ECE ECE145B (undergrad) and ECE218B (graduate)
Final Exam. March 22, 2013

Do not open exam until instructed to.
Open notes, open books, etc
You have 3 hrs.

Use all reasonable approximations (5% accuracy is fine.) , ***AFTER STATING THEM.***
<u>Hint: Stop and think before doing complicated calculations.</u> ***For some problems, there is an easier way.***

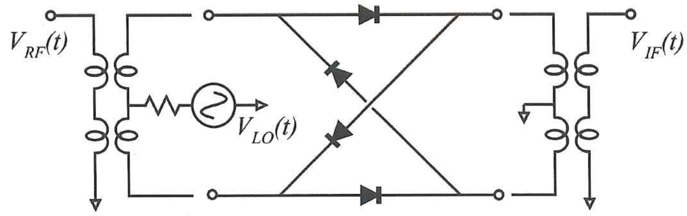
Problem	Points Received	Points Possible
1a		10
1b		10
1c		10
2a		5
2b		10
2c		10
2d		10
3a		10
3b		5
4a		5
4b		5
4c		5
4b		5

Name: Solution

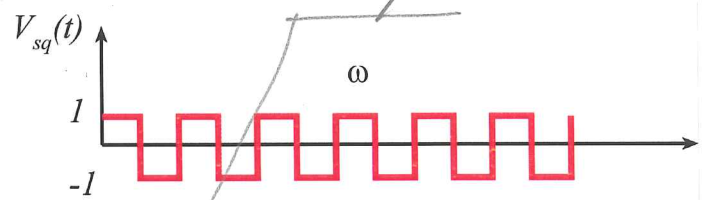
Problem 1, 30 points
mixers and frequency conversion.

part a, 10 points
In the mixer circuit to the right, we simplify analysis by assuming a zero-Ohm generator impedance for $V_{RF}(t)$ and $V_{LO}(t)$, an infinite-impedance load on the IF port, and a very large LO drive amplitude, such that the diodes are driven into zero Ohms impedance on forward bias and infinity Ohms on reverse bias.

Hint: the Fourier series of a squarewave is given to the right.



sorry!



$$V_{sq}(t) = \frac{4V}{\pi}\left[\cos(\omega t) + \frac{\cos(3\omega t)}{3} + \frac{\cos(5\omega t)}{5} + ...\right]$$

If $V_{RF}(t)$ is a 1 mV RMS amplitude ~~square~~ sine wave at 1.1 GHz, and $V_{LO}(t)$ is a 1.0 GHz sinewave, $V_{IF}(t)$ will have a series of Fourier components. What frequencies do the strongest four of these lie at ? What are their RMS amplitudes ?

1 frequency = 100 MHz RMS amplitude = $1mV \cdot \frac{2}{\pi} = 0.64mV$ 1
1 frequency = 2.1 GHz RMS amplitude = $1mV \cdot 2/\pi = 0.64mV$ 1
1 frequency = 4.1 GHz RMS amplitude = $1mV\ 2/3\pi = 0.21\ mV$ 2
1 frequency = 1.9 GHz RMS amplitude = 0.21 mV 2

$$V_{RF}(t) = V_{RF}\cos(\omega_{rf}t)\cdot\left(\frac{4}{\pi}\cos\omega_{LO}t + \frac{4}{3\pi}\cos 3\omega_{LO}t\right)$$

$$V_{IF}(t) = \frac{V_{RF}\cdot 4}{\pi}\,\frac{1}{2}\left(\cos((\omega_{rf}+\omega_{LO})t)\right) + \frac{V_{RF}\cdot 4}{\pi}\cdot\frac{1}{2}\cos((\omega_{RF}-\omega_{LO})t)$$

$$+\frac{V_{RF}\,4}{3\pi}\,\frac{1}{2}\cos((\omega_{RF}+3\omega_{LO})t) + \frac{V_{RF}\,4}{3\pi}\,\frac{1}{2}\cos((\omega_{RF}-3\omega_{LO})t)$$

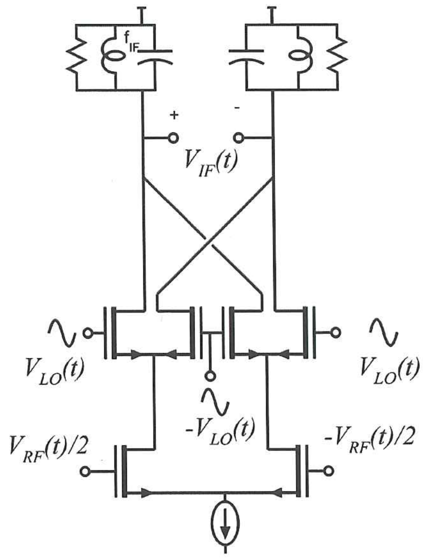
$\omega_{RF} \pm \omega_{LO} \rightarrow$ 100 MHz, 2.1 GHz

$\omega_{RF} \pm 3\omega_{LO} \rightarrow$ 4.1 GHz, 1.9 GHz

part b, 10 points

The two FETs each have 2 mS transconductance. Each IF LC filter is loaded externally with 1 kOhm resistance. The IF is tuned at 1 GHz, the LO is at 20 GHz, and the RF is at 61 GHz.

Treat the mixer quad as a set of ideal switches. If the mixer differential input voltage V_{RF} is 1 mV R.M.S., find the RMS IF output voltage



IF RMS output voltage = 0.42 mV.

this repeats the previous problem.

2 $V_{IF}(t) = 2k\Omega(1mS) \cdot \frac{4}{3\pi \cdot 2} \cos(\omega_{IF} \cdot t) \cdot V_{RF,peak}$

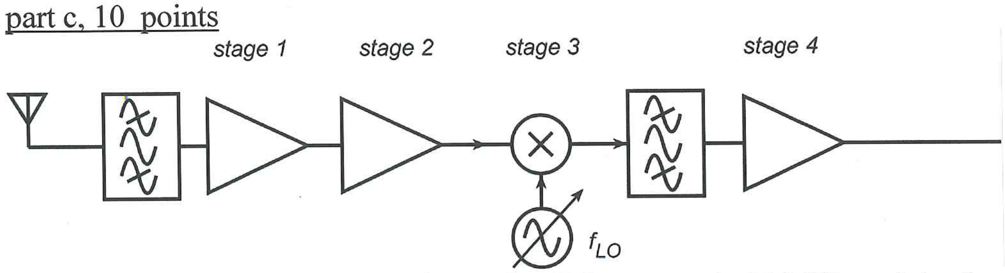
2 factor of 2 from 3rd harmonic — factor of 2 from $\cos\omega_1 \cos\omega_2$ 2

mixer gain $= 2k\Omega \; 1mS \cdot \frac{4}{3\pi \cdot 2}$

$= 2 \cdot \frac{4}{6\pi} = \frac{4}{3\pi}$ 2

$= 0.42$ 2

part c, 10 points



Given the radio block diagram above, the IF frequency is 10 MHz and the (tunable) LO frequency is placed *below* the input RF frequency. This is a very low-data-rate receiver, hence the IF filter is very narrow, only +/- 10 kHz bandwidth. The RF preselect filter is not tunable. If the receiver's RF tuning range has a lower limit of 500 MHz, what it the highest frequency to which the receiver can be tuned if image responses are to be avoided ?

Upper limit of frequency tuning= 520 MHz.

look @ low end of tuning range

[RF = 500 MHz, LO = 490 MHz. 1 IF = 10MHz 1

an RF signal of ~~51~~ 480 MHz would ALSO mix to 10MHz.

Now consider an RF signal @ 520 MHz 2 → requires a 510MHz LO. 2

there is an image response 2 @ ~~490~~ 500 MHz. in this case.

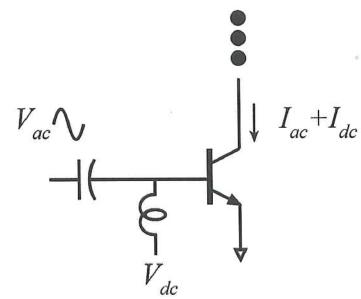
this indicates a maximum preselect bandwidth of 20 MHz 2
(500-520 MHz), and an equal maximum tuning range.

If we consider also the IF bandwidth, the tuning range is 20 kHz smaller, i.e. 19.98 MHz tuning range

Both answers are acceptable

Problem 2, 35 points
distortion and intermodulation.

part a, 5 points
computing nonlinearities
The bipolar transistor amplifier has collector current
$I_c = I_s \exp(qV_{be}/kT)$, from which we find
$I_{c,ac} = I_{c,dc}\exp(qV_{ac}/kT)$



If we write a Taylor series for $I_{c,ac}$, i.e.

$I_{c,ac} = a_1V_{ac}^1 + a_2V_{ac}^2 + a_3V_{ac}^3 + ...$,

find a_1, a_2, and a_3, being careful to give the correct units.

$a_1 =$ I_{cdc}/V_T $a_2 =$ $(I_{dc}/2)/V_T^2$, $a_3 =$ $(I_{dc}/6)\,(1/V_T)^3$ where $V_T = \frac{kT}{q}$

$$I_{cac} = I_{cdc}\cdot \exp(qV_a/kT)$$

but $e^x \cong 1 + x + x^2/2 + x^3/6 + \ldots$ 2

$$= I_{cdc} + \underbrace{I_{cdc}\cdot\frac{q}{kT}}_{\substack{a_1\\ A/V}} V_{ac} + \underbrace{\frac{I_{cdc}}{2}\left(\frac{q}{kT}\right)^2}_{\substack{a_2\\ A/V^2}} V_{ac}^2 + \underbrace{\frac{I_{cdc}}{6}\left(\frac{q}{kT}\right)^3}_{\substack{a_3\\ A/V^3}} V_{ac}^3$$

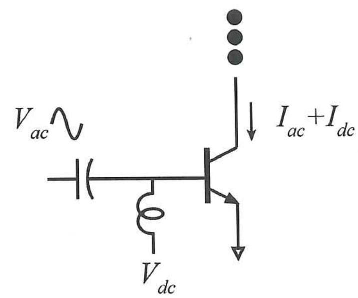
1 1 1

we use notation $V_T = \frac{kT}{q}$ for brevity.

part b, 10 points

computing Fourier components.

Continuing with the circuit of part (a), if we now have $V_{ac} = V_0 \cos(\omega_1 t) + V_0 \cos(\omega_2 t)$, compute the magnitude of the Fourier components of $I_{ac}(t)$ at ω_1, at ω_2, at $(2\omega_1 - \omega_2)$, and at $(2\omega_2 - \omega_1)$. Hint: In order to avoid errors in part (a) passing through to part b, give your answers in terms of a_1, a_2, a_3, and V_0



amplitudes at

ω_1: $a_1 V_0 + a_2 V_0^3 (1/2)$ ω_2: $a_1 V_0 + a_3 V_0^3 \cdot 1/2$,

$(2\omega_1 - \omega_2)$: $V_0^3 a_3 / 4$ $(2\omega_2 - \omega_1)$: ______,

a_2 term gives no responses at the indicated frequencies.

a_1 term: $V_0 \cdot \underbrace{\frac{I_c}{V_T}}_{a_1} [\cos\omega_1 t + \cos\omega_2 t]$ /

a_3 term

2

$$= a_3 V_0^3 [\cos\omega_1 t + \cos\omega_2 t]^3$$

$$= a_3 V_0^3 [(\cos\omega_1 t)^3 + 3\cos^2\omega_1 t \cos\omega_2 t + 3\cos\omega_1 t \cos^2\omega_2 t + \cos^3\omega_2 t]$$

$$(\cos\omega_1 t)^3 = (z_1 + z_1^{-1})^3/8 = 1/8[z_1^3 + z_1 + z_1^{-1} + z_1^{-3}] = 1/4[\cos\omega_1 t + \cos 3\omega_1 t]$$

$$(\cos\omega_2 t)^3 = 1/4[\cos\omega_2 t + \cos 3\omega_2 t]$$

2

$$8\cos^2\omega_1 t \cos\omega_2 t = (z_1 + z_1^{-1})^2 (z_2 + z_2^{-1}) = (z_1^2 + 2 + z_1^{-2})(z_2 + z_2^{-1})$$

$$= z_1^2 z_2 + z_1^2 z_2^{-1} + 2z_2 + 2z_2^{-1} + z_1^{-2} z_2 + z_1^{-2} z_2^{-1}$$

$$= \underbrace{z_1^2 z_2^{-1} + z_1^{-2} z_2}_{2\omega_1 - \omega_2 \sim 2\cos((2\omega_1 - \omega_2)t)} + 2\underbrace{(z_2 + z_2^{-1})}_{\omega_2} + \text{terms @ other frequencies.}$$

2

terms @ ω_1: $V_0 a_1 + V_0^3 a_3 [\overbrace{1/4 + 1/4}^{1/2}]$

2

terms @ $2\omega_1 - \omega_2$: $\frac{V_0^3 a_3}{4} \cos((2\omega_1 - \omega_2)t)$

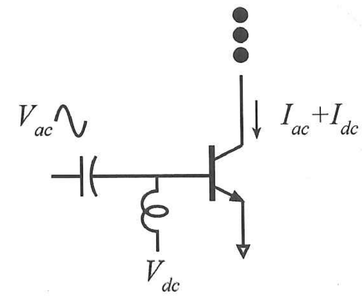
part c, 10 points

computing IP3.

Continuing with the problem of parts (a) and (b), what is the input-referred IP3 in units of Volts ?

First, give your answer in terms of a_1, a_2, a_3,

Then give your answer in terms of q, kT, $I_{c,dc}$



IIP3, written in terms of a_1, a_2, a_3: $2\sqrt{a_1/a_3}$

IIP3, written in terms of q, kT, $I_{c,dc}$: $2\cdot\sqrt{6}\cdot kT/q$

} peak quantities; RMS is $\frac{1}{\sqrt{2}}$ smaller

a_1 [illegible]:

amplitude @ ω_1, ω_2 : $a_1 V_0$] 2

amplitude @ $(2\omega_1-\omega_2), (2\omega_2-\omega_1)$: $a_3 V_0^3/4$] 2

— set these equal: $a_1 V_0 = a_3 V_0^3/4 \equiv$

2

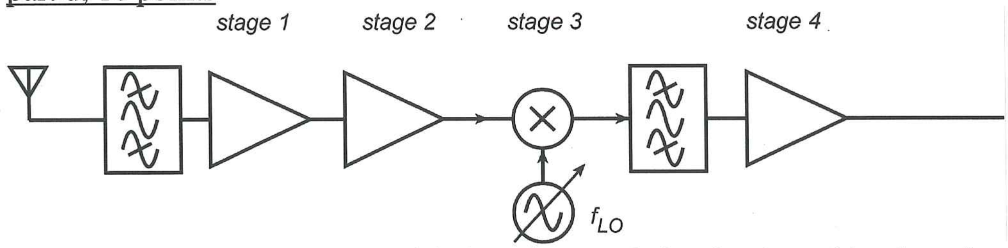
$$4a_1/a_3 = V_0^2$$

2 [$V_0 = 2\sqrt{a_1/a_3}$ peak voltage

but $a_1 = I_{c,dc}/V_T$, $a_3 = I_{c,dc}/6V_T^3$

so $V_0 = 2\sqrt{a_1/a_3} = 2\sqrt{6V_T^3/V_T}$

$= 2\sqrt{6}\cdot V_T = 2\sqrt{6}\cdot kT/q$] 2 part d, 10 points



The radio receiver has intermodulation characteristics dominated by that of stage 1, which has an input-referred third-order intercept of -10 dBm. You are attempting to receive a weak signal at 1 GHz, but there are strong incident received signals at 800MHz and 900MHz, both of which lie within the RF preselect filter bandwidth but outside the IF bandwidth. Ignoring any filter attenuation, if the two interfering signals are both -40 dBm amplitude, and we require a minimum 10 dB signal/interference ratio, what is the minimum received signal power at 1 GHz ?

power @ $(2\omega_1-\omega_2)$, $(2\omega_2-\omega_1)$ varies as cube of 1 input power; is equal to fundamental response when input = IIP_3. 1

the two signals lie 30 dB below IIP_3. 2

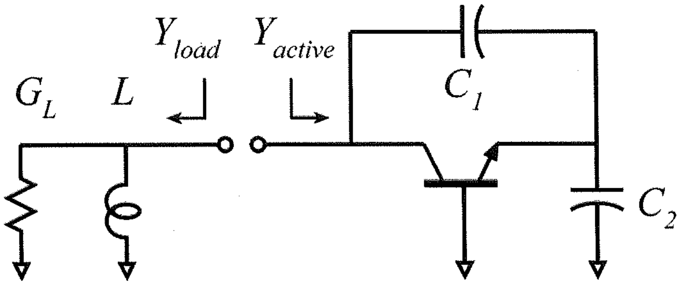
⇒ intermodulation responses are @ 90 dB below IP_3 2

⇒ intermodulation responses are @ −100 dBm 2

10 dB signal/interference → $\boxed{-90 \text{ dBm}}$ 2

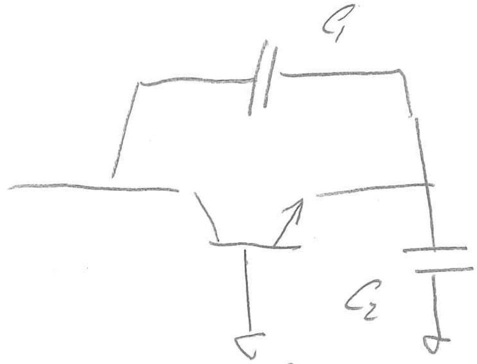
Problem 3, 15 points
oscillators

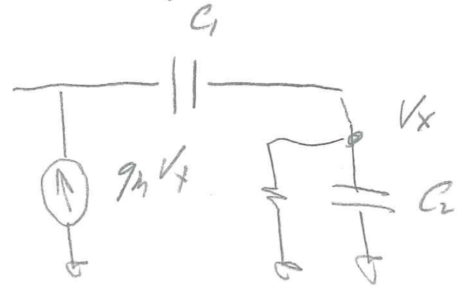
part a, 10 points
colpitts oscillator
The bipolar transistor is modeled with an idea common-base model (infinite beta, zero transit time, zero access resistances, zero depletion capacitances) and a transconductance of 10 mS.



You must design a 1.0 GHz oscillator.
Find the $C_1:C_2$ ratio which gives the most negative output conductance:
$C_1:C_2=$ 1, i.e. $C_1 = C_2$
Find the values of C_1 and C_2 such that $\text{Im}\{Y_{active}\}=100^*\,\text{Re}\{Y_{active}\}$ at 1.0 GHz
$C_1=$ 80pF $C_2=$ 80pF
Find the value of L required for 1.0 GHz oscillation.
L= 0.633 nH
Find the maximum allowable shunt load conductance G_L
G_L,max= 2.5 mS , hence R_L, min= 400Ω.
Now assume that the tuning inductor is no longer ideal. If G_L =0 S, what is the maximum allowable parasitic series resistance to the inductor L ?
Maximum parasitic series resistance = 0.04Ω

2)





we have specified that $\omega C_2 >> r_E = 1/g_m$

⇒ oscillator conductance

2 $G_{active} = -g_m \dfrac{C_1}{C_1+C_2} + g_m \left(\dfrac{C_1}{C_1+C_2}\right)^2 = -g_m \cdot x + g_m (x^2)$

$G_{active} = -g_m(x - x^2)$ $\quad \dfrac{1}{-g_m}\dfrac{\partial G}{\partial x} = 1 - 2x = 0 \Rightarrow x = 1/2$ 2

$\Rightarrow C_1 = C_2$ 1

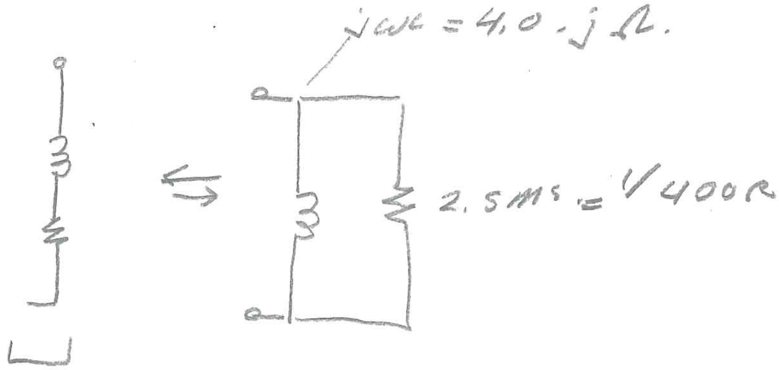
2 $G_{active} = -g_m(1/2) + g_m(1/4) = -g_m/4 = \underline{-2.5\ mS}$

So we need $\omega \dfrac{C_1}{2} = \omega \dfrac{C_2}{2} = 2.5\,mS \cdot 100 = 250\,mS$

2 $\Rightarrow C_1 = C_2 = 79.6\,pF$

$16\,MHz = \dfrac{1}{2\pi\sqrt{LC}}$ where $C = 40\,pF$

2 $\Rightarrow L = 0.633\,nH$



2 $$Y = \frac{1}{j\omega L + R} = \frac{j\omega L - R}{(j\omega L + R)(j\omega L - R)} = \frac{j\omega L - R}{-\omega^2 L^2 - R^2}$$

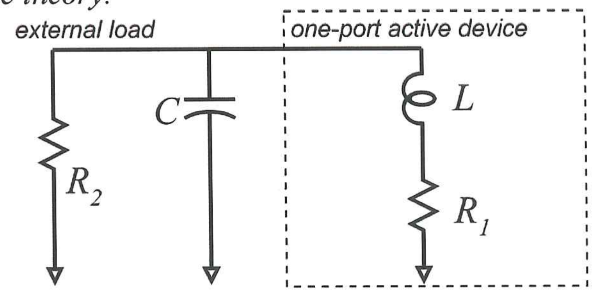
$$= \frac{R}{\omega^2 L^2 + R^2} - \frac{j\omega L}{\omega^2 L^2 + R^2} \cong \underbrace{\frac{R}{\omega^2 L^2}}_{G} + \frac{1}{j\omega L}$$

2 $$\Rightarrow R_{series} = G \cdot \omega^2 L^2 = \frac{(4\Omega)^2}{400\Omega} = \underline{\underline{0.04\Omega}}$$

part b, 5 points

small-signal negative resistance/conductance theory.

The one-port active device has $R_1 = -10$ Ohms and L=3.2 nH. Find the value of C necessary for a 10.0 GHz oscillator. Find the minimum load resistance R_2 if we want to ensure that the circuit will oscillate.



2' $$Y = \frac{R}{\omega^2 L^2 + R^2} - \frac{j\omega L}{\omega^2 L^2 + R^2}$$

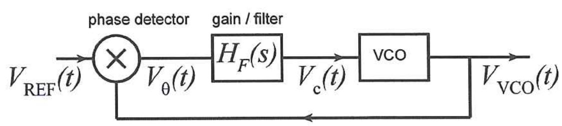
2 $$G = \frac{10\Omega}{(2\pi \cdot 10\,GHz \cdot 3.2\,nH)^2 + (10\Omega)^2} = \underline{0.247\,mS}$$

$$= [4{,}052\Omega]$$

1 $R_2 > 4.05\,k\Omega.$

Problem 4, 20 points
Phase-lock-loops :

part a, xx points
Loop Transfer function
In the PLL to the right, the mixer has
$V_\theta(t) = A_{mixer} \cdot V_{REF}\left(\sin(\Delta\omega \cdot t + \Delta\theta\right)$
with A_{mixer}=1.0 and



$V_{REF}(t) = V_{REF}\cos(\omega_{REF}t + \theta_{REF})$, with V_{REF} =100 mV . The VCO oscillates at 100 MHz, and a 1.0 Volt change in its control voltage changes its frequency by 1 MHz. The loop filter has $H_F(s) = (1 + s\tau_z)/s\tau_i$

part a, 5 points
Defining ω_x as the frequency at which the loop trasmission would be unity if the zero time constant were set to zero,find the necessary integrator time constant if we want $\omega_x / 2\pi$ to be 10 kHz.
τ_i = 0.16 mS

1/2 $\left[T(s) = K_\theta H_f(s) \cdot \frac{K_{vco}}{s} \right.$ where 1/2 $\left[K_\theta = V_{ref} \cdot A_{mixer} = 100\,mV \right.$

1 $\left[T(s) = \frac{100mV}{rad} \cdot \frac{1+s\tau_z}{s\tau_i} \cdot \frac{2\pi \cdot 10^6}{s} \frac{rad}{sec \cdot V} \right]$ $\left. K_{vco} = 2\pi \cdot 10^6 \frac{rad}{sec \cdot V} \right]$ 1/2

ignore τ_z & set $|T(j\omega_x)| = 1$

$1 = \frac{0.1V}{\omega_x \tau_i} \cdot \frac{2\pi \cdot 10^6}{\omega_x \cdot sec \cdot V} \rightarrow$ 1 $\left[\tau_i = \frac{0.1 \cdot 2\pi \cdot 10^6 \, sec^{-1}}{(2\pi \cdot 10^4 \, rad/sec)^2} \right.$

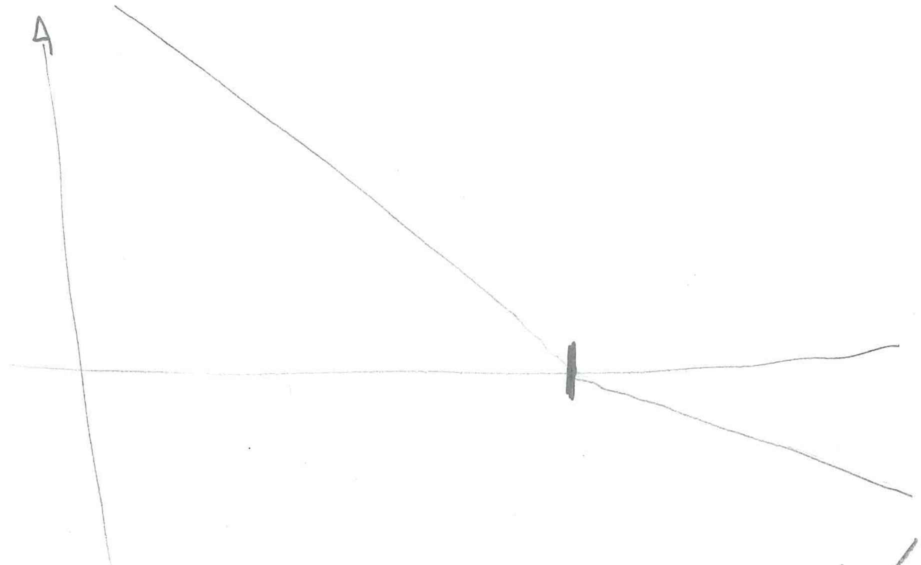
1/2 $\left[\tau_i = 0.159\,mS = 1.6 \cdot 10^{-4} \, sec \right.$

part b, 5 points

Now, pick a zero frequency (in Hz) equal to $\omega_x / 2\pi$. Now find the loop bandwidth and the loop phase margin.

loop bandwidth = 10 kHz

phase margin= 45°



2 although loop bandwidth is slightly larger if we compute exactly, bandwidth is approximately 10 KHz.

3 The zero gives us 45° phase margin.

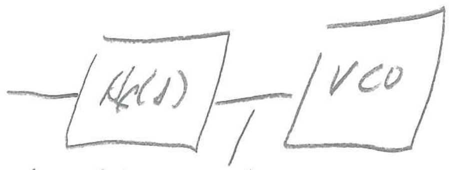
part c, 5 points

Find the maximum frequency scan rate. If the PLL is scanned in frequency at 1/10 its maximum rate, find the resulting phase error.

maximum frequency scan rate = ___625.___ (MHz / second)

phase error = ___0.1 radians or 5.74°___

1 maximum output of phase detector is 100mV (!)



$$\frac{1+s\tau_2}{s\tau_1} \qquad V_c$$

1 From this, $\frac{dV_c}{\partial t} = \frac{100mV}{\tau_1} = 625$ Volts/second.

$\tau_1 \leftarrow 0.16ms$

$K_{vco} = 1 MHz/Volt.$

1 $\Rightarrow \partial f/\partial t = 1\frac{MHz}{V} \cdot 625\frac{V}{sec} = 625$ MHz/second.

2 at 1/10 this rate, the output of the phase detector is reduced 10:1

$\rightarrow \sin(\Delta\theta) = 0.1 \Rightarrow \Delta\theta = \arcsin(0.1)$

$\cong 0.10067$ radians

$\cong 5.74°$

part d, 5 points

What is approximately the maximum frequency acquisition bandwitth ?

$\cong f_{loop,1}$ $\pm 10\,\text{kHz}$