ECE 145B / 218B, notes set 9: Oscillators, Part 1.

Mark Rodwell

University of California, Santa Barbara

rodwell@ece.ucsb.edu 805-893-3244, 805-893-3262 fax

Stability: LaPlace Transform / Eigenvalue Method

Physical system (circuits, etc.) in small - signal limit $\rightarrow$ transfer function.

$$\frac{V_{out}(s)}{V_{gen}(s)} \text{ or } \frac{b(s)}{a(s)} \text{ or } \frac{V_{in}(s)}{I_{in}(s)} \text{ etc} = H(s)$$

$$H(s) = c_1 \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + \ldots} = c_2 \frac{(s - s_{z1})(s - s_{z2})(s - s_{z3})\ldots}{(s - s_{p1})(s - s_{p2})(s - s_{p3})\ldots}$$

Impulse response :

$$h(t) = k_1 \exp(s_{p1}t) + k_2 \exp(s_{p2}t) + k_3 \exp(s_{p3}t) + \ldots$$

Poles are (generally) complex :

$$s_{pi} = \sigma_{pi} + j\omega_{pi}$$

If any poles lie in right half of s - plane (σ_{pi} positive)

then $k_i \exp(s_{pi}t)$ will grow without limit.

$\rightarrow$ unstable system.

Unstable system if any pole has positive real part

$s_{pi} = \sigma_{pi} + j\omega_{pi}$; negative σ_{pi}

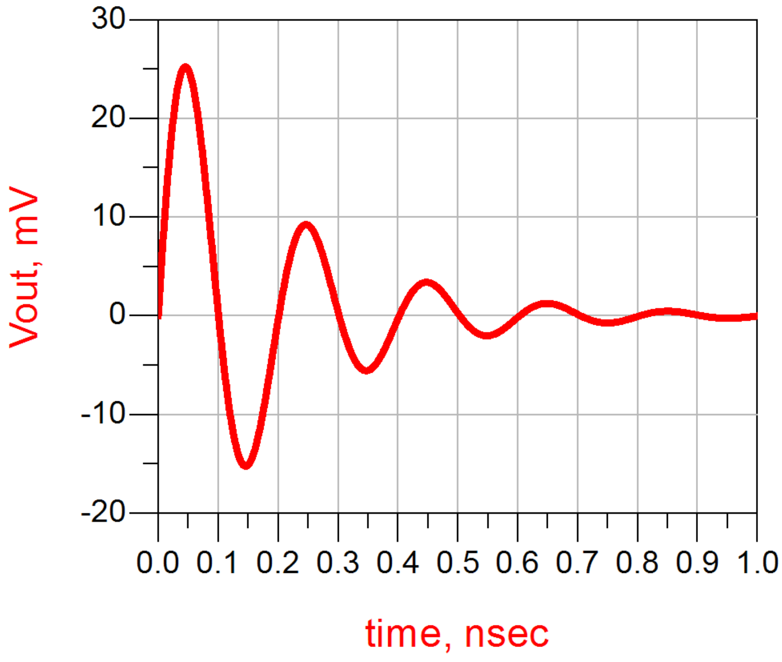
$\rightarrow \exp(s_{pi}t)$ decays.

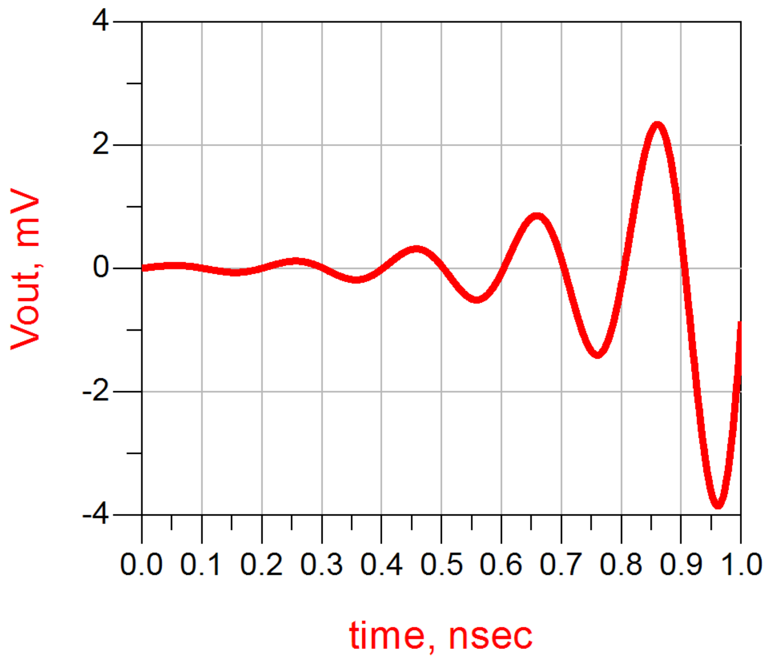
$\rightarrow$ stable system.

$s_{pi} = \sigma_{pi} + j\omega_{pi}$; positive σ_{pi}

$\rightarrow \exp(s_{pi}t)$ grows.

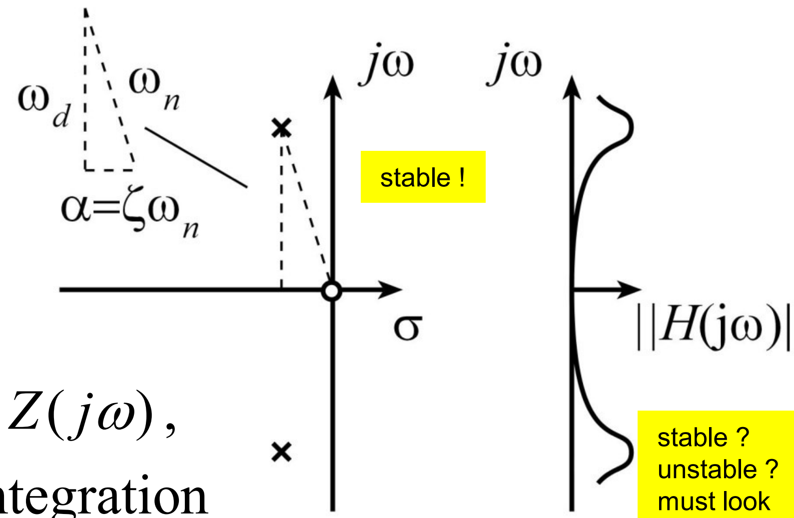
$\rightarrow$ unstable system.



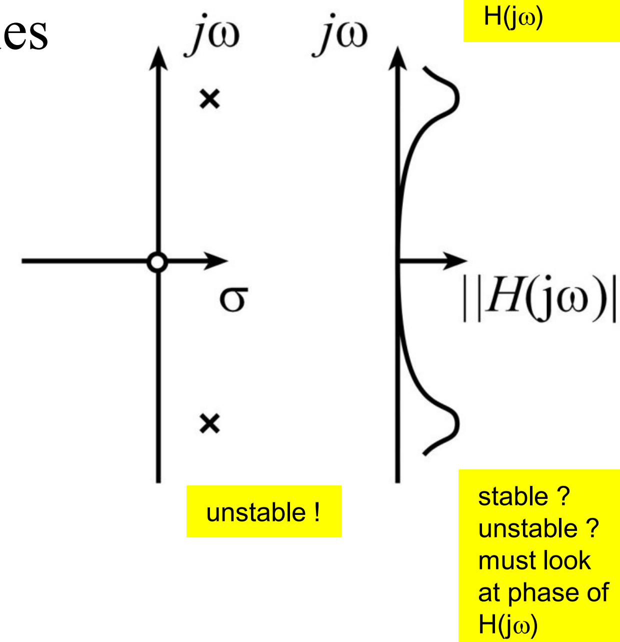


LaPlace vs. Fourier Analysis of Oscillators

In computing transfer functions $H(s)$ or impedances $Z(s)$, location of the pole frequencies in the RHP or LHP is clear.



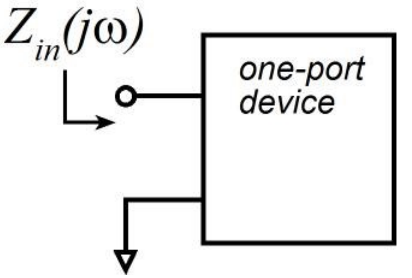
If we restrict ourselves to computing $H(j\omega)$ or $Z(j\omega)$, we must use Theorems derived from contour integration to be certain in which side of the complex plane the poles lie. This is studied in complex analysis.



When we speak of an impedance being composed of resistance and reactance, ie. $Z(j\omega)=R(j\omega)+jX(j\omega)$ we have already resricted ourselves to computing $Z(j\omega)$, not $Z(s)=Z(\sigma+j\omega)$

We must be careful.

Unconditional stability, Potential instability



From impedance viewpoint:

A one-port is unconditionally stable if :
$\mathrm{Re}\{Z_{in}(j\omega)\} > 0$ for all frequencies.

Alternatively, a one-port is unconditionally stable if :
$\mathrm{Re}\{Y_{in}(j\omega)\} > 0$ for all frequencies.

If $\mathrm{Re}\{Z_{in}(j\omega)\} < 0$ for some frequencies,
then the device is potentially unstable.

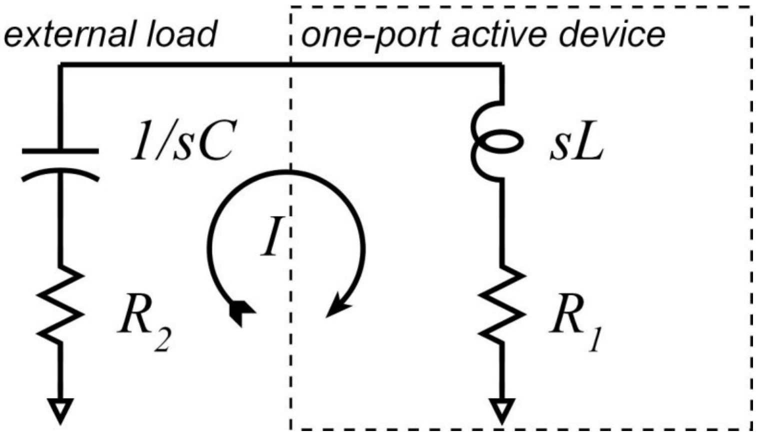
One-Port Oscillator Analysis: Series Impedance

$$I \cdot (sL + R_1 + R_2 + 1/sC) = 0$$

$$1 + sC(R_1 + R_2) + s^2 LC = 0$$

$$1 + (2\zeta / \omega_n)s + s^2 / \omega_n^2 = 0$$

Oscillates if $(R_1 + R_2) < 0$



If $R_1 < 0$, then oscillation requires

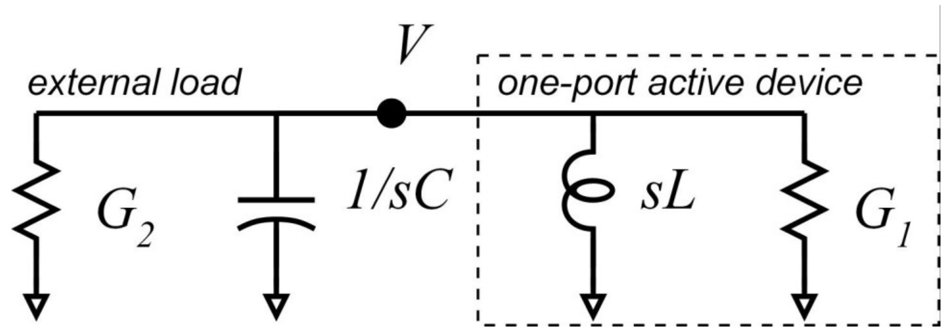
$R_2 < \| R_1 \|$, i.e. $G_2 > \| G_1 \|$

One-Port Oscillator Analysis: Parallel Admittance

$V \cdot (sC + G_1 + G_2 + 1/sL) = 0$

$1 + sL(G_1 + G_2) + s^2 LC = 0$

Oscillates if $(G_1 + G_2) < 0$



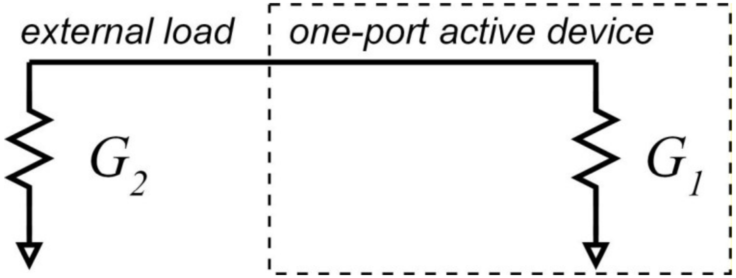
If $G_1 < 0$, then oscillation requires

$G_2 < \| G_1 \|$, i.e. $R_2 > \| R_1 \|$

Stability: A Paradox

Parallel: If $G_1 < 0$, then oscillation requires
$G_2 < \| G_1 \|$, i.e. $R_2 > \| R_1 \|$ (????)

Series: If $R_1 < 0$, then oscillation requires
$R_2 < \| R_1 \|$, i.e. $G_2 > \| G_1 \|$ (???)



Which ??? We cannot tell if the network is series or parallel.

Unless we include the reactive elements, there is no distinction between series and parallel networks.

Unless we include the reactive elements, the network has no variation with frequency, and we cannot calculate behavior with either * frequency * or with * time *.

It is meaningless to ask whether the above circuit is stable.

Impedance Point of View

Active device at frequency ω_0 :

$$Z_{\text{active}}(j\omega_0) = R_{active} + jX_{active}$$

Load at frequency ω_0 :

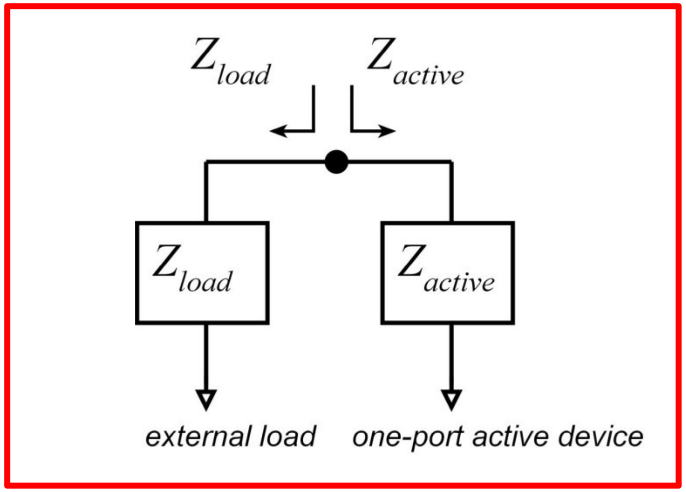
$$Z_{\text{load}}(j\omega_0) = R_{\text{load}} + jX_{\text{load}}$$

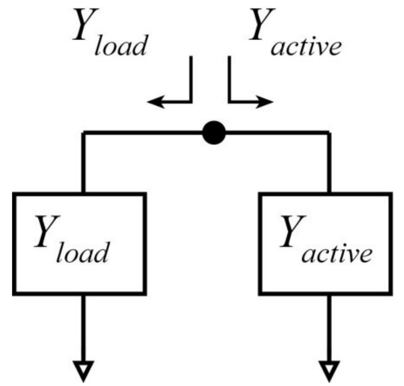
Total impedance

$$Z_{total} = (R_{active} + R_{\text{load}}) + j(X_{active} + X_{\text{load}})$$

Oscillation frequency defined by $(X_{active} + X_{\text{load}}) = 0$

Oscillates if $(R_{active} + R_{\text{load}}) < 0$





Admittance Point of View

Active device at frequency ω_0 :

$$Y_{\text{active}}(j\omega_0) = G_{active} + jB_{active}$$

Load at frequency ω_0 :

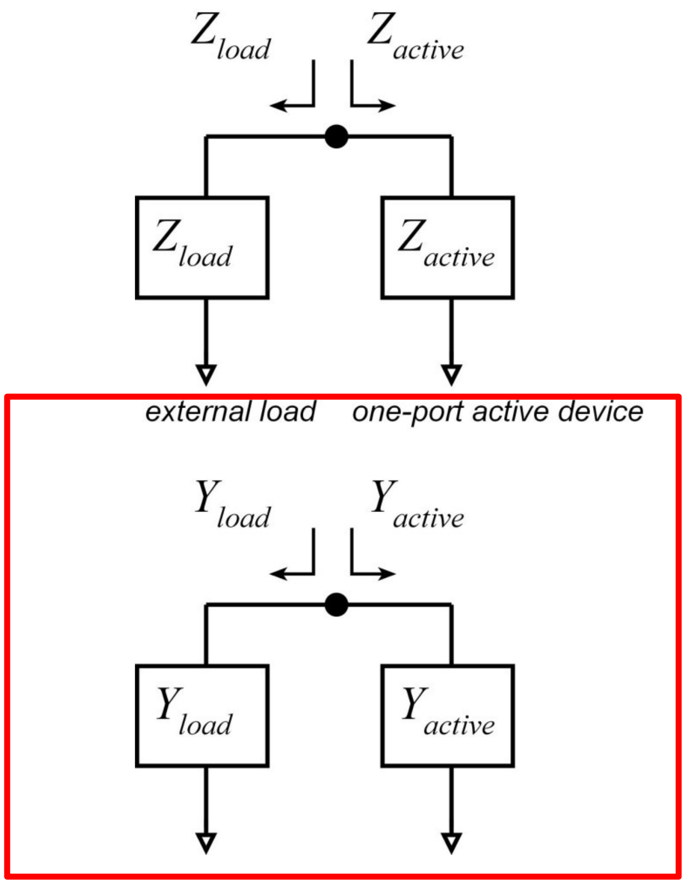
$$G_{\text{load}}(j\omega_0) = G_{\text{load}} + jG_{\text{load}}$$

Total admittance

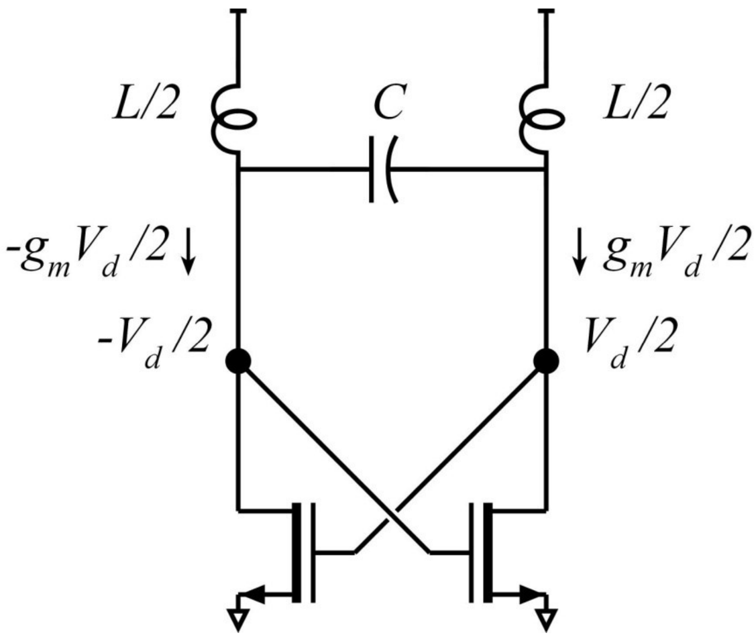
$$Y_{total} = (G_{active} + G_{\text{load}}) + j(B_{active} + B_{\text{load}})$$

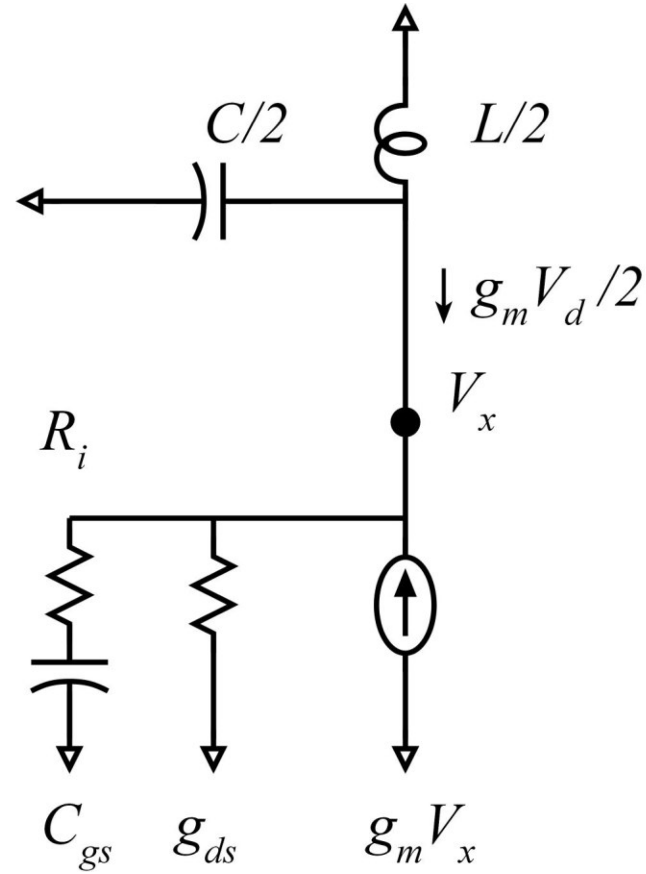
Oscillation frequency defined by $(B_{active} + B_{\text{load}}) = 0$

Oscillates if $(G_{active} + G_{\text{load}}) < 0$



Simple Negative-Resistance Oscillator





Over - simplified half - circuit model is shown.

Full differential circuit must be drawn in order to model C_{gd}, etc.

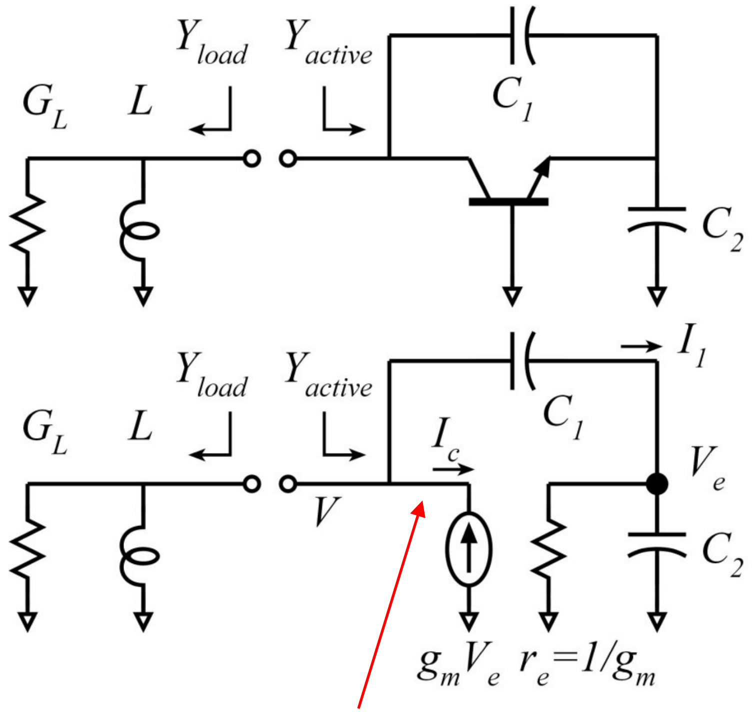
Common-Base Colpitts Oscillator

General analysis is tedious.

Assume:

(1) simplified BJT T-model

(2) $r_e << \omega_0 C_2$



$$\frac{V_e}{V_1} = \frac{j\omega C_1}{j\omega C_1 + j\omega C_2 + g_m} = \frac{C_1/(C_1+C_2)}{1 + g_m/(j\omega C_1 + j\omega C_2)}$$

$$\cong \frac{C_1}{C_1+C_2}\left(1 - \frac{g_m}{j\omega(C_1+C_2)}\right)$$

$$= \left(\frac{C_1}{C_1+C_2}\right) - \frac{g_m}{j\omega C_1}\cdot\left(\frac{C_1}{C_1+C_2}\right)^2$$

$$\frac{I_c}{V_1} = -g_m\frac{V_e}{V_1} = \frac{-C_1 g_m}{C_1+C_2} + \frac{g_m^2}{j\omega C_1}\cdot\left(\frac{C_1}{C_1+C_2}\right)^2 \approx \frac{-C_1 g_m}{C_1+C_2}$$

Negative conductance

small inductive component (we will ignore)

Common-Base Colpitts Oscillator

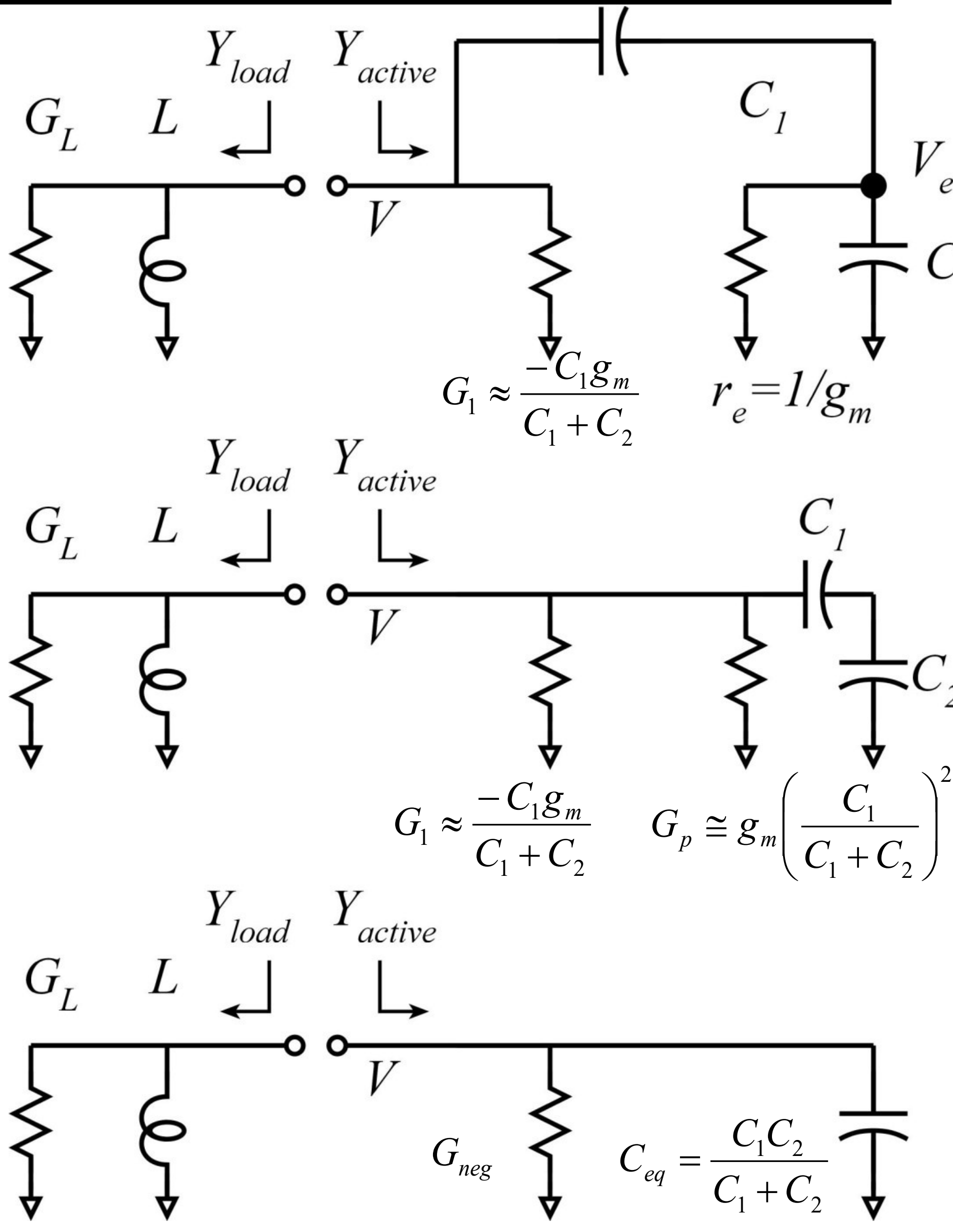
$$G_1 \approx \frac{-C_1 g_m}{C_1 + C_2}$$

The network involving C_1, C_2, and r_e can be analyzed by a series-parallel and then a parallel-series transform.

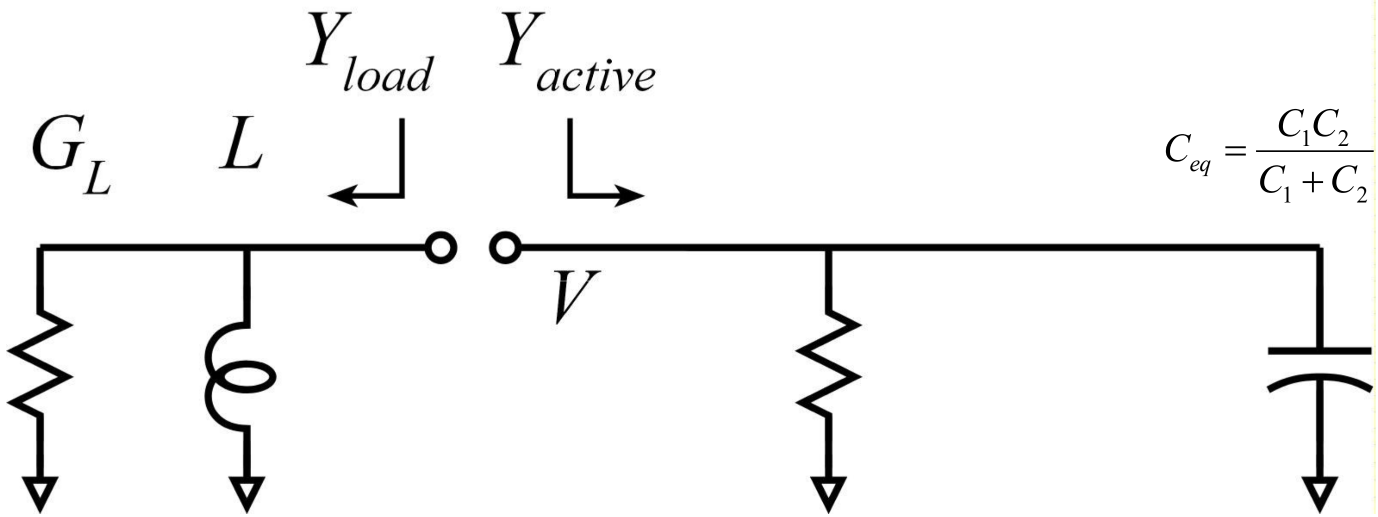
$$\rightarrow G_p \cong g_m \left(\frac{C_1}{C_1 + C_2} \right)^2$$

The overall negative conductance provided by the transistor is

$$G_{neg} \approx -g_m \left(\frac{C_1}{C_1 + C_2} \right) + g_m \left(\frac{C_1}{C_1 + C_2} \right)^2$$



Common-Base Colpitts Oscillator



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$G_{neg} \approx -g_m\left(\frac{C_1}{C_1 + C_2}\right) + g_m\left(\frac{C_1}{C_1 + C_2}\right)^2$$

Analysis was extremely approximate.

Good for first understanding

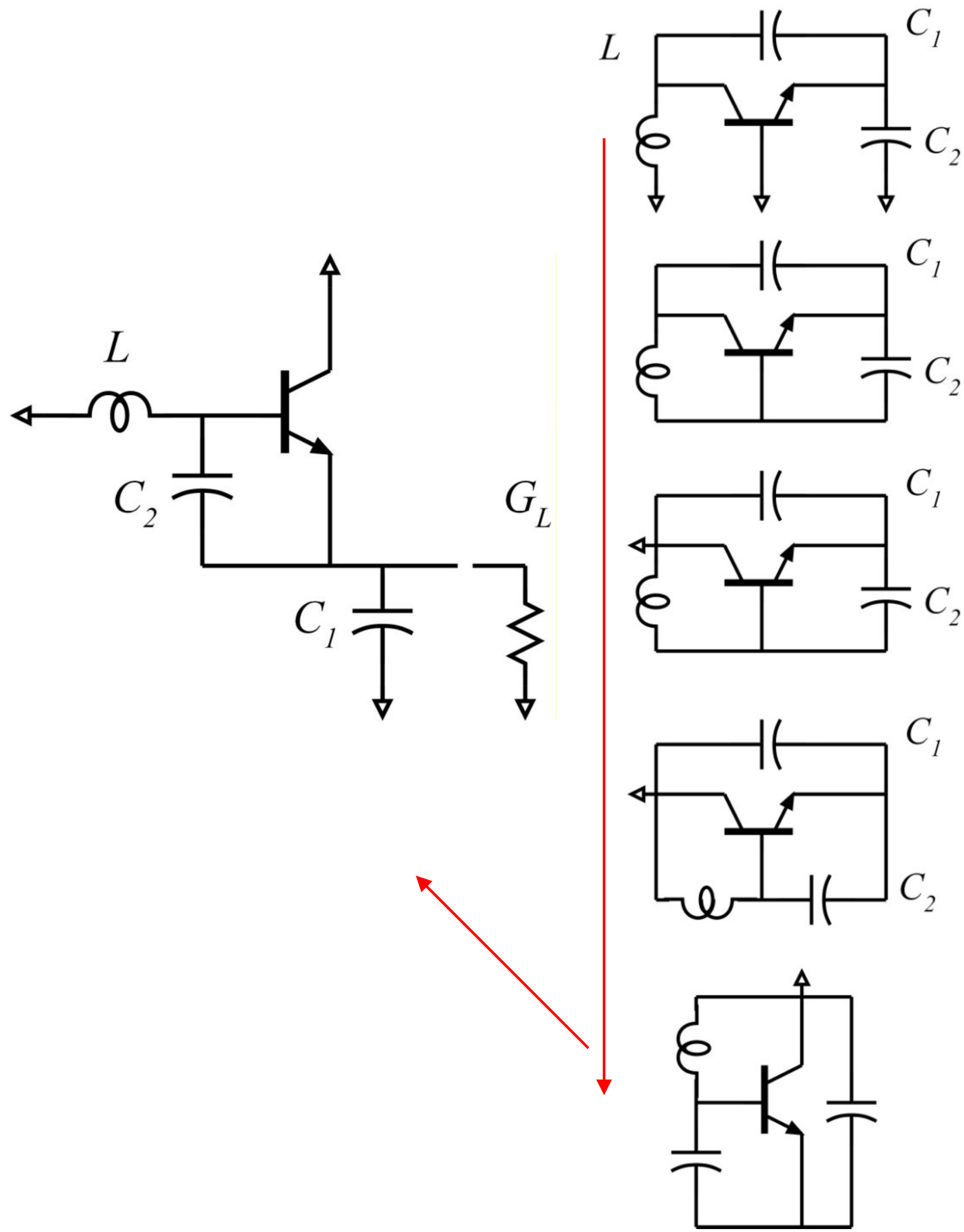
Common-Collector Colpitts Oscillator

Location of the ground node is abritrary in the analysis.

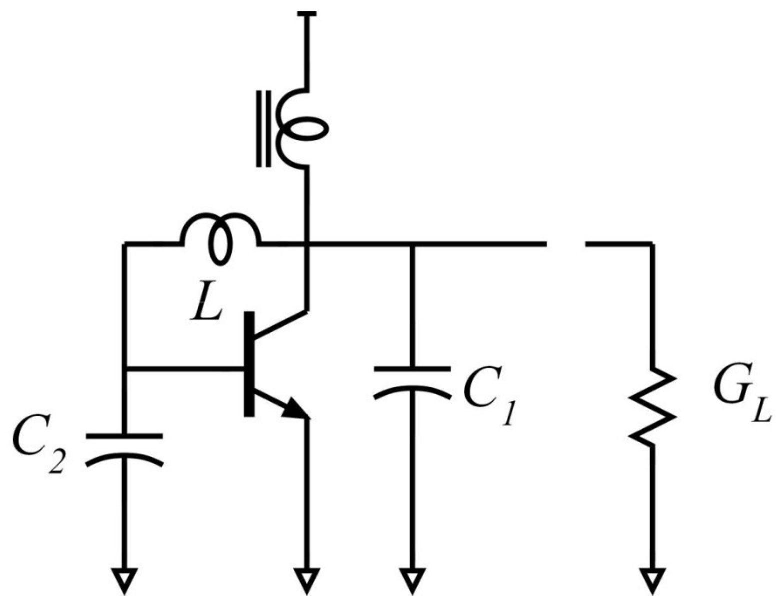
Moving the ground to the collector results in the common - collector Colpitts

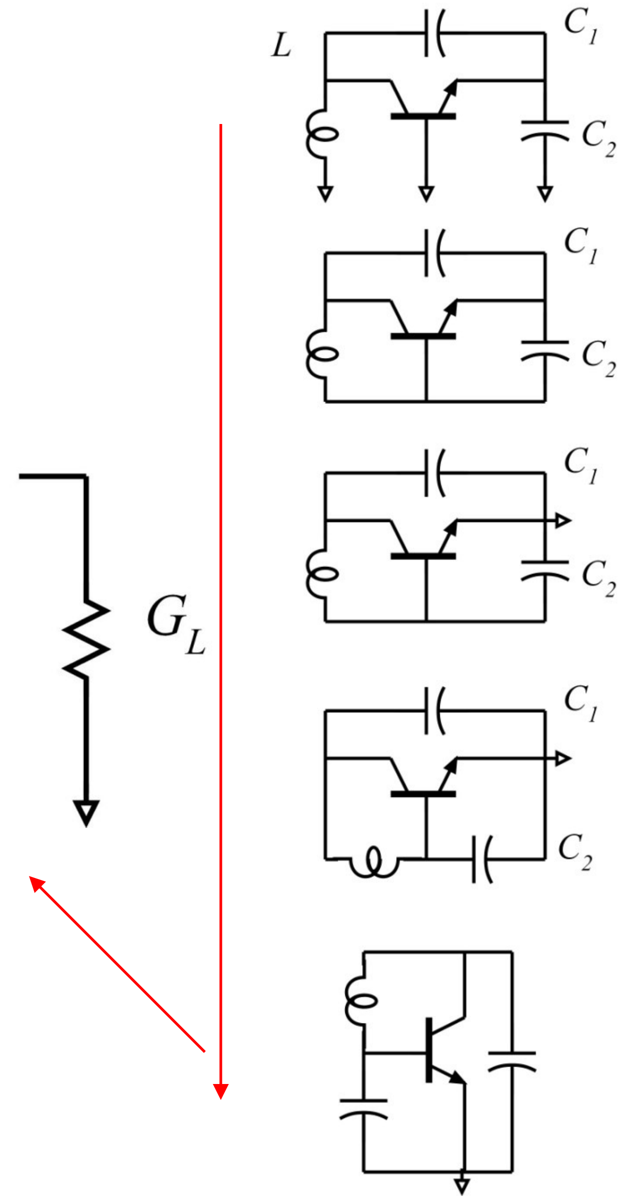
The load * is * however connected differently.

We can apply the impedance/ admittance criterion at either the base or the emitter.



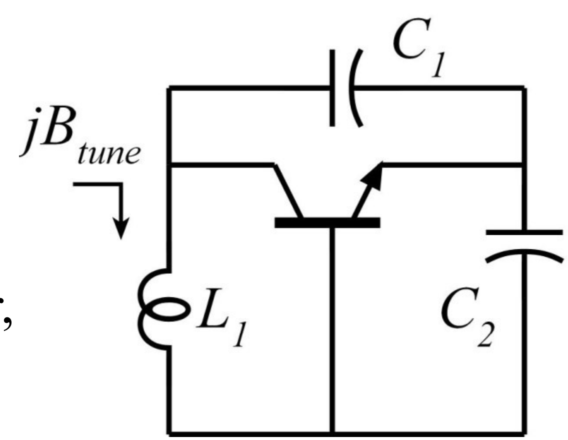
Common-Emitter Colpitts Oscillator

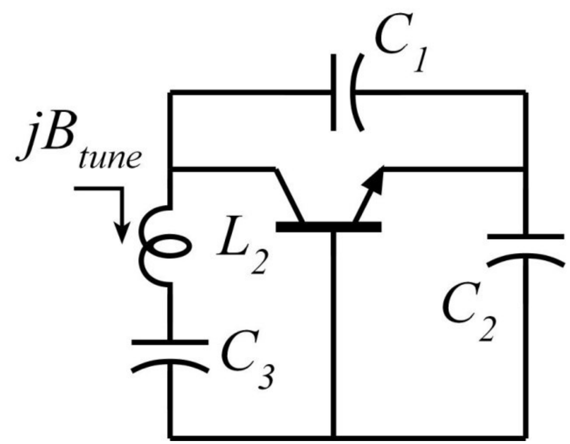




Similar comments

Clapp Oscillator





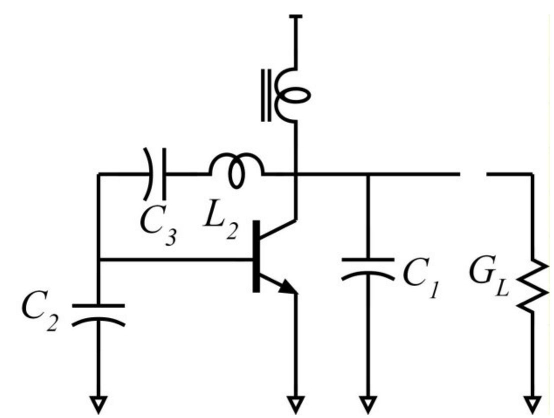
Start with the Colpitts oscillator,

with $jB_{tune} = 1/\, j\omega_o L_1$

Replace with the series combination L_2 and C_3.

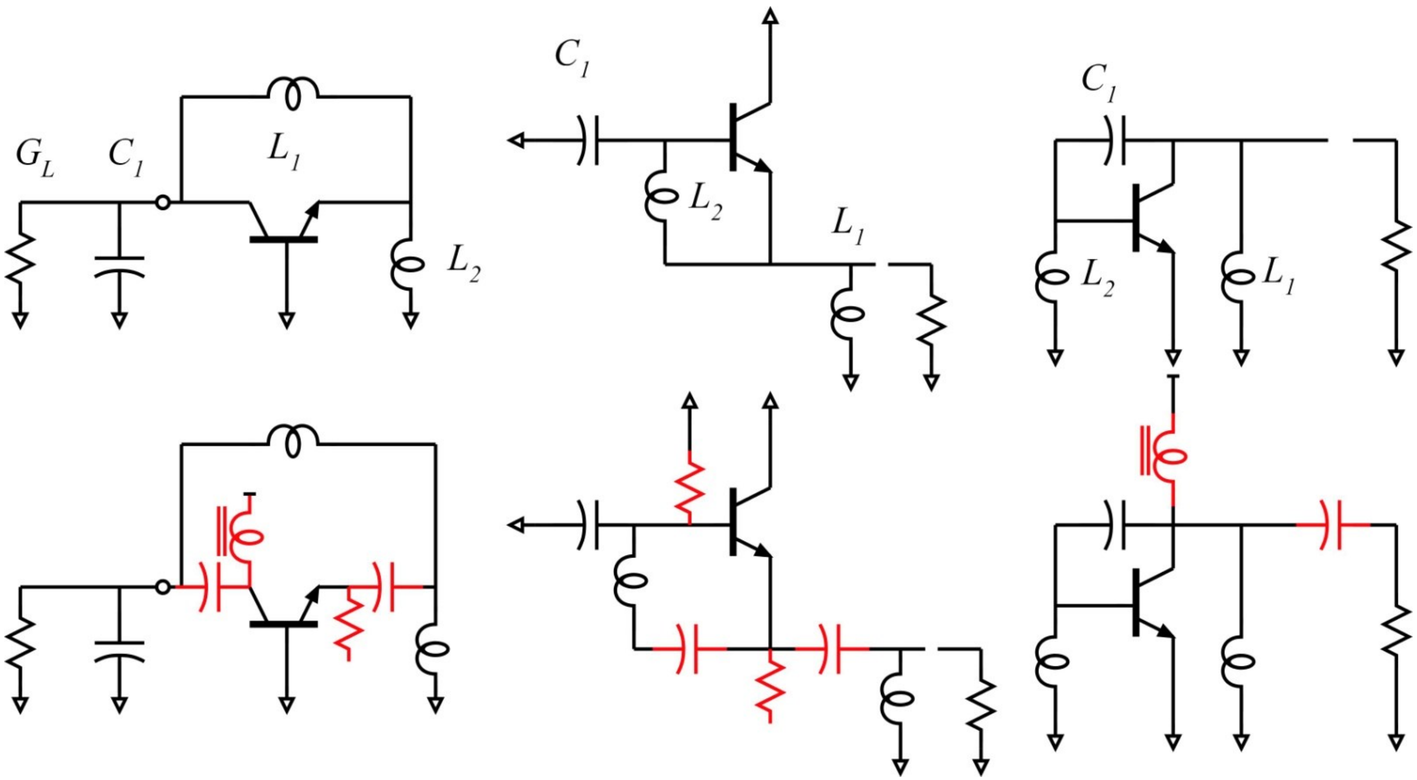
We now have $jB_{tune} = (j\omega_o L_1 - 1/\, j\omega_o C_3)^{-1}$

If B_{tune} is unchanged, then the impedance network presented to the transistor has not changed $\rightarrow$ same oscillation frequency



The common - emitter configuration (to the right) becomes the Pierce oscillator if C and L_2 are replaced by a (series resonant) quartz crystal.

Hartley Oscillator



The AC small-signal network is shown above in common-(base, collector, emitter) form.
Feeback is through inductors L_1 and L_2.

The inductors short-circuit the transistor DC bias, requiring addition of DC blocking.

Watch for parasitic resonance associated with these bias elements