Course content - what are we trying to accomplish?

- Refer to course syllabus -

* specifically -

  high frequency transistor circuit design.

  use of high frequency instruments.

* Key to course content will be systematically treating the effects of conductors (transmission lines) on circuit performance.
Course content will involve

* significant review of transmission line theory - with emphasis on circuit-element analogs.
* significant review of transistor models for high frequency design.

We will try to cover this review material in 3-4 weeks. Experience has shown that the material cannot be skipped.
Transmission Lines

Types:

- Coaxial
- Microstrip

Microstrip:

Signal, Coax, Conductor
... and many others...

The common features are:

1) A pair of conductors

2) A geometry which does not change with distance

- These characteristics are sufficient for the propagation of a guided wave.
Review of transmission-line theory.

We will approach this from a strictly circuit point of view, ignoring the normal electromagnetic approach.

Justification: develop models for transmission lines which strongly parallel lumped electrical elements. This will aid as in circuit design.
Circuit model of transmission lines:

\[ C = \text{capacitance/length} \]

\[ L = \text{inductance/length} \]

Circuit analysis gives us

\[ \frac{\partial V}{\partial t} = -L \frac{\partial I}{\partial t} \quad \text{and} \quad \frac{\partial I}{\partial t} = -C \frac{\partial V}{\partial t} \]

These are called the telegrapher's equations.

From which the wave equation is found:

\[ \frac{\partial^2 V}{\partial t^2} = \frac{1}{C} \frac{\partial^2 V}{\partial x^2} \]
The solutions are

\[ V(3, t) = V^+(t-3/v) + V^-(t+3/v) \]

\[ I(3, t) = \frac{V^+(t-3/v)}{Z_0} - \frac{V^-(t+3/v)}{Z_0} + I_0 \]

where \( V^+ \) and \( V^- \) are the forward and reverse voltage waves, the velocity is

\[ v = \sqrt{1/c^2}, \]  

the characteristic impedance \( Z_0 = \sqrt{1/c} \), and \( I_0 \) is a d.c. current.
These are simple and familiar relationships.

**Key point:**

**Voltage at any point:**

\[ V(3, t) = V^+(3, t) + V^-(3, t) \]

**Current at any point:**

\[ I(3, t) = \frac{V^+(3, t) - V^-(3, t)}{Z_0} + I_0 \]
It is standard in microwave texts to define normalized wave amplitudes:

\[ a = \text{forward wave amplitude} = V^+ \sqrt{Z_0} \]

\[ b = \text{reverse wave amplitude} = V^- \sqrt{Z_0} \]

Power in forward wave = \( a^*a \)

Power in reverse wave = \( b^*b \)

caveats: Power will be \( aa^* \) (\( bb^* \)) and we deal with phases. Also, definitions must be generalized if we are to deal with complex characteristic impedances.
Reflections off simple impedances:

\[ \text{Line, } Z_0 \]

\[ v^+ - v^- \]

\[ \frac{v^+ - v^-}{Z_0} = \frac{I}{V} \]

\[ v^+ + v^- = V \]

\[ V = R_L I \]

Boundary conditions:

Combining these:

\[ v^-(z = 0, t) = v^+(z = 0, t) \cdot \Gamma_2 \]

\[ \Gamma_2 = \frac{R_L}{Z_0 - 1} = \text{reflection coefficient} \]
Cases:

open circuit \((R_L \rightarrow \infty)\)

\[ I_L = +1 \]

\[ V^+ \]

\[ V^- \]

short circuit \((R_L \rightarrow 0)\) \(\Rightarrow I_L = -1\)

\[ V^+ \]

\[ V^- \]

Matched load: \( R_L = R_0 \) \(\Rightarrow I_L = 0\)

No reflection.
Similar considerations can be applied at the sending end of a transmission line:

\[
\begin{align*}
V(\gamma = 0) &= V_s - R_s I(\gamma = 0) \\
V &= V^+ + V^- \quad \text{and} \quad I = (V^+ - V^-)/Z_0
\end{align*}
\]

To obtain:

\[
V^+(\gamma = 0, t) = \int_{\gamma} V(\gamma, t) \, d\gamma + V_s(t) T_s
\]
\[ T_s = \text{"source transmission coefficient"} \]
\[ = \frac{E_0}{E_0 + R_s} \]
\[ \Gamma_s = \text{source reflection coefficient} \]
\[ = \frac{R_s/E_0 - 1}{1 + R_s/E_0} \]

Junctions between lines of dissimilar impedance can be similarly treated, starting with the boundary condition of the equality of voltages at junctions.
$V_2^+ = T_{12} V_1^+ + I_{22} V_2^-$

where

$T_{12} = \frac{2Z_{02}}{Z_{01} + Z_{02}}$

$I_{22} = \frac{Z_{01}/Z_{02} - 1}{Z_{01}/Z_{02} + 1}$
More comments about transmission lines:

Lines fall into two classes:

**Unbalanced lines**

- Have a signal conductor and ground.
- Ground is at zero potential relative to distant objects.
- True unbalanced: coaxial line.
- Almost unbalanced: microstrip.

Coplanar waveguide

With these, there is a small potential on the nominal ground plane.
balanced lines

* have 2 symmetric conductors
* conductor potentials are symmetric with respect to distant objects.

* Twin-lead (TV) antenna cable

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+V/2
\-V/2
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* Twisted-pair (wires twisted by drill)

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+V/2
\-V/2
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* Coplanar strip
Simple but important transmission line laws:

1) Source and load impedances should be equal to the characteristic impedance if reflections are to be avoided.

2) Think about the voltages on transmission line conductors before connecting them.

3) Think about the currents on transmission line conductors before connecting them.
Example T-junction in coplanar waveguide circuit diagram

Incorrect Implementation.

T-junction interrupts the current in the ground plane!

Correct Implementation.

Air-bridge connection between grounds.
Example

balanced line
Coplanar strips

unbalanced line
Coplanar waveguide

This is a problem because the balanced line has potentials (+\sqrt{2} and -\sqrt{2}) on both conductors while the Coplanar line has potential = zero on the ground conductor.
There are a large number of ways to go wrong - and a simple rule to avoid them: think: "Where do the currents go, and where are the conductor voltages?"

If we do make such a mistake - what happens? Generally there are more conductors involved than shown in the above examples. Sets of propagating voltage waves between the various conductors will be excited. The solution involves the use of an impedance matrix method.
Viewing transmission lines as approximate lumped elements:

We want to be able to think of transmission lines with the same level of visualization which (we hope) we have developed for lumped circuits.

For any transmission line:

\[ Z_0 = \text{impedance} = \sqrt{\frac{L}{C}} \]

\[ v = \text{velocity} = \frac{1}{\sqrt{LC}} \]

Suppose the line has length \( L \).
Then the total inductance is

$$L_T = \frac{Z \rho}{\nu} = Z_0 \gamma$$

and the total capacitance is

$$C_T = \frac{\rho}{Z_0 \nu} = \frac{1}{Z_0}$$

where \( \gamma = \frac{\nu}{Z_0} \) is the time delay or time-of-flight on the line.

Note that in terms of electrical properties that the physical line length is irrelevant; properties depend upon impedance and delay time.
A true transmission line is distributed:

But:

- For lines much shorter than a wavelength \( \ell \ll \lambda \)

  - or equivalently -

- For lines whose delay times are much shorter than a pulse-waveform risetime \( \tau_T \ll T_{\text{rise}} \)

- The transmission line can be replaced by a lumped-element equivalent
For longer lines, just break into $N$ sections before using the $T$ or $TT$ equivalents:

\[ L_T/4 \quad L_T/2 \quad L_T/4 \]

\[ C_T/2 \quad C_T/2 \quad C_T/2 \]

etc.
The parallel between lumped and distributed elements is helpful:

- A high-impedance line—i.e., short—is approximately an inductor.
  \[ L_T = \frac{Z_0}{T} \text{ big} \]
  \[ C_T = \frac{T}{Z_0} \text{ small} \]

- A low-impedance line—i.e., short—is approximately a capacitor.
  \[ L_T = \frac{Z_0}{T} \text{ small} \]
  \[ C_T = \frac{T}{Z_0} \text{ big} \]
It is often said that if a line is much smaller than a wavelength that it can be neglected - This is not true.

\[ 10 \text{MS} \]

\[ \text{50-ohm coaxial cable} \]

\[ 1 \text{ meter} \]

\[ v = \frac{2}{3} c \]

\[ T = 545 \text{ns} \]

\[ 10 \text{MS} \]

\[ 1 \text{kHz} \]

\[ \text{signal} \]

\[ 10 \text{MS} \]

\[ \text{LT} = 250 \mu\text{H} \]

\[ C_T = 100 \mu\text{F} \]

while \( LT \) is negligible (\[ LT/20 \text{MS} \) is small)

\[ C_T \] is significant (\( C_T \cdot 5 \text{MS} = 500 \mu\text{s} \))!
Time-domain reflectometry - Echo testing:

... cable system under test.

To monitor points
For oscilloscope

The voltage observed at the monitor point is the sum of the incident and reflected waves...

Since the incident signal is a \( \square \),
the reflected signal is readily identified.
Example load resistor at end of line

\[ \text{Delay } T \]
50Ω line

\[ \text{reflection} \]
\[ \text{magnitude } I_L = \frac{R_L}{Z_0 - 1} \]
\[ \frac{R_L}{Z_0 + 1} \]

Voltage monitored:
\[ I_L V_0 \]

\[ V_0 \]
\[ 2T \]

incident reflected.

From the magnitude of \( I_L \), \( R_L \) can be determined.
Before continuing with the TDR, let's discuss lattice diagrams.

- Handy tool for analysis of transmission line systems with pulsed excitation and resistive source & load terminations.

- Particular applicability for analysis of data buses & interconnects in high speed digital systems.

Best to analyze these present by an example.
Example: lattice diagram:

\[ V(t) \quad T_i = D_i / v \quad T_2 = D_2 / v \]

Analyze for impulse response first, then use convolution for general response.

\[ \text{distance} \]

\[ \text{time} \]

\[ \text{etc} \]
We can now represent the impulse response at point B:

Voltage at B: \[ T_s \cdot \delta(t) \]

So if the input is a step-function:

Then the voltage at point B will be:

\[ \ldots \]
Comment:

depending upon the values of $R$ and $L$, the impulse response may look like so:

![Diagram]

exponentially decaying envelope

or like so:

![Diagram]

You are advised to compare these to the impulse responses of lumped circuits, e.g.

![Diagram]

... as this further illustrates the lumped-distributed relationships.
TDR responses with Reactive Loads

\[ \tau = \frac{1}{R_1 Z_0} \]

\[ \tau = \frac{1}{Z_0 (Z_0 + R)} \]

\[ \tau = \frac{1}{R_1 Z_0} \]

\[ \tau = \frac{1}{R_1 Z_0 + Z_0} \]
In using the above responses to characterize circuit components, be warned of the following:

Instrument + cable risetime

Monitored Voltage

decrease in reflection magnitude

... peak observed reflection magnitude is affected by instrument rise time as well as by the electrical characteristics of the device under test.
TDR of transmission lines:

\[ 50 \Omega \quad 50 \Omega \, T_1 \, \text{Repeat} \quad T_2 \quad 50 \Omega \, T_3 \]

\[ Z_0 = 50 \Omega \]

This can be analyzed exactly using a lattice diagram. The result is generally complex.

If \( Z_0 \) of the second line is reasonably close to 50 \( \Omega \), then the observed response is like so:

\[ Z_1 = \frac{Z_0/50 \Omega - 1}{Z_0/50 \Omega + 1} \]

...and the instrument gives us a plot of \( Z_0 \) as a function of distance.
If, on the other hand, the line under test has a characteristic impedance far from 50 Ω, then the observed response will show significant multiple reflections:

\[ \text{Multiple round trips on line } T_2. \]

... and the display no longer can be exactly interpreted as a plot of \( E_0 \) vs position.

The key point is that the secondary reflections vary in amplitude to to third order in \( (E_{0,\text{so}} - 1) \) while the primary reflection varies to first order in \( (E_{0,\text{so}} - 1) \), so for \( E_0 \approx 50 \Omega \), the display is a plot of \( E_0 \) vs position.