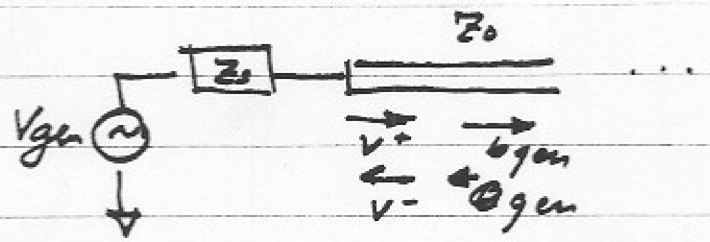
ECE 202A 11/6/89

Recall from last time:



$$v^+ = T_s V_{gen} + \Gamma_s v^-$$

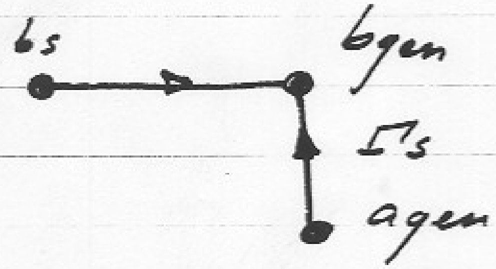
$$\left\{ \begin{aligned} T_s &= Z_0/(Z_0+Z_s) \\ \Gamma_s &= \frac{Z_s - Z_0}{Z_s + Z_0} \end{aligned} \right.$$

so $$v^+/\sqrt{Z_0} = \frac{T_s}{\sqrt{Z_0}} V_{gen} + \frac{\Gamma_s v^-}{\sqrt{Z_0}}$$

$$\Rightarrow b_{gen} = b_s + \Gamma_s a_{gen}$$

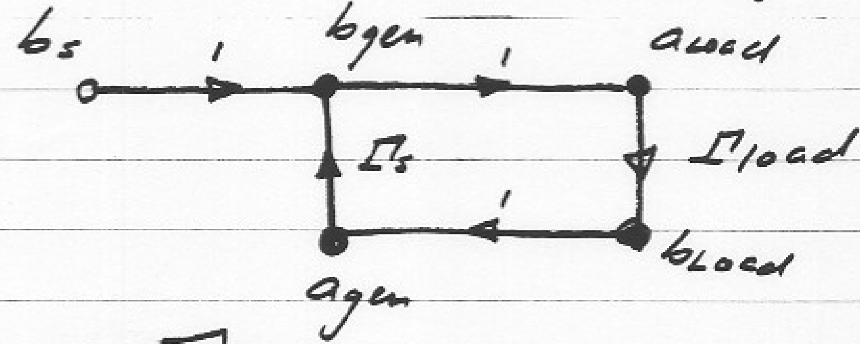
$$b_s = \frac{\sqrt{Z_0}}{Z_0 + Z_s} V_{gen}$$

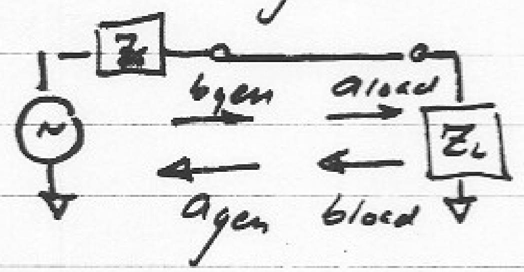
open-circuit voltage



$$\Rightarrow P_{available} = \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$ (rms quantities)

source connected to arbitrary load:





$$P_{Load} = |a_{Load}|^2 - |b_{load}|^2$$

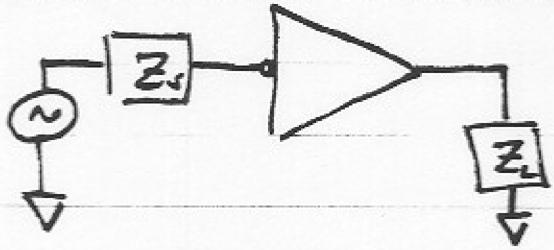
$$\frac{a_{Load}}{b_s} = \frac{1}{1 - \Gamma_s \Gamma_L} \qquad \text{(Mason)}$$

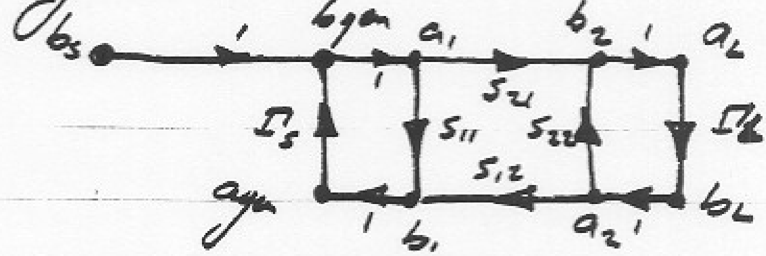
$$\frac{b_{Load}}{b_s} = \frac{\Gamma_L}{1 - \Gamma_s \Gamma_L}$$

$$P_{Load} = \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_s \Gamma_L|^2} \cdot |b_s|^2$$

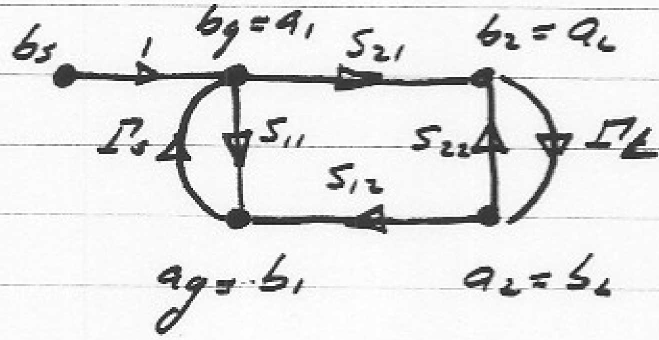
(max when $\Gamma_L = \Gamma_s^*$)

Back to transducer gain





$\Longrightarrow$



(from last time)

$$\frac{a_L}{b_s} = \frac{S_{21}}{(1-\Gamma_s S_{11})(1-\Gamma_L S_{22}) - S_{21} S_{12} \Gamma_s \Gamma_L}$$

$\equiv$

Transducer gain: $G_T = \dfrac{P_{Load}}{P_{AVS}}$ P_L

$$P_{AVS} = \frac{|b_s|^2}{1-|\Gamma_s|^2}$$

while $P_{Load} = |a_L|^2 - |b_L|^2$

$$= |a_L|^2 \left[1 - |\Gamma_L|^2\right]$$

so

$$G_T = \frac{|a_L|^2}{|b_s|^2} \cdot [1 - |\Gamma_s|^2][1 - |\Gamma_L|^2]$$

$$\boxed{G_T = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|(1 - \Gamma_s S_{11})(1 - \Gamma_L S_{22}) - S_{21} S_{12} \Gamma_s \Gamma_L|^2}}$$

Pause to take a breath. I have always looked upon the expressions above as mysterious, and nasty... But the derivation actually was pretty easy, wasn't it?

but we also have

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}$$

$\Longrightarrow$

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in} \Gamma_S|^2} \, |S_{21}|^2 \, \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2}$$

<u>or</u>

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \Gamma_S|^2} \, |S_{21}|^2 \, \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out} \Gamma_L|^2}$$

These expressions are used later.

Operating Power Gain

$$G_P = \frac{P_L}{P_{IN}} = \frac{P_L}{P_{AVS}} = G_T \quad \underline{\underline{\text{iff}}} \quad P_{IN} = P_{AVS}$$

Input matched

$\Gamma_S = \Gamma_{in}^*$

$$G_P = G_T \text{ with } \Gamma_S = \Gamma_{in}^*$$

$$= \frac{1 - |\Gamma_{in}^*|^2}{|1 - |\Gamma_{in}|^2|^2} \; |S_{21}|^2 \; \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= \frac{1}{[1 - |\Gamma_{in}|^2]} \; |S_{21}|^2 \; \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

similarly for the available power gain

$$G_A = G_T \text{ with } \Gamma_L = \Gamma_{out}^*$$

$$= \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \; |S_{21}|^2 \; \frac{1}{1 - |\Gamma_{out}|^2}$$

In general, the analysis is very messy and mathematical. So far we are following the text almost exactly. But at this point the text attacks "stability" (sec. 3.3). Lets' leave this alone for a little while, and look to generate some familiarity with S-parameter design first.

To do this, we will first concentrate on the highly simplified case of the unilateral amplifier: $S_{12} = 0$ (or ≈ 0)

Unilateral design:

suppose $S_{12} = 0$

$$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - \Gamma_S S_{11})(1 - \Gamma_L S_{22}) - S_{12} S_{21} \Gamma_S \Gamma_L|^2}$$

if $S_{12} = 0$:

$$= \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2} \quad |S_{21}|^2 \quad \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L S_{22}|^2}$$

Aha! simple, and easy to recognize.

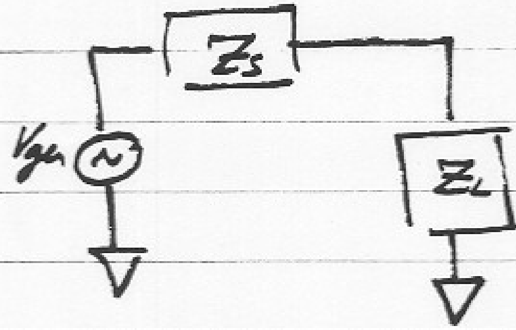
$$= G_S \; |S_{21}|^2 \, G_L$$

G_S (expression above) is the fraction of gain lost / gained due to input mismatch / match

G_L is similarly the fraction of power gain lost / gained due to output match mismatch.

To see this, compare to:

(as before)



$$P_{load} = |b_S|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_S \Gamma_L|^2}$$

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S S_{11}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L S_{22}|^2} \quad \text{if } S_{12} = 0$$

what is the maximum available gain??

Since $S_{12} S_{21} = 0$, $S_{11} = \Gamma_{in}$ & $S_{22} = \Gamma_{out}$

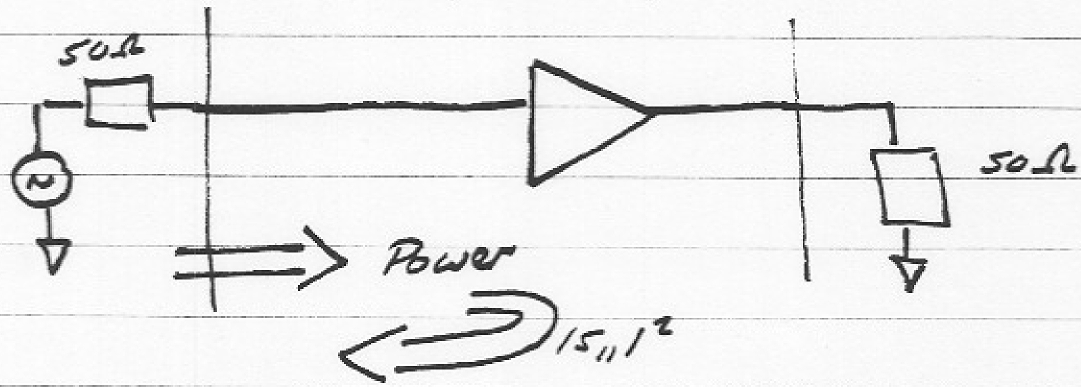
so for simultaneous input/output conjugate match, we want $\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$

$$\Rightarrow \boxed{G_{max} = \left(\frac{1}{1 - |S_{11}|^2}\right) |S_{21}|^2 \left(\frac{1}{1 - |S_{22}|^2}\right) \quad \text{iff } S_{12} = 0}$$

(Assuming $S_{12}=0$)

This is a very intuitive equation!!!

Lets' do a thought experiment (Gerdanken)



Fraction of input power reflected back = $|S_{11}|^2$

In this case $P_{out} = P_{avs} \cdot |S_{21}|^2$.

but a fraction $(1-|S_{11}|^2)$ of the input power

is reflected back from the amplifier to the load.

If we could recycle the reflected power,

then the output power would be $\dfrac{P_{avs}\,|S_{21}|^2}{1-|S_{11}|^2}$

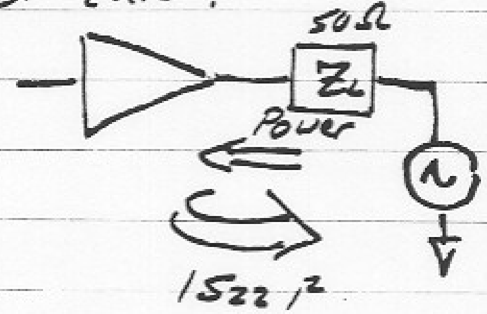
Recycle all power = send all source power to be

absorbed in amplifier = input conjugate matched.

assuming $S_{12}=0$

How about output?

If we did this:



then a fraction of the power directed at the amplifier output would be reflected back.

The coupling would be inefficient by the fraction $\frac{1}{1-|S_{22}|^2}$, just as the input coupling was inefficient by $\frac{1}{1-|S_{11}|^2}$. By reciprocity, this means that the amplifier (when loaded with $Z_0 = 50\Omega$) is inefficiently coupled to the load by the fraction $\frac{1}{1-|S_{22}|^2}$

Remove the inefficiency by matching, and the gain increases by $(1-|S_{22}|^2)^{-1}$

$S_{12} = 0$

Gain Circles in the unilateral Case

$$G_T = G_S \, |S_{21}|^2 \, G_L$$

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

so how do we represent the variation in transducer gain with source and load reflection coefficients?

$\Rightarrow$ RePresent graphically!

Note that $\left.\begin{aligned} G_S &= F(\Gamma_S) \\ G_L &= F(\Gamma_L) \end{aligned}\right\}$ for a given device.

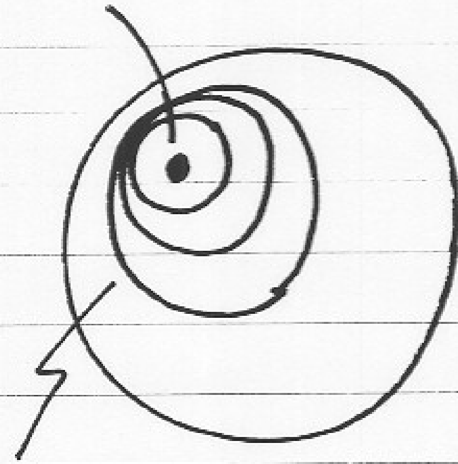
so plot G_S vs Γ_S on smith chart
and plot
G_L vs Γ_L on smith chart.

$\boxed{S_{12} = 0}$

$G_S = G_{S\,max}$ when $\Gamma_S = S_{11}^*$

G_S circles

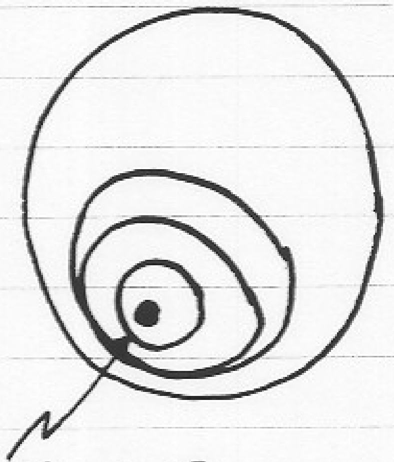
Γ_S plane



Circles of successively less gain (G_S)
1 dB contours, typically.

G_L circles

Γ_L plane



$G_L = G_{L\,max}$ when $\Gamma_L = S_{22}^*$

These plots graphically indicate the penalty in gain for failing to attain a perfect input/output match.

$\boxed{S_{12}=0}$

Gonzales shows that the contours are indeed circles... you may want to read this... or take it as a given (its' just the properties of conformal transformations coming up again)

More important: datasheets and Touchstone don't give circles of G_S & G_L -- they give circles (in the Γ_S plane) of the transducer power gain with the output matched ($= G_A$) as a function of the source reflection coefficient.

$\boxed{S_{12} = 0}$

Ex

Γ_S - plane

Could also (less common) give contours of constant G_P (ie G_T with input matched) as a function of Γ_L.

Γ_L - plane

So this is really about all that can be said about the unilateral amplifier (easy, isn't it?)

Except for one thing: when can we consider a device unilateral???

Gonzales compares the expressions for Transducer power gain with $S_{12} = 0$ (G_{Tu}) and $S_{12} \neq 0$ (G_T).

He finds:

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{Tu}} < \frac{1}{(1-U)^2}$$

where $$U = \frac{|S_{12}|\,|S_{21}|\,|S_{11}|\,|S_{22}|}{(1-|S_{11}|^2)\,(1-|S_{22}|)^2}$$

is called the unilateral figure of merit, which says that if U is small (say 0.1) you can pretend that the device is unilateral with reasonable (±20% in power gain) accuracy.

so this is a way of quantifying "how unilateral a device is" (a lot, a little...)

Danger: There is also a unilateral power gain G_u, which is the power gain (maximum available) of a device with a feedback network connected between input & output designed to make s_{12} for the overall device $= 0$. Due to a confusion of notation, several books, including sze, mistakenly give the expression for u (unilateral figure of merit → how unilateral) as the expression for G_u (unilateral gain → how much power gain if I use feedback to kill s_{12}?) watch for this.