stability: stable (unconditionally) if
$|\Sigma_i < 0$ for all possible $\Sigma_i$.

equivalently: unconditionally stable if
$|\Sigma_i < 1$ for all possible $\Sigma_i$. 
unconditionally stable: if

\[ k > 1; \quad K = \frac{1 - 15_{11}^2 - 15_{22}^2}{\det S} \left[ \frac{1}{2} \right] \]

\[ \text{and} \quad \det S < 1 \]

\[ \text{and} \quad 15_{11}, 15_{22} < 1 \]

---

Ok, so now let's calculate the maximum available power gain (MAG):

To attain this, must have match on both input & output:

\[ I_3 = I_{in}^* \quad \text{and} \quad I_2 = I_{out}^* \]

but we know that:

\[ I_{in} = 5_{11} + \frac{5_{12}5_{21}I_2}{1 - 5_{22}I_2} \]

\[ I_{out} = 5_{22} + \frac{5_{12}5_{21}I_3}{1 - 5_{11}I_3} \]

Solve these simultaneously (or ask Tuckster to)
\[ E_{1}^{*} = S_{1} + \frac{S_{12}S_{21}E_{2}}{1 - S_{22}E_{2}} \]

\[ E_{2}^{*} = S_{22} + \frac{S_{12}S_{21}E_{2}}{1 - S_{11}E_{2}} \]

... \(
\text{answer given in Gonzales pp 113.} \)

Maximum available gain - \( \text{MAG} = G_{T} \) with \( \Delta_{1} = E_{1}^{*}, \Delta_{2} = E_{2}^{*} \)

so, the values we calculate from above for the matched \( E_{1}, E_{2} \) are substituted into the relationship for \( G_{T} \): (page 4 of 116 notes, remember

\[ G_{T} = \frac{15z_{1}^{2} (1 - \Delta_{1}^{2}) (1 - \Delta_{2}^{2})}{1 - (\Delta_{1}^{2} + \Delta_{2}^{2}) (1 - \Delta_{2}^{2}) - S_{21}S_{12}E_{1}E_{2} \Delta_{1}^{2} \Delta_{2}^{2}} \]

\( \Rightarrow \) expression for \( G_{\text{max}} \):

\[ G_{\text{max}} = \frac{15z_{1}}{113z_{1}} \left( k - \sqrt{k^{2} - 1} \right) \]
oh, so, there is clearly some serious algebra in these steps, which I have skipped. I don't really feel that it is important that you do the algebra... just that you know the process by which the expression comes about:

\[ E_{\text{in}} \text{ depends on } S, E_3 \]
\[ E_{\text{out}} \text{ depends on } S, E_5 \]

Solve simultaneously to find \( E_3 \) and \( E_5 \) for simultaneous Conjugate match.

\[ \{ G_4 \text{ depends on } S, E_5, E_3 \} \]
\[ G_4 = G_{\text{max}} \text{ when input } & \text{ output are matched} \]

expression for \( G_{\text{max}} = \frac{1521}{1512} \left( k - \sqrt{k^2 - 1} \right) \]

note that if \( k < 1 \) (unstable)

\[ G_{\text{max}} \text{ doesn't exist.} \]

\( \Rightarrow \) of course! unstable = can build an oscillator
stabilization how to make sure it does not oscillate

\[ |E_{out}| = 1 \]

\( \mathbb{S} \text{-plane} \)

Circle corresponding to constant normalized resistance = \( r \)

If we put a resistor in series with input...

(\( R = r \mathbb{Z}_0 \))

\[ \begin{aligned}
    \mathbb{Z}_s &\quad R \\
    &\downarrow ^{Z_s} \\
    &\mathbb{Z}_s
\end{aligned} \]

... no matter what \( Z_s \) is, \( \mathbb{Z}_s \) has to lie within the bold circle above

\[ \Rightarrow \text{can't oscillate.} \]
Circle corresponding to constant normalized conductance $g$

If we put a resistor in parallel with output...

$(R = Z_0/g)$

... no matter what $Y_L$ is, $\tilde{Y}_L$ has to lie within the bold circle above.

$\Rightarrow$ can't oscillate.
There are, in principle, 4 possible methods

... feasibility simply depends upon where the stability circles lie... solutions 1 & 3 are usually most appropriate.

Note that

- or -

is like increasing $R_9$ or $R_6$.

and that

- or -

is like decreasing $R_2$ or $R_6$.

... think of effect of this on the output of the composite device (goes down).
so stability factor \( \xi \) must now = 1!

where \( \xi \) now refers to a new "device",

the combined resistor & transistor.
Before stabilization:

\[ G_{\text{max}} = \frac{15\omega_1}{15\omega_1} \left( K - \sqrt{K^2 - 17} \right) \]

= undefined (unstable)

After stabilization:

\[ G_{\text{max}} = \frac{15\omega_1}{15\omega_1} \frac{K - \sqrt{K^2 - 17}}{1} \]

\[ = 1 \frac{\omega_1}{15\omega_1} \]

Note that \( \tilde{S}_1 \approx S_2 \) and \( \tilde{S}_2 \approx S_1 \). But if the series stabilization element is \( \ll Z_0 \) (or if the shunt stabilization element is \( \gg Z_0 \)) then \( S_2 \approx S_2 \) won't have changed much. Then \( G_{\text{max}} \) after adding just enough stabilization so that it is just impossible to find any \((\tilde{I}_s, \tilde{I}_l)\) which cause oscillation is \( \approx 1 \frac{\omega_1}{15\omega_1} \).

"Maximum stable gain" = MSG = \( 15\omega_1 / 15\omega_1 \)

...if the device is potentially unstable.
Now wait... the texts state $MSG = |S_{21}|/|S_{11}|$, not approximately.

Clearly, adding stabilization changes the $S$-parameters; maybe it doesn't change the $S_{21}/S_{11}$ ratio.

Check this:

$S_R = \begin{bmatrix} S_{11R} & S_{12R} \\ S_{21R} & S_{22R} \end{bmatrix}$

$S_{22R} = S_{11R} = \frac{R_{44}}{R_{22}}$; $S_{12R} = S_{21R} = 1 - S_{11R}$

$1 + R_{22}R_{44}$
\[ \frac{b_2}{a_2} = \frac{R}{5_{z1}} = \frac{S_{z1}R S_{z1}T}{1 - S_{z2}S_{\Pi T}} \]

\[ \frac{b_1}{a_2} = \frac{R}{5_{z2}} = \frac{S_{\Pi T} S_{z2}R}{1 - S_{z2}S_{\Pi T}} \]

So:
\[ \frac{b_{z1}}{5_{z1}} = \frac{S_{z1}R S_{z1}T}{S_{\Pi T} S_{z2}R} \quad \text{let resistor have } 5_{z2} = 5_{z1} \]

So:
\[ \frac{b_{z1}}{5_{z2}} = \frac{5_{z2}}{5_{z1}} \quad \frac{b_{z1}}{5_{z2}} = \frac{5_{z2}}{5_{z1}} \]
How about the other 3 cases?

Well note that the denominators in \( S_{12} \) & \( S_{21} \) are always the same.

\[
\begin{align*}
1 & \quad \boxed{A} \quad \boxed{B} \quad 2 \\
S_B & \quad S_A 
\end{align*}
\]

In general: 
\[ \tilde{S}_{21} = \frac{S_{21A} S_{21B}}{\text{denominator}} \]
\[ \tilde{S}_{12} = \frac{S_{12A} S_{12B}}{\text{denominator}} \]

Also, a general property of two-ports containing only passive reciprocal elements (e.g., C's, R's, Hx lines...)
is that 
\[ S_{21} = S_{12} \]

So adding a series "black box" to the transistor input & output won't change ratio of \( S_{12}/S_{21} \).
So, always:

\[ \text{Maximum stable gain} = \frac{1521}{1521} \] (if it was unstable)

this means that:

- Given that you have an unstable transistor
- You add stabilization resistance so you can just guarantee unconditional (overall) stability.
- You then provide matching on input & output

\[ \Rightarrow \text{You will then attain a power gain of} \frac{1521}{1521} \]

Trick Question: Suppose \[ K > 1 \], \[ 1521 = 100 \text{dB} \]
\[ 1521 = -10 \text{dB} \].

What is MSG?
Touchstone plots MAG (when called For) at all frequencies where the device is unstable. It then defaults to MSG.

---

Practical Hint: In the TWA, terrible danger of oscillation at or near the lines' breagg frequencies.

Solution: \[ \text{capacitor is added so that } \frac{1}{2\pi f} \approx \text{breagg} \] so that cap the network only has effect at the high end of the amplifier band cost of increased drain, loading, and danger of
Ok. So we have just stabilized the transistor. Is this enough? Not really. We will find that the amplifier is matched at one port with a reflection coefficient of exactly one.

The shape of either the operating (Gp) or available (Ga) power gain will change as we add stabilization. If we have just added a critical level of stabilization, the gain is very sensitive to the match.

⇒ "Taisy" Match
Before Stabilization

Circles of Constant operating power gain $G_p$

$|\Gamma_{in}| = 1$

Circle of infinite $G_p$

$\Gamma$-plane

After Stabilization - with just the critical stabilization added

Circles of Constant operating power gain $G_p$

$|\Gamma_{in}| = 1$

Circle (point) for $G_p = G_{ms}$

$\Gamma$-plane

After Stabilization - with more than critical stabilization added

Circles of Constant operating power gain $G_p$

$|\Gamma_{in}| = 1$

Circle (point) for $G_p = G_{ms}$

$\Gamma$-plane

Similar construction on input ($\Gamma$-plane, $G_p$-circles)
So to design a narrowband matched amplifier: first decide what gain, bandwidth, (noise, power) required?

Is the device stable or unstable at the design frequency?

If the device is stable, determine the source and load reflection coefficients required for simultaneous input/output matching. To do this, solve the simultaneous equations (Gonzales page 113) or use Touchstone to look at the required matches conditions.

Ga circles show how the gain varies with input (mis)match assuming that the output is matched.

Gp circles show how the gain varies with output (mis)match assuming that the input is matched.

[Diagram showing Ga and Gp circles for input and output with optimum reflection coefficients marked]
Then design input and output matching networks: (here I am assuming 50Ω source and load)

Input: Γ's plane, Gα circles
Output: Π plane, Gp circles

So, implementation looks like:

=> Then replace with microstrip elements.

Now simulate over frequency: is the desired bandwidth attained?

How much does performance change with small changes in parameters? Look at Gα and Gp circles of the overall amplifier.
If either the bandwidth is too small or the design too parameter-sensitive (amplifier Gm Gp circles), need to either use alternative matching networks (\( \frac{m}{m} \) vs \( \frac{n}{n} \) vs \( \ldots \)), use broadband matching networks (next time) or use a lossy match.

If transistor is unstable at the match frequency:

1) add series or shunt stabilization resistance at input or output

\[ \text{or} \]

\[ \text{or} \]
2) Look at resulting $G_d, G_p$ circles: If gain varies very rapidly with match, or if the optimum matches are at edge of chart, add greater stabilization.

3) Design input & output matching networks.

4) Again look at parameter sensitivity: $G_i$ ($G_d, G_p$ circles of overall amplifier).
Biasing: need to bring in DC:

Small-signal design:

Adding bias:

Block DC - Chip capacitor

Z >> Z₀
λ/4
gate bias

Why not here?

Might be better to do this:

λ/4
2nd example:

Small-signal design:

Adding bias:

gate bias

drain bias.
Final point: having designed amplifier + matched so that we get desired gain, bandwidth: we need to look at the stability factor from DC to finite, of the overall amplifier.

Is the overall system unconditionally stable?

Note that stabilisation resistor was picked at design frequency. It then depends upon the input & output networks whether $I_i$, $I_e$ can enter the regions of potential instability.
How to actually provide bias:

If you need to operate device at specified $V_{ds}, V_g$

Bipolar:

$V_{cc}$
or feedback:

Can also be used with fet if control of drain current is desired: