ECE202A Notes set 2

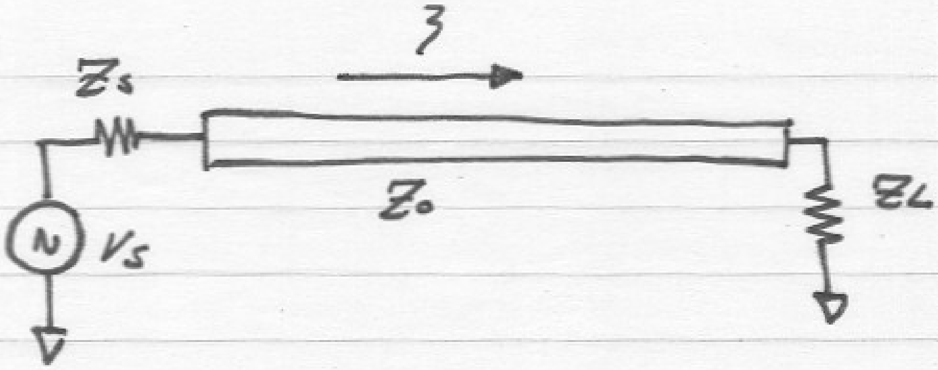
Transmission Lines in the frequency domain and the smith chart.

Time-domain analysis is intellectually clearer, the picture being forward & reverse waves propagating, reflecting, and re-reflecting.

This analysis becomes <u>intractable</u> as soon as we introduce <u>reactive</u> (-mm- & -||-) impedances as multiple convolutions will be required for time-domain reflection analysis

→ Analyze in the frequency domain instead

This is the classical approach.



assume: $V_s = Re\{V_0 e^{-j\omega t}\}$

V_0 is complex: $V_0 = \|V_0\| \cdot \exp\{-j\phi_0\}$

so $v_s(t) = \|V_0\| \cdot \cos(\omega t + \phi_0)$

on transmission line, waves travel as

$(z \pm vt)$ or equivalently $(t \pm z/v)$

For a wave traveling at velocity v,

$$\cos(\omega t + \phi_0) \rightarrow \cos(\omega(t \pm z/v) + \phi_0)$$

$$= \cos(\omega t + \phi_0 \pm z \cdot \omega/v)$$

$$= \cos(\omega t + \phi_0 \pm \beta z)$$

where β is the phase constant $\beta = \omega/v$

$$\beta = \omega / v = 2\pi / \lambda$$

... where λ is the wavelength

For exponential waves, the form will thus be

$$v_0 e^{j\omega t} e^{j\phi_0} e^{\pm j\beta z}$$

and, as always with phasor notation, the $e^{j\omega t}$ time dependence is taken as implicit:

<u>Voltage on the transmission line:</u>

$$V(z) = V^+(z) + V^-(z)$$

$$= V^+(0) e^{-j\beta z} + V^-(0) e^{+j\beta z}$$

$$Z_0 I(z) = V^+(z) - V^-(z)$$

$$= V^+(0) e^{-j\beta z} - V^-(0) e^{+j\beta z}.$$

Wave parameters, again:

define wave amplitudes such that if a=1 then wave power = 1 watt

$\rightarrow$ $a(z) = V^{+}(z) / \sqrt{Z_0}$ forward wave amplitude

$b(z) = V^{-}(z) / \sqrt{Z_0}$ reverse wave amplitude.

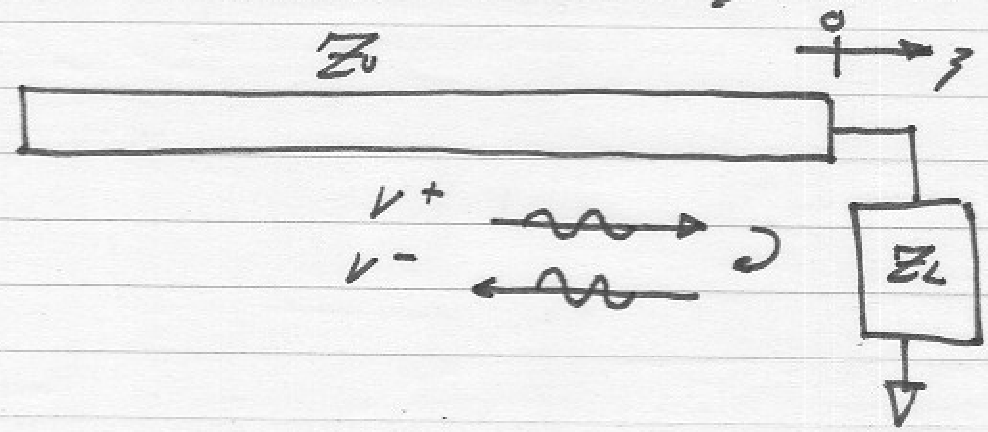
Power in forward wave $= aa^*$

" " reverse " $= bb^*$

It is assumed throughout that we are using R.M.S. quantities.

*Note that the relationships must be generalized if Z_0 is complex.

Reflections in the frequency domain:

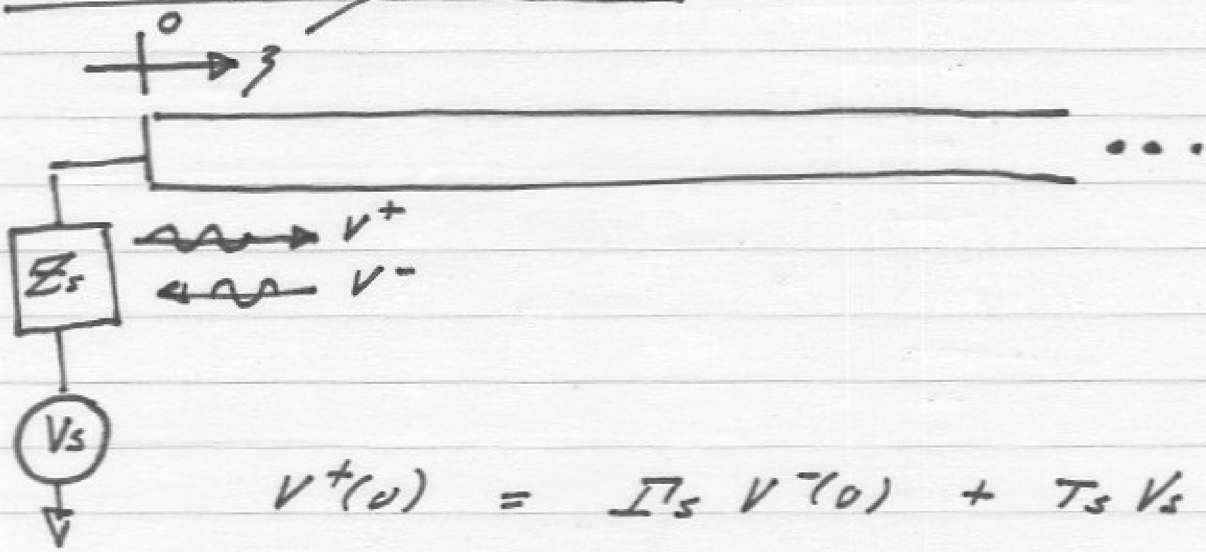


$$V^-(0) = V^+(0)\,\Gamma_L$$

$$\Gamma_L = \frac{z_L - 1}{z_L + 1} \quad \text{where}$$

$z_L = Z_L / Z_0$ is the normalized load impedance

From a generator:



$$V^+(0) = \Gamma_s V^-(0) + T_s V_s$$

T_s = source transmission coefficient

$$= \frac{Z_0}{Z_s + Z_0}$$

$$\Gamma_s = \frac{z_s - 1}{z_s + 1}$$ source reflection coefficient

$z_s = Z_s / Z_0$ normalized source impedance.

Movement of reference plane

$$V(z) = V^+(z) + V^-(z)$$

$$= V^+(z)\left[1 + \Gamma(z)\right]$$

where $\Gamma(z) \triangleq V^-(z) / V^+(z)$

is the position-dependent ~~impedance~~. reflection-coefficient

$$V(z) = V^+(0)\, e^{-j\beta z}\left[1 + \Gamma(0)\, e^{+2j\beta z}\right]$$

because

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{V^-(0)\, e^{+j\beta z}}{V^+(0)\, e^{-j\beta z}} = \Gamma(0)\, e^{2j\beta z}$$

So:

$\Gamma(-\ell)$ = the reflection coefficient at a distance ℓ from the load

$$= \Gamma(0)\, e^{-2j\beta\ell}$$

So, the reflection coefficient has gone through a phase shift of

<u>minus</u> $\frac{\ell}{\lambda} \cdot 2 \cdot 2\pi$ radians

or

<u>minus</u> $2\beta\ell$ radians

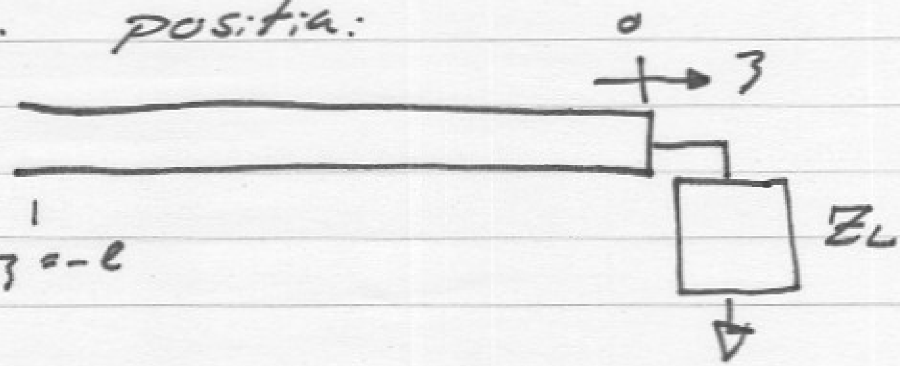
or

<u>minus</u> $\frac{\ell}{\lambda} \cdot 2 \cdot 360$ degrees.

... reflection coefficient changes <u>phase</u> with position.

Impedance vs. position:

Same picture:



Impedance at any point:

$$Z(z) \triangleq V(z) / I(z)$$

$$= \left[V^+(z) + V^-(z)\right] \Big/ \left[\frac{1}{Z_0}\left(V^+(z) - V^-(z)\right)\right]$$

$$= Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

Normalized impedance at any point

$$z(z) \triangleq Z(z)/Z_0 = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Input impedance of line at $z=-\ell$

$$z(-\ell) = \frac{1+\Gamma(-\ell)}{1-\Gamma(-\ell)} \qquad \text{normalized}$$

$$Z(-\ell) = Z_0\, z(-\ell) \qquad \text{unnormalized.}$$

So: Impedance depends upon position.

summarizing:

$$\left.\begin{aligned} V(z) &= V^+(z) + V^-(z) \\ Z_0 I(z) &= V^+(z) - V^-(z) \end{aligned}\right\} \text{waves}$$

$$\left.\begin{aligned} \Gamma(z) &= V^-(z) / V^+(z) \\ &= \Gamma(0)\, e^{2j\beta z} \end{aligned}\right\} \text{reflection coefficient}$$

$$\left. z(z) = \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right\} \text{Normalized impedance.}$$

While this is conceptually very simple, it is a great deal of math.

$\Rightarrow$ A graphical representation would help

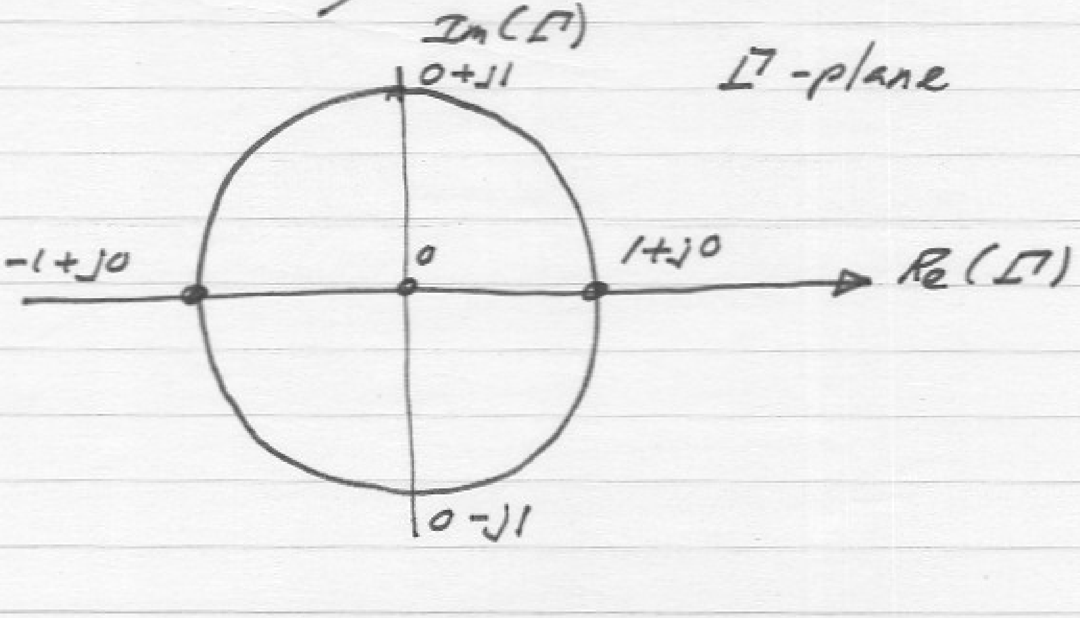
this is the Smith chart.

The relationship

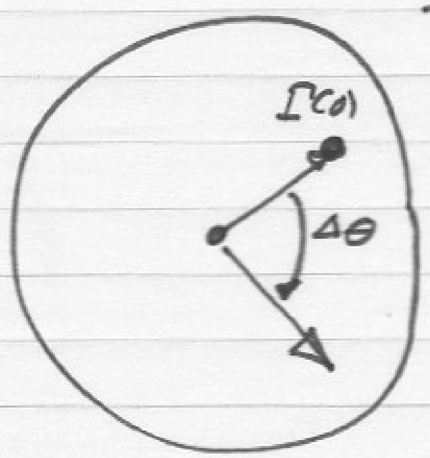
$$z = \frac{1+\Gamma}{1-\Gamma} \qquad \Gamma \text{ and } z \text{ both complex.}$$

Is key. It is a 1:1 mapping between complex numbers, and is in fact an analytic function and a conformal transformation. Review your complex analysis if necessary and interested.

In the 2-dimensional plane of values of Γ – the Γ-plane – a reflection coefficient Γ is represented by a point:



Γ- plane



as we move away from the load by a distance ℓ on the transmission line, Γ rotates by an angle $\Delta\theta$

$$\Delta\theta = -2\beta\ell$$

$$= -360^\circ \times \frac{\ell}{\lambda} \cdot 2$$

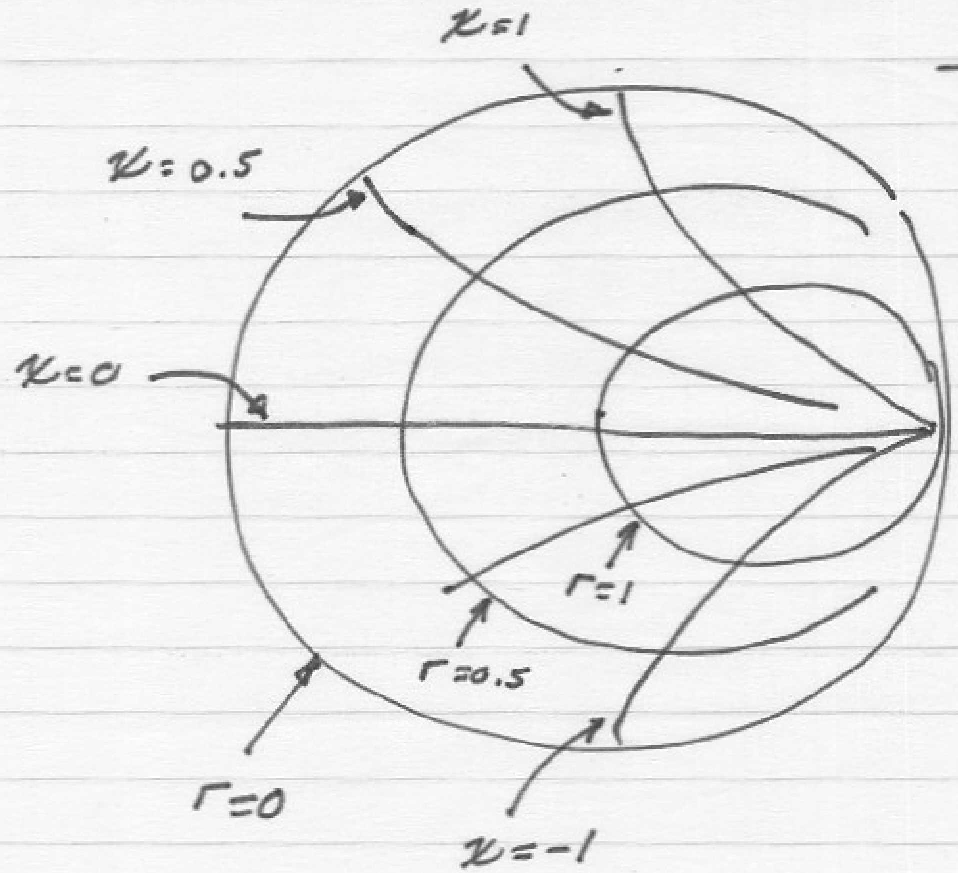
= one whole rotation in the Γ plane for each half-wavelength movement on the line.

$$z = \frac{1+\Gamma}{1-\Gamma}$$

This is a 1:1 relationship between Γ (magnitude & phase) and z (real & imaginary)

Plot the units of normalized impedance on the Γ-plane!

Γ plane



$$Z = R + jX$$
$$z = r + jx$$

This is the smith chart.

So, given some load impedance z, we can calculate Γ just by looking up the z-coordinates of the point and reading off the magnitude and phase of Γ with a straightedge and protractor.

Change of impedance vs position: just rotate the Γ-vector clockwise around the chart for the distance ℓ, at a rate of one ~~wavelength~~ rotation for every half-wavelength of movement.

- Then read off impedance from the chart.

Impedance - Admittance Chart

Impedance

$$\underset{\text{resistance}}{Z = R} + \underset{\text{reactance}}{jX}$$

Normalized Impedance

$$z = Z/Z_0 = r + jx$$

Admittance

$$Y = 1/Z = \underset{\text{Conductance}}{G} + \underset{\text{susceptance}}{jB}$$

Normalized admittance

$$y = Y/Y_0 = Y \cdot Z_0 = g + jb$$

We can plot the values of

r and x

- and -

g and b

on the Γ-plane. This is the impedance-admittance chart.

Among several advantages,

If $Z = 50 + j50\,\Omega$ 

$Z_0 = 50\,\Omega$

so that $z = 1 + j1$

then $Y = ?$

while we could use complex math, we can just use the plot:

draw the point $z = 1 + j1$ and read off the normalized admittance as $y = 0.5 - j0.5$

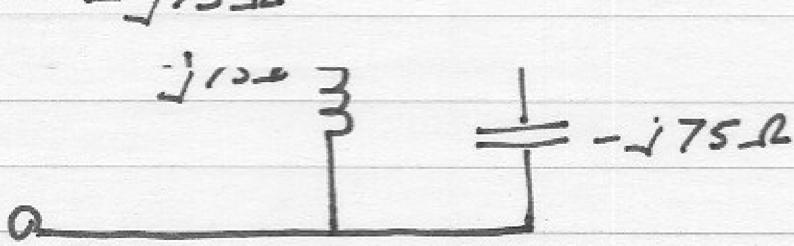
un-normalize: $Y = \dfrac{0.5 - j0.5}{50\,\Omega}$

This is convenient.

What if there is no transmission line?

We can still use the chart, just define whatever Z_0 is convenient.

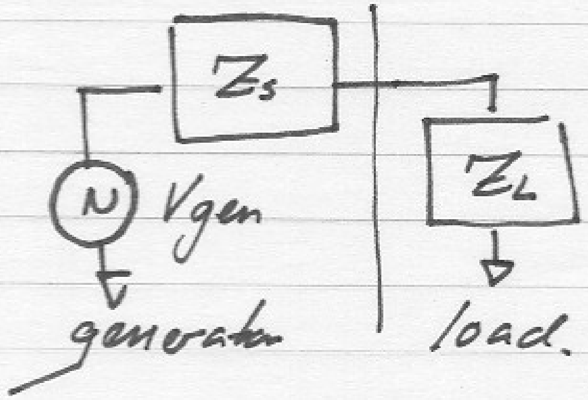
example:



* Solution on Smith chart (next page) gives $Z_{in} = 75 + j0\ \Omega$.

+ This is also an introductory example of impedance matching.

First statement of impedance matching



$$Z_s = R_s + j X_s$$

Maximum power delivered to load if

$Z_L = Z_s^*$ (Conjugate matching)

$$P_{max} = \| V_{gen} \|^2 / 4 \, Re [Z_s]$$

$$= \| V_{gen} \|^2 / 4 R_s$$

R.M.S. quantities are assumed.

This is called the power available from the generator.

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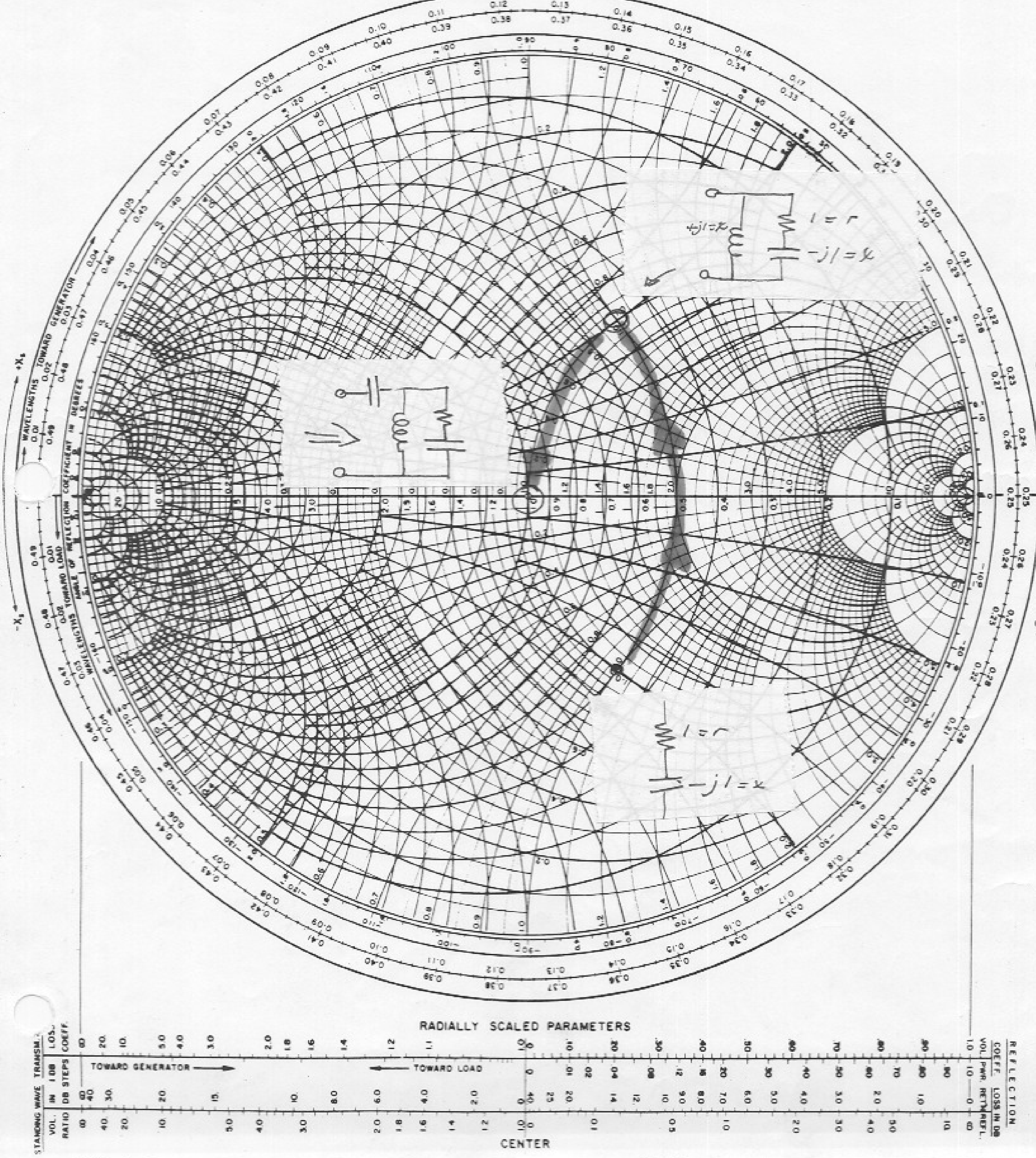
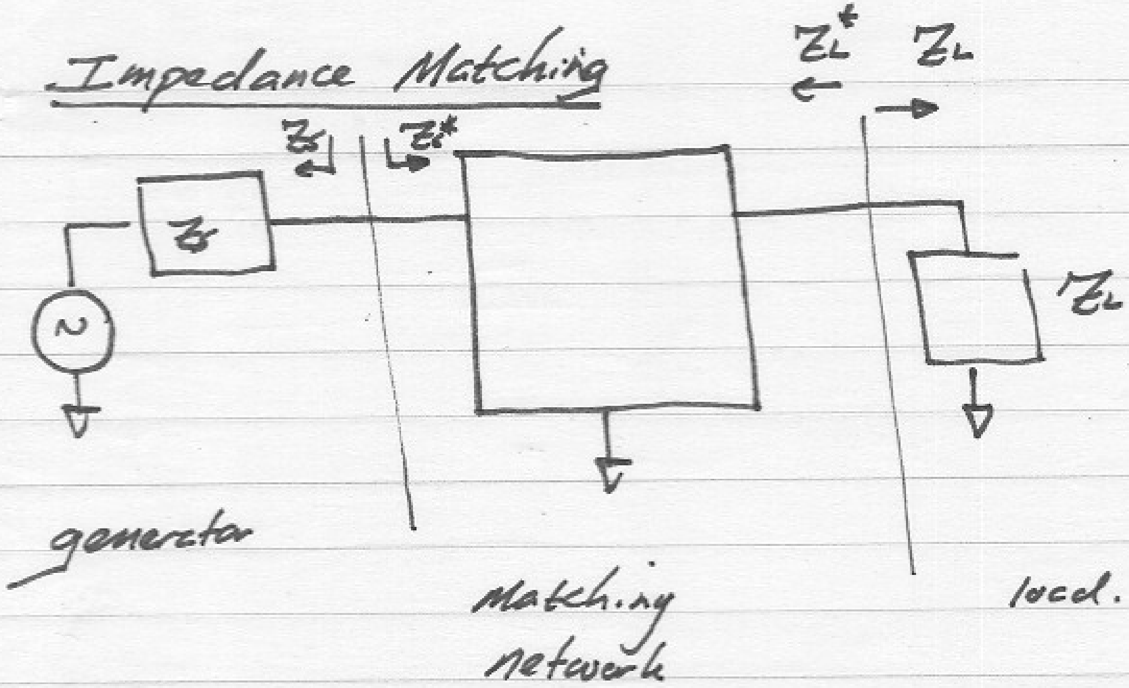
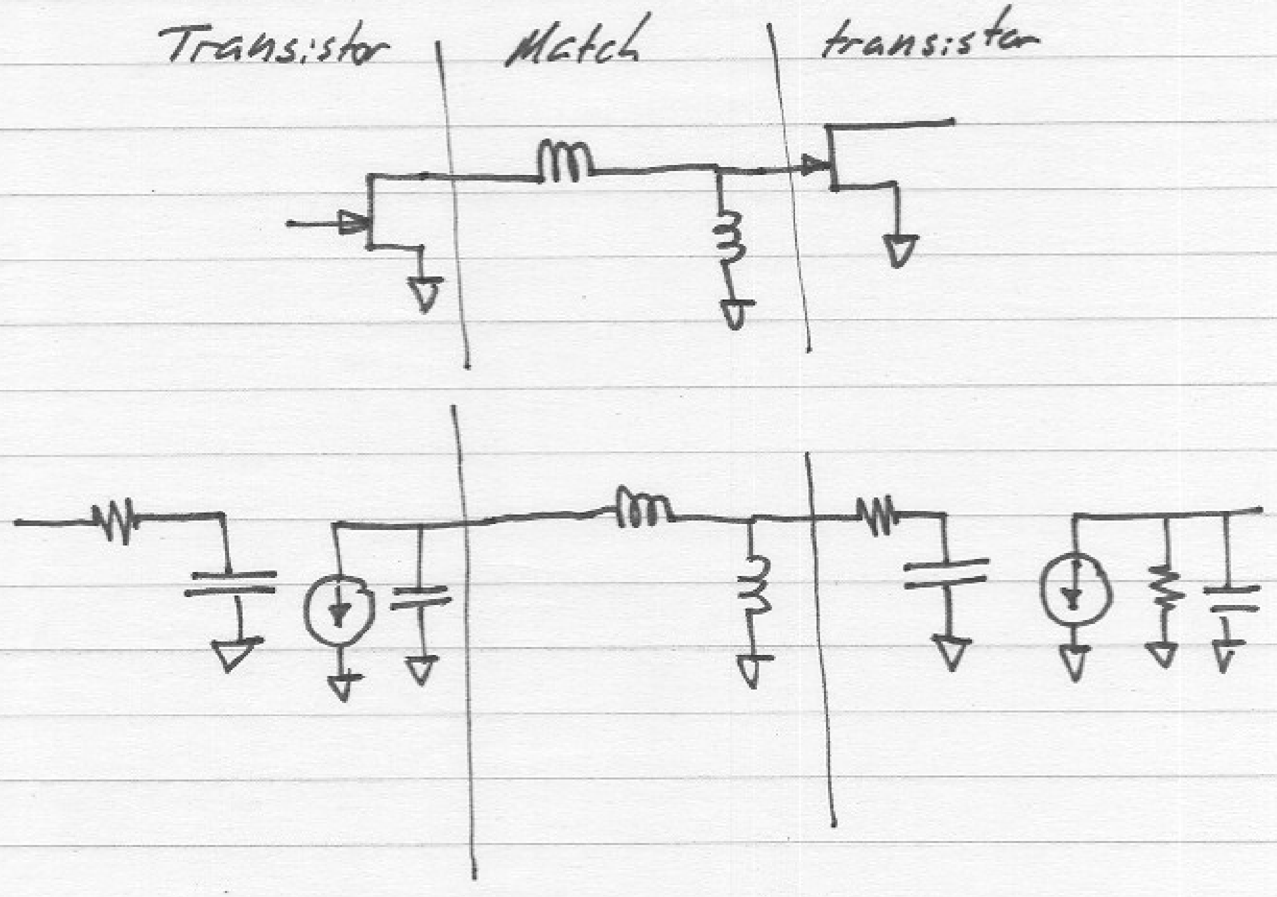
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Impedance Matching



ex:



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