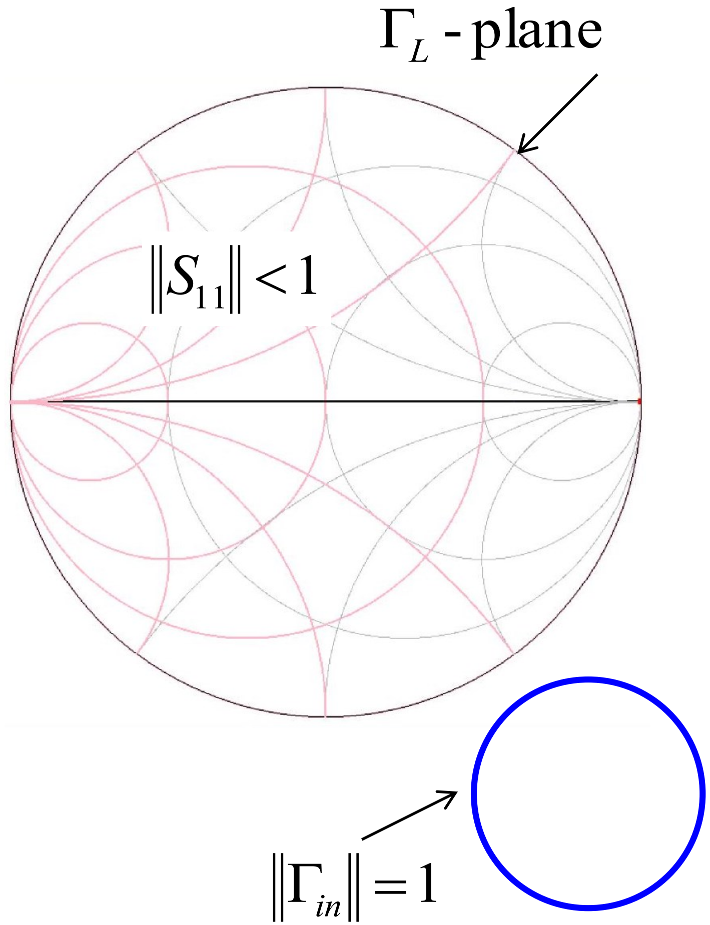
This is a test at one specific frequency; must test at all frequencies.

Unconditionally stable Amplifier

Source stability circle

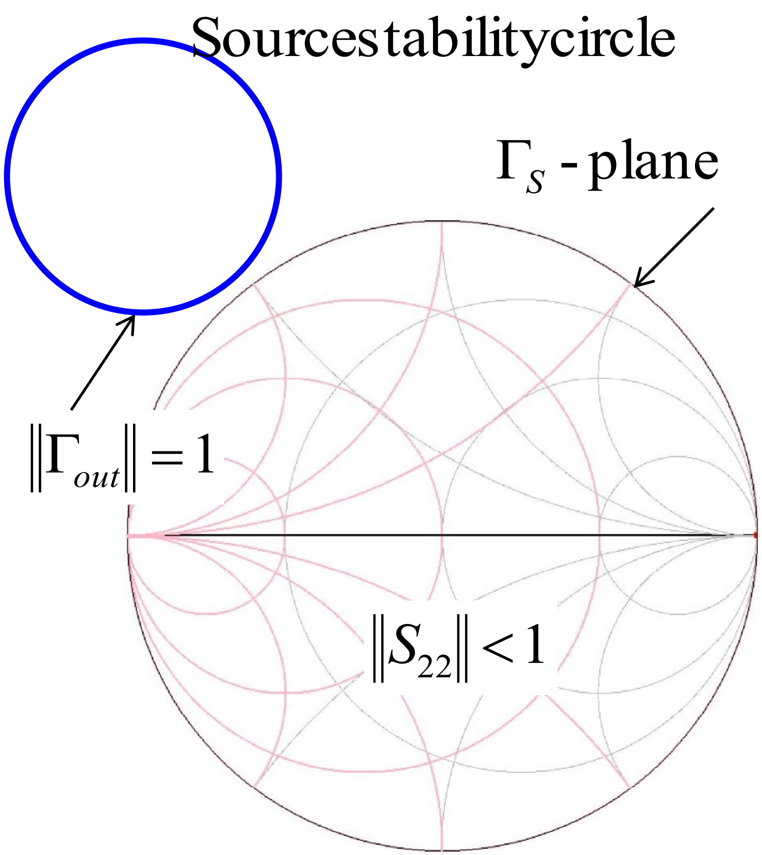


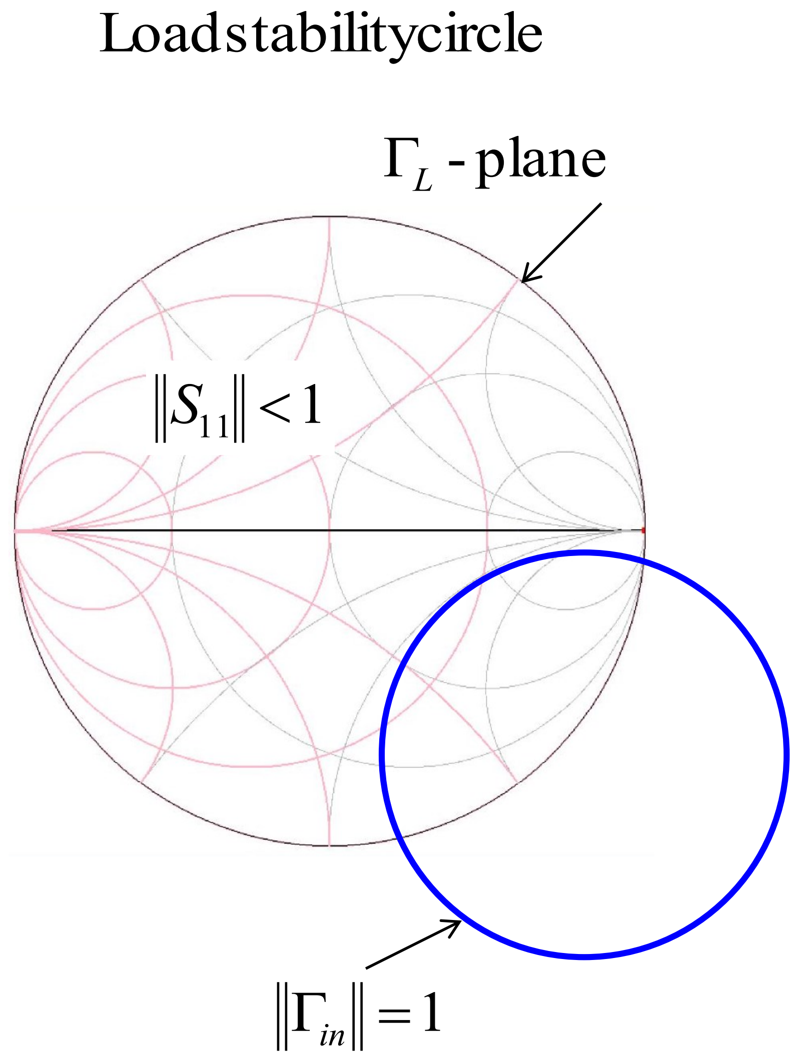
Load stability circle



This is a test at one specific frequency; must test at all frequencies.

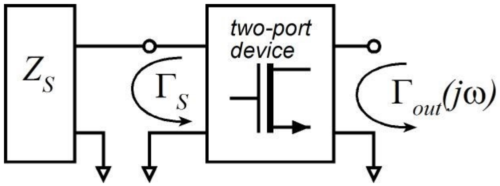
Is this possible ?????

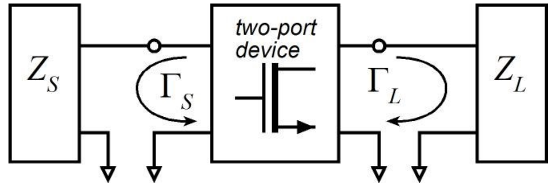


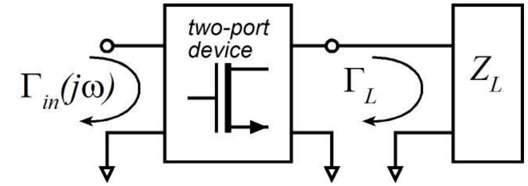


Stop and think clearly...

We need check only one stability circle







If there is no Γ_L for which $\|\Gamma_{in}\| > 1$,

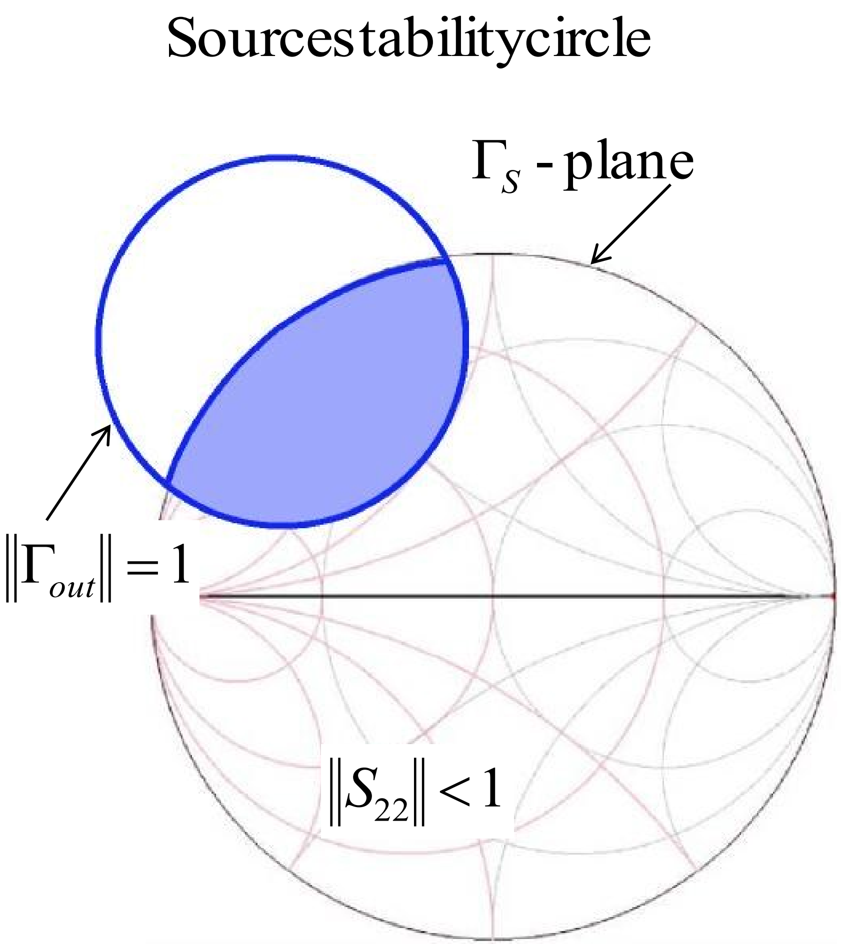
then no combination of Γ_S and Γ_L can cause oscillation.

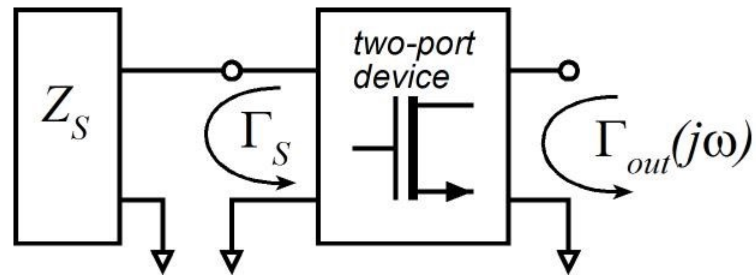
If there is no Γ_S for which $\|\Gamma_{out}\| > 1$,

then no combination of Γ_S and Γ_L can cause oscillation.

If one stablity circle passes the stablity test, then so must the other.

"Safe" and "Unsafe" Impedances



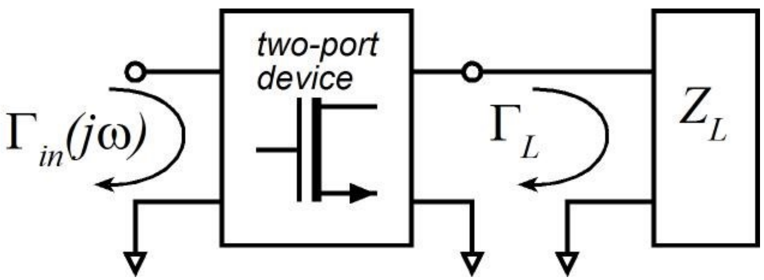


$$\Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

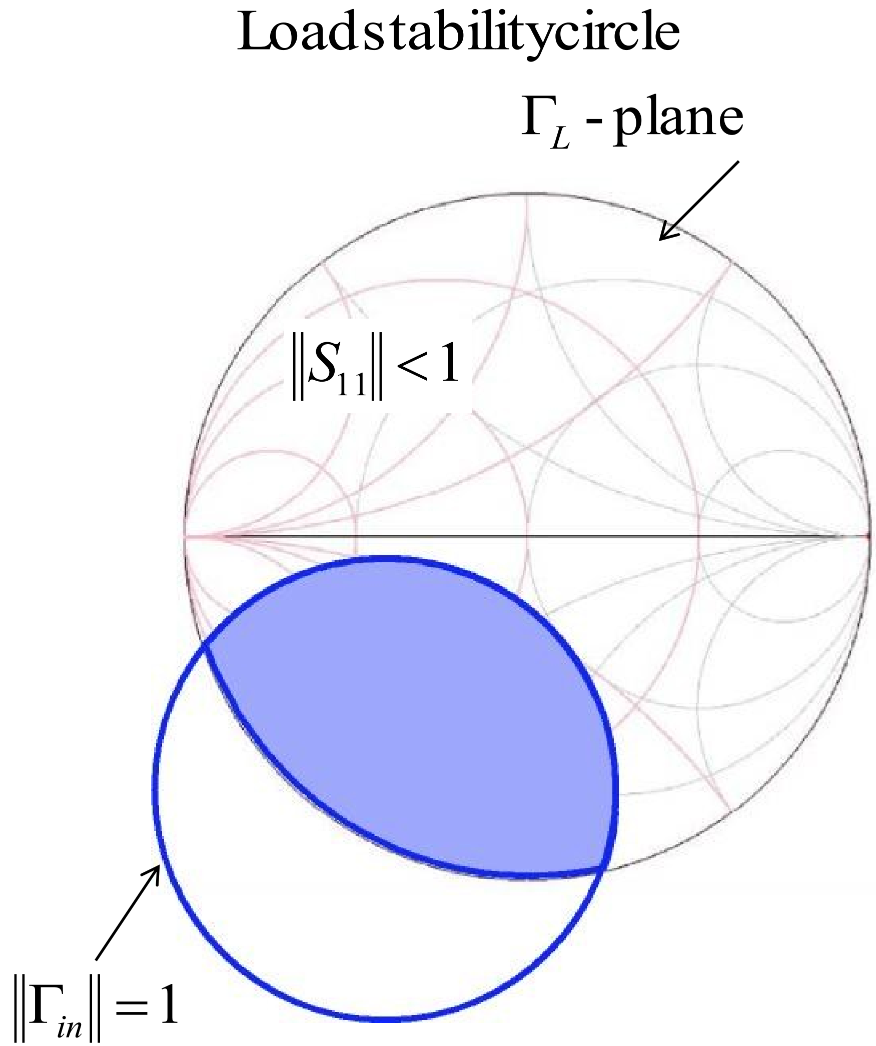
Γ_S outside the shaded region gives $\|\Gamma_{out}\| < 1$, cannot oscillate with any Γ_L.

Γ_S inside in the shaded region gives $\|\Gamma_{out}\| > 1$, might oscillate given with wrong Γ_L.

"Safe" and "Unsafe" Impedances



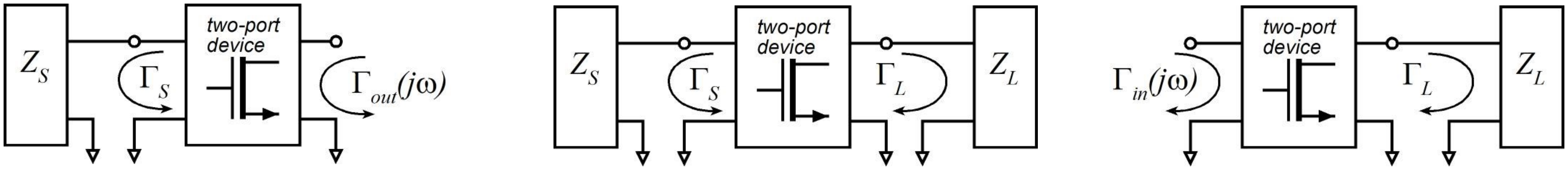
$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L}$$



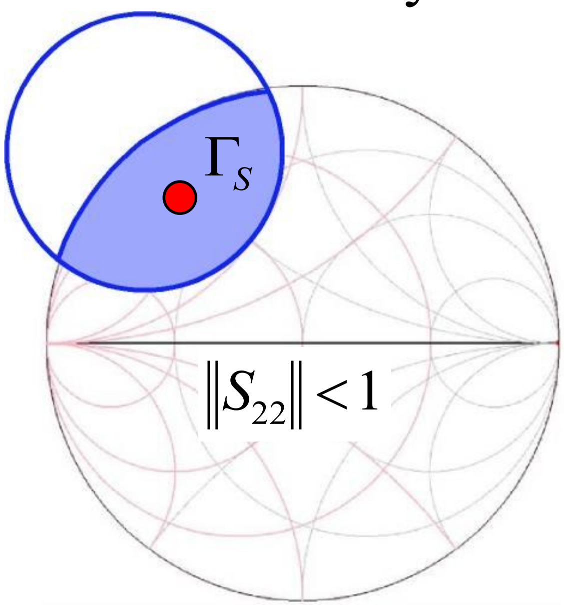
Γ_L outside the shaded region gives $\|\Gamma_{in}\| < 1$, cannot oscillate with any Γ_S.

Γ_L inside in the shaded region gives $\|\Gamma_{in}\| > 1$, might oscillate given with wrong Γ_S.

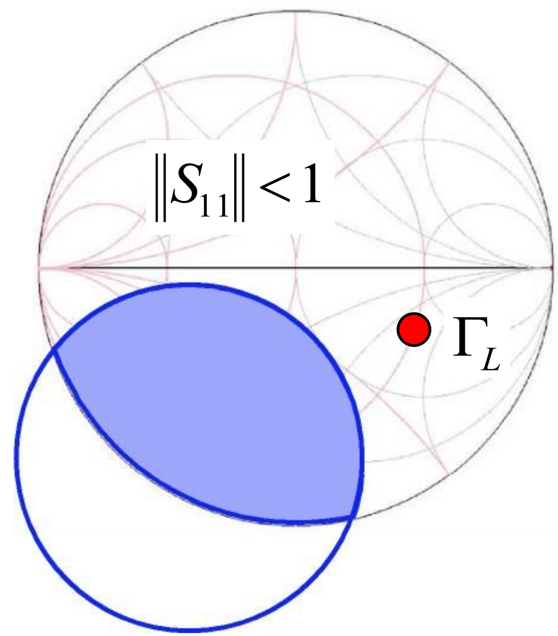
Stable Interfaces to a Potentially Unstable Device



Source stability circle



Load stability circle



If either Γ_S or Γ_L lies outside the danger zones, the circuit will be stable.

Stability Factors

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \qquad \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$

If there is no Γ_L for which $\|\Gamma_{in}\| > 1$, the network is unconditionally stable.

If there is no Γ_S for which $\|\Gamma_{out}\| > 1$, the network is unconditionally stable.

A 2-port is unconditionally stable if the Rollet stability factor K > 1, where

1) $K = \dfrac{1 - \|S_{11}\|^2 - \|S_{22}\|^2 + \|S_{11}S_{22} - S_{12}S_{21}\|^2}{2 \cdot \|S_{12}S_{21}\|}$

and

2a) $\|S_{11}S_{22} - S_{12}S_{21}\| < 1.$

An alternative 2nd condition is that

2b) $B_1 > 0$...this is graphed in ADS, but the B_1 definition given in ADS is wrong