ECE145a / 218a
Tuned Amplifier Design
-basic gain relationships
-design in the (simple) unilateral limit

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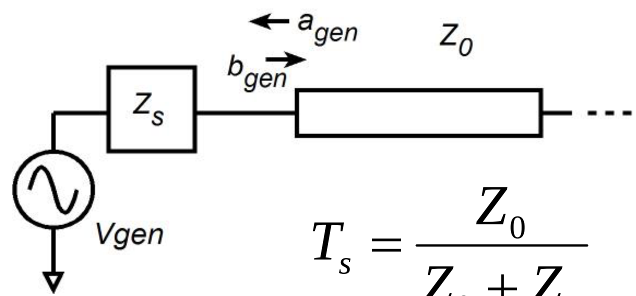
Generator Wave Relationships

Recall from last notes set :

$$V^+ = T_s V_{gen} + \Gamma_s V^-$$

$$\rightarrow V^+ / \sqrt{Z_0} = T_s V_{gen} / \sqrt{Z_0} + \Gamma_s V^- / \sqrt{Z_0}$$

$$\rightarrow b_{gen} = b_s + \Gamma_s a_{gen}$$



$$T_s = \frac{Z_0}{Z_0 + Z_s}$$

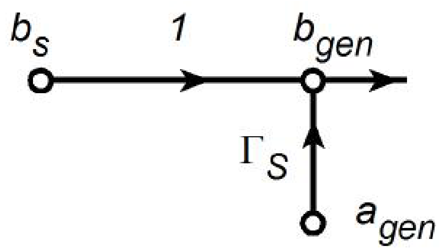
$$b_{gen} = b_s + \Gamma_s a_{gen}$$

where $b_s = \dfrac{\sqrt{Z_0}}{Z_0 + Z_s} \cdot V_{gen}$

and $\Gamma_s = \dfrac{Z_s - Z_0}{Z_s + Z_0}$

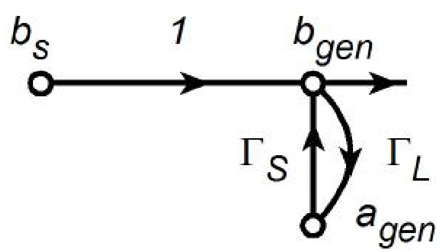
Generator Wave Relationships: Load Power

$$b_{gen} = b_s + \Gamma_s a_{gen}$$



Power delivered to Z_0 : $P_{Z_0} = \left\| b_{gen} \right\|^2$

Power delivered to Γ_L : $P_{Load} = \left\| b_{gen} \right\|^2 - \left\| a_{gen} \right\|^2$



$b_{gen} = b_s / (1 - \Gamma_S \Gamma_L)$ and $a_{gen} = \Gamma_L b_{gen}$

so : $\left\| b_{gen} \right\|^2 = \dfrac{\left\| b_s \right\|^2}{\left\| 1 - \Gamma_S \Gamma_L \right\|^2}$ and $\left\| a_{gen} \right\|^2 = \left\| \Gamma_L \right\|^2 \cdot \left\| b_{gen} \right\|^2$

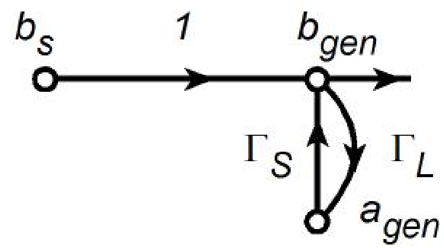
Power delivered to an arbitary load

$$P_{load} = \frac{1 - \left\| \Gamma_L \right\|^2}{\left\| 1 - \Gamma_S \Gamma_L \right\|^2} \cdot \left\| b_s \right\|^2 = \frac{1 - \left\| \Gamma_L \right\|^2}{\left\| 1 - \Gamma_S \Gamma_L \right\|^2} \cdot P_{Z_0}$$

Generator Wave Relationships: Available Power

Power delivered to an arbitary load

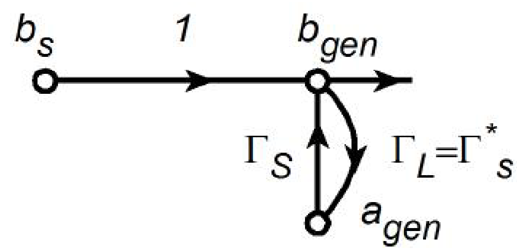
$$P_{load} = \frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_S\Gamma_L\|^2}\cdot\|b_s\|^2 = \frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_S\Gamma_L\|^2}\cdot P_{Z_0}$$



Power delivered to a matched load $\Gamma_L = \Gamma_S^*$

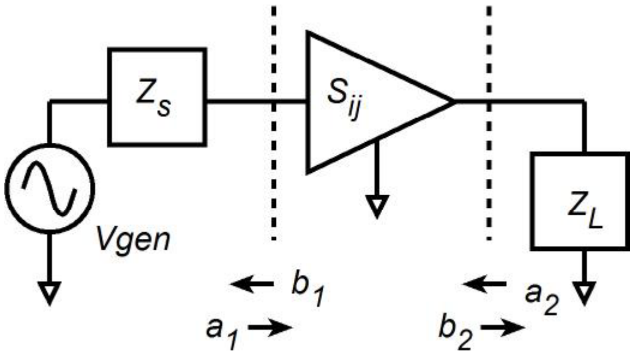
$$P_{AVG} = \frac{1-\|\Gamma_S\|^2}{\|1-\Gamma_S\Gamma_S^*\|^2}\cdot\|b_s\|^2 = \frac{1-\|\Gamma_S\|^2}{\left\|1-\|\Gamma_S\|^2\right\|^2}\cdot\|b_s\|^2$$

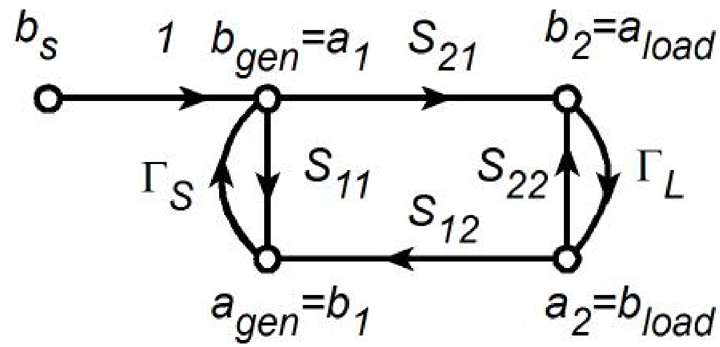
$$= \frac{\|b_s\|^2}{1-\|\Gamma_S\|^2}$$



$$P_{AVG} = \frac{\|b_s\|^2}{1-\|\Gamma_S\|^2} = \frac{P_{Z_0}}{1-\|\Gamma_S\|^2}$$

Transducer Gain





$$\frac{a_{load}}{b_s}=\frac{S_{21}}{1-\Gamma_s S_{11}-\Gamma_l S_{22}-\Gamma_s\Gamma_l S_{21}S_{12}+\Gamma_s S_{11}\Gamma_l S_{22}}=\frac{S_{21}}{(1-\Gamma_s S_{11})(1-\Gamma_l S_{22})-\Gamma_s\Gamma_l S_{21}S_{12}}$$

Transducer Gain: $G_T=P_{load}/P_{AVG}$

$$P_{AVG}=\frac{\|b_s\|^2}{1-\|\Gamma_s\|^2}$$

while $P_{load}=\|a_{load}\|^2-\|b_{load}\|^2=\|a_{load}\|^2\cdot\left[1-\|\Gamma_{load}\|^2\right]$

Transducer Gain

$$G_T = \frac{P_{load}}{P_{AVG}} = \frac{\left[1-\|\Gamma_s\|^2\right]}{\|b_s\|^2}\cdot\|a_{load}\|^2\cdot\left[1-\|\Gamma_{load}\|^2\right] = \frac{\|a_{load}\|^2}{\|b_s\|^2}\cdot\left[1-\|\Gamma_{load}\|^2\right]\cdot\left[1-\|\Gamma_s\|^2\right]$$

$$\text{but } \frac{a_{load}}{b_s} = \frac{S_{21}}{(1-\Gamma_s S_{11})(1-\Gamma_l S_{22}) - \Gamma_s \Gamma_l S_{21} S_{12}}$$

$$\Rightarrow G_T = \frac{\|S_{21}\|^2\cdot\left[1-\|\Gamma_{load}\|^2\right]\cdot\left[1-\|\Gamma_s\|^2\right]}{\|(1-\Gamma_s S_{11})(1-\Gamma_l S_{22}) - \Gamma_s \Gamma_l S_{21} S_{12}\|^2}$$

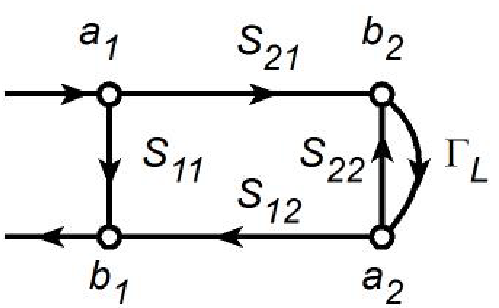
This is the general bilateral expression for transducer gain.

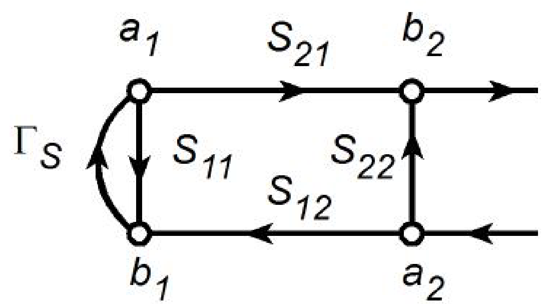
It depends upon, Γ_s, Γ_{load} , and $\{S_{21}, S_{12}, S_{11}, S_{22}\}$.

Input and Output Impedance Relationships

Recall that

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \qquad \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S}$$





Input and Output Impedance Relationships

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \quad \text{and } G_T = \frac{\|S_{21}\|^2 \cdot \left[1 - \|\Gamma_{load}\|^2\right] \cdot \left[1 - \|\Gamma_s\|^2\right]}{\|1 - \Gamma_s S_{11} - \Gamma_l S_{22} + \Gamma_s S_{11}\Gamma_l S_{22} - \Gamma_s \Gamma_l S_{21}S_{12}\|^2} = \frac{N}{D}$$

$$D = \left\| 1 - \Gamma_s \left(\Gamma_{in} - \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \right) - \Gamma_l S_{22} + \Gamma_s \Gamma_l S_{22} \left(\Gamma_{in} - \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} \right) - \Gamma_s \Gamma_l S_{21}S_{12} \right\|^2$$

$$= \left\| 1 - \Gamma_s \Gamma_{in} + \Gamma_s \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} - \Gamma_l S_{22} + \Gamma_s \Gamma_l S_{22}\Gamma_{in} - \Gamma_s \Gamma_l S_{22}\Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} - \Gamma_s \Gamma_l S_{21}S_{12} \right\|^2$$

$$= \left\| 1 - \Gamma_s \Gamma_{in} - \Gamma_l S_{22} + \Gamma_s \Gamma_l S_{22}\Gamma_{in} + \Gamma_s \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} - \Gamma_s \Gamma_l S_{22}\Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} - \Gamma_s \Gamma_l S_{21}S_{12} \right\|^2$$

$$= \left\| 1 - \Gamma_s \Gamma_{in} - \Gamma_l S_{22} + \Gamma_s \Gamma_l S_{22}\Gamma_{in} + \Gamma_s \Gamma_L S_{21}S_{12} \left(\frac{1}{1 - S_{22}\Gamma_L} - \frac{S_{22}\Gamma_L}{1 - S_{22}\Gamma_L} - 1 \right) \right\|^2$$

$$= \|1 - \Gamma_s \Gamma_{in} - \Gamma_l S_{22} + \Gamma_s \Gamma_l S_{22}\Gamma_{in}\|^2 = \|(1 - \Gamma_s \Gamma_{in})(1 - \Gamma_l S_{22})\|^2$$

$$= \|1 - \Gamma_s \Gamma_{in}\|^2 \cdot \|1 - \Gamma_l S_{22}\|^2 \quad \text{....tedious, but not hard.}$$

Transducer Power Gain

Therefore

$$G_T = \frac{1-\|\Gamma_s\|^2}{\|1-\Gamma_{in}\Gamma_s\|^2}\cdot\|S_{21}\|^2\cdot\frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2}$$

where $\Gamma_{in} = S_{11} + \Gamma_L \dfrac{S_{21}S_{12}}{1-S_{22}\Gamma_L}$

By a similar substitution

$$G_T = \frac{1-\|\Gamma_s\|^2}{\|1-S_{11}\Gamma_s\|^2}\cdot\|S_{21}\|^2\cdot\frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_{out}\Gamma_L\|^2}$$

where $\Gamma_{out} = S_{22} + \Gamma_S \dfrac{S_{21}S_{12}}{1-S_{11}\Gamma_S}$

Look carefully at these expressions :

the input - output bilateral interaction is hidden in either the Γ_{in} or Γ_{out} terms.

We will use these to derive various power gains.

Operating Power Gain: Gain with Input Matched

$$G_p = \frac{P_{load}}{P_{in}} \xrightarrow{\text{iff } P_{in}=P_{AVG}} \frac{P_{load}}{P_{AVG}} = G_T$$

$$P_{in} = P_{AVG} \text{ implies } \Gamma_s = \Gamma_{in}^*$$

$$\text{Since } G_T = \frac{1-\|\Gamma_s\|^2}{\|1-\Gamma_{in}\Gamma_s\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2}$$

$$G_P = \frac{1-\|\Gamma_{in}^*\|^2}{\|1-\Gamma_{in}\Gamma_{in}^*\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2}$$

$$G_P = \frac{1}{1-\|\Gamma_{in}\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1-\|\Gamma_L\|^2}{\|1-S_{22}\Gamma_L\|^2}$$

$$\text{where } \Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1-S_{22}\Gamma_L}$$

G_p is used to examine the variation in transistor gain as we vary the output match.

G_p is computed with the input matched.

Available Power Gain: Gain with Output Matched

$$G_A = \frac{P_{AVA}}{P_{AVG}} \xrightarrow{\text{iff } P_{AVA}=P_{load}} \frac{P_{load}}{P_{AVG}} = G_T$$

$$P_{AVA} = P_{load} \text{ implies } \Gamma_L = \Gamma_{out}^*$$

$$\text{Since } G_T = \frac{1-\|\Gamma_s\|^2}{\|1-S_{11}\Gamma_s\|^2}\cdot\|S_{21}\|^2\cdot\frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_{out}\Gamma_L\|^2}$$

$$G_A = \frac{1-\|\Gamma_s\|^2}{\|1-S_{11}\Gamma_s\|^2}\cdot\|S_{21}\|^2\cdot\frac{1-\|\Gamma_{out}^*\|^2}{\|1-\Gamma_{out}\Gamma_{out}^*\|^2}$$

$$G_A = \frac{1-\|\Gamma_s\|^2}{\|1-S_{11}\Gamma_s\|^2}\cdot\|S_{21}\|^2\cdot\frac{1}{1-\|\Gamma_{out}^*\|^2}$$

$$\text{where } \Gamma_{out} = S_{22} + \Gamma_S\frac{S_{21}S_{12}}{1-S_{11}\Gamma_S}$$

G_A is used to examine the variation in transistor gain as we vary the input match.

G_A is computed with the output matched.

How About Maximum Available Gain ?

$$G_{\max} = \frac{P_{AVA}}{P_{in}} \xrightarrow{\text{iff } P_{AVA}=P_{load} \text{ and } P_{in}=P_{AVG}} \frac{P_{load}}{P_{AVG}} = G_T$$

we now need $\Gamma_L = \Gamma_{out}^*$ and $\Gamma_s = \Gamma_{in}^*$

we must simultaneously solve

$$\Gamma_{in} = S_{11} + \Gamma_L \frac{S_{21}S_{12}}{1 - S_{22}\Gamma_L} = \Gamma_S^* \text{ and } \Gamma_{out} = S_{22} + \Gamma_S \frac{S_{21}S_{12}}{1 - S_{11}\Gamma_S} = \Gamma_L^*$$

and then substitute into

$$G_{\mathrm{T}} = \frac{1 - \|\Gamma_s\|^2}{\|1 - \Gamma_{in}\Gamma_s\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1 - \|\Gamma_L\|^2}{\|1 - S_{22}\Gamma_L\|^2}$$

This is, at a minimum, mathematically tedious.

Worse, there may be no solution... ...more on this later.

Unilateral Amplifier Design

Unilateral Amplifier Design

We will now consider the various gain relationships under the simplifying case of $S_{12} = 0$.

We do this to develop better our understanding of these gains.

The unilateral expressions are not to be used for real amplifier design, or real transistor gain calculations.

Warning : the expressions are gains for a device which happens to *be* unilateral, not the gain of a bilateral device after we provide feedback to *make it* unilateral. The latter quantity is Mason's invariant, the unilateral gain, and is described elsewhere.

Unilateral Amplifier Design

Suppose we have a device with $S_{12} = 0$.

$$G_{\mathrm{T}} = \frac{\|S_{21}\|^2 \cdot \left[1 - \|\Gamma_{load}\|^2\right] \cdot \left[1 - \|\Gamma_s\|^2\right]}{\|(1-\Gamma_s S_{11})(1-\Gamma_l S_{22}) - \Gamma_s \Gamma_l S_{21} S_{12}\|^2} \Rightarrow \frac{\cdot [\;] \cdot \left[1 - \|\Gamma_s\|^2\right]}{\|(1-\Gamma_s S_{11})(1-\Gamma_l S_{22})\|^2}$$

$$G_T = \frac{1 - \|\Gamma_S\|^2}{\|1 - \Gamma_S S_{11}\|^2} \cdot \|S_{21}\|^2 \cdot \frac{1 - \|\Gamma_L\|^2}{\|1 - \Gamma_L S_{22}\|^2}$$

Simple and easy to recognize!

$$G_T = G_S \cdot \|S_{21}\|^2 \cdot G_L \quad \text{where } G_S = \frac{1 - \|\Gamma_S\|^2}{\|1 - \Gamma_S S_{11}\|^2} \text{ and } G_L = \frac{1 - \|\Gamma_L\|^2}{\|1 - \Gamma_L S_{22}\|^2}$$

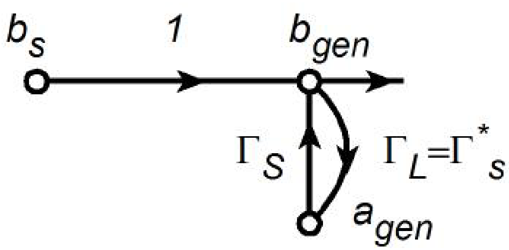
G_S is the fraction of gain lost/gained due to input mismatch/match

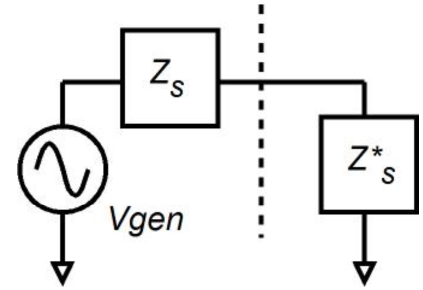
G_L is the fraction of gain lost/gained due to output mismatch/match

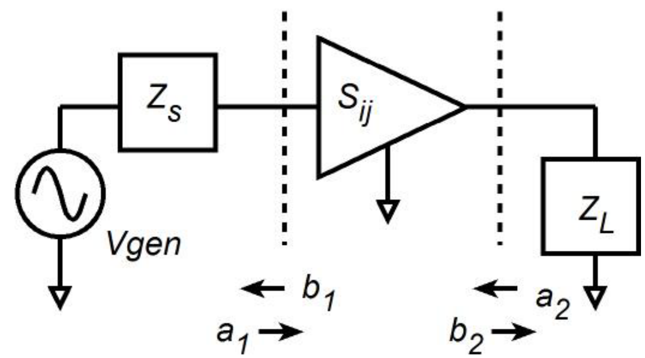
Power Wasted Due to Mismatch

Recollect the power relationships for a generator

$$P_{load} = \frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_S\Gamma_L\|^2}\cdot\|b_s\|^2 = \frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_S\Gamma_L\|^2}\cdot P_{Z_0}$$







Recognize the same terms in

$$G_T = G_S\cdot\|S_{21}\|^2\cdot G_L \quad \text{where } G_S = \frac{1-\|\Gamma_S\|^2}{\|1-\Gamma_S S_{11}\|^2} \text{ and } G_L = \frac{1-\|\Gamma_L\|^2}{\|1-\Gamma_L S_{22}\|^2}$$

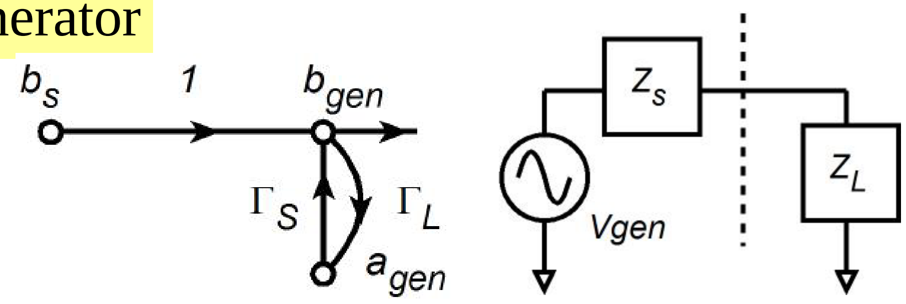
Given that $S_{12}S_{21}=0$, S_{11} and S_{22} are the input and output reflection coefficients.

Hence G_S and G_L just represent impedance mismatch terms at input and output.

Maximum Power Transfer, Maximum Available Gain

Recollect the power relationships for a generator

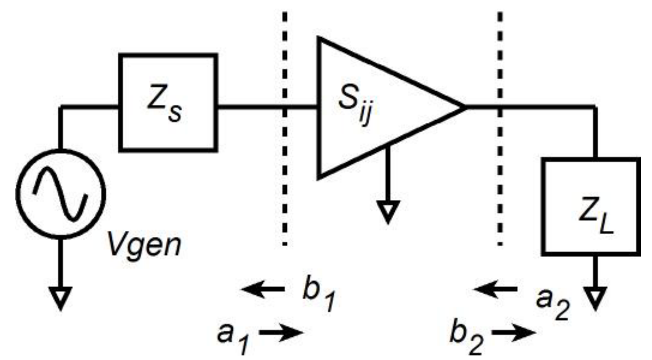
$$P_{AVG}=\frac{\|b_s\|^2}{1-\|\Gamma_S\|^2}=\frac{P_{Z_0}}{1-\|\Gamma_S\|^2}$$



We must now match on input and output : $\Gamma_S=S_{11}^*$ and $\Gamma_L=S_{ss}^*$

Recognize the same terms in

$$G_{\max}=\frac{1}{1-\|S_{11}\|^2}\cdot\|S_{21}\|^2\cdot\frac{1}{1-\|S_{22}\|^2}$$



Can we make sense of this expression ?

Impedance matching = Recyling the Reflections !

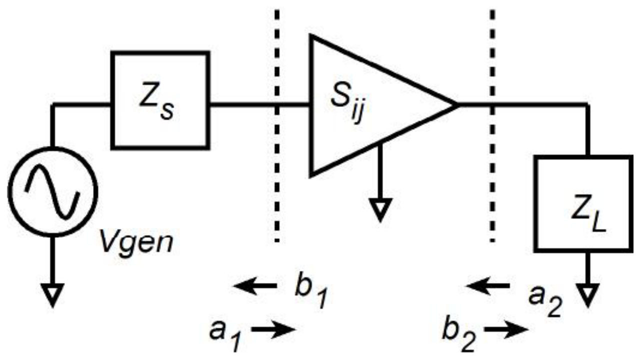
Suppose we have an amplifier with $S_{11} = 0.5 + j0$.

Given a 50Ω generator, 25% of the available input power is reflected and lost.

If we could recycle this power,

the input power would be increased in proportion to $\dfrac{1}{1-0.25}$

which is to say, in proportion to $\dfrac{1}{1-\left\|\Gamma_{in}\right\|^2}$.



Impedance - matching recycles the reflected power.