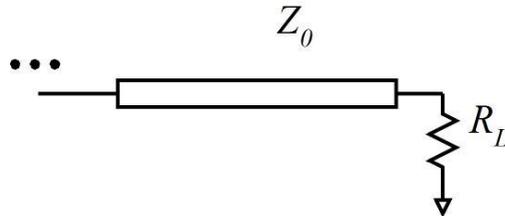


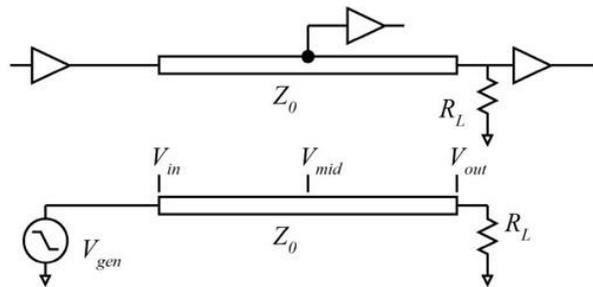
**ECE 145a /218A first problem set**  
**(basics of transmission lines and lumped elements)**

**Problem 1:** A transmission line has 50 Ohms characteristic impedance and a load impedance of (a) 25 Ohms (b) 100 Ohms (c) 50 Ohms. Compute in each case the voltage reflection coefficient.



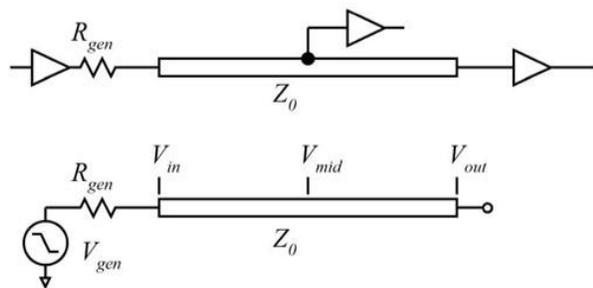
**Problem 2:** : lattice diagrams:

(a) A logic gate with low (zero ohms) output impedance drives a 50 Ohm transmission line of 1 m length and  $2 \cdot 10^8$  m/s propagation velocity. The equivalent circuit is in the lower image. The load impedance is 50 Ohms and  $V_{gen}$  is a 1V step-function occurring at time = zero..



Draw clean plots of  $V_{in}$ ,  $V_{mid}$ , and  $V_{out}$  as a function of time

(b) a logic gate with 50 Ohms output impedance drives a 50 Ohm transmission line of 1 m length and  $2 \cdot 10^8$  m/s propagation velocity. The equivalent circuit is in the lower image. The load impedance is infinity Ohms (an open circuit) and  $V_{gen}$  is a 1V step-function occurring at time = zero..



Draw clean plots of  $V_{in}$ ,  $V_{mid}$ , and  $V_{out}$  as a function of time

c) Comment on the relative utility of the 2 schemes for distributing logic signals using a transmission-line bus

**Problem 3:**

lumped-distributed relationships

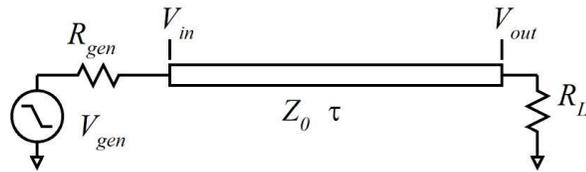
$Z_o = 50 \text{ Ohms}$ ,  $\tau = l/v = 100 \text{ ps}$ , and

$V_{gen}$  is a 1 V step-function.

a)  $R_L = 250 \text{ } \Omega$ ,  $R_{gen} = 250 \text{ } \Omega$ : Compute and plot  $V_{in}(t)$  using lattice diagram methods

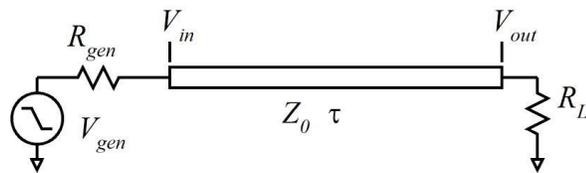
b)  $R_L = 10 \text{ } \Omega$ ,  $R_{gen} = 10 \text{ } \Omega$ . Compute and plot  $V_{in}(t)$  using lattice diagram methods

c)  $R_L = 10 \text{ } \Omega$ ,  $R_{gen} = 250 \text{ } \Omega$ . Compute and plot  $V_{in}(t)$  using lattice diagram methods

**Problem 4:** lumped-distributed relationships

$Z_o = 50 \text{ Ohms}$ ,  $\tau = l/v = 100 \text{ ps}$ , and

$V_{gen}$  is a 1 V step-function.



In each of the cases (a,b,c) below, replace the transmission-line with a T or Pi equivalent circuit, compute and compute and plot  $V_{in}(t)$  using basic circuit analysis. Hint: it is easiest to use a T equivalent for (a), a Pi equivalent for (b), and a Pi equivalent for (c). To make the analysis simpler, in cases (a) and (b) first compute L/R and RC time constants, and ignore either L or C if the associated time constant is more than 5:1 less than the dominant time constant. In case (c), the Pi equivalent has two capacitors, but one has a much shorter time constant  $(C/2)R_L$  than the other  $(C/2)R_{gen}$ ; therefore ignore the capacitor  $(C/2)$  connected between the output and ground.

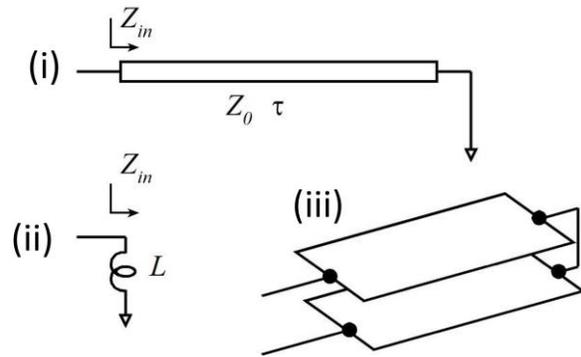
a)  $R_L = 250 \text{ } \Omega$ ,  $R_{gen} = 250 \text{ } \Omega$ :

b)  $R_L = 10 \text{ } \Omega$ ,  $R_{gen} = 10 \text{ } \Omega$ .

c)  $R_L = 10 \text{ } \Omega$ ,  $R_{gen} = 250 \text{ } \Omega$ .

**Problem 5:** lumped-distributed relationships

A transmission line (iii) has length  $l = 10$  cm and propagation velocity and  $v$  equal to the speed of light in Teflon (look up its dielectric constant).  $Z_o = 50$  Ohms.



a) The plate vertical separation is 1 mm. Using approximate microstrip-line formulas, compute the *width* of the conductor.

b) Using the relationship  $L = Z_o \tau = Z_o l / v$ , compute (ii) the line inductance. Compute the impedance  $Z_{in} = j\omega L$  from DC to a frequency of  $f_{high} = v / l$

c) The transmission-line is short-circuited (i). Using the relationships  $Z_{load} = 0\Omega$ ,

$$\Gamma_{load} = \left( \frac{Z_L}{Z_o} - 1 \right) / \left( \frac{Z_L}{Z_o} + 1 \right),$$

$$\Gamma_{in} = \Gamma_L \exp(-j2\beta l) = \Gamma_L \exp(-j2\omega\tau) = \Gamma_L \exp(-j4\pi l / \lambda)$$

$$\text{and } Z_{in} = Z_o \left( \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \right)$$

....derive an algebraic expression for  $Z_{in}$ .

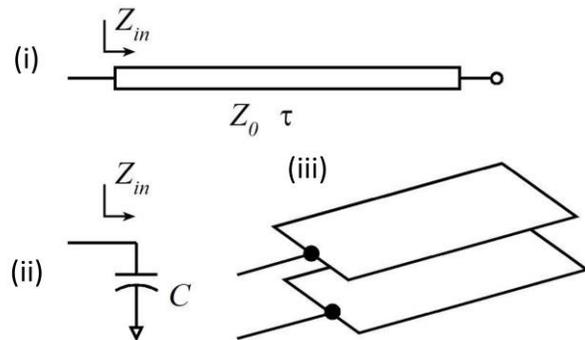
d) Using the Smith chart, compute  $Z_{in}$  at the following frequencies:

$$f = 0, v/8l, 2v/8l, 3v/8l, 4v/8l, 5v/8l, 6v/8l, 7v/8l,$$

e) Using \*cartesian\* axes (straight lines, not a Smith chart), make a plot of the imaginary part of  $Z_{in}$ , plotting the answers from parts (b), (c), and (d). Comment on the similarities and differences between the three curves

**Problem 6:** lumped-distributed relationships

A transmission line (iii) has length  $l = 10$  cm and propagation velocity, and  $v$  equal to the speed of light in Teflon (look up its dielectric constant).  $Z_o = 50$  Ohms.



a) Using the relationship  $C = \tau / Z_o = l / vZ_o$ , compute (ii) the line capacitance. Compute the impedance  $Z_{in} = 1 / j\omega C$  from DC to a frequency of  $f_{high} = v / l$

b) The transmission-line is short-circuited (i). Using the relationships  $Z_{load} = \infty \Omega$ ,

$$\Gamma_{load} = \left( \frac{Z_L}{Z_o} - 1 \right) / \left( \frac{Z_L}{Z_o} + 1 \right),$$

$$\Gamma_{in} = \Gamma_L \exp(-j2\beta l) = \Gamma_L \exp(-j2\omega\tau) = \Gamma_L \exp(-j4\pi l / \lambda)$$

and  $Z_{in} = Z_o \left( \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \right)$

....derive an algebraic expression for  $Z_{in}$ .

c) Using the Smith chart, compute  $Z_{in}$  at the following frequencies:

$$f = 0, v/8l, 2v/8l, 3v/8l, 4v/8l, 5v/8l, 6v/8l, 7v/8l,$$

e) Using \*cartesian\* axes (straight lines, not a Smith chart), make a plot of the imaginary part of  $Z_{in}$ , plotting the answers from parts (a), (b), and (c). Comment on the similarities and differences between the three curves