ECE 202A Notes set 7

A review of circuit design methods

* Here we come to a decision point in the course: serve all students, or only those expert in circuit design?

* Lets try to serve all students, including those who are rusty in circuit design.

* The next six pages neatly summarize the things which you must review if unfamiliar with the topic.

Method of time constants:

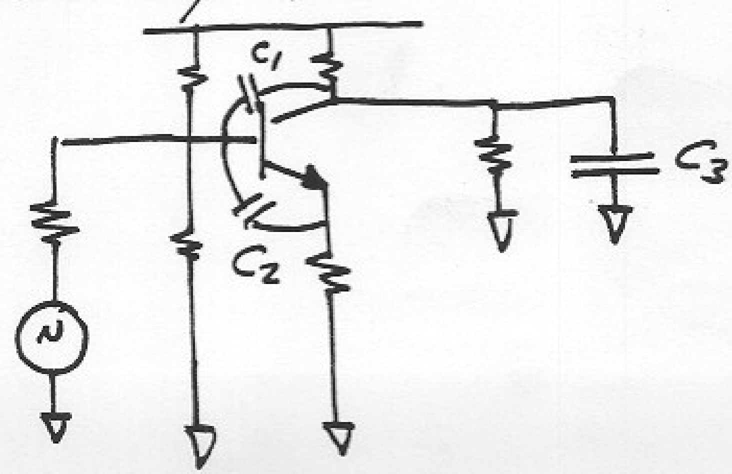
General frequency response of circuit:

$$\frac{V_o(s)}{V_g(s)} = \frac{1 + b_1 s^2 + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots} \cdot A_{dc}$$

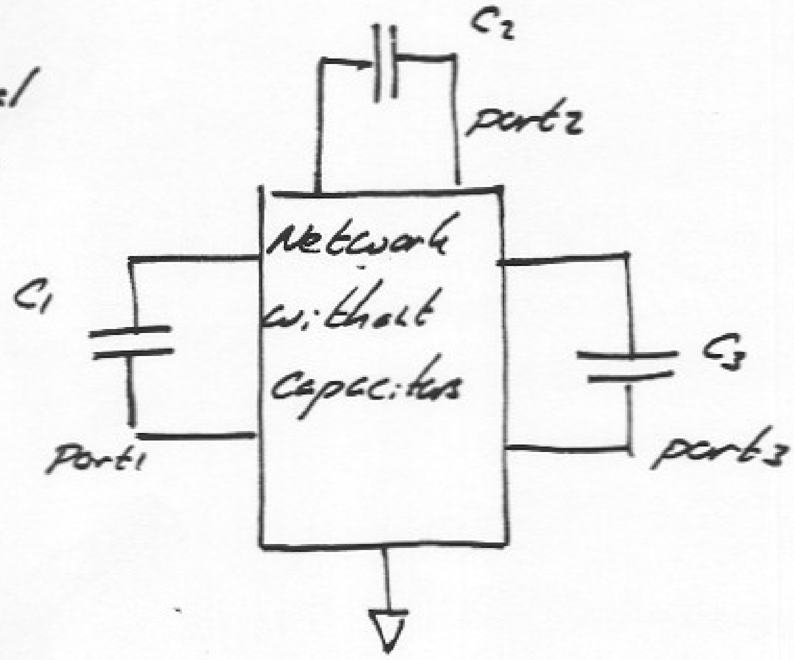
Methods of solution of circuit:

- Nodal Analysis: hard - and unrecognizable answers.
- Miller Approximations: often quite wrong.
- Time Constant Method: accurate, fairly easy.

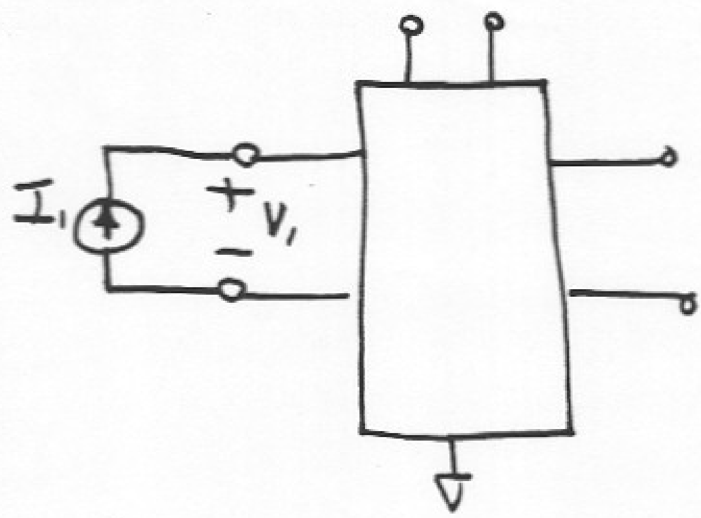
Network - An amplifier:



General
Picture



define R_{11}^{0} - Resistance at port 1 with all capacitors removed:



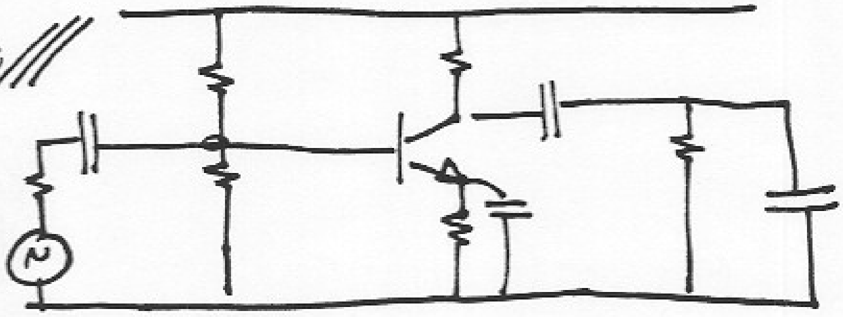
$R_{11}^{0} \triangleq V/I$

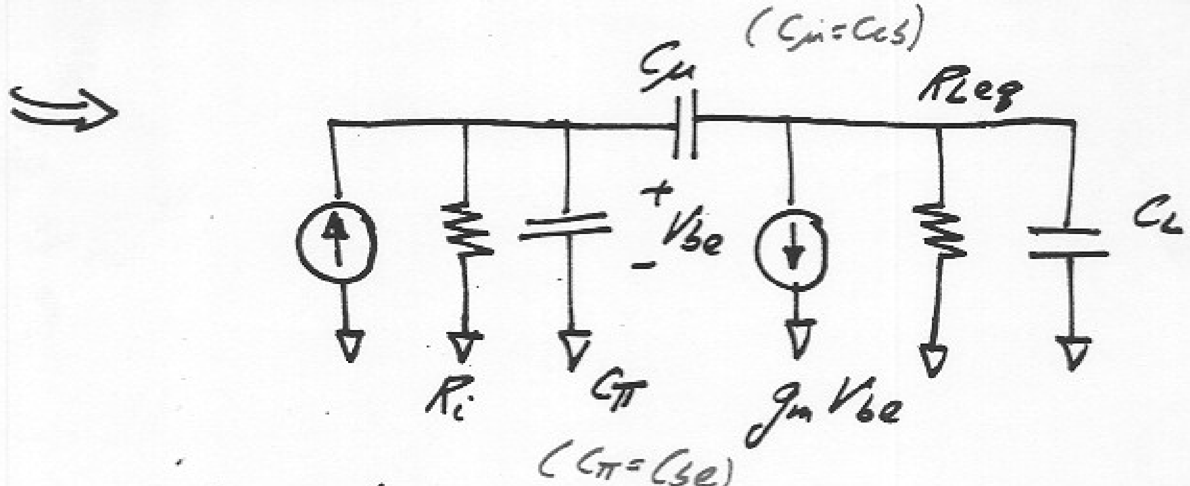
Method of time constants gives:

$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 + \dots$$

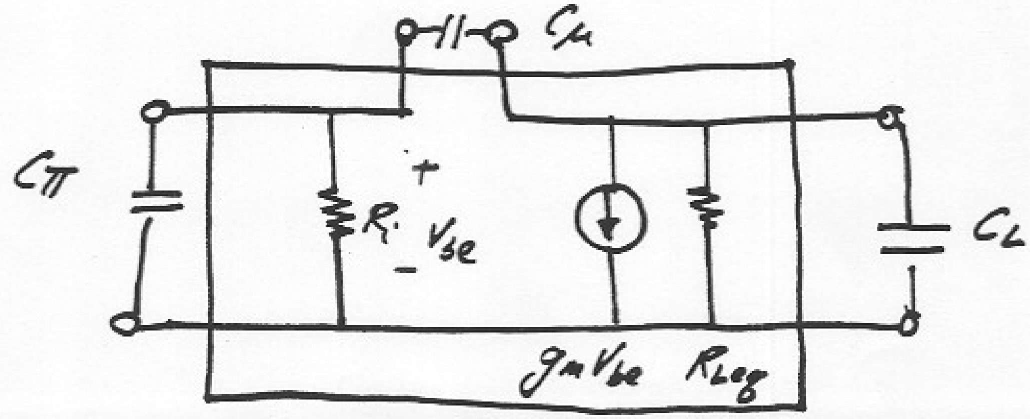
these are called the open-circuit time constants

Ex///

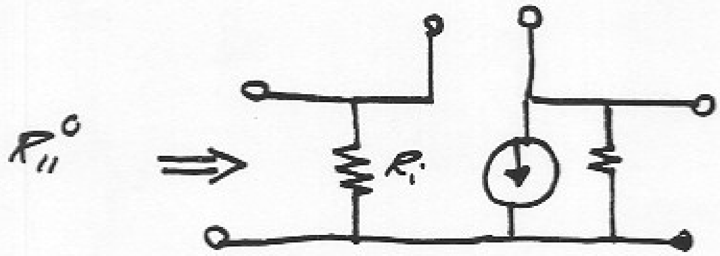




Network without capacitors:



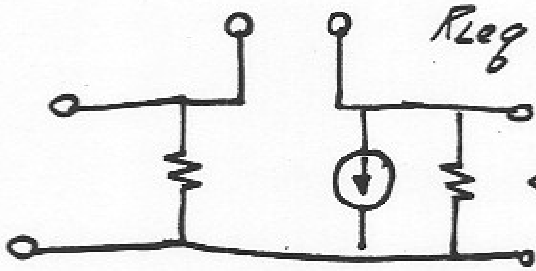
What is R_{11}^0? Easy!



$R_{11}^0 = R_i$

- so first time constant term is $C_\pi R_i$

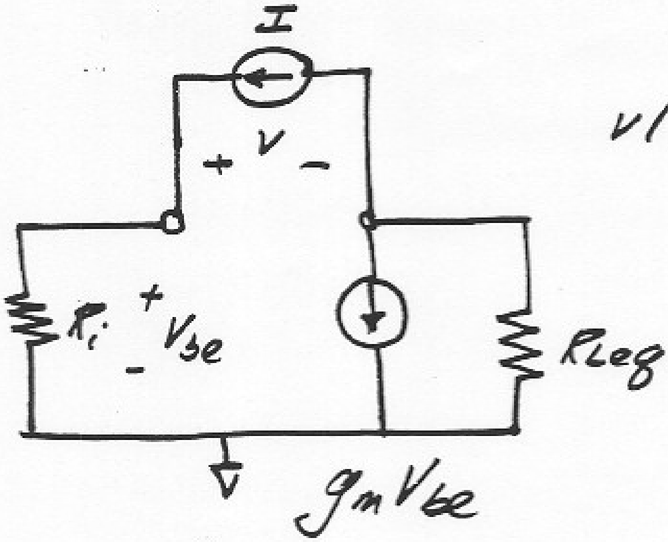
What is R_{33}^0? Easy!



$\Leftarrow$ $R_{33}^0 = R_{Leq}$

- So second time constant term is $C_L R_{Leq}$.

What is R_{22}^{0}? – Harder



$V/I = R_{22}^{0}$.

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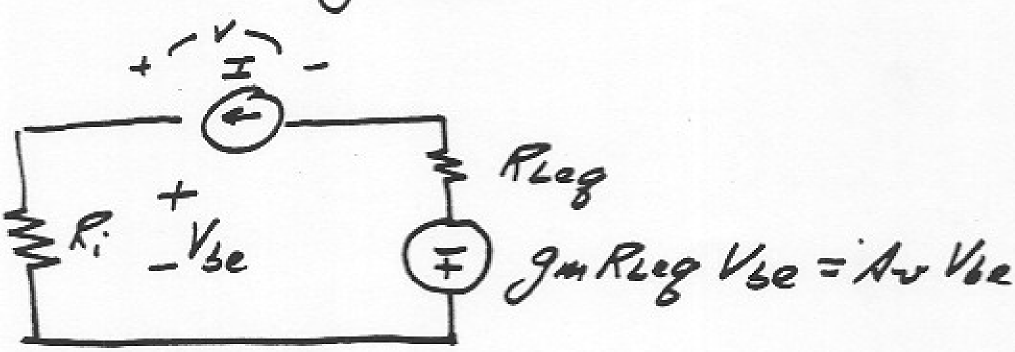


$$V = V_{be} + A_v V_{be} + I R_{Leq}$$
$$= I R_i (1 + A_v) + I R_{Leq}$$

$$R_{22}^{0} = R_i (1 + A_v) + R_{Leq}$$

– so third term in time constant is

$$C_\mu \left[R_i (1 + A_v) + R_{Leq} \right]$$

Sum of time constants is

$$a_1 = R_i C_\pi + R_{Leq} C_L + C_\mu [R_i(1+A_v) + R_{Leq}]$$

or grouping terms differently:

$$a_1 = R_i [C_\pi + C_\mu(1+A_v)] + R_{Leq}[C_L + C_\mu]$$

this we recognize as the same answer as given by both Nodal analysis & the Miller Effect.

What about higher-order poles?

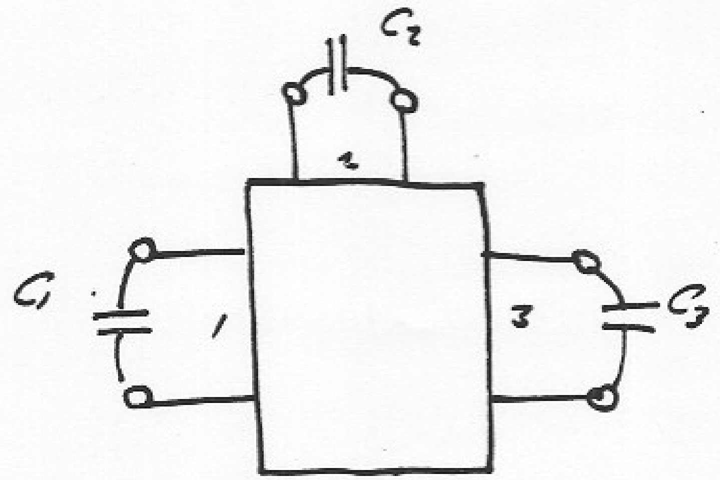
$$\frac{V_o(s)}{V_g(s)} = A_{MB} \frac{\text{Numerator (zeros)}}{1 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots}$$

If a_3 and higher terms are small, and: $f\ a_2/a_1 << a_1$
then:

$$\cong A_{MB} \frac{\text{Numerator}}{(1 + a_1 s)(1 + s\, a_2/a_1)}$$

(separated-pole approximation)

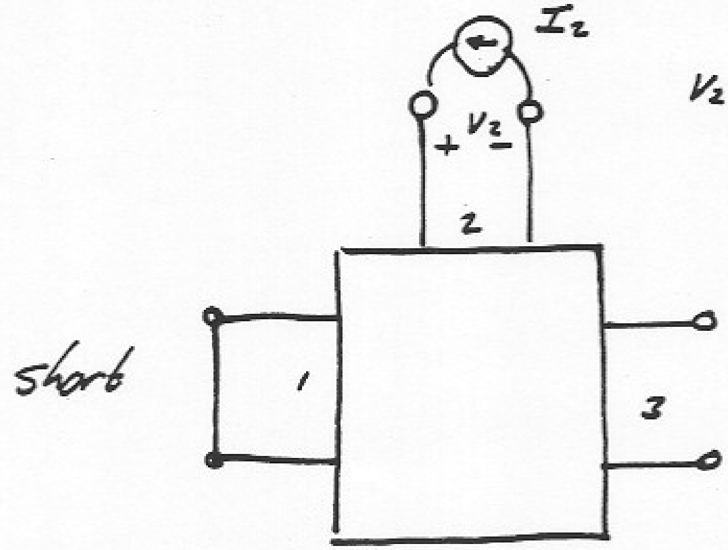
How do we find a_2?



$$a_2 = R_{11}^0 C_1 R_{22}^1 C_2 + R_{11}^0 C_1 R_{33}^1 C_3 + R_{22}^0 C_2 R_{33}^2 C_3$$

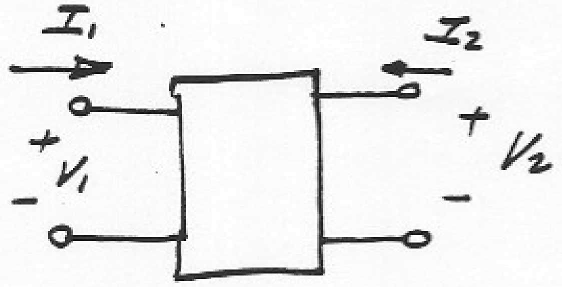
where we define R_{22}^1 as below:

"Resistance seen at port 2 with port 1 shorted"



$$V_2 / I_2 = R_{22}^1$$

Now Consider a 2-port for a moment:



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$R_{11}^{0} = Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \& \quad R_{11}^{2} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

If $V_2 = 0 \Rightarrow Z_{21} I_1 + Z_{22} I_2 = 0 \Rightarrow \frac{V_1}{I_1} = \boxed{Z_{11} + \frac{Z_{12}(-Z_{21})}{Z_{22}} = R_{11}^{2}}$

Similarly: $R_{22}^{1} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11}}$

so: $R_{11}^{0} R_{22}^{1} = Z_{11}\left[Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11}}\right] = Z_{11}Z_{22} - Z_{12}Z_{21} = \Delta Z$

and $R_{11}^{2} R_{22}^{0} = \left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right] Z_{22} = \ldots = \Delta Z$

So: $\boxed{R_{11}^{0} R_{22}^{1} = R_{22}^{0} R_{11}^{2} = \Delta Z}$

So

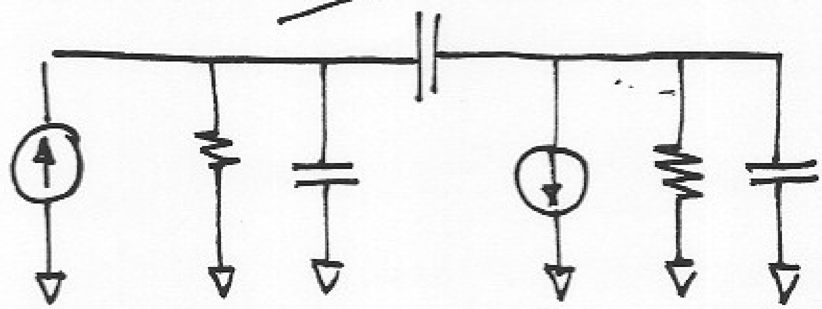
$$a_2 = R_{11}^0 R_{22}^1 C_1 C_2 + R_{11}^0 R_{33}^1 C_1 C_3 + R_{22}^0 R_{33}^2 C_2 C_3$$

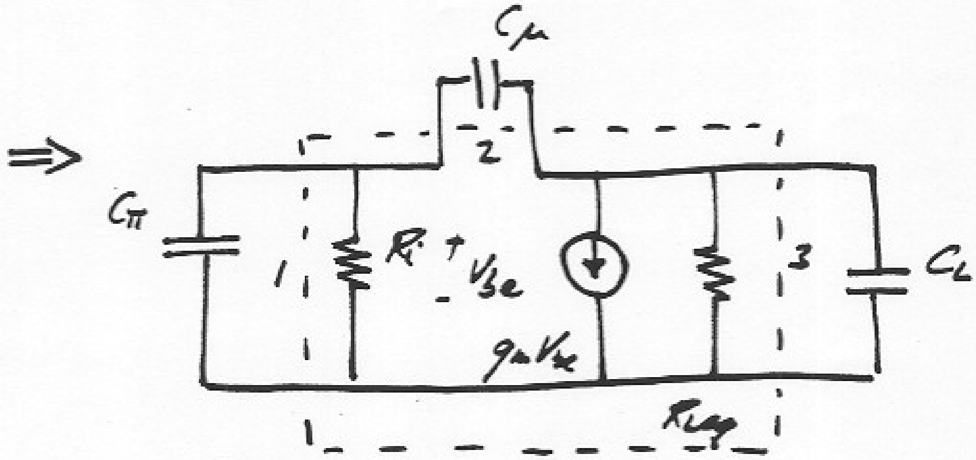
$$= R_{22}^0 R_{11}^2 C_1 C_2 + R_{33}^0 R_{11}^3 C_1 C_3 + R_{33}^0 R_{22}^3 C_2 C_3$$

each term can be calculated 2 ways

EKW

Common-emitter stage





$\underline{R_{11}^{0} \, R_{22}^{1}}$:

$$R_{11}^{0} = R_i \, , \quad R_{22}^{1} = R_{Leq}$$

$\underline{R_{11}^{0} R_{33}^{1}}$ $\quad R_{11}^{0} = R_i \;;\; R_{33}^{1} = R_{Leg}$

$\underline{R_{22}^{0} R_{33}^{2}} = R_{33}^{0} R_{22}^{3}$

$R_{33}^{0} = R_{Leg} \;;\; R_{22}^{3} = R_i \,.$

so: $\quad R_{11}^{0} R_{22}^{1} = R_{11}^{0} R_{33}^{1} = R_{22}^{0} R_{33}^{2} = R_i R_{Leg} \,.$

$\Rightarrow \quad a_2 = \left[C_{\mu} C_L + C_{\mu} C_{\pi} + C_{\pi} C_L \right] R_i R_{Leg}$

so:

$$\frac{V_o(s)}{V_g(s)} = A_{mb} \frac{\text{Numerator}}{1 + a_1 s + a_2 s^2}$$

$$a_1 = R_i\left[C_\pi + C_\mu(1 + g_m R_{Leq})\right] + R_{Leq}\left[C_\mu + C_L\right]$$

$$a_2 = R_i R_{Leq}\left[C_\mu C_\pi + C_\mu C_L + C_\pi C_L\right]$$

- same answer as by Nodal Analysis.

ECE137B: High-Frequency Amplifier Crib Sheet, Page 1

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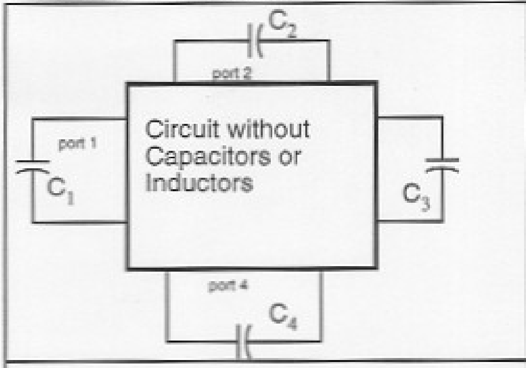
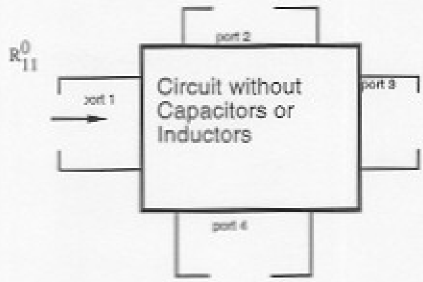
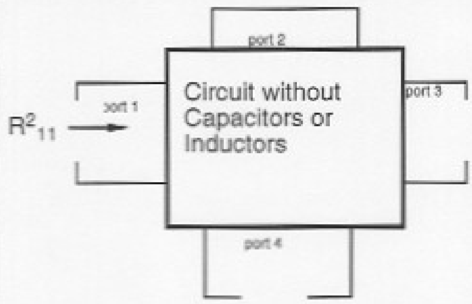
Method of Time Constants





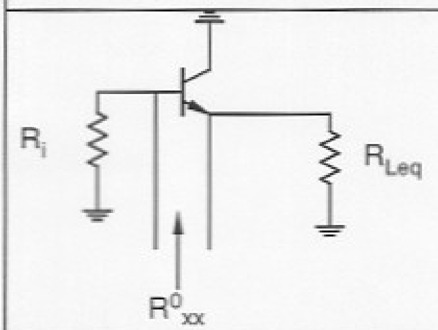


$$\frac{V_{out}}{V_{gen}} = \left(\frac{V_{out}}{V_{gen}}\right)_{mb} \frac{1+b_1 s+b_2 s^2+\cdots}{1+a_1 s+a_2 s^2+\cdots}$$

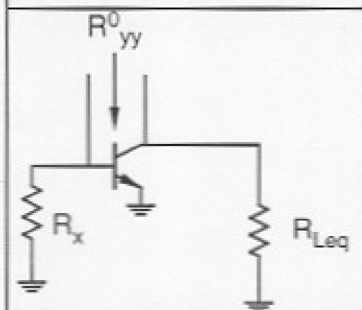
$$a_1 = R^0_{11}C_1 + R^0_{22}C_2 + R^0_{33}C_3 + R^0_{44}C_4$$

$$a_2 = R^0_{11}R^1_{22}C_1C_2 + R^0_{11}R^1_{33}C_1C_3 + R^0_{11}R^1_{44}C_1C_4 + R^0_{22}R^2_{33}C_2C_3 + R^0_{22}R^2_{44}C_2C_4 + R^0_{33}R^3_{44}C_3C_4$$

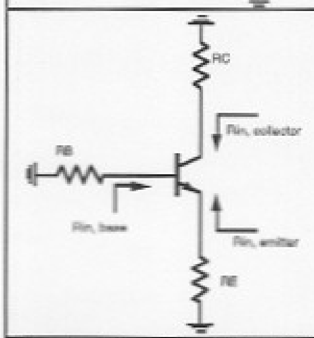
notethat $R^0_{xx}R^x_{yy} = R^y_{xx}R^0_{yy}$



$$R^0_{xx} = R_\pi \| \left(r_e \| R_{Leq} + R_i(1-A_{vmb}) \right)$$
$$A_{vmb} = \left(R_{Leq} / (r_e + R_{Leq}) \right)$$



$$R_i = R_x \| R_\pi$$
$$R^0_{yy} = R_i(1+g_m R_{Leq}) + R \; + R_{Leq}$$



For the bipolar

$$R_{in,base} = (\beta+1)(R_E + r_e)$$

$$R_{in,emitter} = \left(r_e + \frac{R_B}{\beta+1}\right)\left(\frac{r_o+R_C}{r_o}\right) \cong \left(r_e + \frac{R_B}{\beta+1}\right)$$

$$R_{in,collector} = r_o\left\{1 + g_m R_E\left(\frac{r_\pi}{r_\pi + R_E + R_B}\right)\right\} \equiv r_o\{1+g_m R_E\}$$

these give almost all the remaining relationships needed for the MOTC

For the fet: THE EQUATIONS ARE THE SAME IF ß is INFINITE!

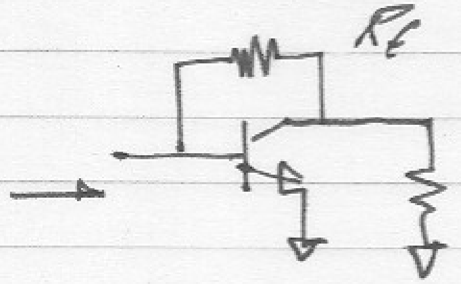
If we refer to the MOTC crib sheet on the previous page, we will find expressions for R_{xx}^{Y} & R_{xx}^{O} for almost every case

It is generally unnecessary with the MOTC to do much math!

The only exception:

networks with feedback, e.g.

$R_{11}^{O} = ?$

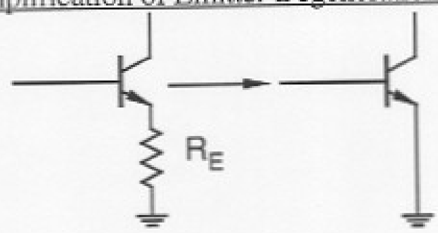


will generally need working through on a case-by-case basis.

Emitter Degeneration

From the crib sheet:

Simplification of Emitter Degeneration



approximate only; check notes for bounds on validity

$$\tilde{C}_\pi = C_\pi\left(r_e/(r_e + R_E)\right)$$

$$\tilde{g}_m = g_m\left(r_e/(r_e + R_E)\right) = \alpha/(r_e + R_E)$$

$$\tilde{C}_\mu = C_\mu$$

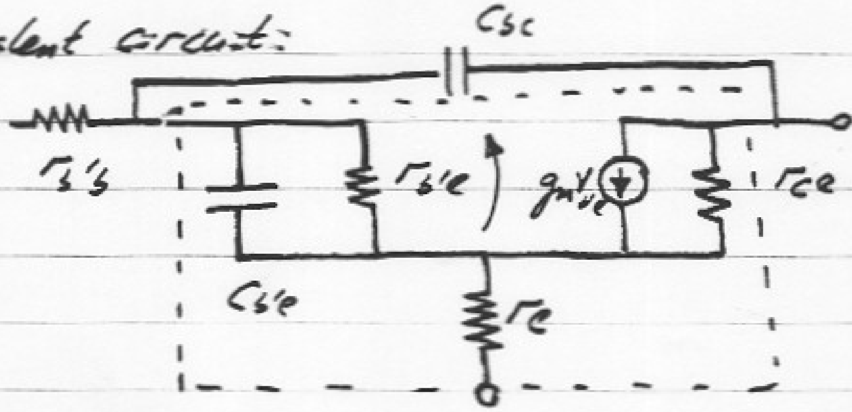
$$\tilde{R}_\pi = R_\pi\left((r_e + R_E)/r_e\right) = (\beta + 1)(r_e + R_E)$$

this is a powerful simplification for the purposes of analysis.

Proof (and limits) of the relationship are given on the following pages, which will not be covered in lecture...

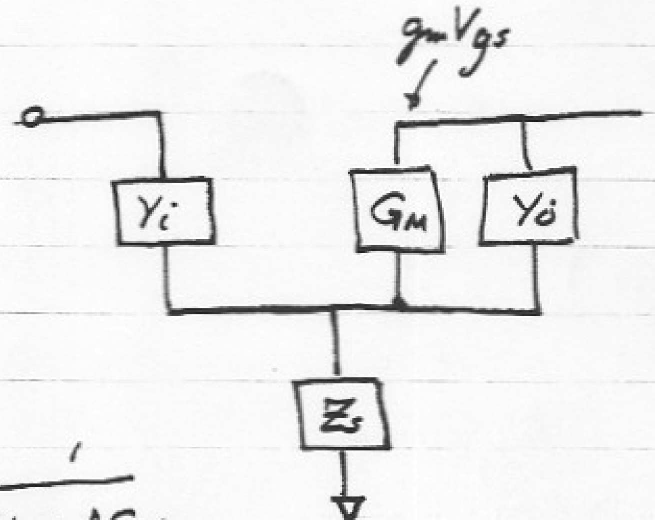
Conversion (first-order) of device equivalent circuit to "absorb" R_S (or R_E)

equivalent circuit:



First: put aside $r_{b'b}$ & C_{cb}; we will put back later

second: some notation:



$$Z_i = \frac{1}{Y_i} = \frac{1}{r_{b'e} + sC_{b'e}}$$

$Y_o = \frac{1}{Z_o} = \frac{1}{r_{ce}}$ (not characteristic impedance)

$Z_s = r_e = 1/Y_s$: tendency to mix notation: $r_s = r_e$

Approximations

We are developing only an approximate equivalent circuit.

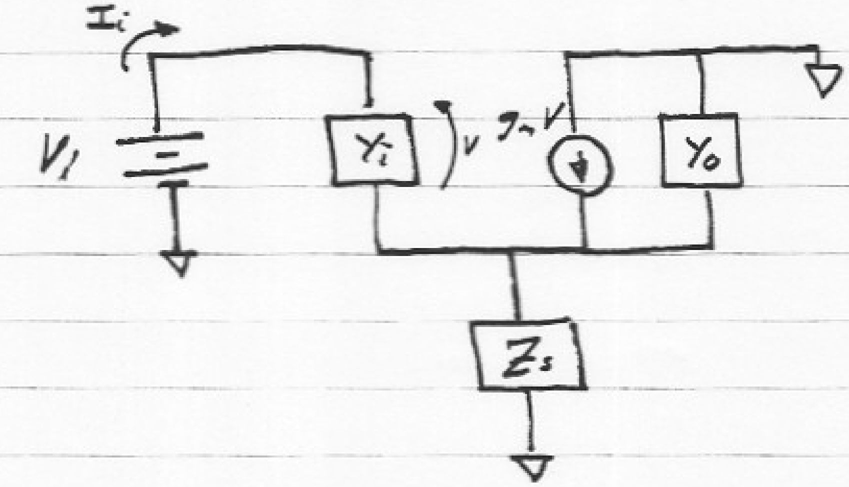
Assuming: $Z_s << Z_i$ otherwise, expressions can be generalized

$g_m >> Y_i$ this implies $f << f_T$

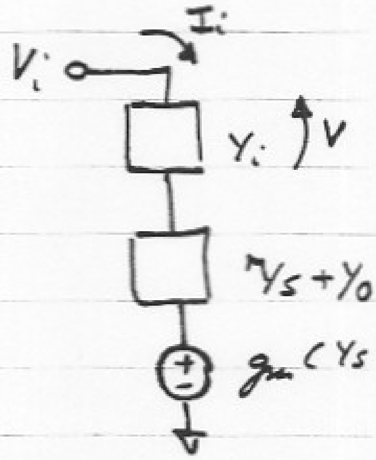
$Z_s << \frac{1}{Y_o}$ reasonable almost always.

Get the Y-parameters:

$$Y_{11} = I_i / V_i \big|_{V_o = 0}$$



$\Rightarrow$

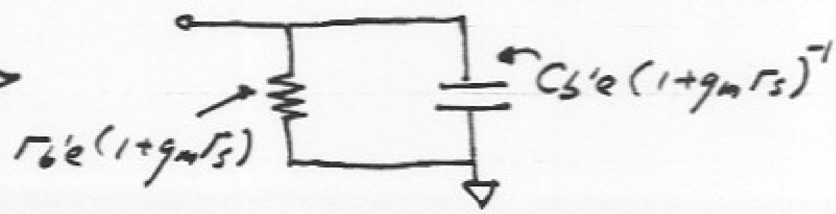


$$g_m (Y_s + Y_o)^{-1} V = g_m (Y_s + Y_o)^{-1} Z_i \cdot I_i$$

$$\frac{1}{Y_{11}} = Z_i + \frac{1}{Y_s + Y_o} + Z_i \frac{g_m}{Y_s + Y_o} \cong Z_i (1 + g_m Z_s) + Z_s$$

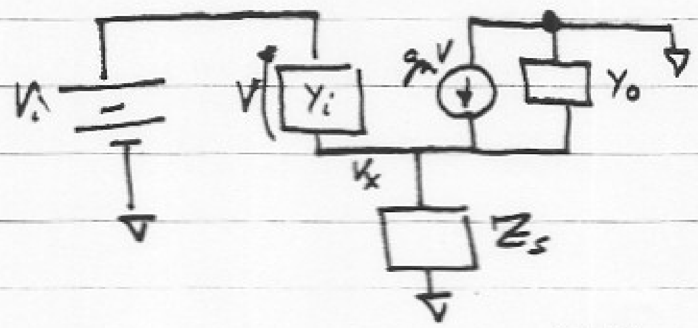
$$\cong Z_i (1 + g_m Z_s)$$

$$Y_{11} \cong \frac{Y_i}{1 + g_m r_s} \Rightarrow$$



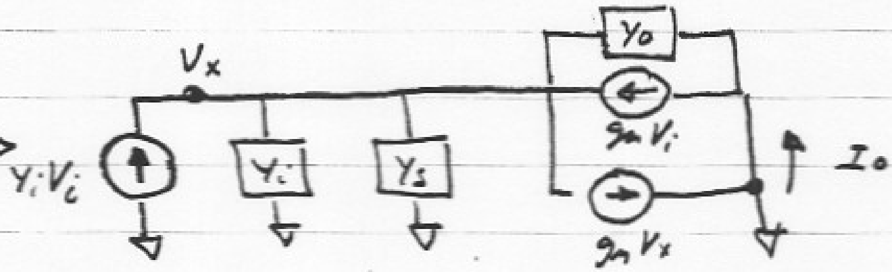
Transconductance

$$Y_{21} = I_o / V_i \big|_{V_o = 0}$$



$$g_m V = g_m V_i - g_m V_x$$

$\Rightarrow$



$$V_x(Y_i + Y_s + Y_o + g_m) - (Y_i + g_m) V_i = 0$$

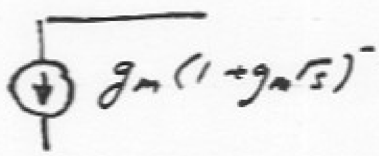
$$V_x = V_i (Y_i + g_m)/(Y_i + Y_s + Y_o + g_m)$$

$$I_o = -(g_m + Y_o) V_x + g_m V_i$$

$$I_o / V_i = \frac{-Y_o Y_i + g_m Y_s}{g_m + Y_o + Y_i + Y_s}$$

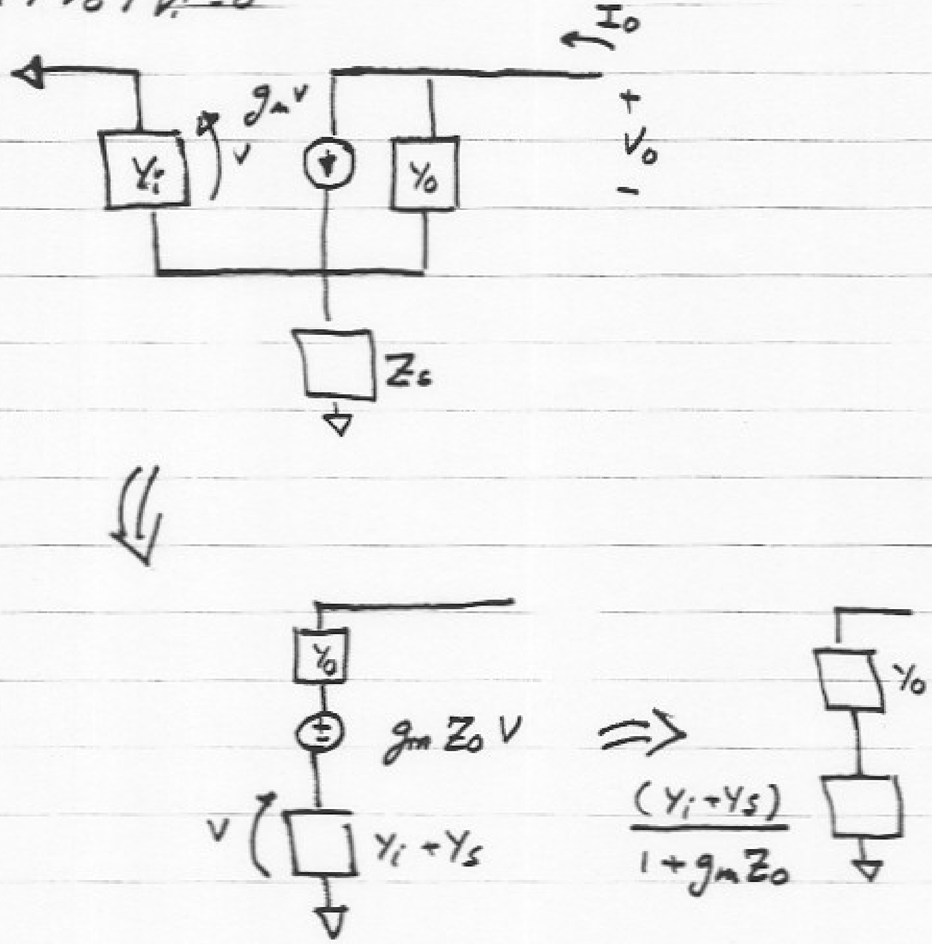
$$\cong \frac{g_m Y_s}{g_m + Y_s} = \frac{g_m}{1 + g_m r_s}$$

$\Rightarrow$



output conductance

$$Y_{22} = I_i / V_o \big|_{V_i = 0}$$



$$Z_{out} = \frac{1}{Y_{22}} = Z_o + (1 + g_m Z_o)/(Y_i + Y_s)$$

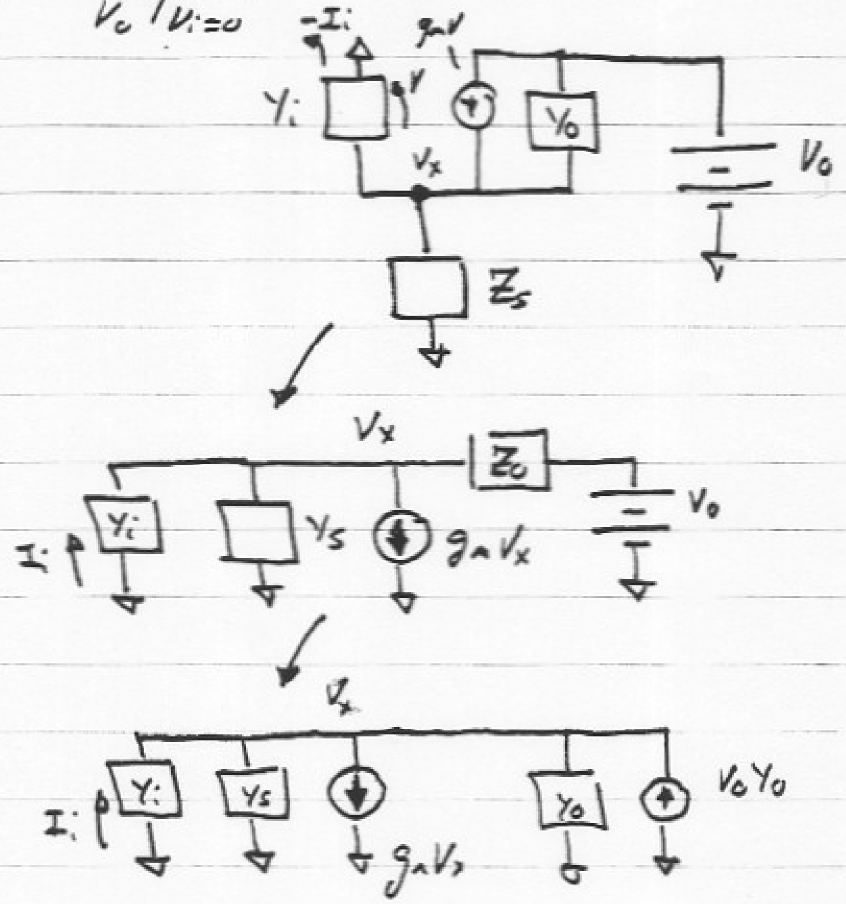
$$\cong Z_o + (1 + g_m Z_o)/Y_s = Z_o + (1 + g_m Z_o) r_s$$

$$= Z_o(1 + g_m r_s) + r_s \cong Z_o(1 + g_m r_s)$$

$r_{ce} \cdot (1 + g_m r_s)$

Reverse Conductance: (Y_{12})

$$Y_{12} = \left.\frac{I_i}{V_o}\right|_{V_i=0} =$$



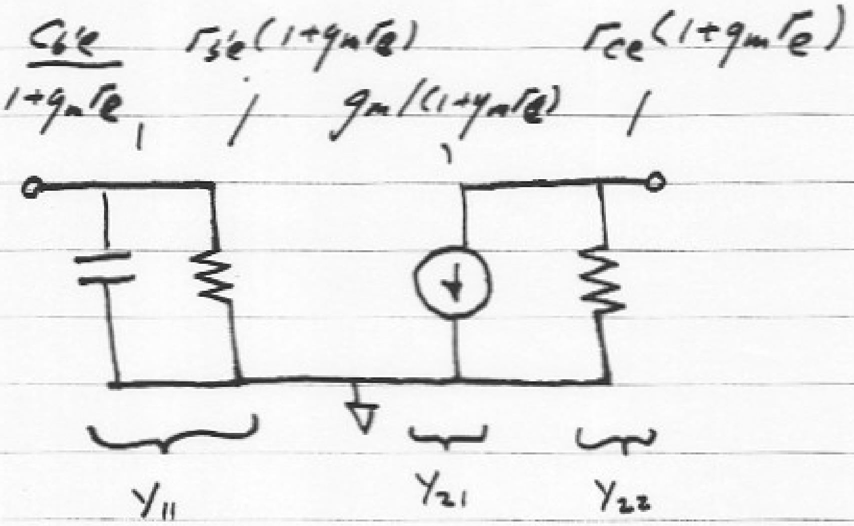
$$V_x (Y_i + Y_s + g_m + Y_o) = V_o Y_o$$

$$I_i = -Y_i V_x$$

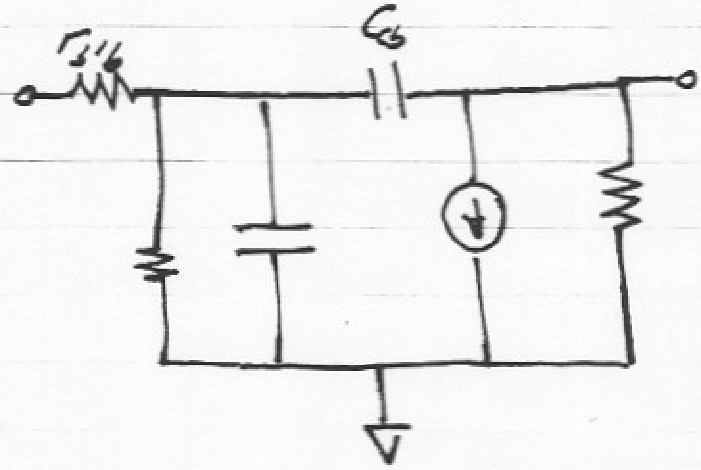
$$\frac{I_i}{V_o} = \frac{-Y_i Y_o}{Y_i + Y_s + g_m + Y_o} \cong -Y_i \frac{Y_o}{g_m + Y_s}$$

but Y_i is fairly small and $Y_o \ll g_m$ => neglect to first order.

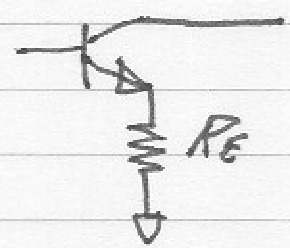
Bringing it all together:



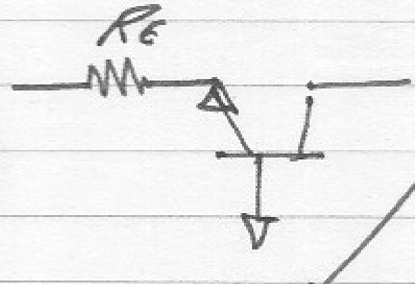
Adding back parasitic C_{cb}, $r_{b'b}$:

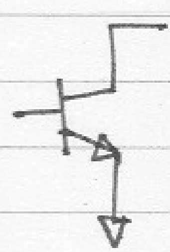


Note that, within the bounds of accuracy of the derivation above, the degeneration method can be used on both common-emitter and common-base stages:

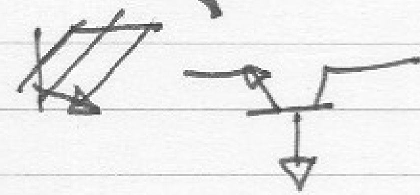


$$
\begin{cases}
\tilde{g}_m = g_m\,(r_e/(r_e+R_E)) \\
\tilde{C}_{be} = C_{be}\,(r_e/(r_e+R_E)) \\
\tilde{C}_{bc} = C_{bc} \\
\tilde{R}_{be} = R_{be}\cdot(r_e+R_E)/R_E.
\end{cases}
$$



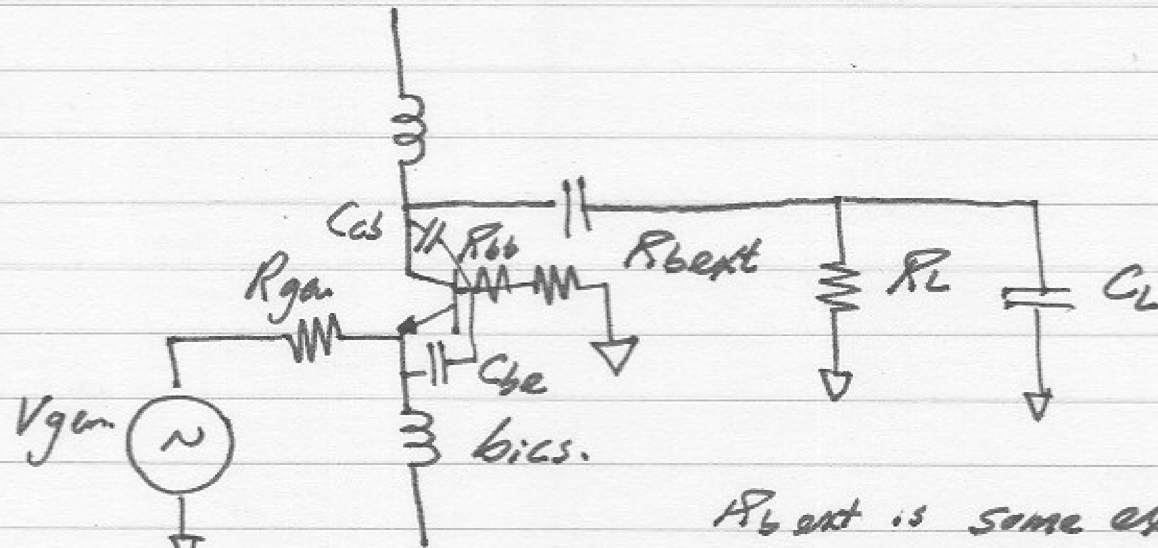


and



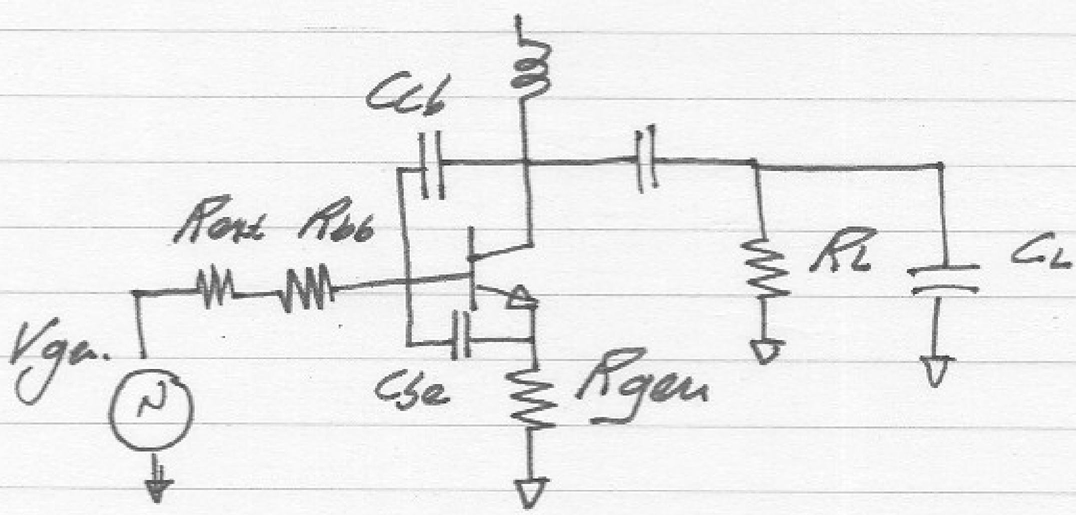
Common-base stage by MOTC:

now lets include a non-zero R_{bb}!



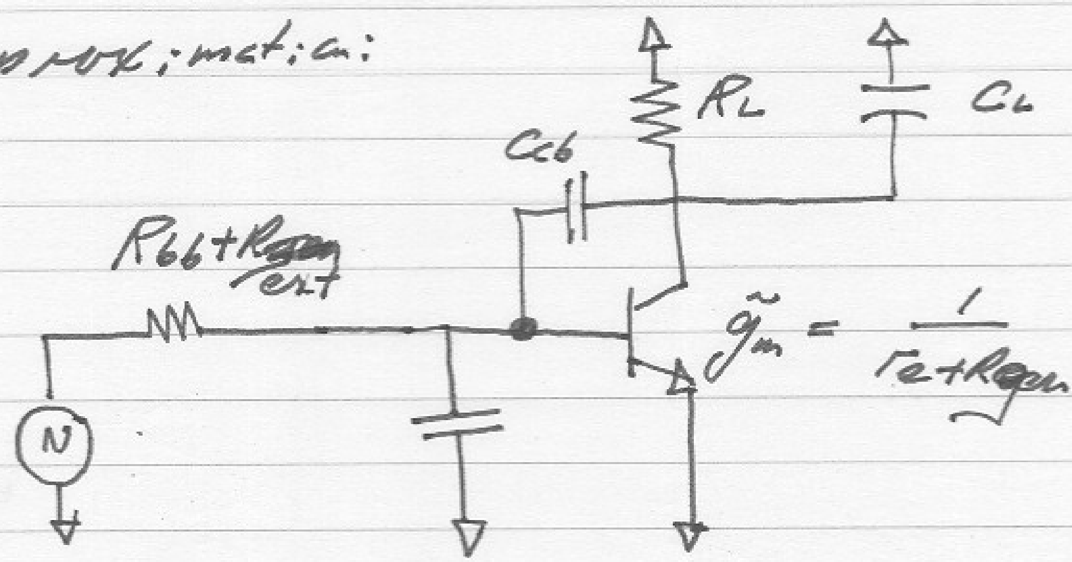
R_b ext is some external resistance...

Before taking this further, consider a common-emitter circuit with peculiar notation:



* Lets first simplify slightly by taking $\beta \rightarrow \infty$.

* Then lets use the degeneration approximation:



$$\tilde{C}_{be} = C_{be} \cdot \frac{r_e}{r_e + R_{gen}}$$

Now we know that

(From nodal analysis or MOTC)

$$H(s) \propto \frac{N(s)}{1 + a_1 s + a_2 s^2}$$

where

$$a_1 = (R_{bb} + R_{ext})\left[\frac{C_{be} r_e}{r_e + R_{gen}} + C_{cb}\left(1 + \frac{R_L}{r_e + R_{gen}}\right)\right]$$

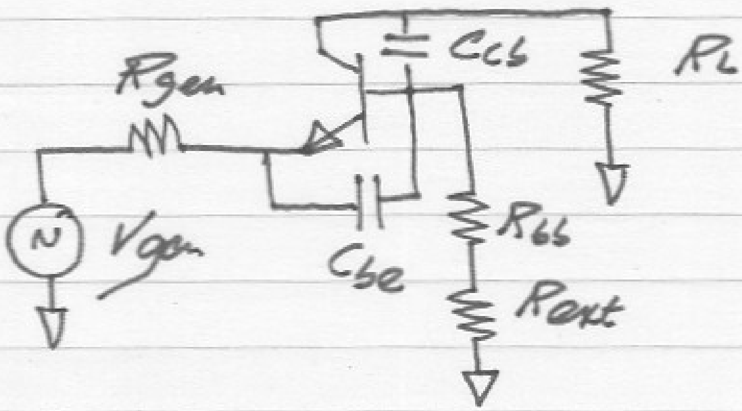
$$+ \; C_{cb} \cdot R_L + C_L \cdot R_L$$

$$a_2 = R_L \cdot (R_{bb} + R_{ext})$$

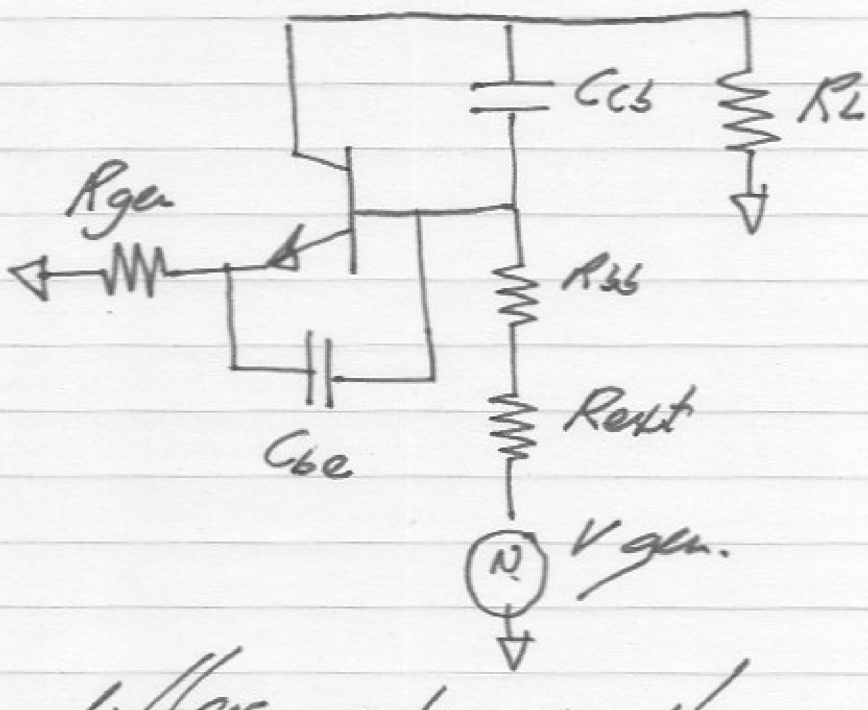
$$\cdot \left[C_{cb} \cdot C_L + \frac{C_{be} r_e}{r_e + R_{gen}} \cdot C_L + \frac{C_{be} r_e}{r_e + R_{gen}} \cdot C_{cb}\right]$$

But wait:

CB circuit:

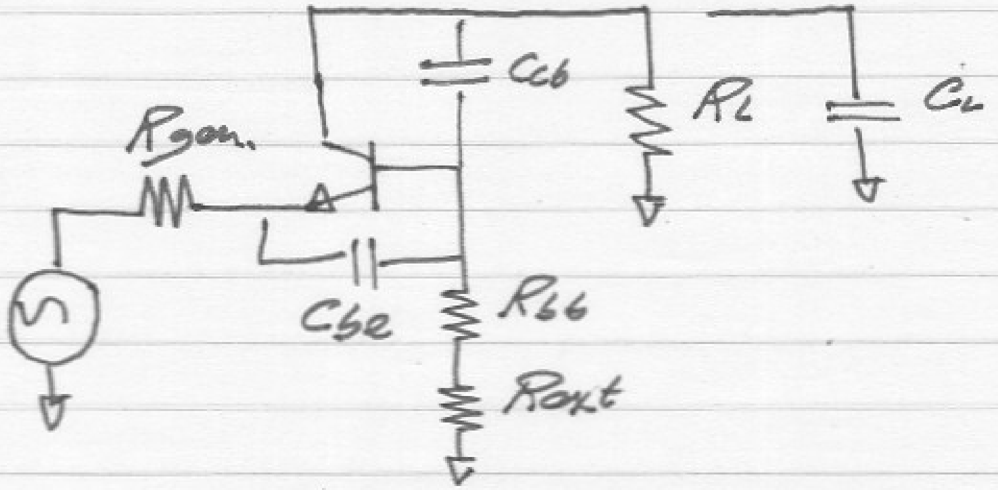


CE circuit



the 2 circuits differ only in the connection of Vgen, and that has no effect on a_1 or a_2 or a_3 etc (only b_1 b_2 b_3)

So for the common base stage:



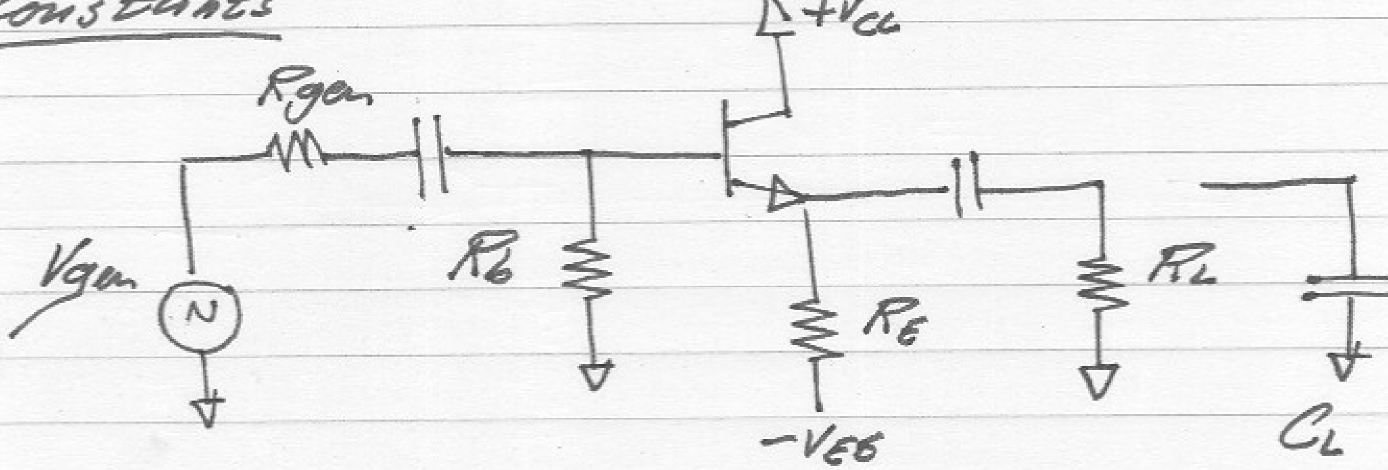
$$a_1 \cong C_{be} \cdot \left[\frac{r_e}{r_e + R_{gen}} \right] \cdot (r_{bb} + R_{ext})$$

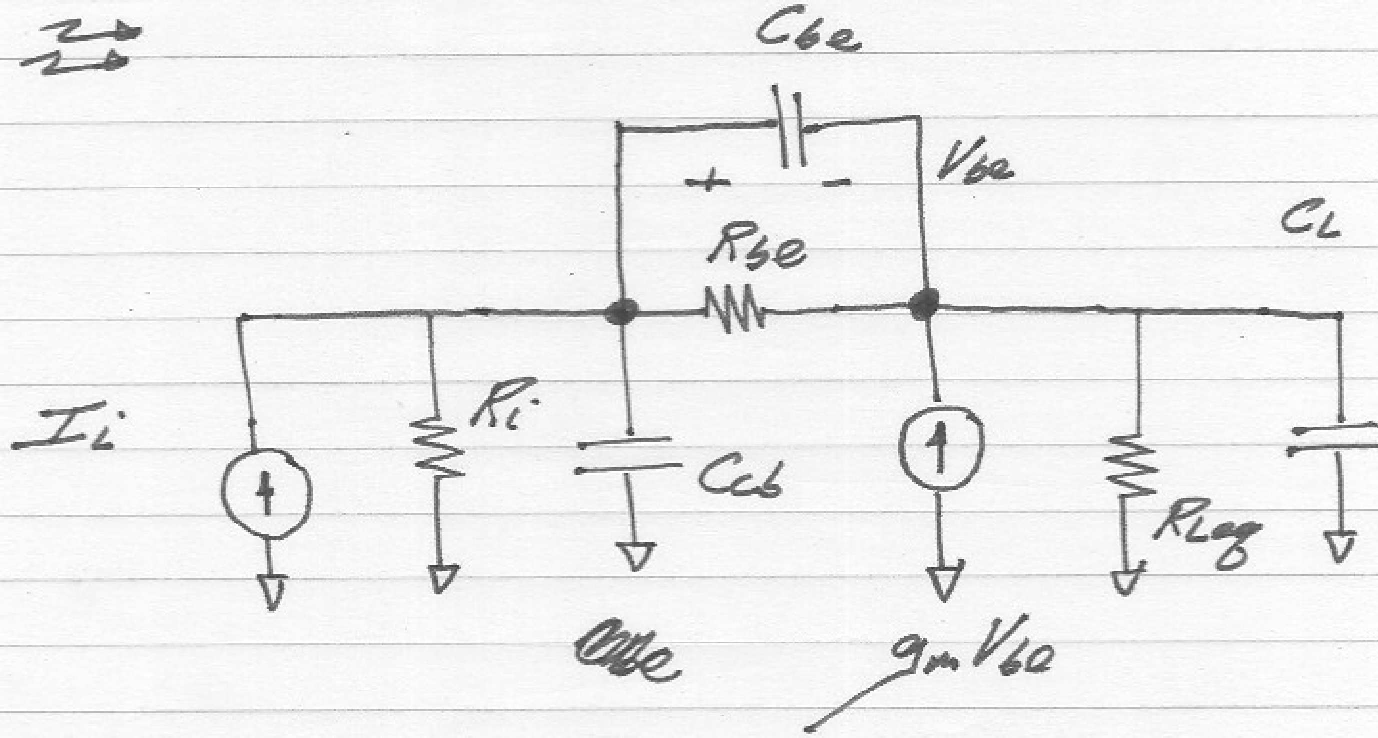
$$+ \; C_{cb} \left[R_L + (R_{bb} + R_{ext}) \underbrace{\left(1 + \frac{R_L}{r_e + R_{gen}}\right)} \right]$$

$$+ \; C_L \cdot R_L$$

this is <u>very</u> similar to the Miller Effect. $(R_{bb} + R_{ext})$ needs to be small.

Emitter follower by Method of time Constants



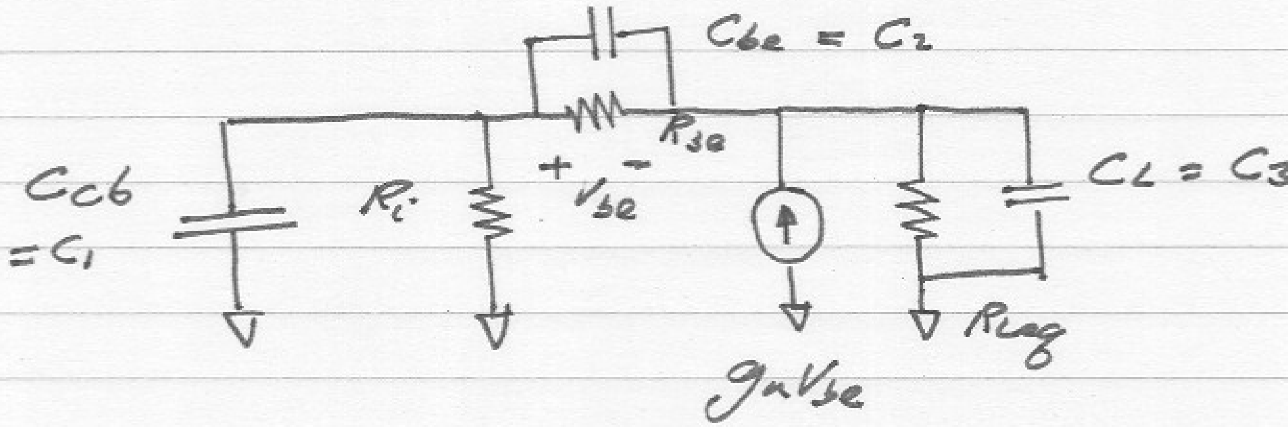


$R_i = R_{b'b} + (R_{gen} \| R_i')$

$R_{Leq} = R_E \| R_L$

Method of time constants:

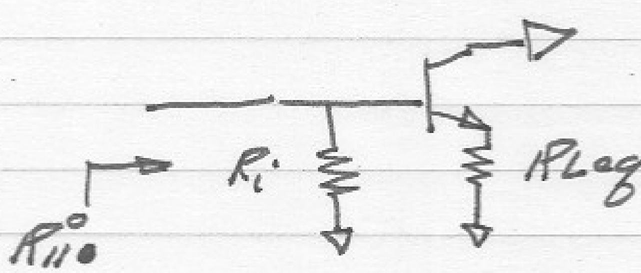
$$H(s) = H_{MB} \cdot \frac{N(s)}{1 + a_1 s + a_2 s^2}$$



$$a_1 = C_1 R_{11}^{\circ} + C_2 R_{22}^{\circ} + C_3 R_{33}^{\circ}$$

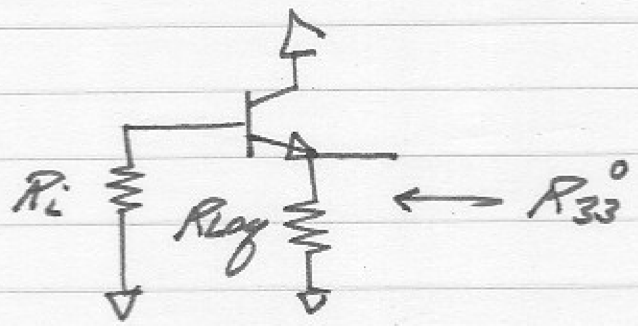
$C_1 R_{11}^{\circ}$

what is R_{11}° ?



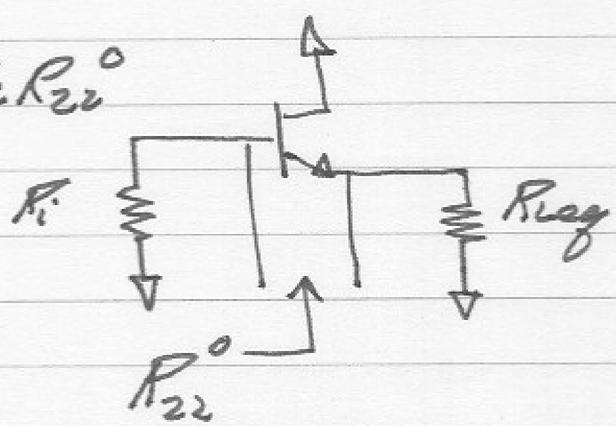
by inspection $R_{11}^{\circ} = R_i \,||\, (\beta+1)(r_e + R_{Leq})$!

$C_3 R_{33}^{o}$



by inspection, $R_{33}^{o} = R_{Leq} \| \left(r_e + \frac{R_i}{\beta+1} \right)$

$C_2 R_{22}^{o}$



this is harder. Requires nodal analysis.

Answer is on Crib sheet.

$$R_{22}^{o} = R_\pi \Big\| \left[(r_e \| R_{Leq}) + R_i \left(1 - \frac{R_{eq}}{r_e + R_{Leq}} \right) \right]$$

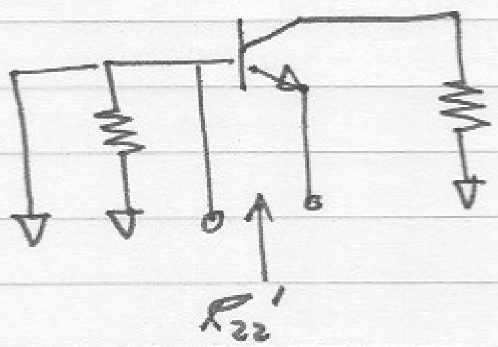
the very alert reader will recognize the miller effect in the last term.

Now second-order time constants.

$$a_2 = C_1 C_2 R_{11}^0 R_{22}^1 + C_1 C_3 R_{11}^0 R_{33}^1 + C_2 C_3 R_{22}^0 R_{33}^2$$

R_{11}^0, R_{22}^0 & R_{33}^0 we have already found.

<u>$R_{22}^1 = ?$</u>



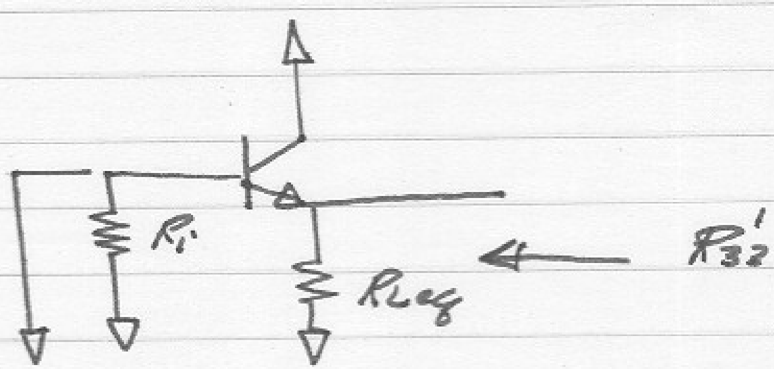
$$R_{22}^0 = R_\pi \,\|\, \left[(r_e \| R_{Leq}) + R_i \left(1 - \frac{R_{Leq}}{r_e + R_{Leq}}\right) \right]$$

we have just <u>shorted</u> part 1 (R_i)

so just set $R_i \to 0$.

$$R_{22}^1 = R_\pi \| r_e \| R_{Leq} \cong r_e \| R_{Leq}$$

R_{33}'



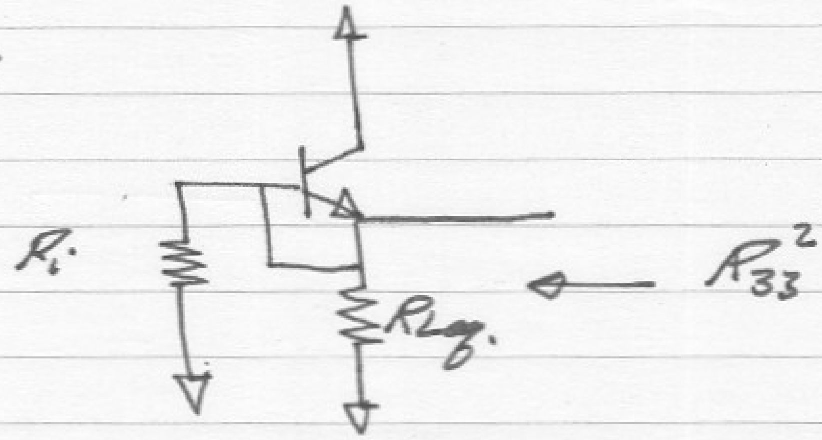
by same argument:

$$R_{33}' = R_{Leq} \,\|\, \left(r_e + \cancel{\frac{R_i}{\beta+1}}\right)$$

$$\cancel{\frac{R_i}{\beta+1}} \to 0$$

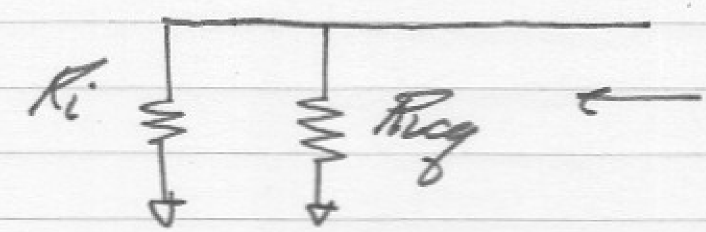
$$= R_{Leq} \,\|\, r_e$$

$\underline{R_{33}^{2}}$:



b.e. junction is ac shorted.

no small-signal base or emitter current flows →



$R_{33}^{2} = R_i \| R_{Leq.}$

Gathering terms:

$$a_1 = C_{be} \cdot \left[R_{be} \,\|\, \left(r_e \| R_{Leq} + R_i \left(1 - \frac{R_{Leq}}{R_{Leq} + r_e}\right)\right)\right]$$

$$+ C_{cb} \left[R_i \,\|\, (\beta+1)(r_e + R_{Leq}) \right]$$

$$+ C_L \left[R_{Leq} \,\|\, \left(r_e + \frac{R_i}{\beta+1} \right) \right]$$

$$a_2 = (r_e \| R_{Leq})(R_i \| (\beta+1)(r_e + R_{Leq}))$$

$$\cdot \left[C_{be} C_{cb} + C_{be} C_L + C_{cb} C_L \right]$$

$\rightarrow$ and the bandwidth is

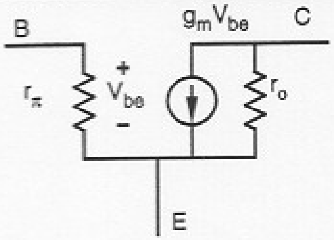
$$f_{p1} \sim \frac{1}{2\pi a_1}$$

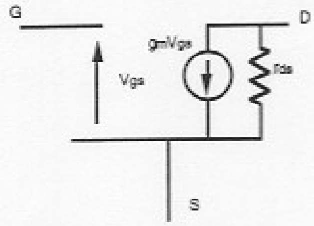
... note there is also a zero @ $f_z = \frac{g_m}{2\pi C_{be}}$

$= f_T$

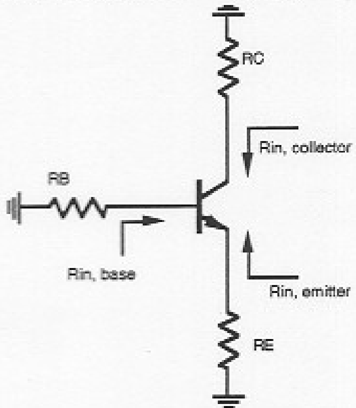
Single stage amplifier crib sheet, page 1

Basics: Amplifiers at Low Frequencies





Left is the equivalent circuit of a bipolar transistor; $g_m = I_E/V_T = 1/r_e$, $V_T = kT/q$, $r_\pi = (\beta+1)r_e = \beta/g_m$. On the right is the FET model. Below is a transistor with ac equivalent load resistances R_c, R_E, and R_B.

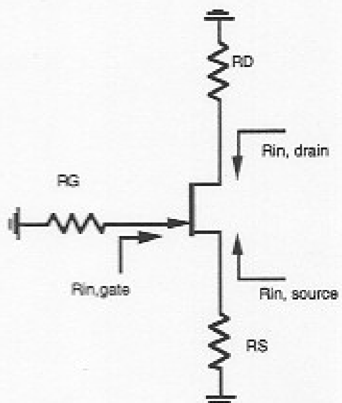


For the bipolar

$$R_{in,base} = (\beta+1)(R_E + r_e)$$

$$R_{in,emitter} = \left(r_e + \frac{R_B}{\beta+1}\right)\left(\frac{r_o + R_C}{r_o}\right) \cong \left(r_e + \frac{R_B}{\beta+1}\right),$$

$$R_{in,collector} = r_o\left\{1 + g_m R_E\left(\frac{r_\pi}{r_\pi + R_E + R_B}\right)\right\} \cong r_o\{1 + g_m R_E\}$$



For the fet: THE EQUATIONS ARE THE SAME IF ß is INFINITE!

$$R_{in,gate} = open$$

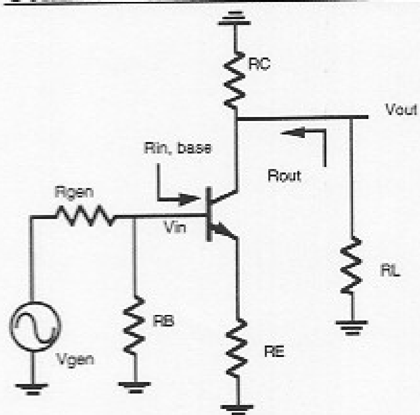
$$R_{in,source} = \left(\frac{1}{g_m}\right)\left(\frac{R_D + r_{ds}}{r_{ds}}\right) \cong \left(\frac{1}{g_m}\right)$$

$$R_{in,drain} = r_{ds}\{1 + g_m R_S\}$$

Be warned that in the equations above Rc, RE and RB are the equivalent resistances seen by the transistor. So in, the amplifier circuits below, THINK: "what is the effective resistance seen by the transistor?", before plugging into the equations.

Amplifier Stages

Common Emitter and Common Source



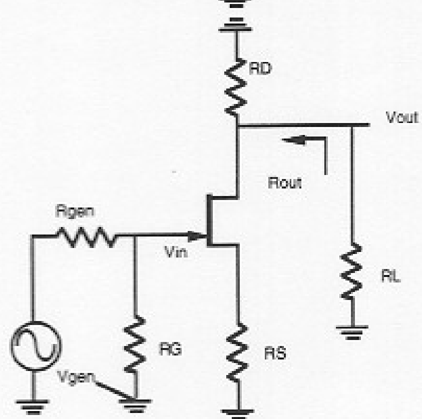
Common emitter:

$$R_{in} = R_B \| R_{in,base} = R_B \| (\beta+1)(R_E + r_e)$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-(R_{Leq})}{r_e + R_E} = \frac{-(R_C \| R_L \| R_{out,collector})}{r_e + R_E}$$

$$R_{out} = (R_C \| R_{out,collector})$$



Common source:

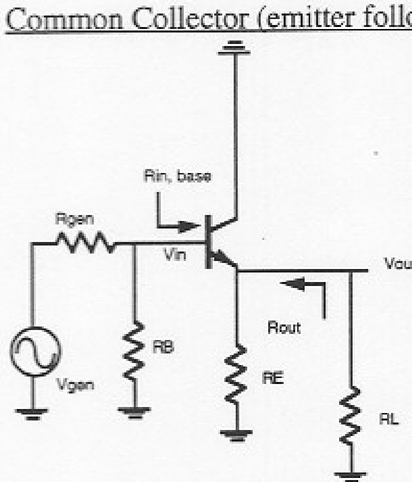
$$R_{in} = R_G$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-(R_{Leq})}{1/g_m + R_S} = \frac{-(R_D \| R_L \| R_{out,drain})}{1/g_m + R_S}$$

$$R_{out} = R_D \| R_{out,drain}$$

Common Collector (emitter follower) and common drain (source follower)



for the emitter follower:

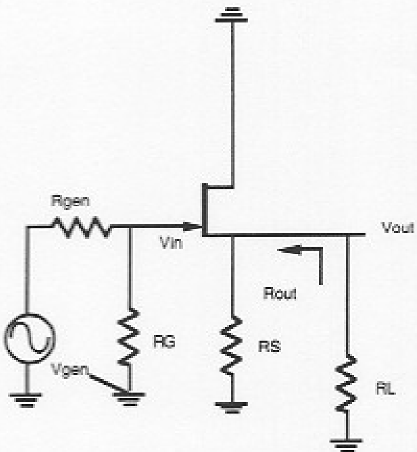
$$R_{in} = R_B \| R_{in,base} = R_B \| (\beta+1)(R_E \| R_L + r_e)$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R_{Leq}}{r_e + R_{Leq}} = \frac{R_E \| R_L}{r_e + R_E \| R_L}$$

$$R_{out} = R_E \| R_{in,emitter} = R_E \left\| \left(r_e + \frac{R_B}{\beta+1} \right) \right.$$

Single stage amplifier crib sheet, page 3



For the source follower:

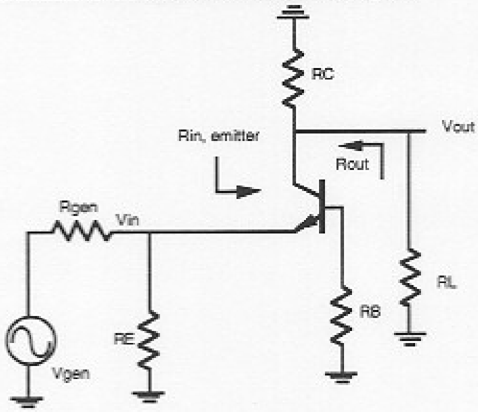
$$R_{in} = R_G$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R_{Leq}}{1/g_m + R_{Leq}} = \frac{R_S \| R_L}{1/g_m + R_S \| R_L}$$

$$R_{out} = R_S \| R_{in,source} = R_S \| \frac{1}{g_m}$$

Common Base and common gate



For the common base:

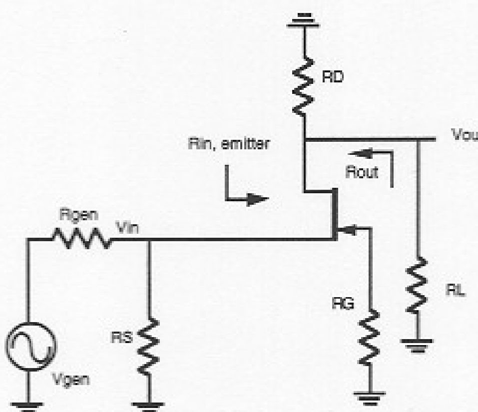
$$R_{in} = R_E \| R_{in,emitter}$$

$$R_{in,emitter} = \left(r_e + \frac{R_B}{\beta+1}\right)\left(\frac{r_o + R_C \| R_L}{r_o}\right) \cong \left(r_e + \frac{R_B}{\beta+1}\right)$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R_{L,eq}}{R_{in,emitter}} = \frac{R_C \| R_L}{R_{in,emitter}}$$

$$R_{out} = (R_C \| R_{out,collector})$$



For the common gate

$$R_{in} = R_S \| R_{in,Source}$$

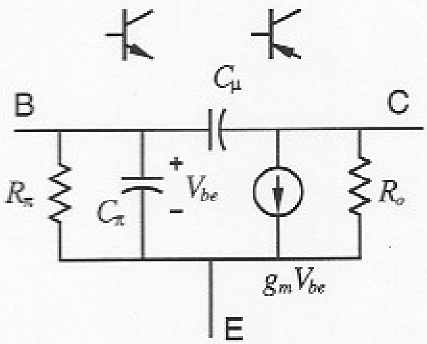
$$R_{in,source} = \left(\frac{1}{g_m}\right)\left(\frac{R_D \| R_L + r_{ds}}{r_{ds}}\right) \cong \left(\frac{1}{g_m}\right)$$

$$\frac{V_{in}}{V_{gen}} = \frac{R_{in}}{R_{in} + R_{gen}}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{R_{L,eq}}{R_{in,source}} = \frac{R_D \| R_L}{R_{in,source}}$$

$$R_{out} = R_D \| R_{out,drain}$$

Basics: Transistor small-signal high-frequency models



Hybrid-pi model:

$C_\mu = C_{bc}$

$C_\pi = C_{be} = C_{\pi,depl} + C_{\pi,diff}$

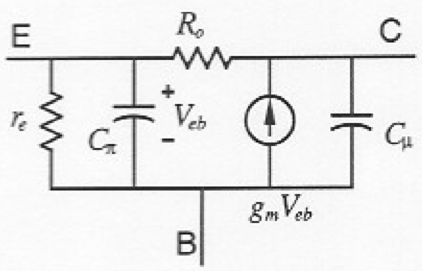
$C_{\pi,diff} = g_m \tau_f$

$f_\tau = g_m / \left(2\pi\left(C_\pi + C_\mu\right)\right)$

$g_m = \alpha / r_e = I_c / V_T$

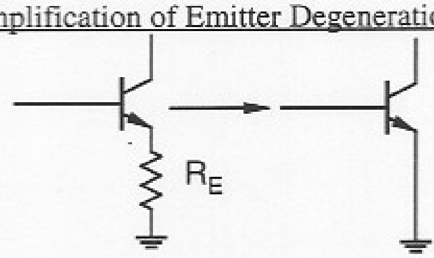
$R_\pi = (\beta + 1) r_e$

$R_o = V_A / I_C$



"T"-model
Makes common-base analysis much easier. Both these models are only approximate, being "good" up to f_t. Additionally $R_{b'b}$ is often an important parasitic (not discussed much in 137B.

Simplification of Emitter Degeneration



approximate only; check notes for bounds on validity

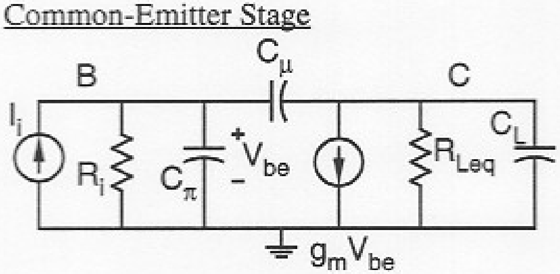
$\tilde{C}_\pi = C_\pi\left(r_e/\left(r_e + R_E\right)\right)$

$\tilde{g}_m = g_m\left(r_e/\left(r_e + R_E\right)\right) = \alpha/\left(r_e + R_E\right)$

$\tilde{C}_\mu = C_\mu$

$\tilde{R}_\pi = R_\pi\left(\left(r_e + R_E\right)/r_e\right) = (\beta + 1)\left(r_e + R_E\right)$

Common-Emitter Stage



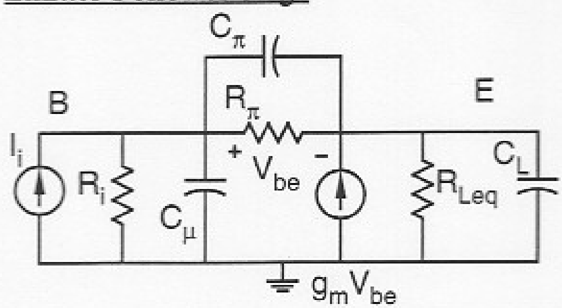
$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2}$$

$$a_1 = R_i\left(C_\pi + C_\mu\left(1 + g_m R_{Leq}\right)\right) + R_{Leq}\left(C_\mu + C_L\right)$$

$$a_2 = R_i R_{Leq}\left(C_\mu C_L + C_\mu C_\pi + C_\pi C_L\right)$$

$$\tau_{zero} = -C_\mu / g_m$$

Emitter-Follower Stage



$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1+s\tau_{zero}}{1+a_1 s + a_2 s^2}$$

given that $A_{vmb} = \left(r_e/\left(r_e + R_{Leq}\right)\right)$:

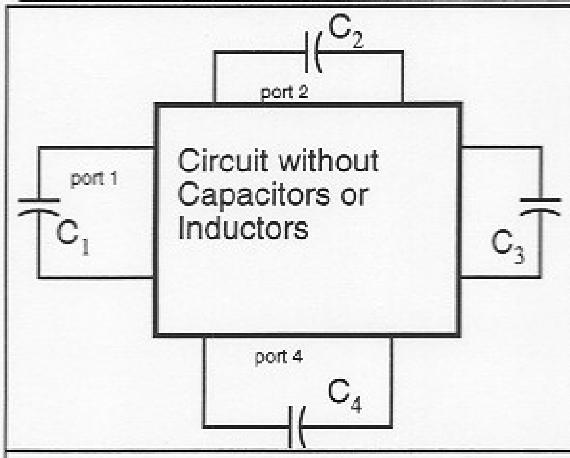
$$a_1 = C_\pi\left(R_\pi \left\| \left(r_e \| R_{Leq} + R_i\left(1-A_{vmb}\right)\right)\right.\right)$$
$$+C_\mu\left(R_i \| \text{transistor input resistance}\right)$$
$$+C_L\left(R_{Leq} \| \text{transistor output resistance}\right)$$
$$a_2 = \left(R_i \| \text{transistor input resistance}\right)\left(R_{Leq} \| r_e\right)$$
$$\times\left(C_\mu C_\pi + C_\mu C_L + C_L C_\pi\right)$$
$$\tau_{zero} = g_m / C_\pi$$

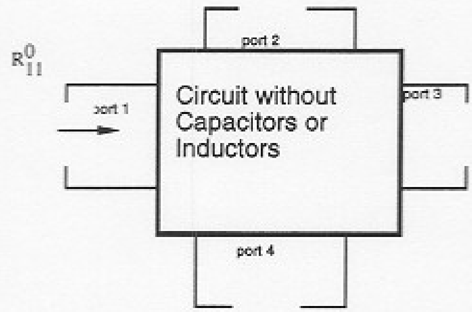
General Solutions of Problems: Nodal analysis (Know how to do this!)

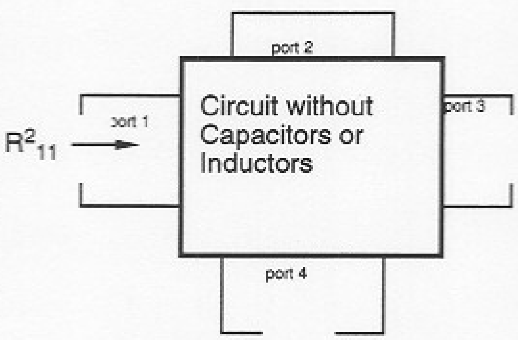
1)Write the nodal equations (sum of the currents=0) at each circuit node, and put the resulting equations in matrix form (the Y's being various combinations of g_m's, 1/R's, and sC's): $\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 = V_{in} \\ V_2 \\ V_3 \\ V_4 = V_{out} \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}$	2) Use Cramer's rule to solve: $\dfrac{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & I_{in} \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ Y_{31} & Y_{32} & Y_{33} & 0 \\ Y_{41} & Y_{42} & Y_{43} & 0 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix}} = V_{out}$
3) This comes out as: $\dfrac{V_{out}}{I_{in}} = s^m \dfrac{c_0 + c_1 s + c_2 s^2 + \cdots}{d_0 + d_1 s + d_2 s^2 + \cdots}$,which is divided through to get: (if present, m is the number of zeros, minus the number of poles, in the transfer function)	4) $\dfrac{V_{out}}{I_{in}} = \left(\dfrac{V_{out}}{I_{in}}\right)_{mb} s^m \dfrac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$ ...and the poles and zeroes are found by factoring the numerator and denominator. The separated-pole approximation, if applicable, makes this factoring easy.
5) To find the *impulse response*, do a partial-fraction expansion and then take the inverse LaPlace transform.	6) To find the sinusiodal frequency response, set $s = j\omega$.

7

General Solutions of Problems: Method of Time Constants





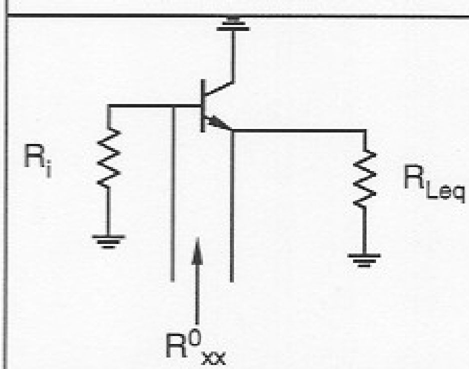


$$\frac{V_{out}}{V_{gen}} = \left(\frac{V_{out}}{V_{gen}}\right)_{mb} \frac{1+b_1 s + b_2 s^2 + \cdots}{1+a_1 s + a_2 s^2 + \cdots}$$

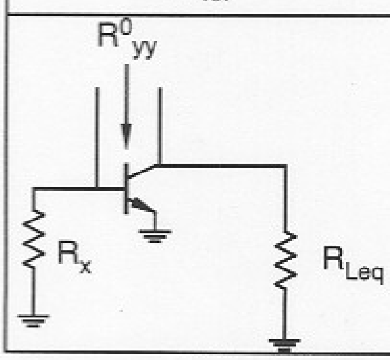
$$a_1 = R^0_{11}C_1 + R^0_{22}C_2 + R^0_{33}C_3 + R^0_{44}C_4$$

$$a_2 = R^0_{11}R^1_{22}C_1C_2 + R^0_{11}R^1_{33}C_1C_3 + R^0_{11}R^1_{44}C_1C_4 + R^0_{22}R^2_{33}C_2C_3 + R^0_{22}R^2_{44}C_2C_4 + R^0_{33}R^3_{44}C_3C_4$$

notethat $R^0_{xx}R^x_{yy} = R^y_{xx}R^0_{yy}$



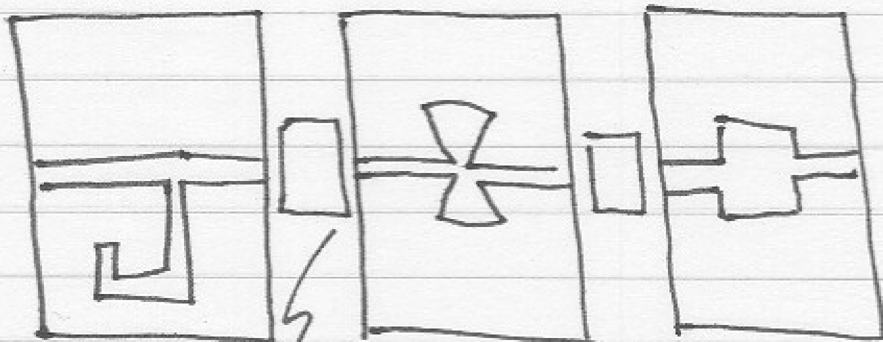
$$R^0_{xx} = R_\pi \| \left(r_e \| R_{Leq} + R_i\left(1 - A_{vmb}\right)\right)$$
$$A_{vmb} = \left(R_{Leq} / \left(r_e + R_{Leq}\right)\right)$$



$$R_i = R_x \| R_\pi$$
$$R^0_{yy} = R_i\left(1 + g_m R_{Leq}\right) + R_i$$

This brings me to my second major point:

Classical Microwave design looks like this.



transistor die

Microwave matching networks on alumina.

Classical Microwave smith-chart design works very well for this.

But industry is moving increasingly to IC's

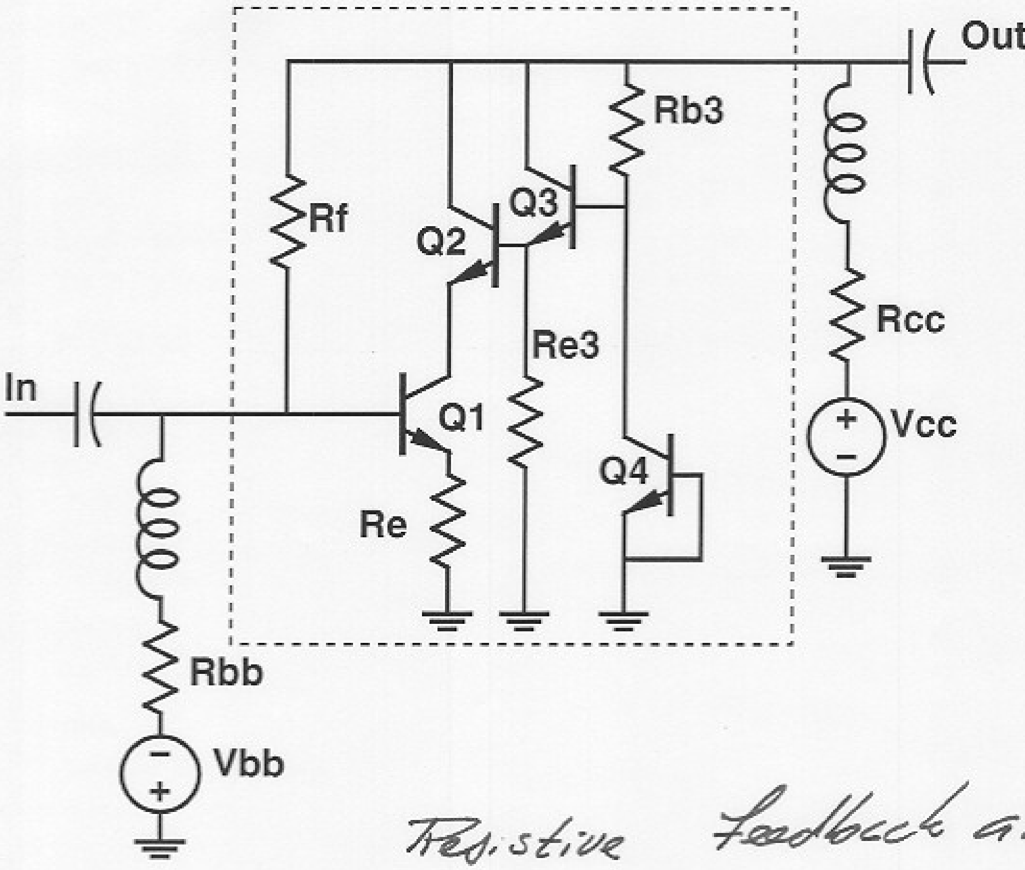
- some look just like the prior example.

- But many look like the examples attached on the next 2 pages.

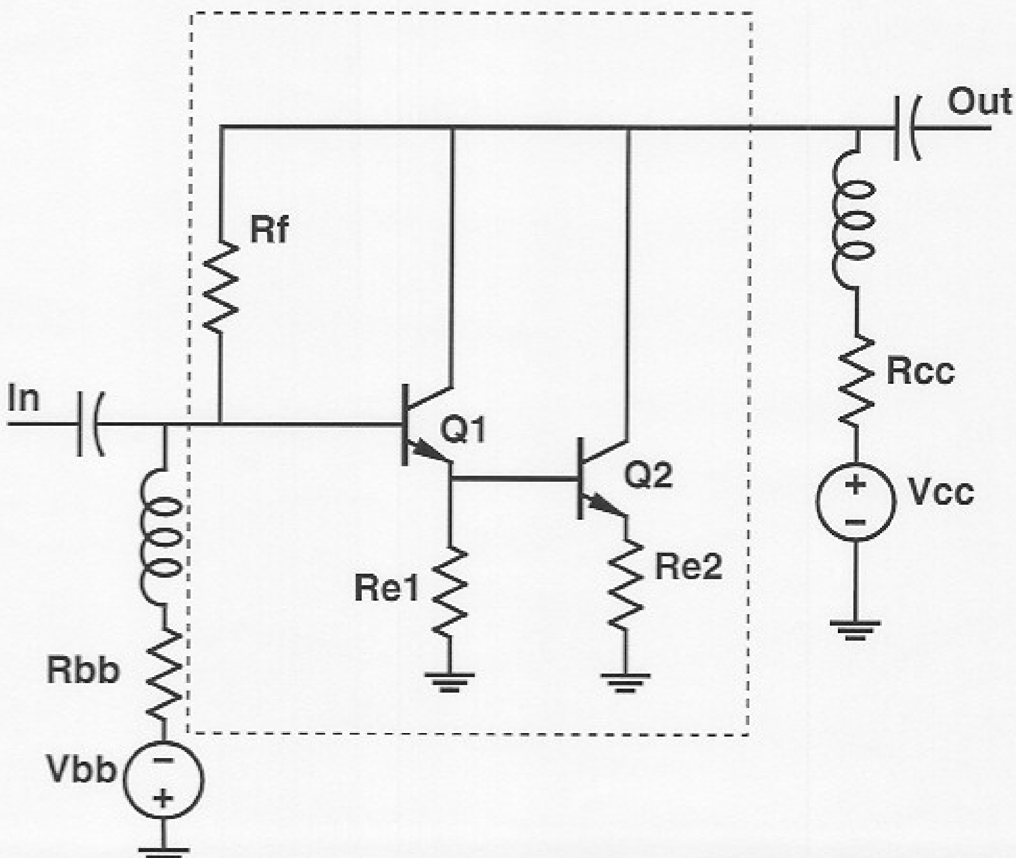
For these, we will need a mixture of classical microwave design and analog IC design methods. Both are important.

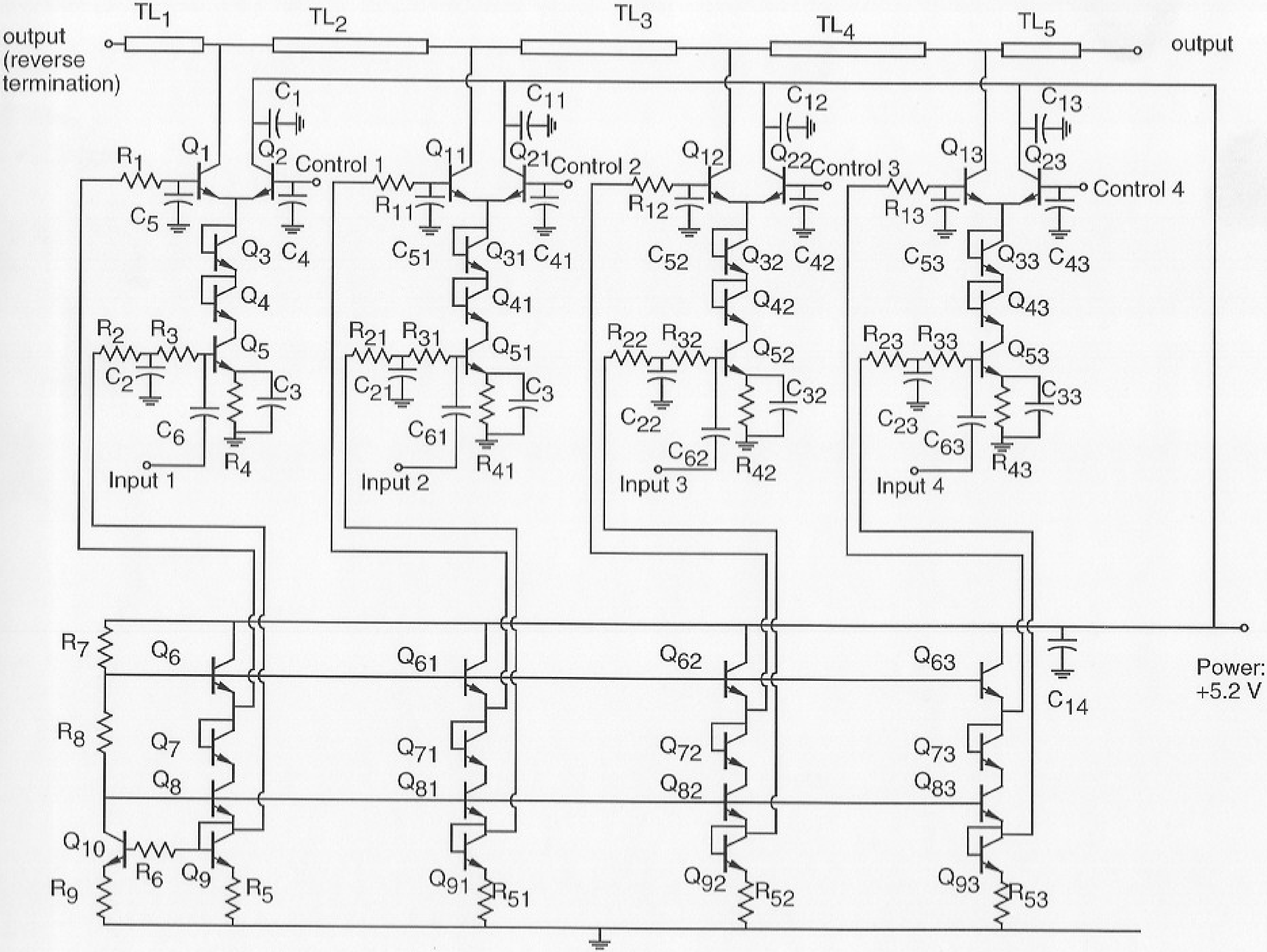
An excellent reference for "analog IC design" is

Grey and Meyer "Analog Integrated Circuits"



Resistive feedback amplifiers ...





Microwave Active Switch.

Many of these circuit can and do operate over bandwidths as high as 30 GHz

→ "Analog Circuit design" techniques are perfectly applicable at microwave frequencies!

Methods for analog design:

* Nodal Analysis

* Method of time constants.

Lets review these below.

Please refer <u>extensively</u> to the Cr.6 sheets on pp. <u>2-7</u>.

Nodal Analysis

General Solutions of Problems: Nodal analysis (Know how to do this!)

1)Write the nodal equations (sum of the currents=0) at each circuit node, and put the resulting equations in matrix form (the Y's being various combinations of g_m's, 1/R's, and sC's): $\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 = V_{in} \\ V_2 \\ V_3 \\ V_4 = V_{out} \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}$	2) Use Cramer's rule to solve: $\dfrac{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & I_{in} \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ Y_{31} & Y_{32} & Y_{33} & 0 \\ Y_{41} & Y_{42} & Y_{43} & 0 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix}} = V_{out}$
3) This comes out as: $\dfrac{V_{out}}{I_{in}} = ks^m \dfrac{c_0 + c_1 s + c_2 s^2 + \cdots}{d_0 + d_1 s + d_2 s^2 + \cdots}$,which is divided through to get: (if present, m is the number of zeros, minus the number of poles, in the transfer function)	4) $\dfrac{V_{out}}{I_{in}} = \left(\dfrac{V_{out}}{I_{in}}\right)_{mb} s^m \dfrac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$ ...and the poles and zeroes are found by factoring the numerator and denominator. The separated-pole approximation, if applicable, makes this factoring easy.
5) To find the *impulse response*, do a partial-fraction expansion and then take the inverse LaPlace transform.	6) To find the sinusiodal frequency response, set $s = j\omega$.

lets give an example of this...

Before we do, what is

"the separated-pole approximation"?

If we have $H(s) = \dfrac{N(s)}{1 + a_1 s + a_2 s^2}$

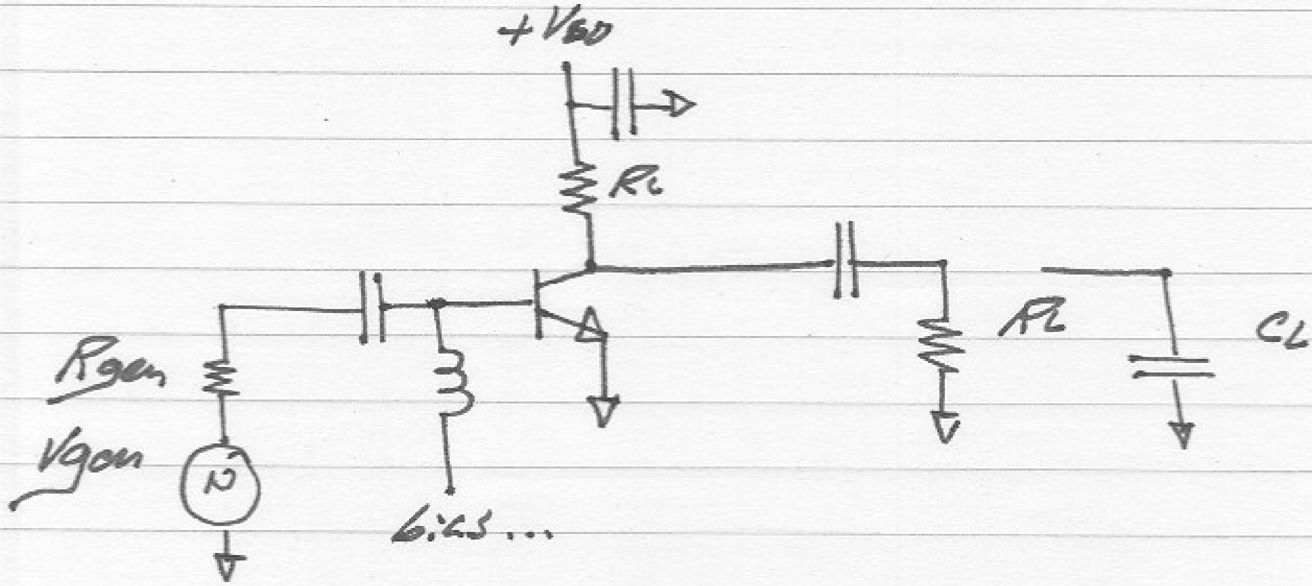
$$H(s) \cong N(s) \cdot \frac{1}{1+a_1 s} \cdot \frac{1}{1+ a_2 s/a_1}$$

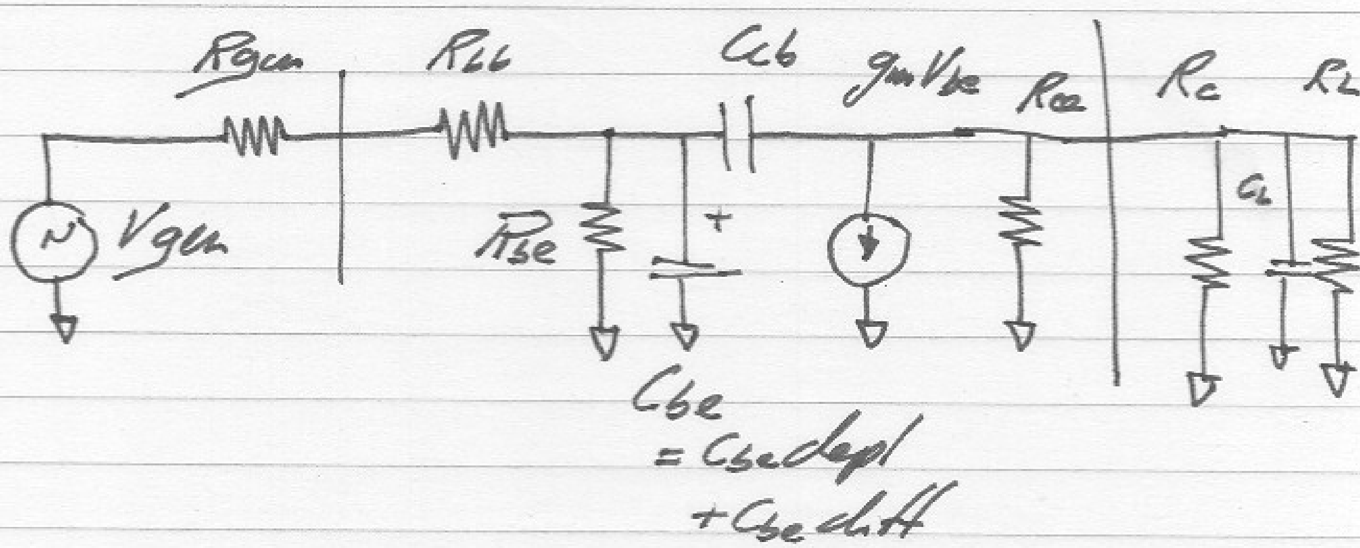
$$= N(s) \frac{1}{1 + (a_1 + \frac{a_2}{a_1}) s + a_2 s^2}$$

If $a_1 >> a_2/a_1$

This a valuable method of finding the roots (poles) of a transfer function without resorting to the quadratic equation...

Example: Common-Emitter stage:





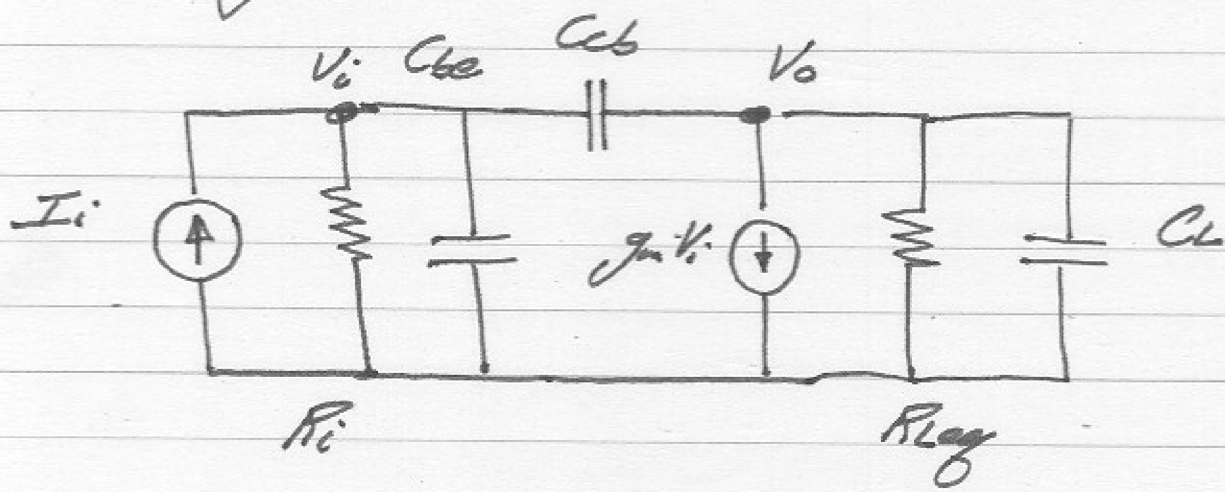
note the simplicity of the models assumed.

I will convert V_{gen}, R_{bb} & R_{gen} with a Thevenin - Norton Transformation.

and write $R_i = R_{be} \| (R_{bb} + R_{gen})$

$$R_{Leq} = R_{ce} \| R_L \| R_c$$

$$I_i = V_{gen} / (R_{bb} + R_{gen})$$



So now we write $\Sigma I = 0$ at each node where the voltage is unknown (V_i & V_o)

$$\begin{cases} V_i\left(\frac{1}{R_i} + sC_{be} + sC_{cb}\right) + V_o(-sC_{cb}) = I_i \\ V_i(-sC_{cb} + g_m) + V_o\left(\frac{1}{R_{eq}} + sC_{cb} + sC_L\right) = 0 \end{cases}$$

we then in general write this in matrix form to solve...

$$\begin{bmatrix} I_i \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_i} + sC_{be} + sC_{cb}\right) & -sC_{cb} \\ (-sC_{cb} + g_m) & \left(\frac{1}{R_{eq}}\right) + sC_{cb} + sC_L \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix}$$

and use cramers-rule to solve:

$$\Rightarrow \frac{V_{out}}{V_{gen}} = \left.\frac{V_{out}}{V_{gen}}\right|_{MB} \quad \frac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2}$$

$$a_1 = R_i\left(C_{be} + C_{cb}(1 + g_m R_{Leq})\right) + R_{Leq}(C_{cb} + C_L)$$

$$a_2 = R_i R_{Leq}\left(C_{cb} C_{be} + C_{cb} C_L + C_{be} C_L\right)$$

$$\tau_{zero} = -C_{cb}/g_m.$$

note that I have glossed over ~ 2 pages of algebra to obtain this.

We now come to use the separated pole approximation:

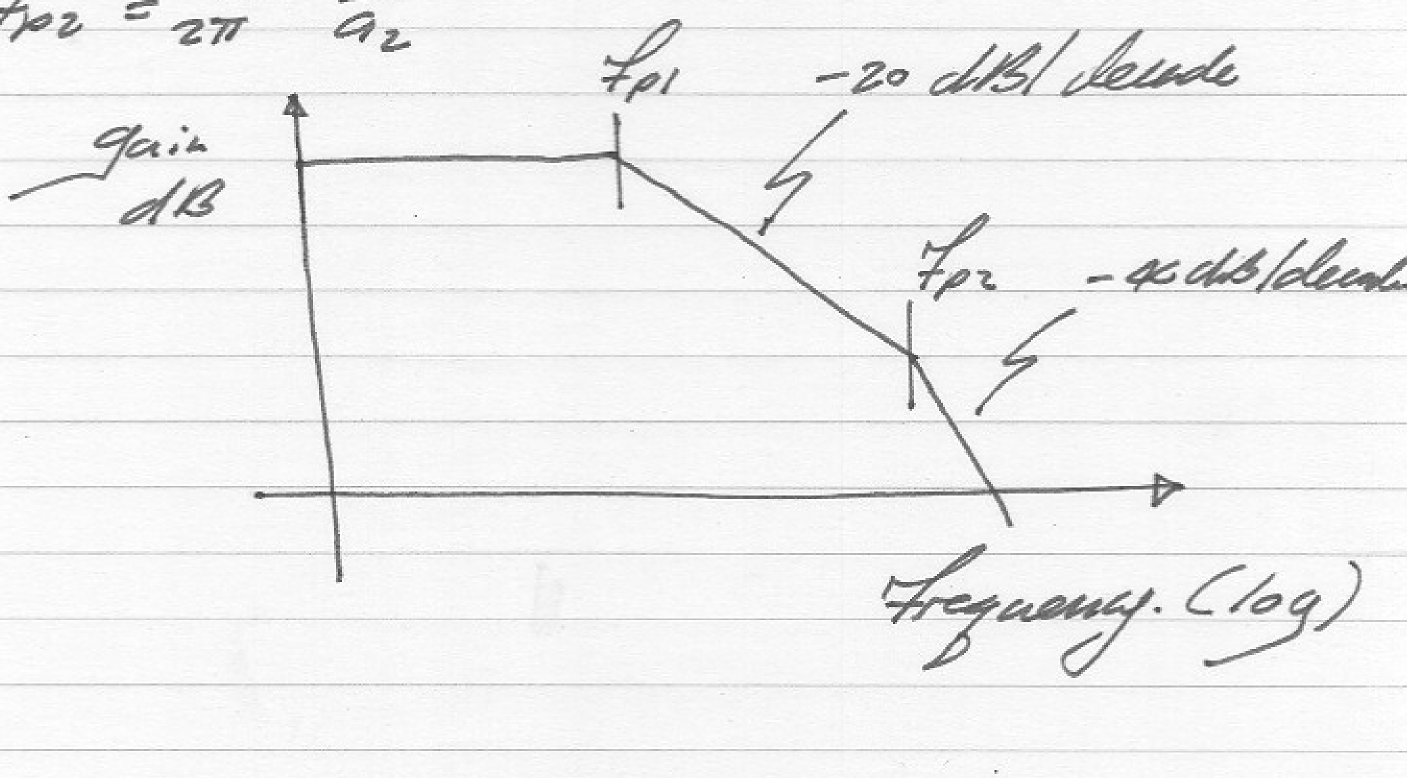
$$\frac{V_{out}}{V_{gen}} \cong \left.\frac{V_{out}}{V_{gen}}\right|_{MB} \frac{1 + s\,\tau_{zero}}{(1 + a_1 s)\left(1 + \frac{a_2}{a_1} s\right)}$$

which gives pole frequencies of

$$f_{p1} = \frac{1}{2\pi a_1}$$

and

$$f_{p2} = \frac{1}{2\pi}\frac{a_1}{a_2}$$



Look at the answer

$$f_{p1} = \frac{1}{2\pi a_1}$$

$$a_1 = \underbrace{R_i\,(C_{be} + C_{cb}(1+g_m R_{eq}))}_{\substack{\text{Input RC time constant} \\ \tau_{in}}} + \underbrace{R_{eq}(C_{cb}+C_L)}_{\substack{\text{output RC} \\ \text{time constant} \\ \tau_{out}}}$$

$$\tau_{in} = R_i \, C_{equivalent}$$

where

$$C_{equivalent} = C_{be} + C_{cb}\,(\,\underbrace{1 + g_m R_{eq}}_{1 - A_v})$$

this is

the familiar Miller Approximation.

<u>Ok</u> so what have we accomplished:

- obtained gain-bandwidth relationships for an elementary C.E. stage.

  - I hope this was just a reminder

- Reviewed <u>How</u> to generally solve frequency response problems

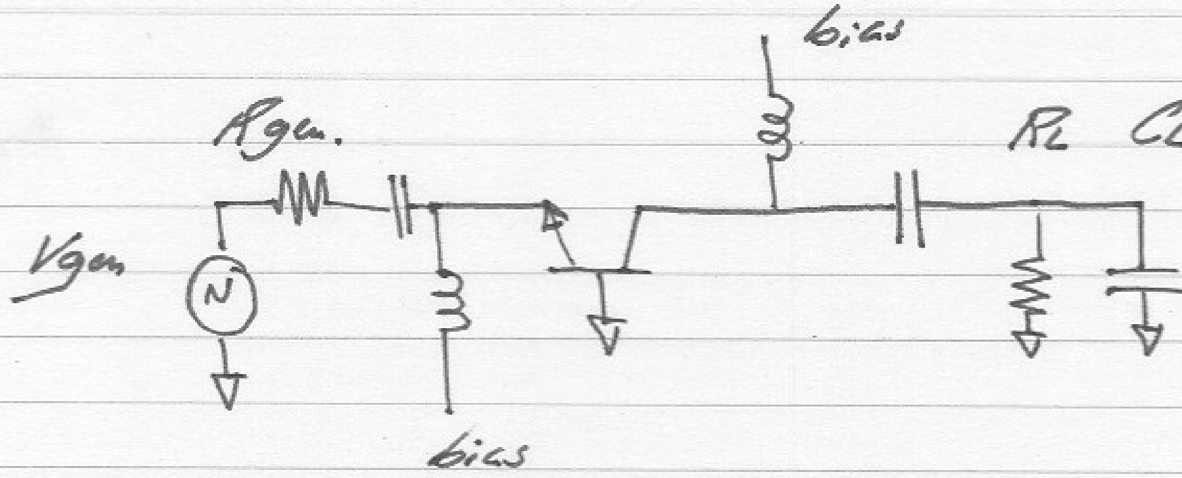
  ↯

This is the message. Although a simple procedure, one has to do this <u>all</u> <u>the</u> <u>time</u> in evaluating complex IC's.

Next Example

Simplified Common-base stage

You are warned that here we ignore R_{bb} which is a very dangerous assumption.

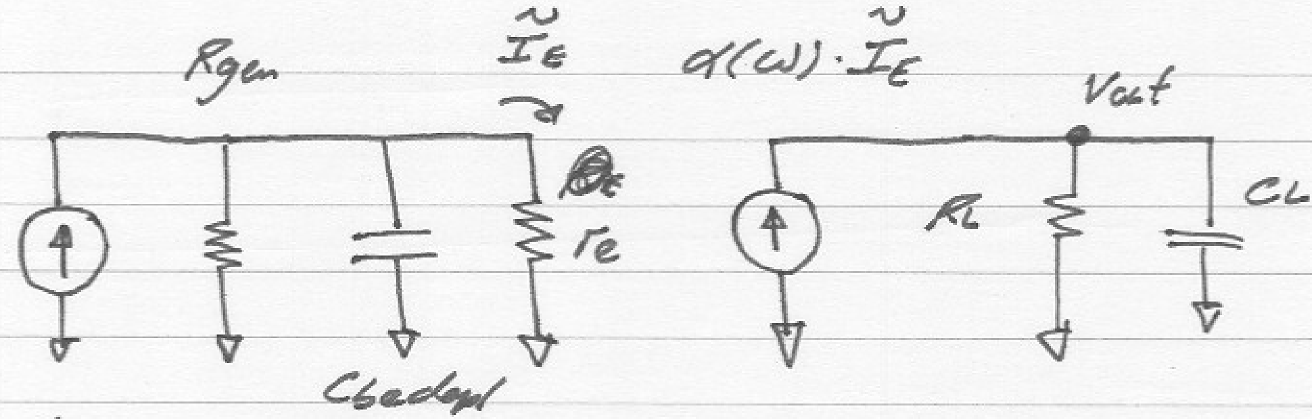


We will use the T-model

Simplify: $R_{bb} \to 0$ dangerous!

$R_{ex} \to 0$ (easy to fix)

$R_{CE} \to +\infty$ reasonable.



I_i
$= V_{gen}/R_{gen}$

by inspection:

$$\frac{V_{out}}{V_{gen}} \cong \left.\frac{V_{out}}{V_{gen}}\right|_{mbs} \cdot \frac{1}{1 + s\, C_{badepl}\, (r_e \| R_{gen})}$$

$$\cdot \frac{1}{1 + j\omega(\tau_b + \tau_c)}$$

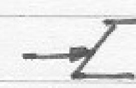
$$\cdot \frac{1}{1 + j\omega R_L C_L}$$

... The third term is usually dominant again, one is warned of the danger of ignoring $R_{bb'}$.

What about the emitter follower stage?

— We will find this hard: lets learn the method of time constants first!

What about Field-Effect Transistors?

The models of  and  are basically similar — only the notation changes ($C_{be} \rightarrow C_{gs}$ etc)