

# ECE 145C / 218C, notes set xx: Class A Power Amplifiers

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# Class A power amplifier

Class A: amplifier that is at least nominally linear for small to moderate-amplitude input signals.

Clearly clips (limits) for large output voltages.

Maximum voltage  $V_{\max}$

Minimum voltage  $V_{\min}$

Maximum current  $I_{\max}$

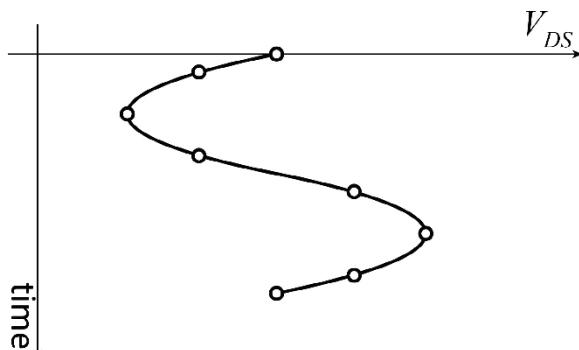
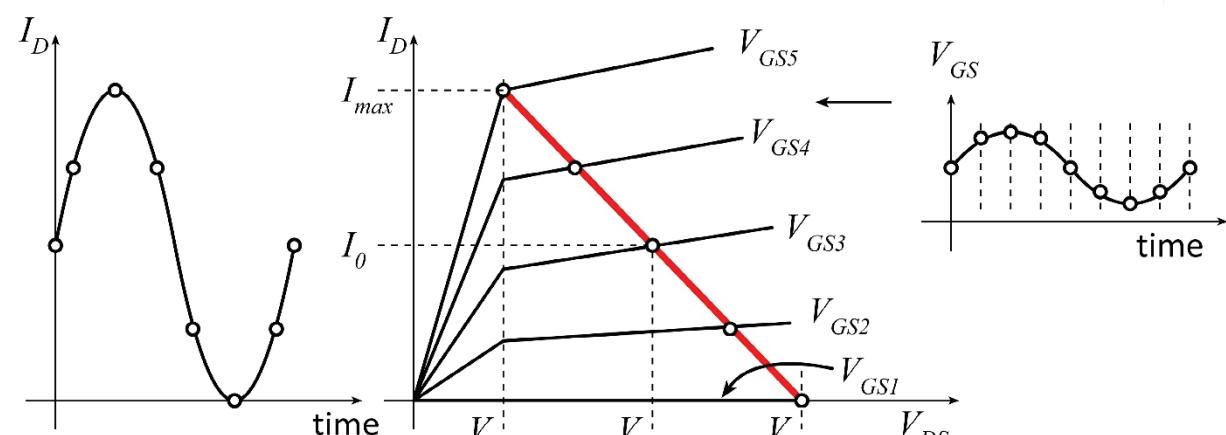
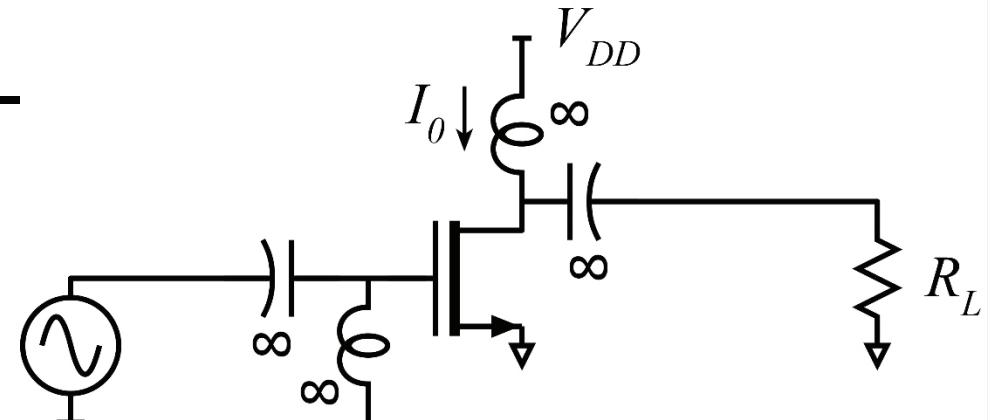
Bias drain current  $I_0$

Bias drain voltage  $V_{DD}$

If FET  $I-V$  characteristics were linear then

$$I_0 = I_{\max} / 2; V_{DD} = (V_{\max} - V_{\min}) / 2.$$

...but real transistors are not perfectly linear



# Transistor Output Characteristics

## Idealized:

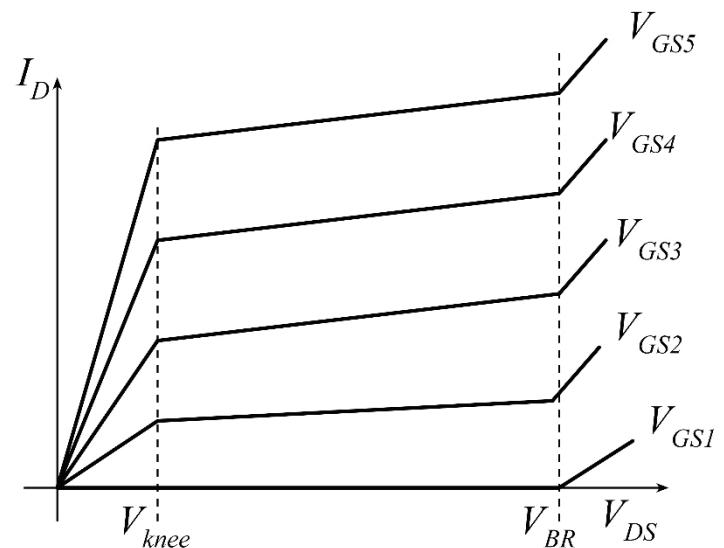
Minimum voltage:  $V_{knee}$

Maximum voltage:  $V_{br}$

Maximum current:  $I_{max}$

FETs:  $I_{max} \propto (\text{gate width})(\# \text{ gate fingers}) = N_g W_g$ ,

bipolars:  $I_{max} \propto (\text{emitter length})(\# \text{ emitter fingers}) = N_e L_e$



## Real

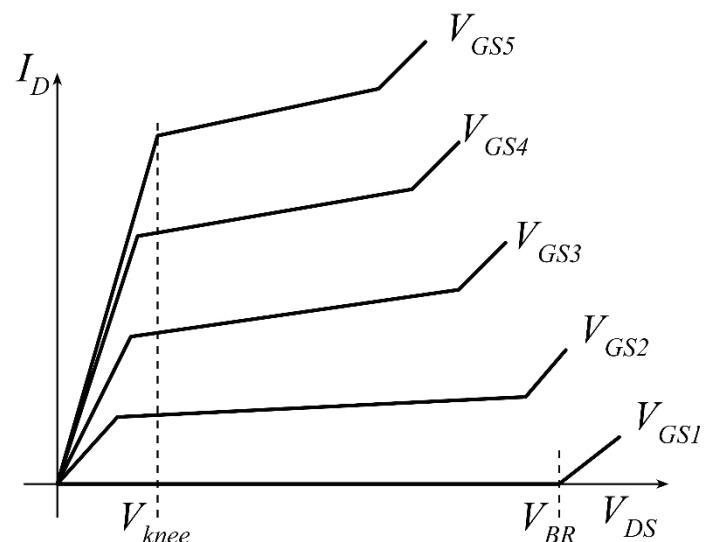
$V_{br}$  depends on  $I_D$  or  $I_C$

$V_{knee}$  depends on  $I_D$  or  $I_C$

## Comments/Caution

Textbooks:  $V_{knee} = V_{sat}$  ; Reality  $V_{knee}$  due to many factors

Textbooks: bipolar  $V_{BR} = V_{BRCBO}, V_{BRCEO}$  etc. ; Reality is much more complicated.

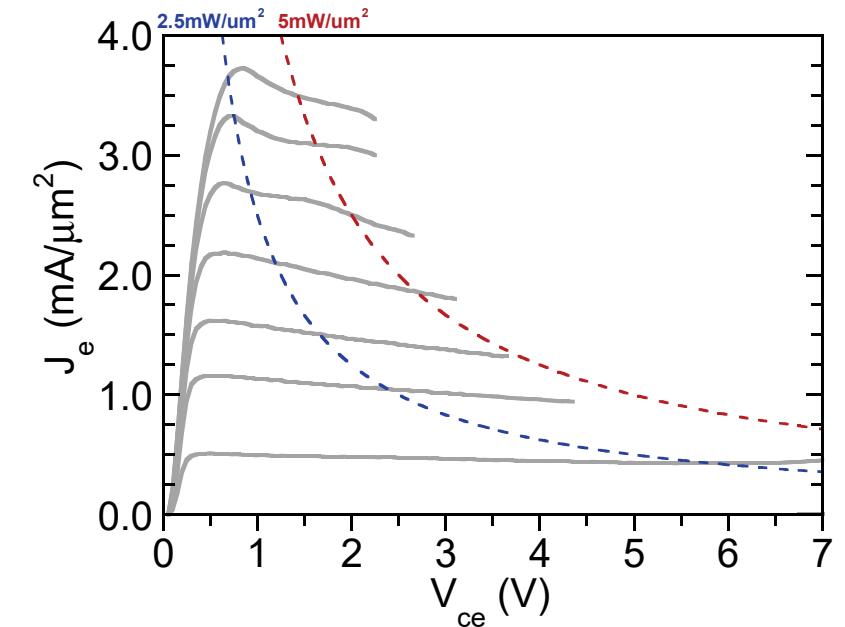


# Transistor Output Characteristics: measured

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250nm InP HBT

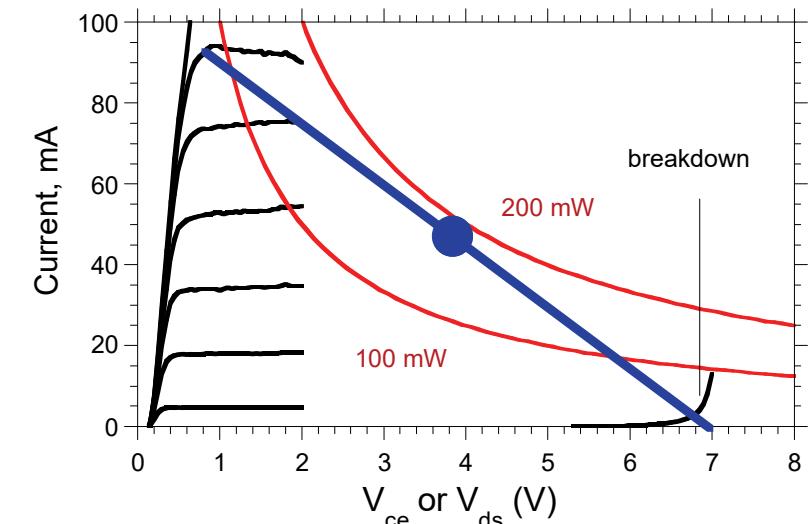
$$\begin{aligned}
 I_E &= (\text{emitter current density})(\text{total emitter area}) \\
 &= J_E (\text{number emitter fingers})(\text{emitter area per finger}) \\
 &= J_E N_E A_E
 \end{aligned}$$



500nm InP HBT

(not sure what emitter area this is)

For I-V curves of state-of-art MOSFETs,  
please see recent IEDM conference digests.



# Transistor Output Characteristics: Instantaneous Power Contours

Instantaneous **dissipated** power =  $V_{ce} I_c$  or  $V_{DS} I_D$

Constant power contours: hyperbolas on  $I_{out}, V_{out}$  plane

Low-frequency power amplifier

signal frequency  $\ll$  (thermal time constant) $^{-1} = \tau_{\text{thermal}}^{-1}$

→ loadline must lie below maximum power density curve

$\tau_{\text{thermal}}$  can be of order of  $\mu\text{s}$  to ns, depending on transistor size.

High-frequency power amplifier

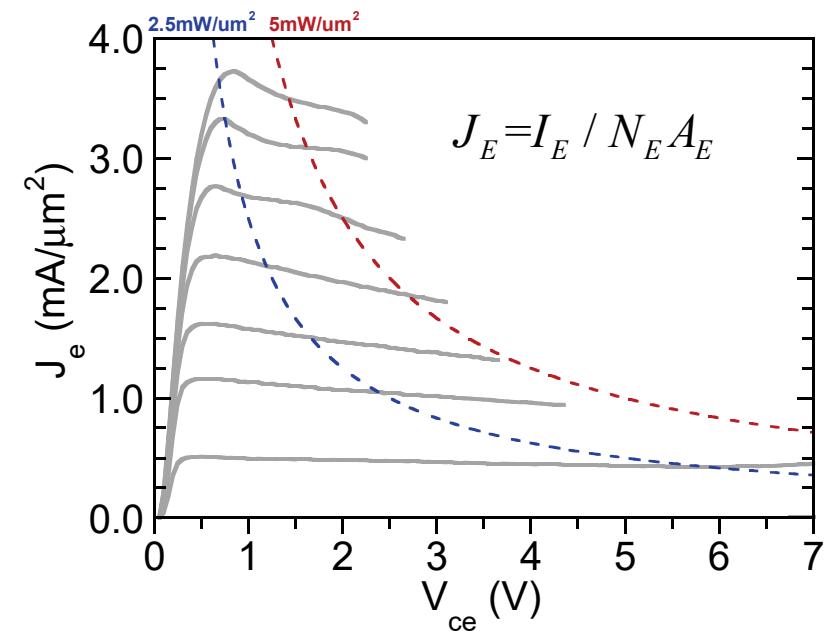
signal frequency  $\gg$  (thermal time constant) $^{-1}$

→ \*bias point\* must lie below maximum power density curve

Note: measured DC (I,V) characteristics are influenced by heating.

Breakdown with  $f_{\text{signal}} \gg \tau_{\text{thermal}}^{-1}$  can be larger than  $V_{BR}$  @DC.

RF loadline can surpass DC breakdown voltage.



# Transistor parameters and cutoff frequencies vary across (I,V) plane

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Bipolar Transistors:

Low voltage and high current:

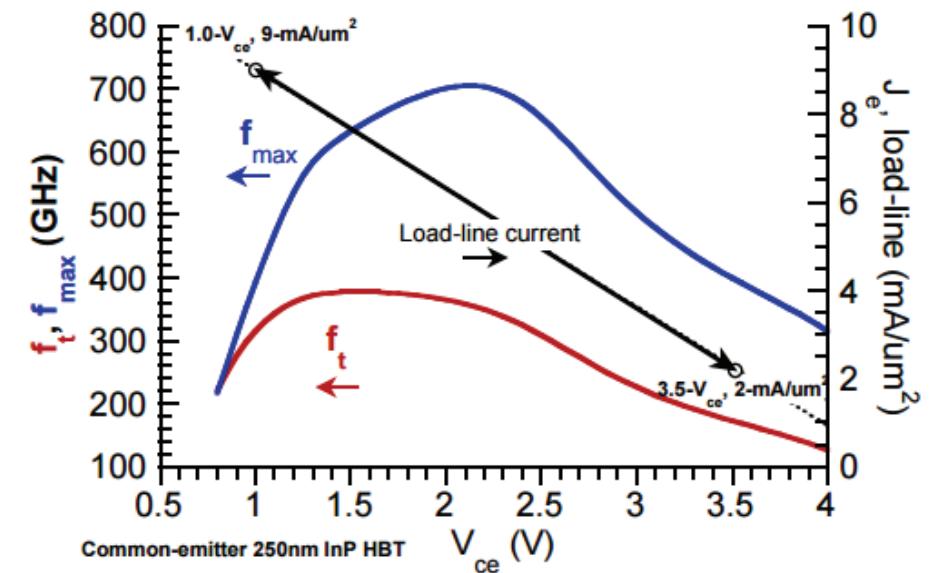
"Kirk effect" = space-charge-limited current

reduction in  $f_\tau$ , increase in  $C_{cb}$   $\rightarrow$  reduced bandwidth

High voltage:

all semiconductors: push-out of collector depletion edge

III-V semiconductors: reduction of electron velocity in collector  
reduction in  $f_\tau$   $\rightarrow$  reduced bandwidth



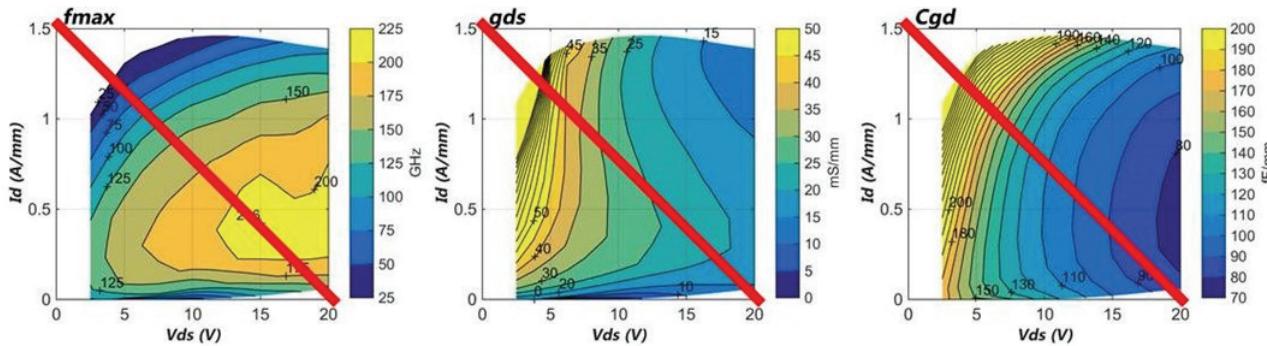
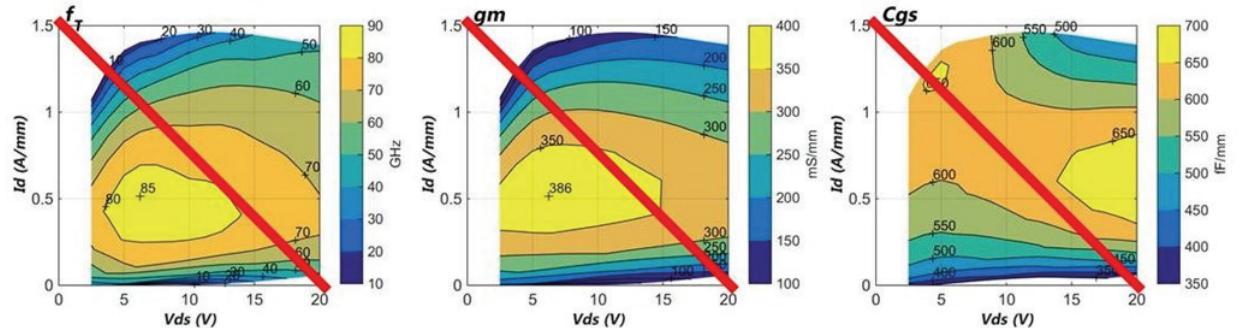
Z. Griffith, 2012 IPRM

# Transistor parameters and cutoff frequencies vary across (I,V) plane

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Field-Effect Transistors:

Device parameters vary strongly across (I,V) plane  
 $(f_\tau, f_{\max})$  vary strongly across (I,V) plane



GaN HEMT experimental data (c.a. 2010) due to Kiesuke Shinohara, Teledyne Scientific

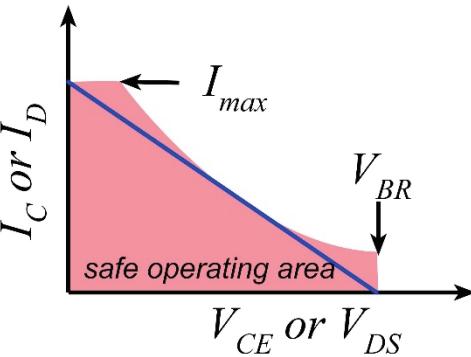
# Safe Operating Area (SOA); "Fast Operating Area" (FOA ?)

## Safe operating area

standard terminology

region bounded by maximum power hyperbola,  $V_{br}$ ,  $I_{max}$

SOA includes other effects beyond the scope of these notes



## "Fast operating area"

not standard terminology

region with adequately high  $f_t, f_{max}$

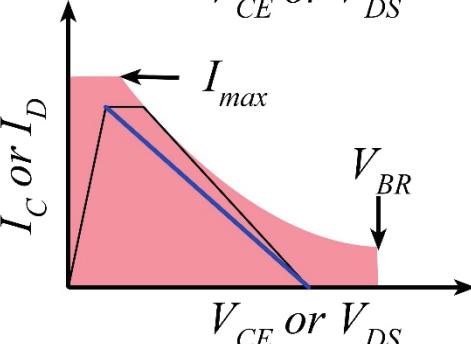
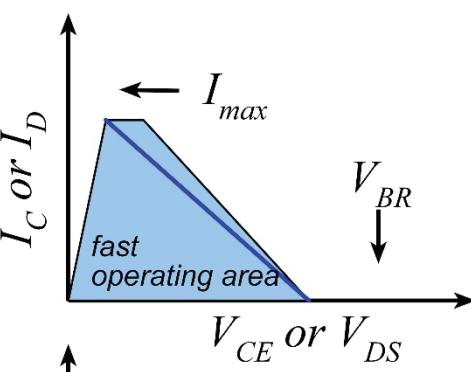
Loadline must lie within both

## Reliability

Particularly with Si MOSFETs,

reliability decreases as  $V_{DS}$  increases.

→ maximum  $V_{DS}$  well below DC breakdown



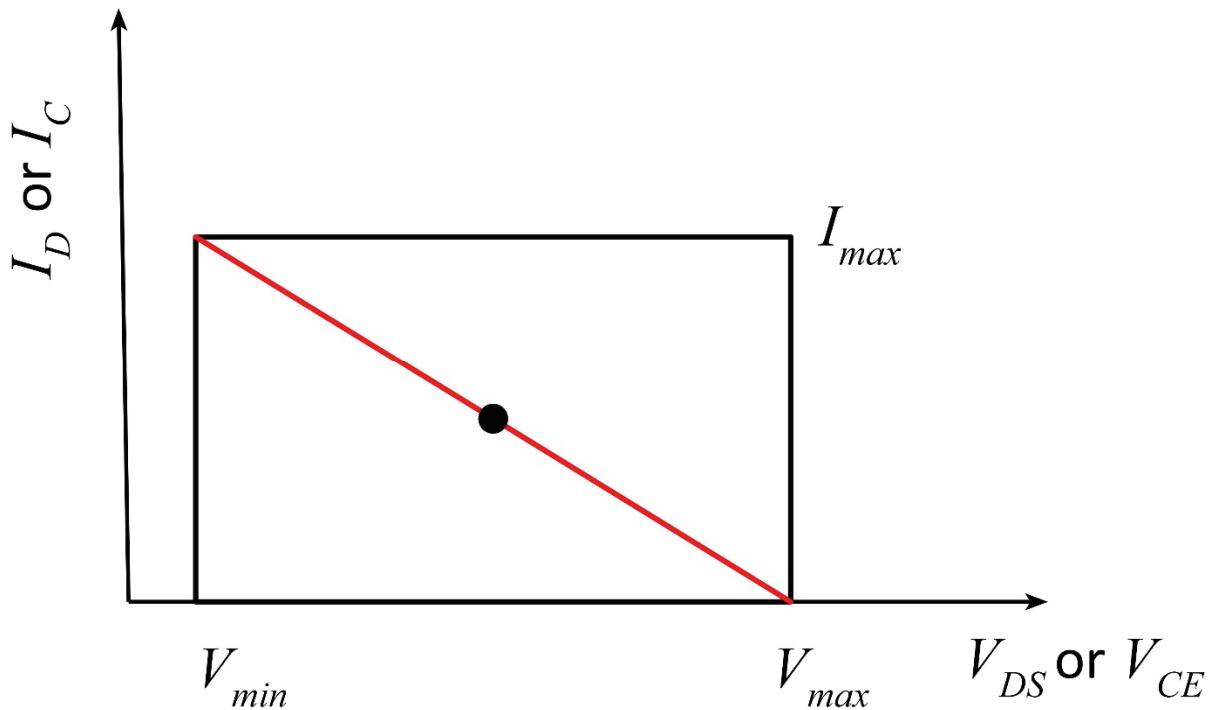
■ safe operating area  
 ■ fast operating area  
— loadline

# Oversimplified SOA/FOA; for class

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Maximum, minimum voltages

Maximum current  $\propto N_g W_g$  or  $N_E L_E$



# Simple class A power analysis

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Bias point in center of rectangle.

Loadline reaches corners of rectangle

$$Z_L = R_L + j0\Omega = (V_{\max} - V_{\min}) / I_{\max}$$

$$P_{DC} = V_{DC} I_{DC} = (V_{\max} + V_{\min}) I_{\max} / 4$$

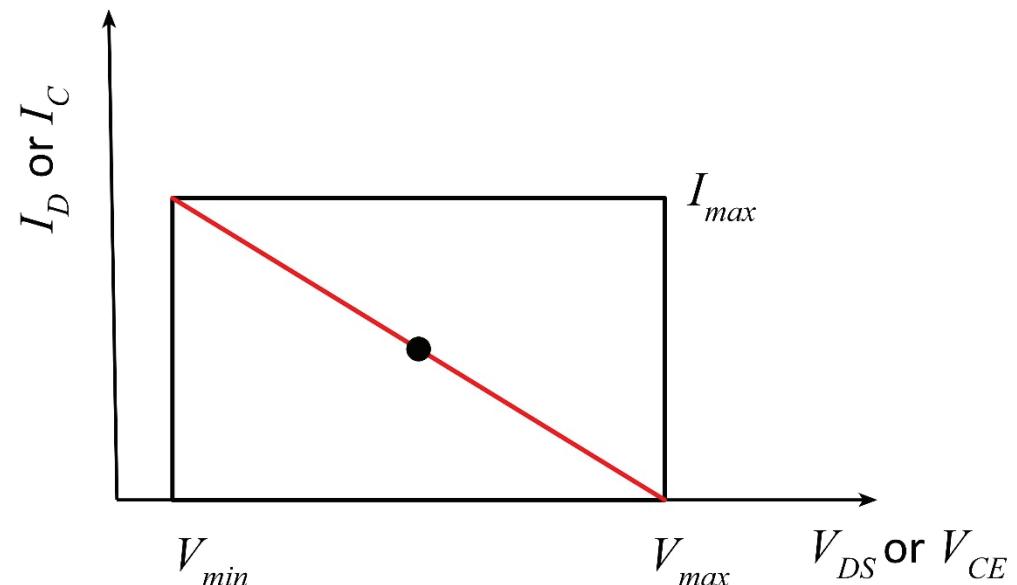
$$P_{RF,\max} = (V_{\max} - V_{\min}) I_{\max} / 8 = (V_{\max} - V_{\min})^2 / 8R_L$$

drain/collector efficiency at peak output power

$$\eta_{\text{drain/collector}} = \frac{P_{RF,\max}}{P_{DC}} = \frac{(V_{\max} - V_{\min}) I_{\max} / 8}{(V_{\max} + V_{\min}) I_{\max} / 4}$$

$$\eta_{\text{drain/collector}} = \frac{1}{2} \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

...all expressions correct if and only if  $Z_L = (V_{\max} - V_{\min}) / I_{\max}$ .

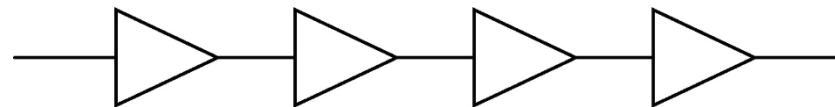


# Power-added efficiency

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$$\text{P.A.E.} \equiv (P_{out} - P_{in}) / P_{DC}$$

$$= \frac{P_{out}}{P_{DC}} \left[ 1 - \frac{P_{in}}{P_{out}} \right] = \eta_{\text{drain/collector}} \cdot \left[ 1 - \frac{1}{\text{gain}} \right] \text{ where } \eta_{\text{drain/collector}} = \frac{P_{out}}{P_{DC}}$$



$$\text{PAE}_{\text{overall}} = \frac{P_{out,N} - P_{in1}}{P_{DC1} + P_{DC2} + \dots + P_{DC,N}} = \frac{(P_{out,N} - P_{in,N}) + (P_{out,N-1} - P_{in,N-1}) + \dots + (P_{out1} - P_{in1})}{P_{DC1} + P_{DC2} + \dots + P_{DC,N}}$$

$$= \frac{1}{P_{DC1} + P_{DC2} + \dots + P_{DC,N}} \left( P_{DC,1} \frac{(P_{out,1} - P_{in,1})}{P_{DC,1}} + P_{DC,2} \frac{(P_{out,2} - P_{in,2})}{P_{DC,2}} + \dots + P_{DC,N} \frac{(P_{out,N} - P_{in,N})}{P_{DC,N}} \right)$$

$$= \frac{1}{P_{DC1} + P_{DC2} + \dots + P_{DC,N}} (P_{DC,1} \text{PAE}_1 + P_{DC,2} \text{PAE}_2 + \dots + P_{DC,N} \text{PAE}_N)$$

Now suppose  $\text{PAE}_1 = \text{PAE}_2 = \dots = \text{PAE}_N$

$$\text{PAE}_{\text{overall}} = \frac{\text{PAE}_1}{P_{DC1} + P_{DC2} + \dots + P_{DC,N}} (P_{DC,1} + P_{DC,2} + \dots + P_{DC,N}) = \text{PAE}_1$$

A PA chain obtains the same overall PAE as that of 1 stage if each PA has the same PAE and if each stage in the chain reaches that PAE under the same overall RF drive. Due to accumulated gain compression, real multistage power amplifiers are rarely designed to meet this criterion.

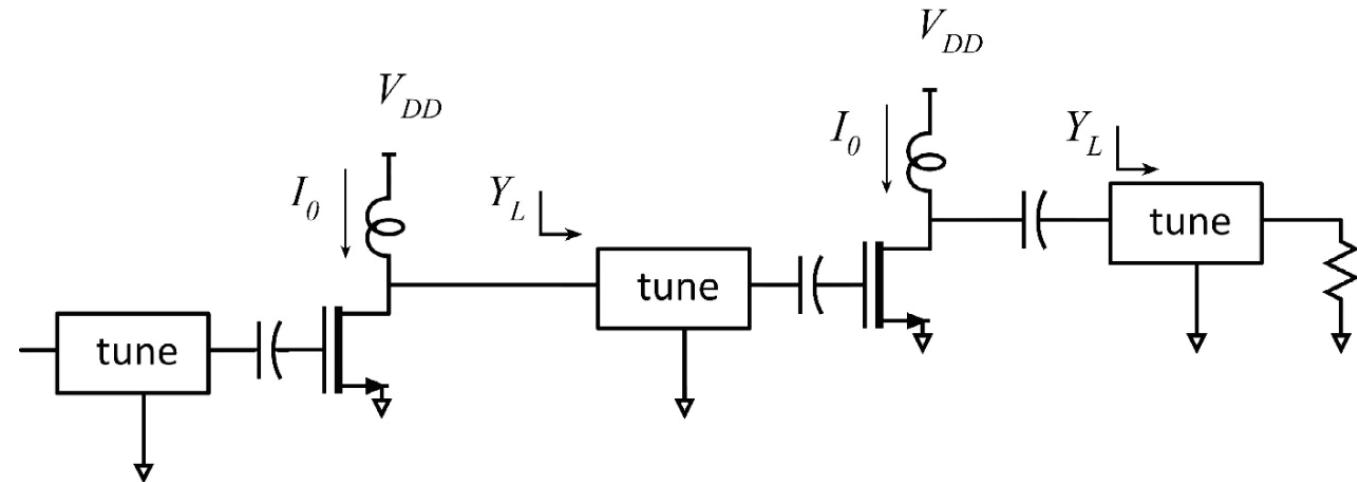
# Power Amplifier Design

For highest efficiency, each stage should be loaded with

$$Z_{L,opt} = (V_{\max} - V_{\min}) / I_{\max}$$

The interstage networks are tuning networks,  
not matching networks.

Thus far, we have neglected transistor parasitics.



# Loadline: inclusive of transistor parasitics

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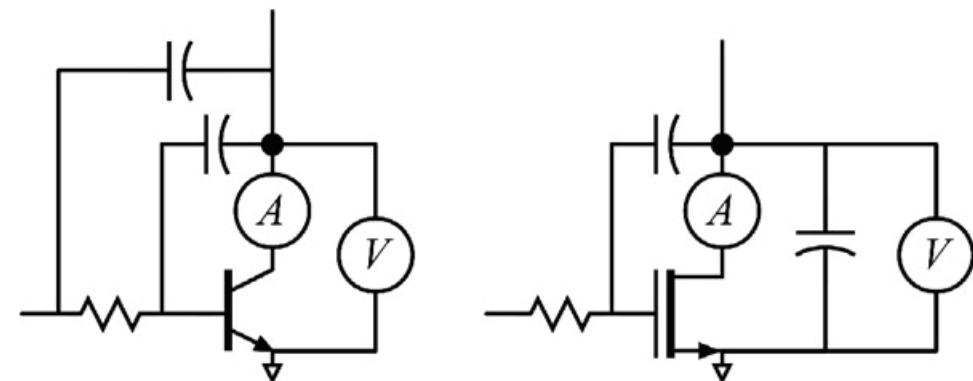
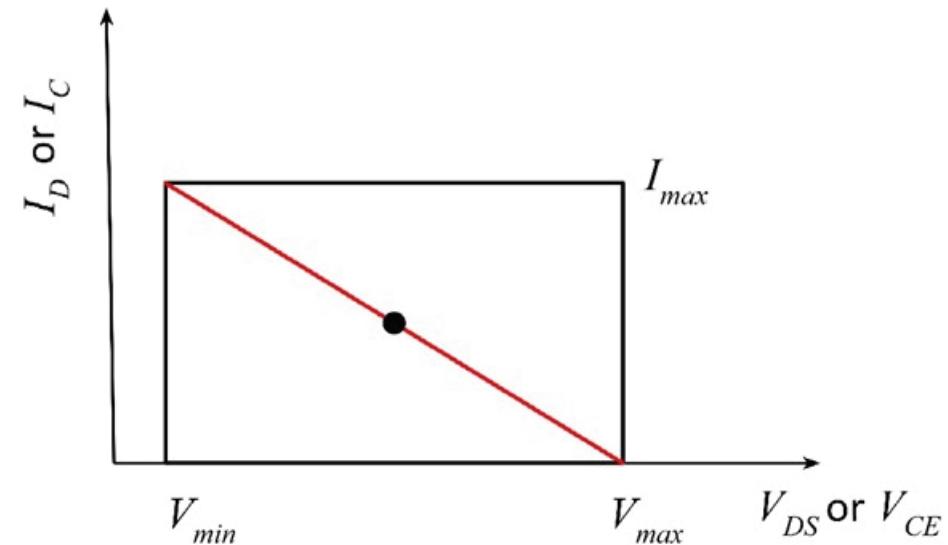
Transistors have resistive, capacitive parasitics.

It is the \*internal\* ( $I_C, V_{CE}$ ) or ( $I_D, V_{DS}$ )  
that must follow this loadline →

Loadline current must be electron current,  
not  $C \cdot dV / dt$  displacement current.

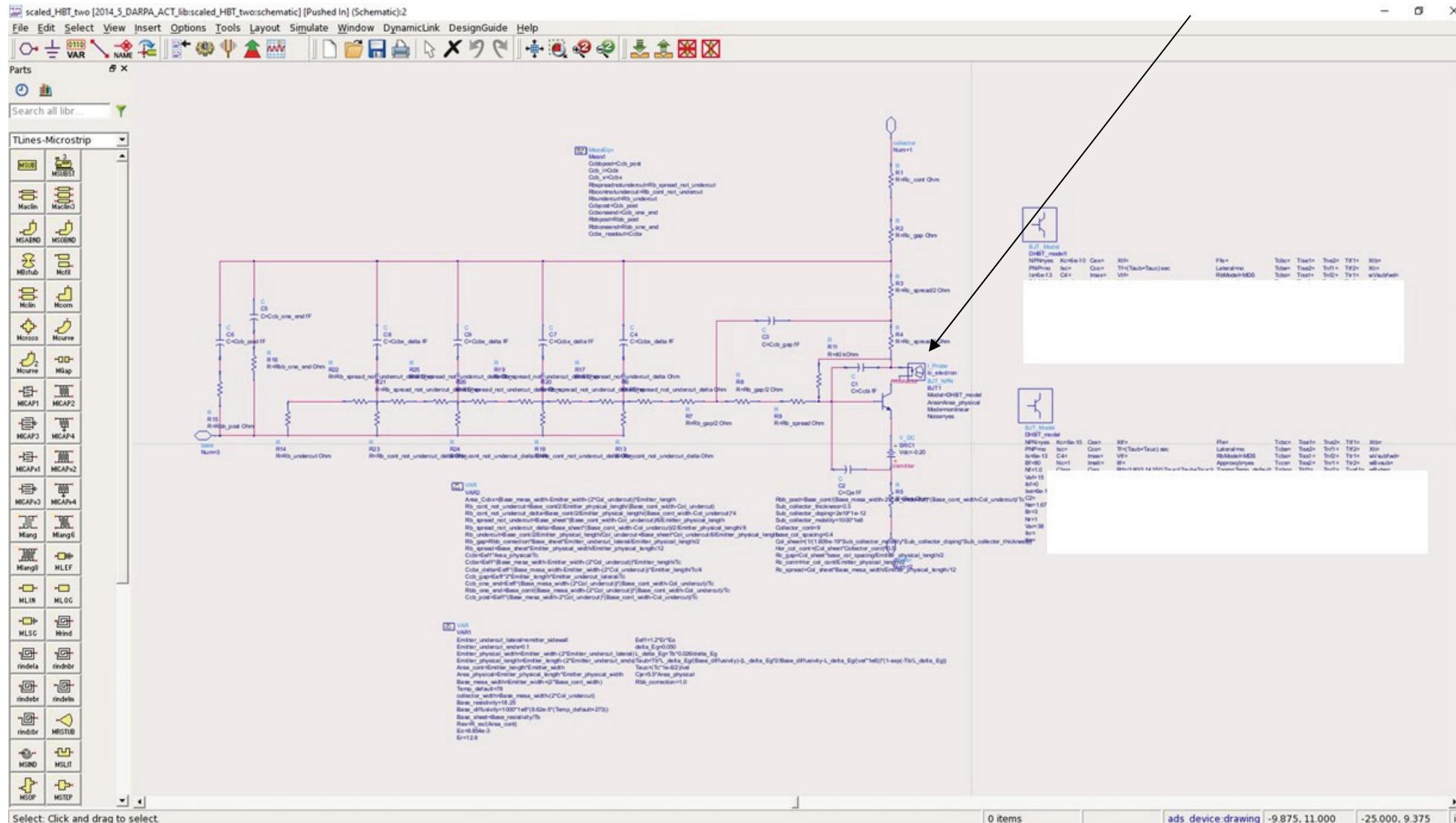
→ Current meters must be placed inside the capacitive parasitics.

How do we do this ?



# Loadline inclusive of transistor parasitics

If the transistor technology originates from the organization you work for, then CAD models may not be encrypted. If so, you can edit the device CAD model to place an internal current meter.



# Loadline inclusive of transistor parasitics

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If the transistor technology does not originate from the organization you work for, then the CAD models may be encrypted.

If so

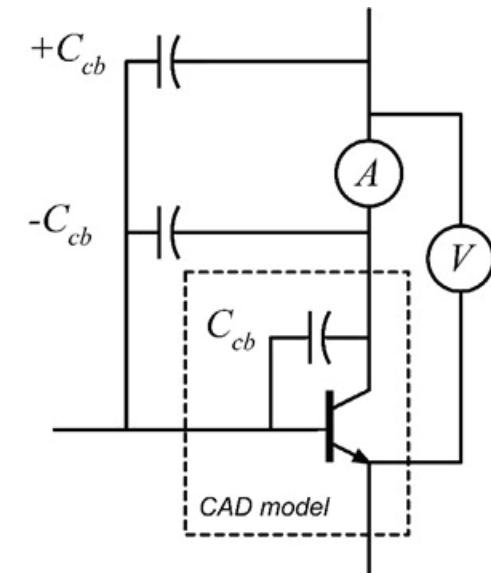
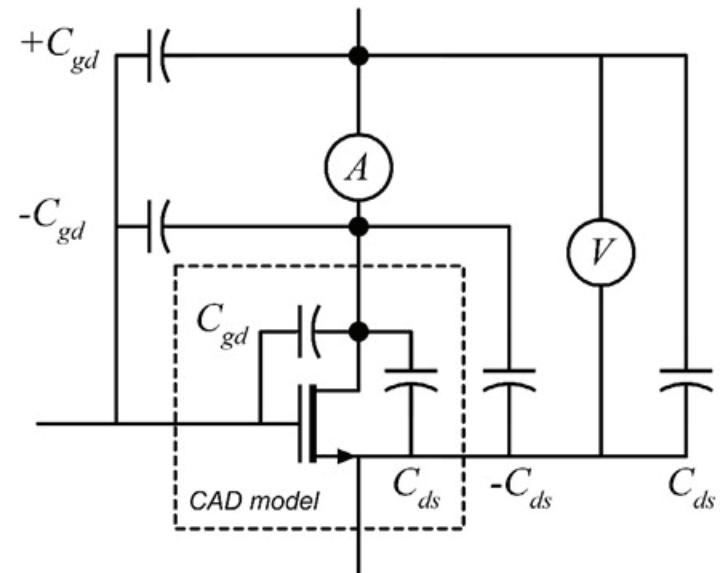
Use design kit CAD device model → simulate device S-parameters  
From the S-parameters,  
use established extraction procedures to determine transistor capacitances

Then, with transistor capacitances known:

- 1) add external negative capacitances to cancel these.
- 2) add voltmeter and current meter.

These correctly measure the loadline

- 3) then add back external positive capacitances.

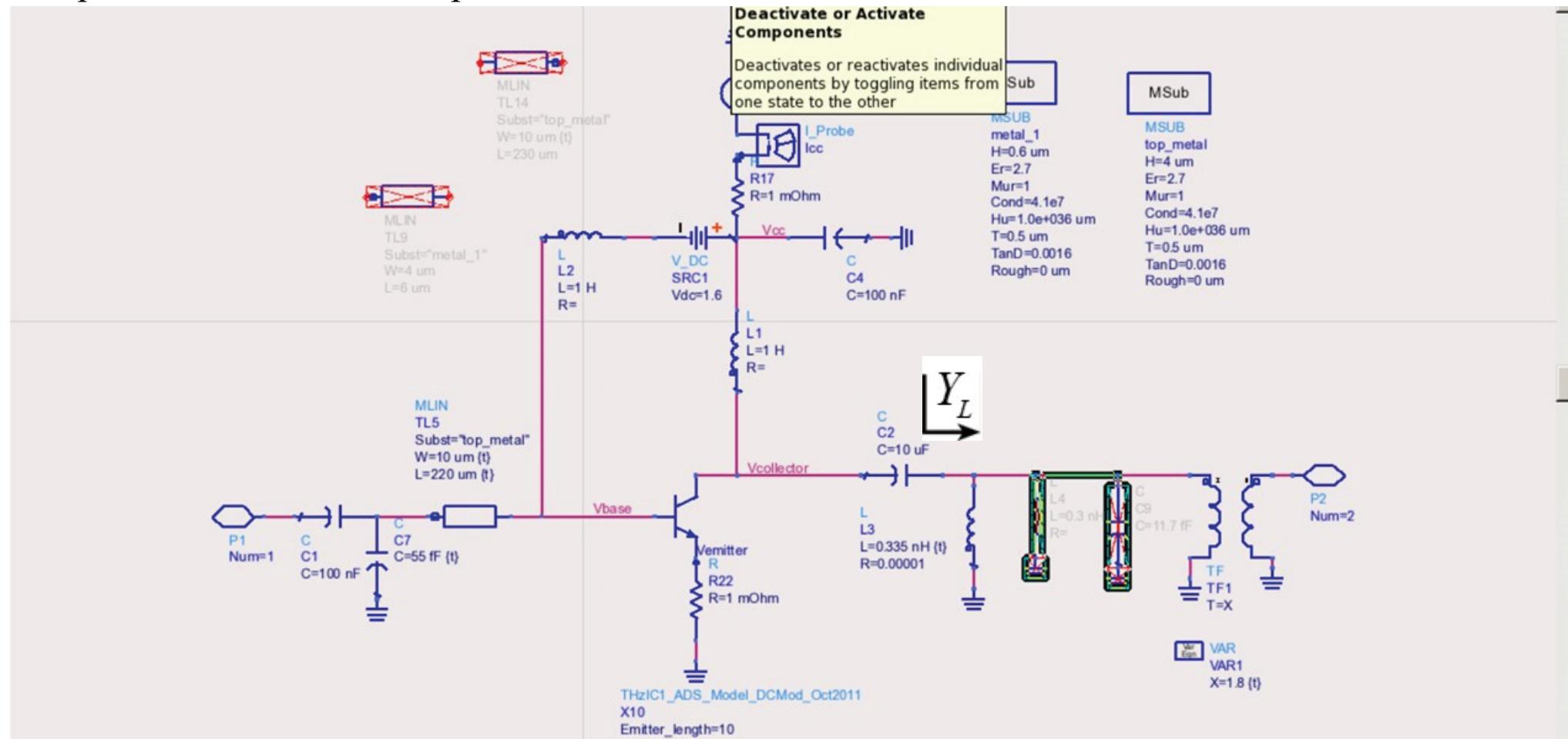


# Power Amplifier Design Example

Power amplifier \*cell\*. 20 micron emitter finger. Bias:  $I_c = 10\text{mA}$ ,  $V_{cb} = 1.86 \text{ Volts} \rightarrow V_{CE} = 2.5 \text{ V}$ .

The biasing technique used is just for CAD experimentation; (not practical bias circuit)

The inactivated output LC network is a bandpass filter



# Power Amplifier Design Example

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First, simulate the transistor S-parameters vs. frequency at the bias point

Determine:

$$f_{\max},$$

stability factors, stability circles at the design frequency.

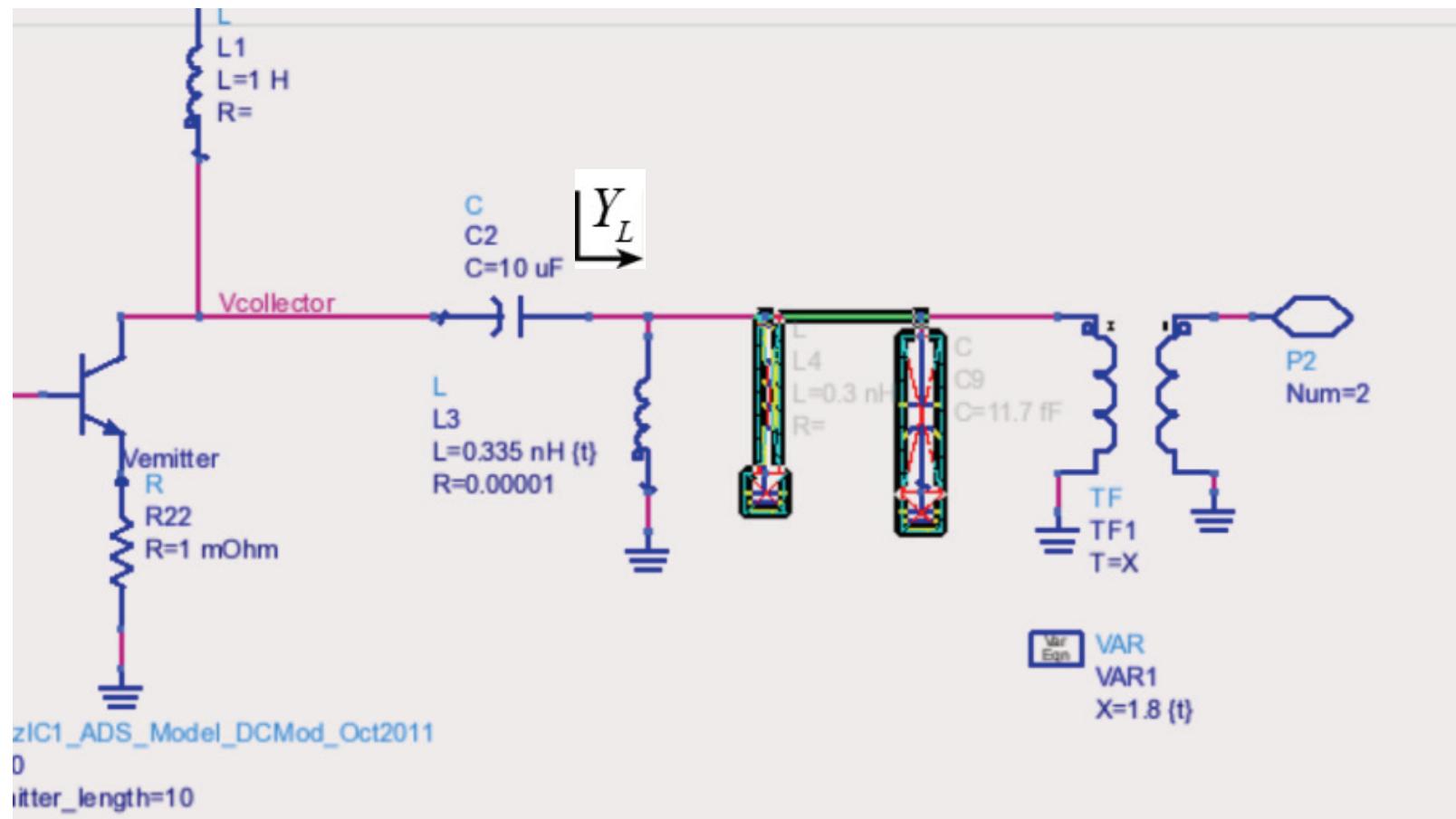
MAG/MSG at the design frequency.

If potentially unstable, stabilize (input resistance or reactive feedback ) at the design frequency

# PA example: output network

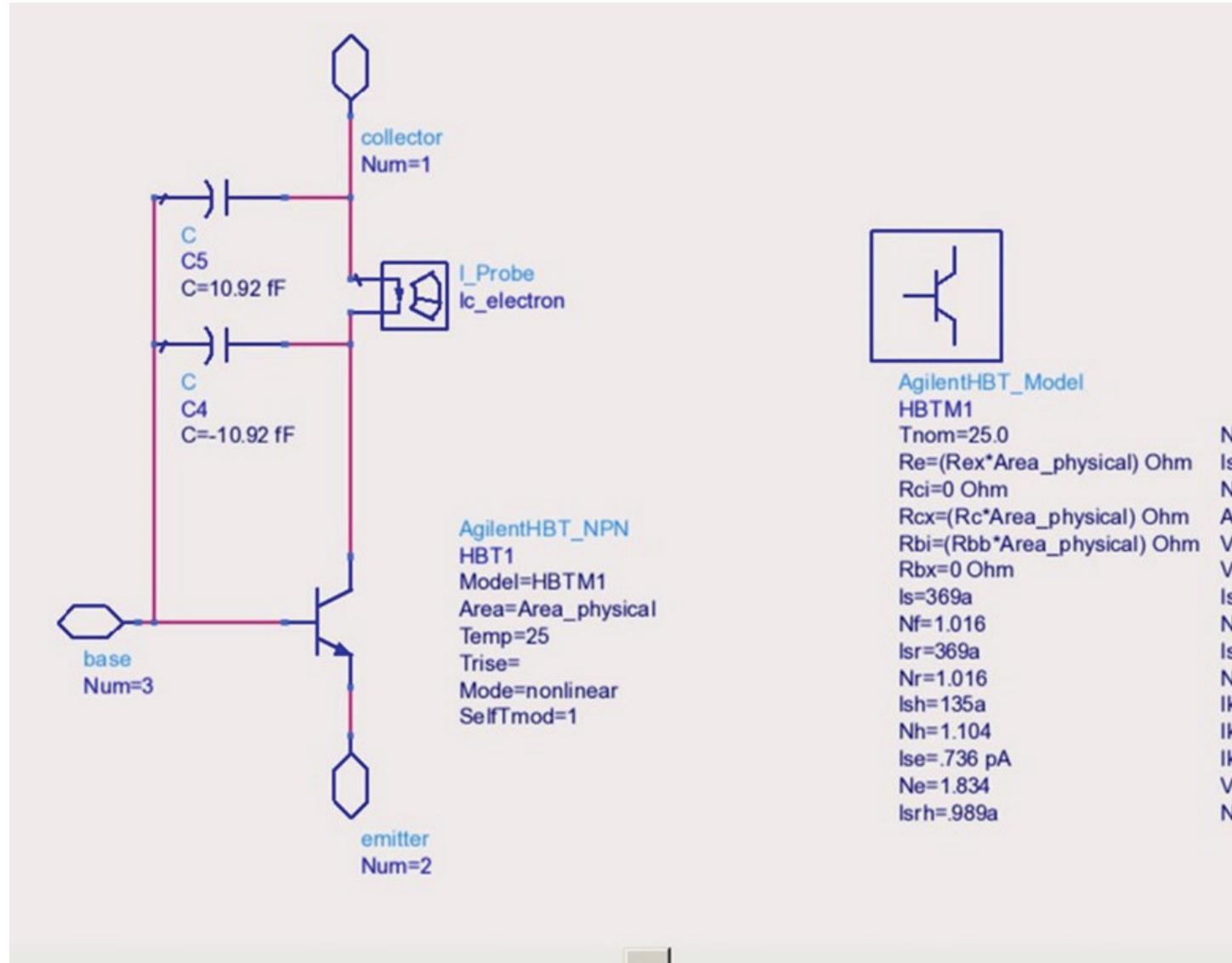
Note the output network: ideal transformer plus parallel inductance.

This is not the final ouput network: it is used to quickly find  $Y_{L,opt}$ .



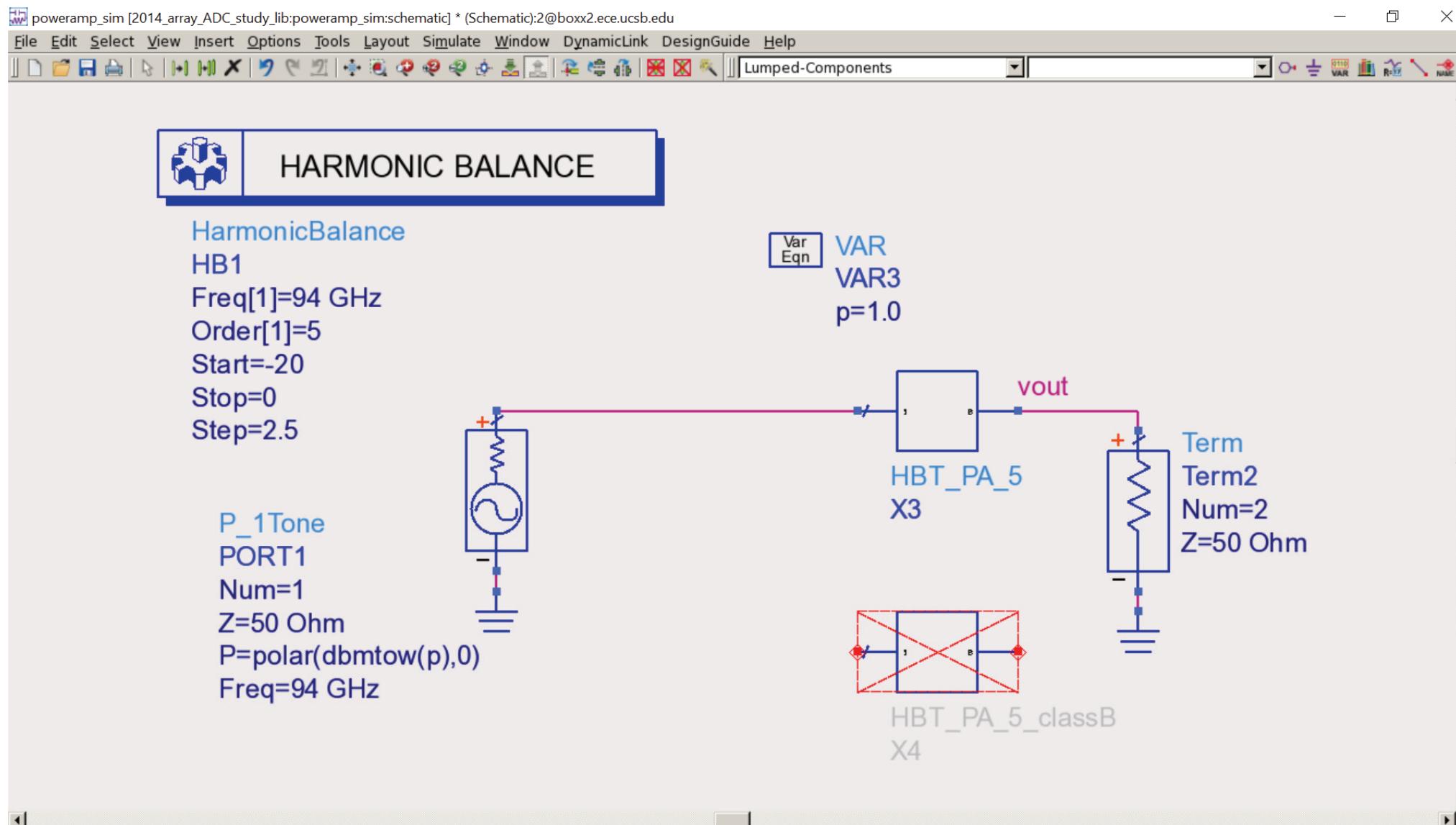
# PA example: measuring the loadline

Dropping down in the hierarchy into the device model, we see the monitoring ammeter plus negative and positive  $C_{cb}$ .



# PA example: finding the optimum load

The input is not yet matched, and the load is not yet tuned. We drive the transistor with a large drive signal and observe the loadline



# Lissajous patterns

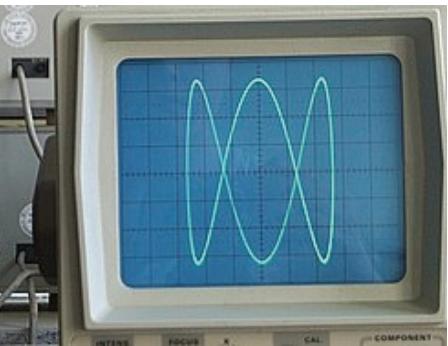
$$x = A \cos(at + \theta)$$

$$y = B \cos(bt)$$

closed patterns if  $a / b$  is rational.

$a / b$  or  $b / a$  sets # of loops.

for  $a=b$ ,  $\theta$  varies the curve between a line and an ellipse.



- Wikipedia +

https://en.wikipedia.org/wiki/Lissajous\_curve

Lissajous curve

A Lissajous curve /lɪsədʒuː/, also known as **Lissajous figure** or **Bowditch curve** /baʊdɪtʃ/, is the graph of a system of parametric equations

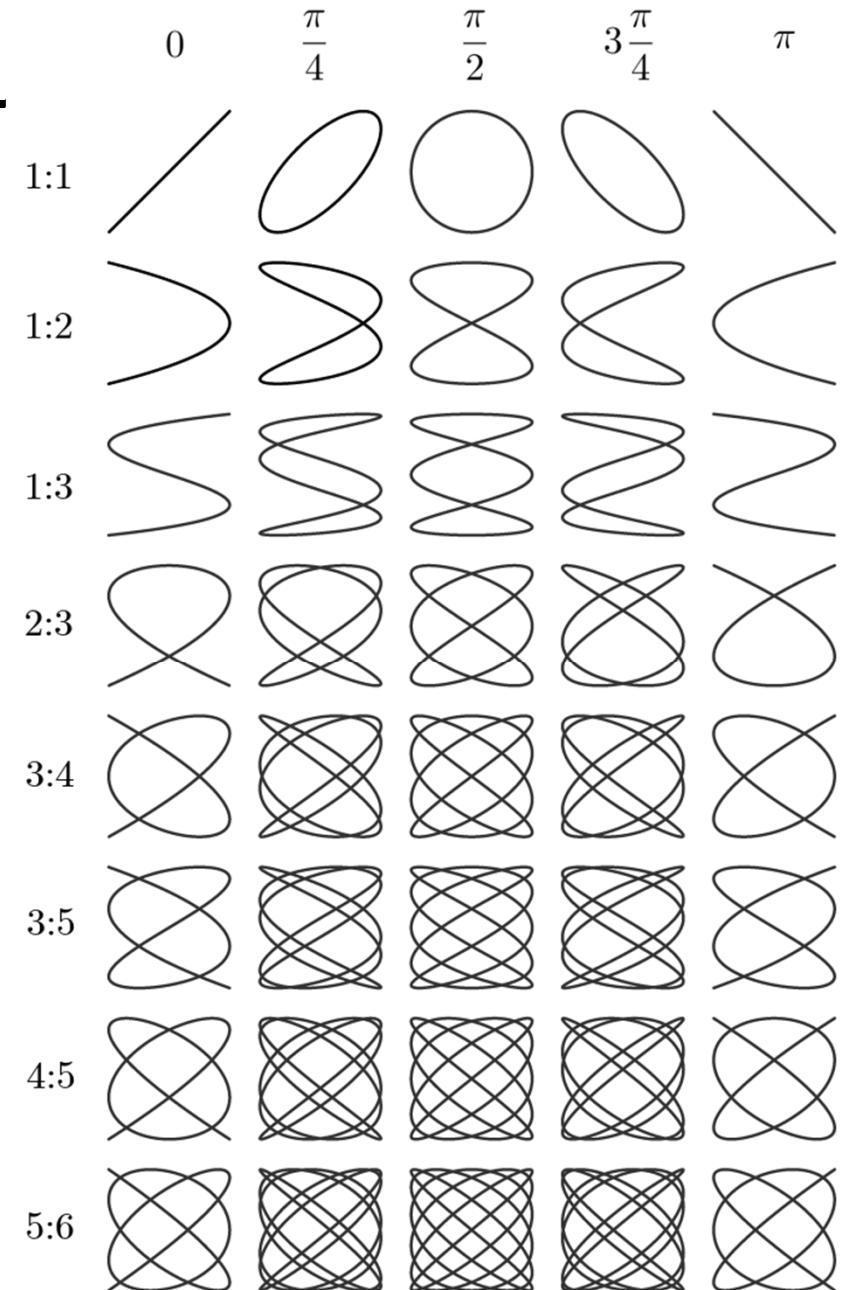
$$x = A \sin(at + \delta), \quad y = B \sin(bt),$$

which describe the superposition of two perpendicular oscillations in x and y directions of different angular frequency ( $a$  and  $b$ ). The resulting family of curves was investigated by Nathaniel Bowditch in 1815, and later in more detail in 1857 by Jules Antoine Lissajous (for whom it has been named). Such motions may be considered as a particular kind of complex harmonic motion.

A Lissajous figure, made by releasing sand from a container at the

[https://en.wikipedia.org/wiki/Lissajous\\_curve](https://en.wikipedia.org/wiki/Lissajous_curve)

[https://en.wikipedia.org/wiki/Lissajous\\_curve#/media/File:Lissajous\\_relaciones.png](https://en.wikipedia.org/wiki/Lissajous_curve#/media/File:Lissajous_relaciones.png)



# Reactive load: I vs. V is ellipse in (V,I) plane.

$$V_{DS}(t) = V_{DC} + V_{peak} \cos(\omega t + \theta)$$

$$I_{D,\text{internal}}(t) = I_{DC} + I_{peak} \cos(\omega t)$$

$\theta = 0^\circ$  or  $180^\circ$

→  $I_{D,\text{internal}}$  vs.  $V_{DS,\text{internal}}$  is straight line in ( $V, I$ ) plane

$$\rightarrow Y_{L,\text{internal}} = (I_{peak} / V_{peak}) \exp(-j\theta) = G_{L,\text{internal}} + jB_{L,\text{internal}}$$

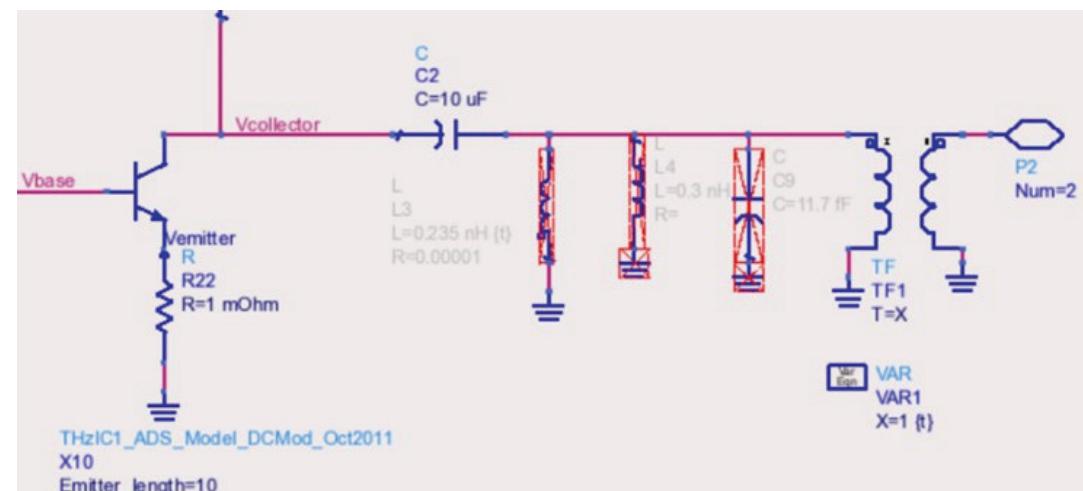
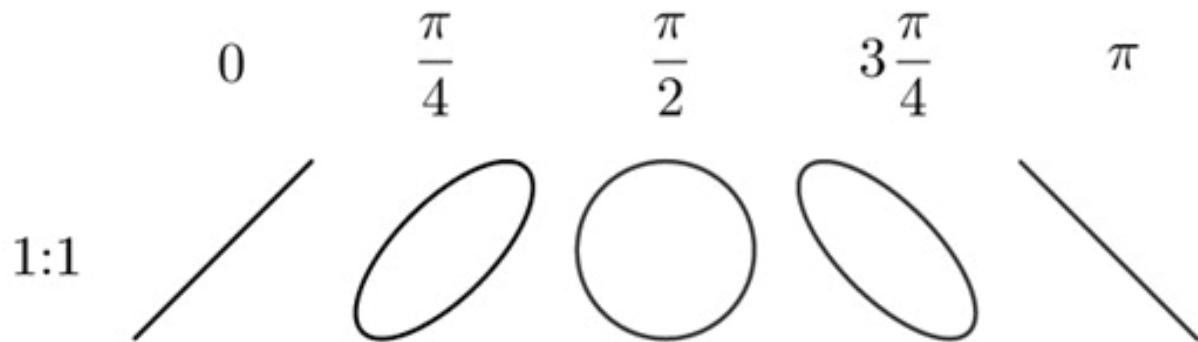
$Y_{L,\text{internal}} = G_{L,\text{internal}} + j0 \text{ S}$  is purely real admittance.

$\theta \neq 0^\circ$  and  $\neq 180^\circ$

→  $I_D$  vs.  $V_{DS}$  is ellipse in ( $V, I$ ) plane

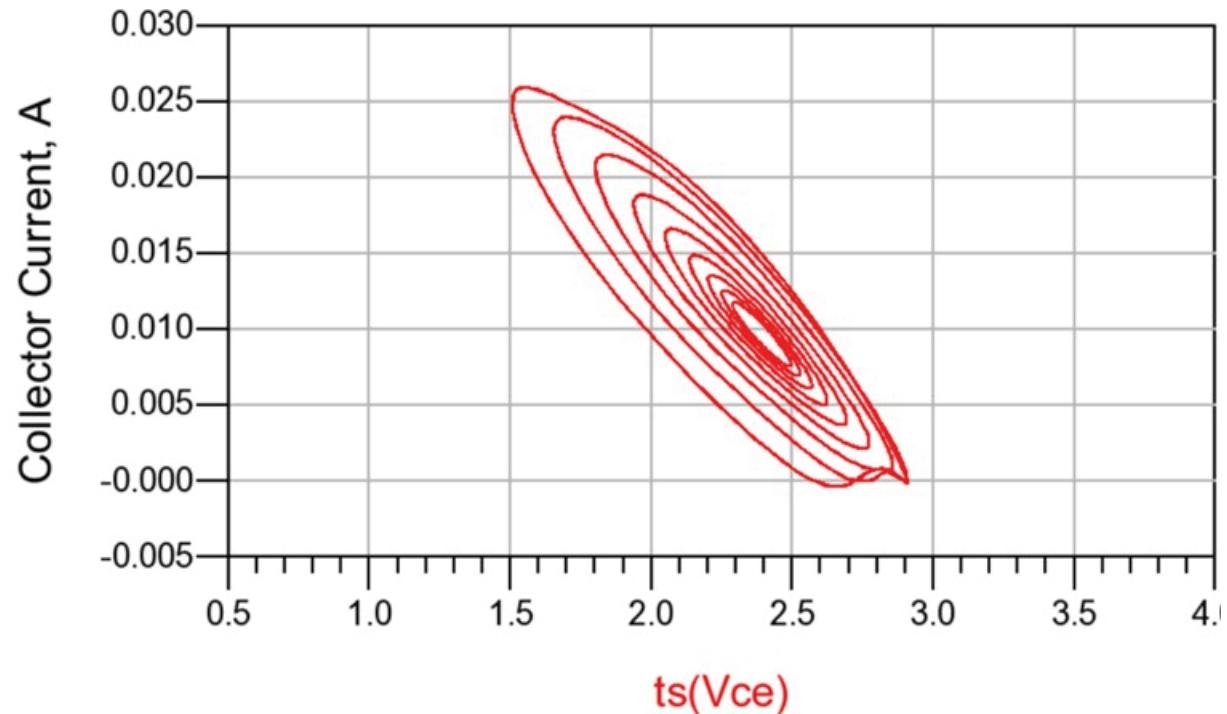
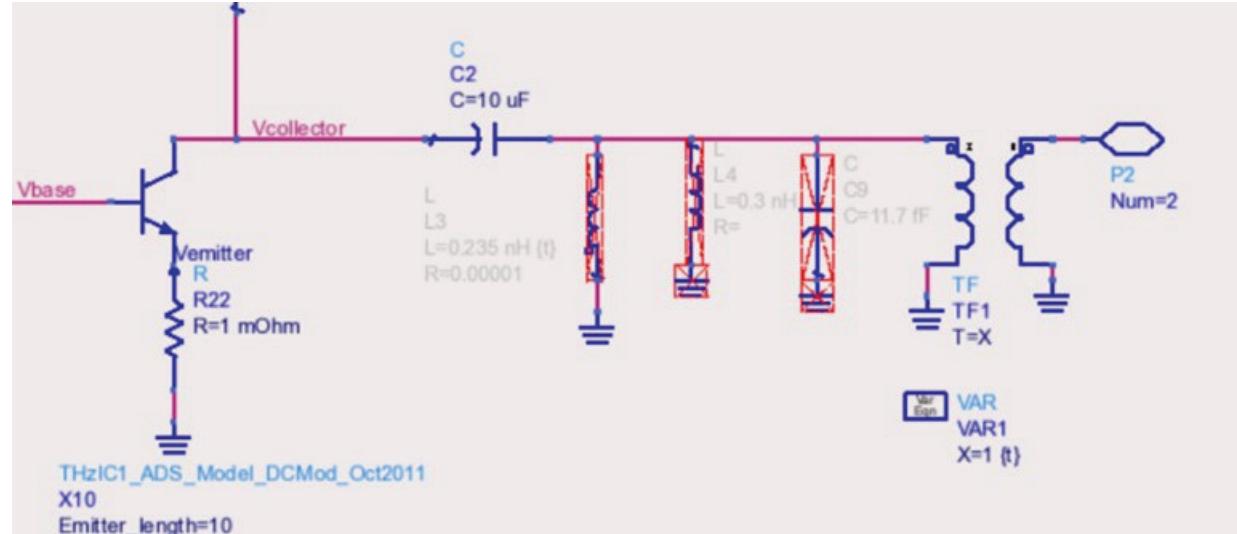
$$\rightarrow Y_{L,\text{internal}} = (I_{peak} / V_{peak}) \exp(-j\theta)$$

$Y_{L,\text{internal}} = G_{L,\text{internal}} + jB_{L,\text{internal}}$  with  $B_{L,\text{internal}} \neq 0 \text{ S}$  ; nonzero load susceptance



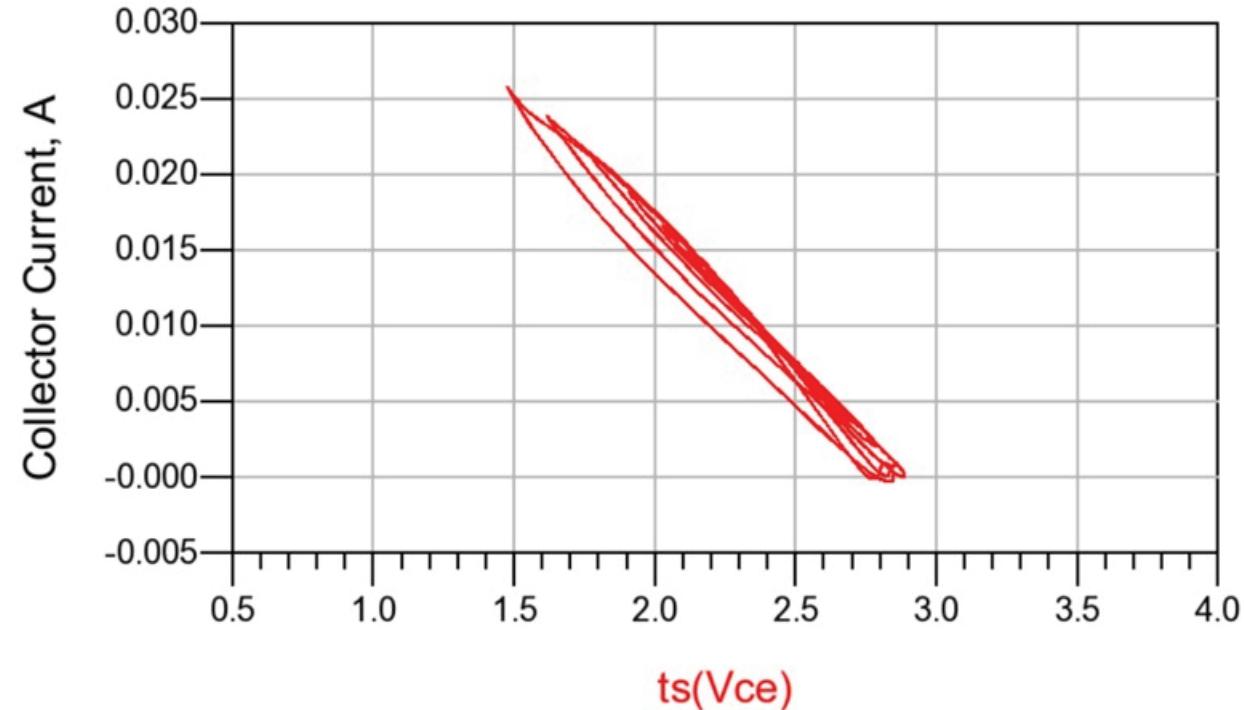
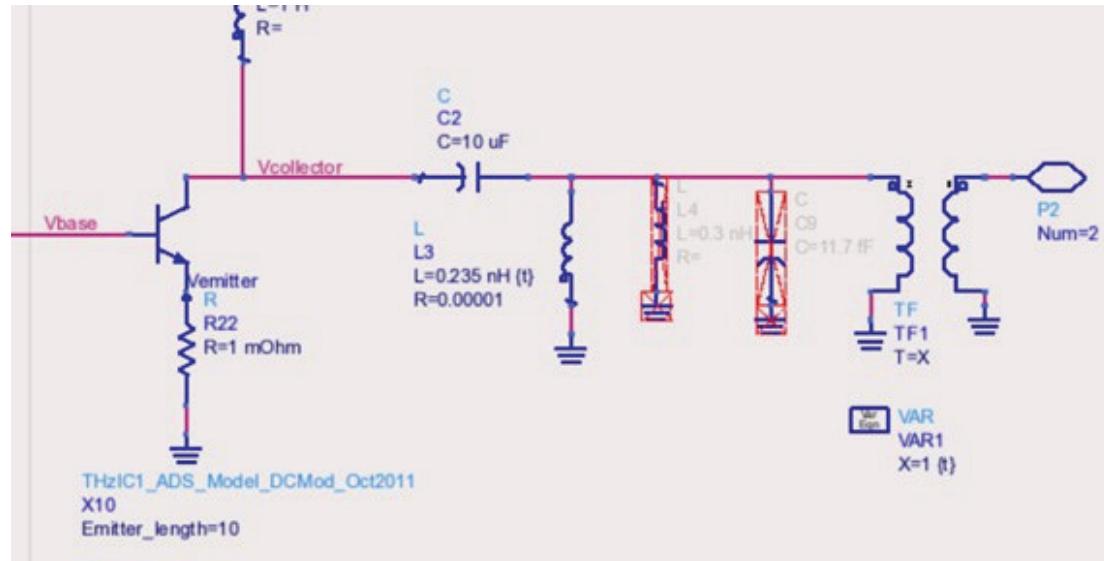
# Power Amplifier Design Example

With no inductive tuning, and with a 1:1 transformer ratio, the loadline initially looks like this:



# Power Amplifier Design Example

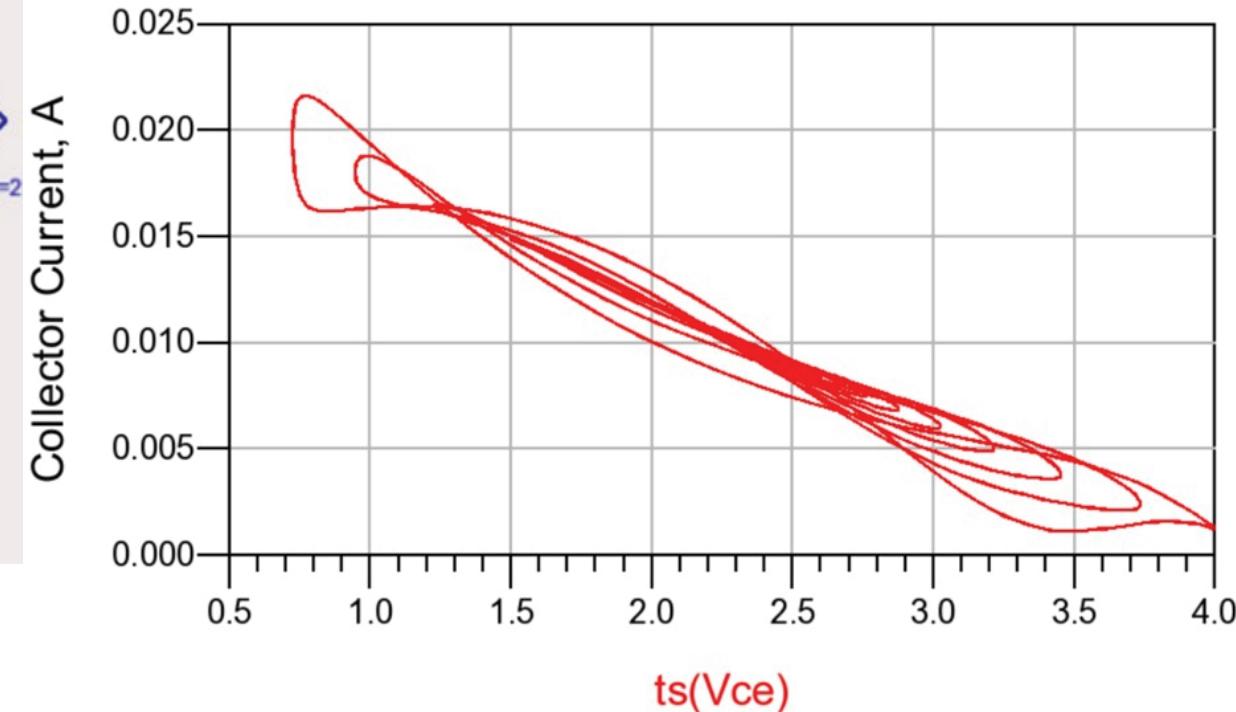
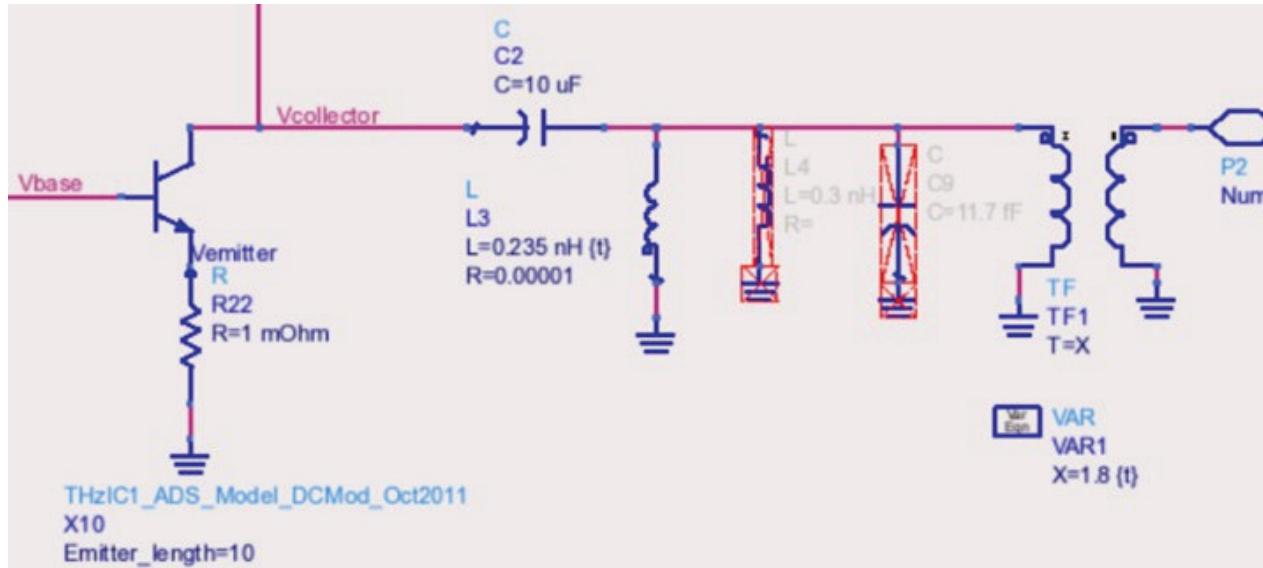
First add the inductive tuning, adjusting the shunt inductance,  $L$ , to eliminate loadline looping



# Power Amplifier Design Example

Then adjust the transformer ratio to obtain a loadline passing through the target endpoints  $(V_{\min}, I_{\max})$  and  $(V_{\max}, I_{\min} = 0A)$

We had expected a straight line. The looping (3 per cycle) is 3rd harmonic generation (consider again the Lissajous patterns)

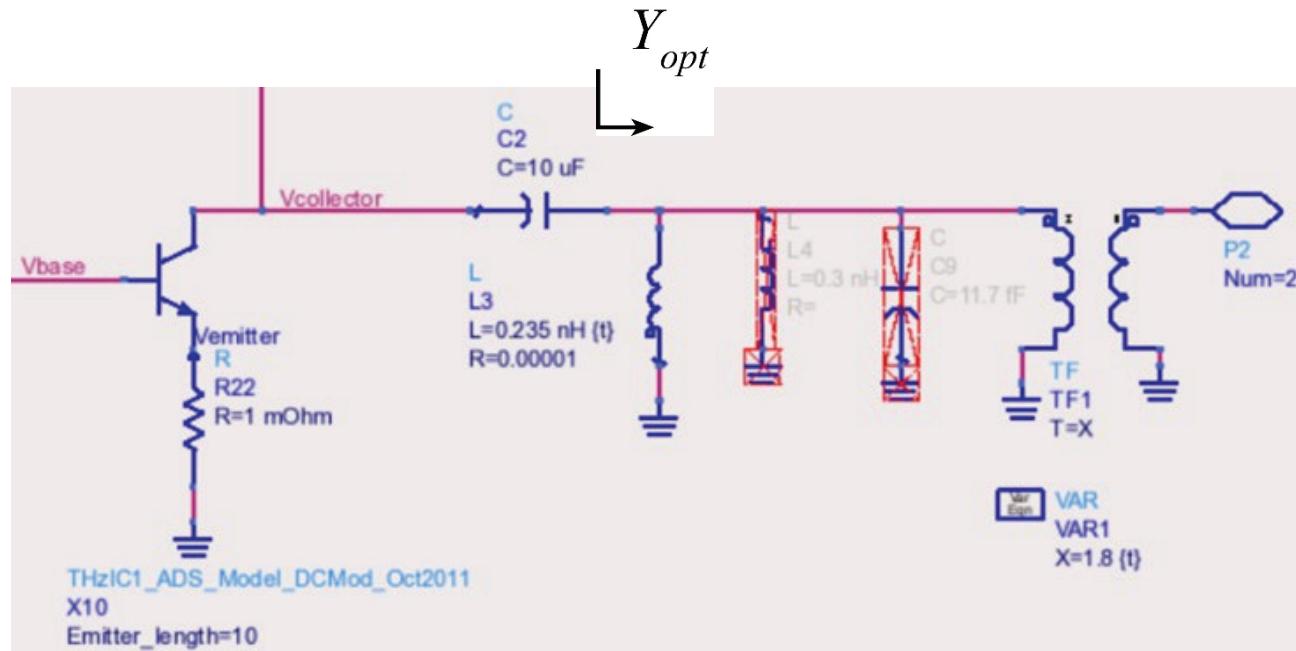


# Power Amplifier Design Example

We have now determined  $Y_{opt}$ .

In this example:

$$\begin{aligned} Y_{opt} &= \frac{1}{j\omega L} + \left( \frac{1}{\text{transformer turns ratio}} \right)^2 \frac{1}{50\Omega} \\ &= \frac{1}{j\omega L} + \left( \frac{1}{X} \right)^2 \frac{1}{50\Omega} \\ &= \frac{1}{j \cdot 2\pi \cdot 94 \text{ GHz} \cdot 0.235 \text{ nH}} + \left( \frac{1}{1.8} \right)^2 \frac{1}{50\Omega} \end{aligned}$$

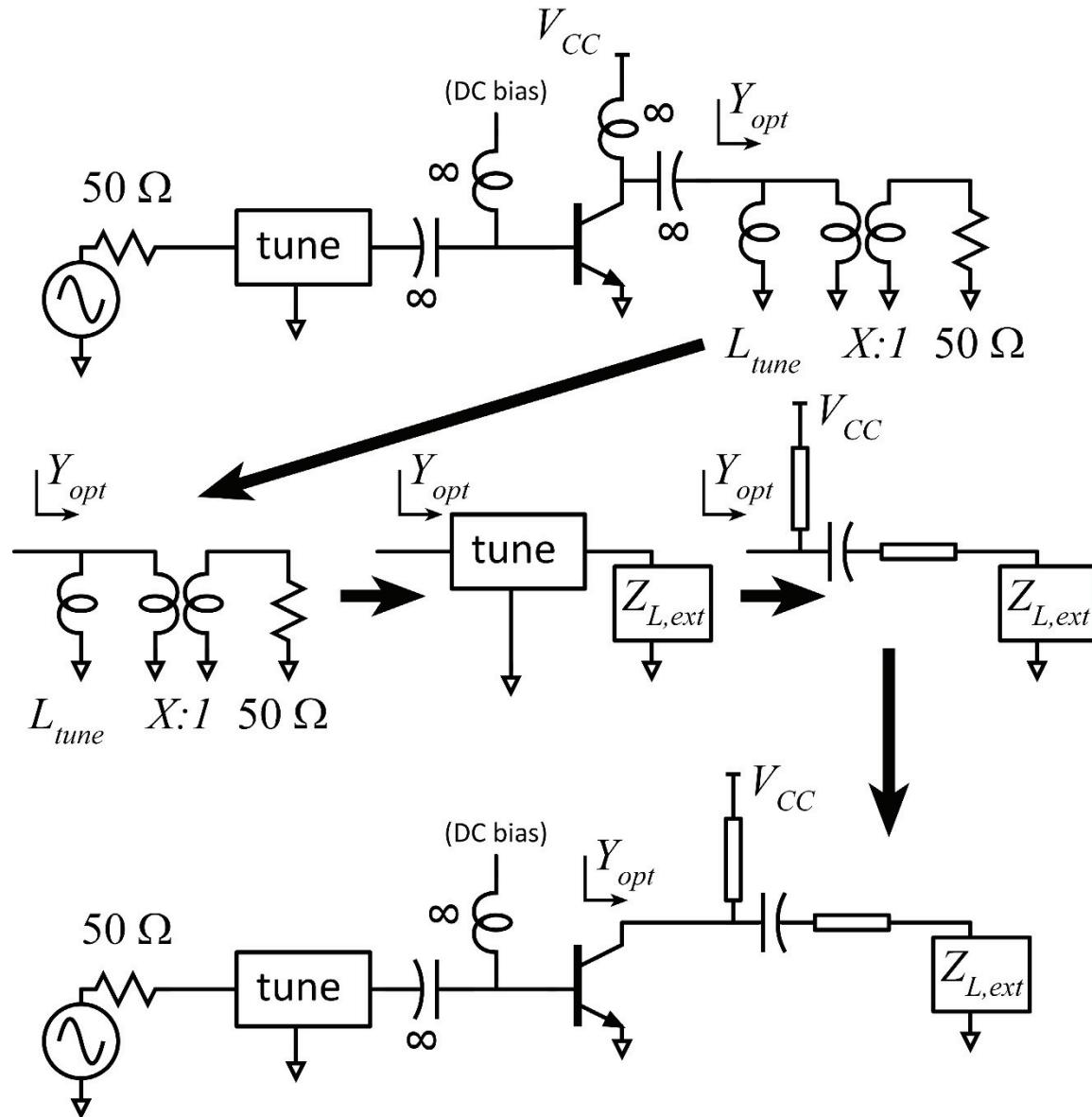


# Power Amplifier Design Example

With  $Y_{opt}$  now determined,

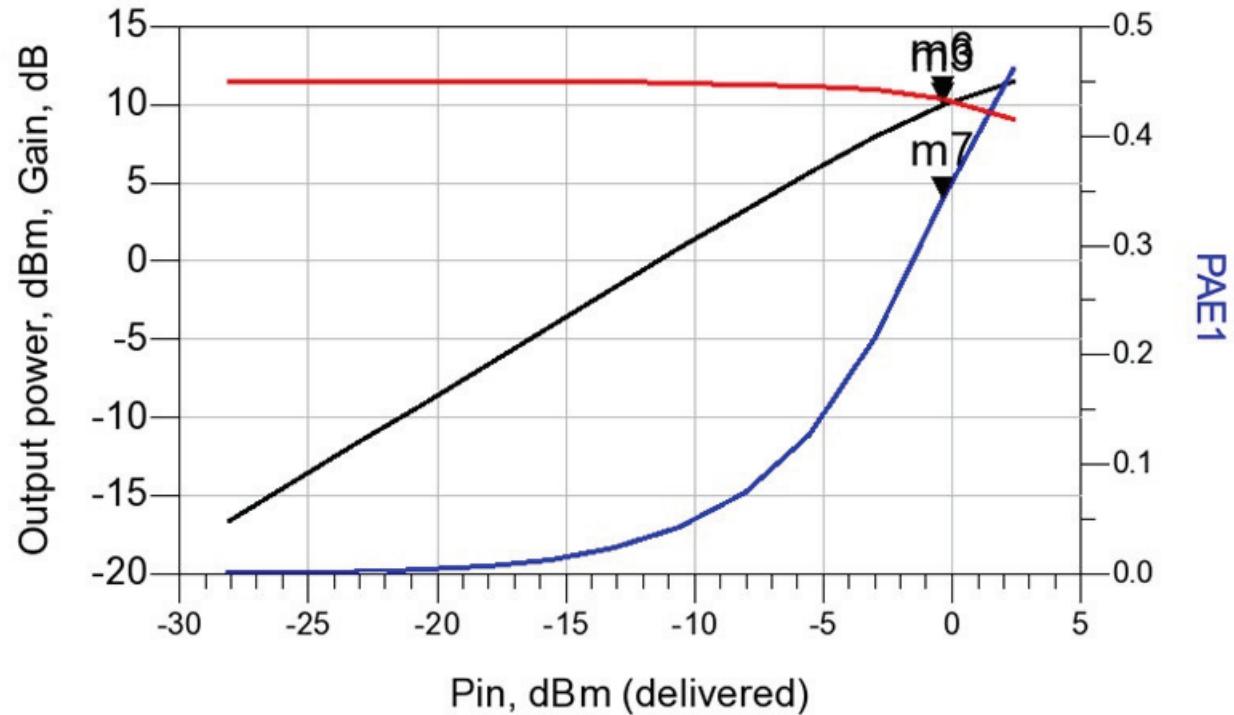
We then design a practical output network which provides this  $Y_{opt}$ .

We then add this to the PA.



# Power Amplifier Design Example

Here is our final simulated performance



```
m6  
indep(m6)=-0.394  
plot_vs(pout-Pin_deliv_dBm, Pin_deliv_dBm)=10.379
```

```
m7  
indep(m7)=-0.394  
plot_vs(PAE1, Pin_deliv_dBm)=0.342
```

```
m3  
indep(m3)=-0.394  
plot_vs(pout, Pin_deliv_dBm)=9.985
```

Other issues:

Input matching network

Out-of-band stabilization

(ece145a)

Often: filters

to suppress 2nd, 3rd harmonics.

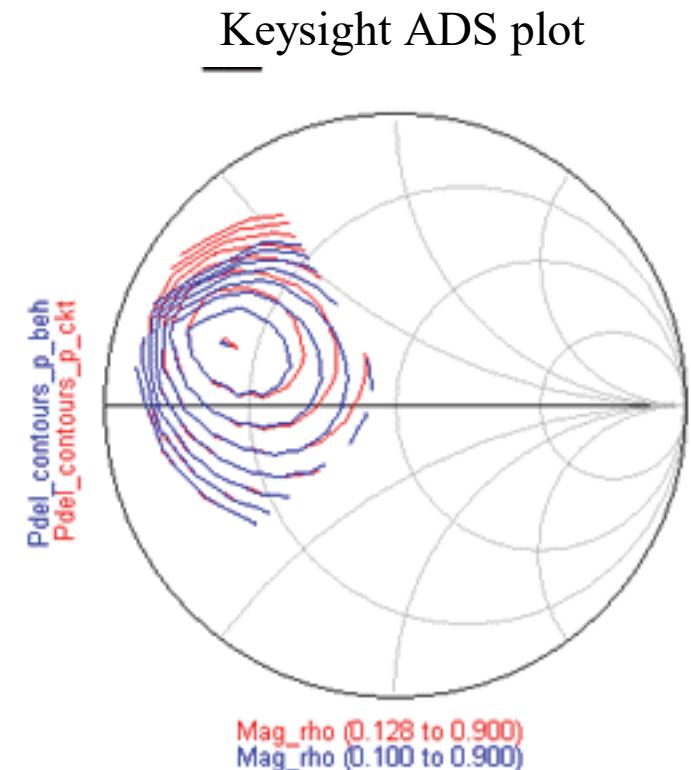
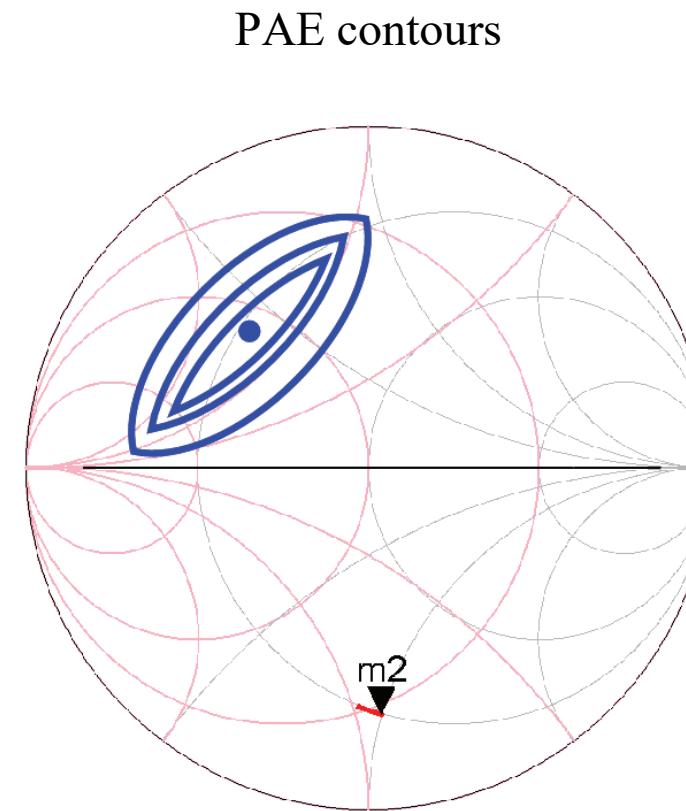
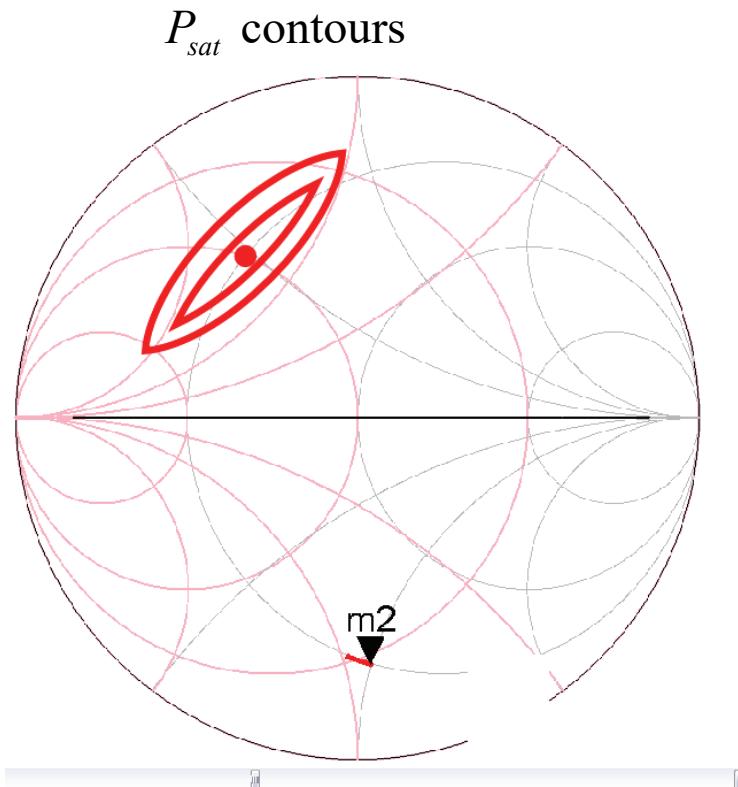
# Load pull method

We can also empirically determine (in CAD, or with instruments) the load impedance giving the largest saturated  $P_{out}$ .

We then use this impedance for our PA design

Empirical method. Simply contour plots of maximum  $P_{out}$  and peak PAE

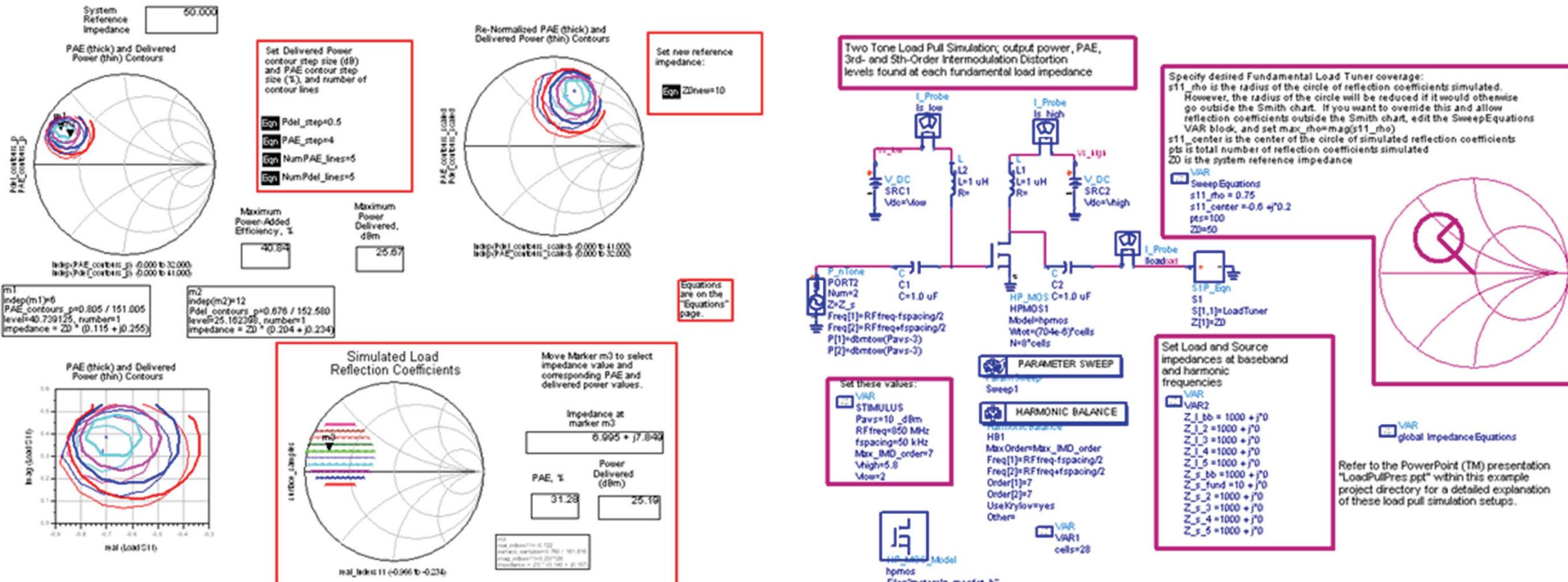
Steven Cripps has a paper that shows that these contours are sets of intersecting ellipses on the Smith chart.



# Load pull method

We can also empirically determine (in CAD, or with instruments) the load impedance giving the largest saturated  $P_{out}$ .

We then use this impedance for our PA design. Keysight ADS and other CAD packages have pre-configured test benches to do this.



# What about the input network ?

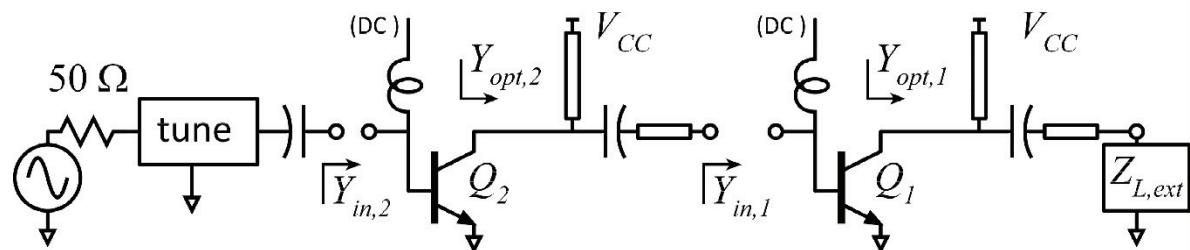
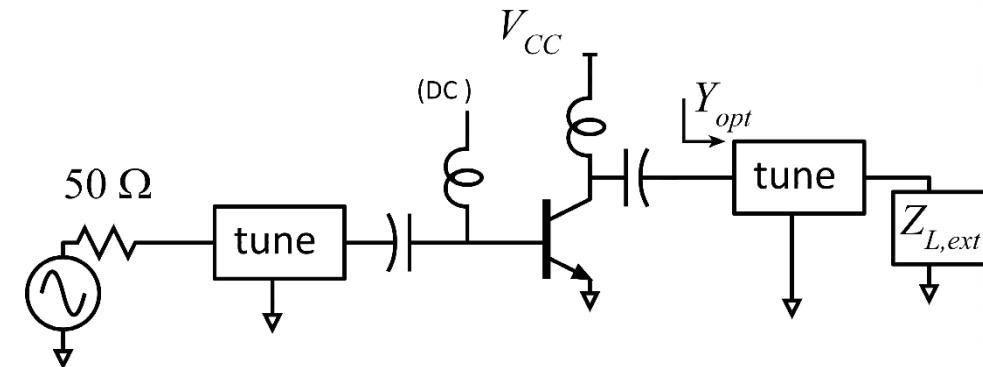
Thus far, we have considered only the output tuning network.

What about the input tuning network ?

To answer this, consider a multi-stage power amplifier.

Each cascaded stage, to be most efficient, must be loaded in its own optimum impedance

The input matching network design of  $Q_1$  is now clear.



Considering a multi-stage power amplifier however immediately raises other concerns...

# Multi-stage power amplifiers: power distribution

Assume for  $Q_1$  :

@  $P_{out1} = 0$  dBm (small signal),  $P_{in1} = -12$  dBm (12 dB gain) and PAE is low.

@  $P_{out1} = 17$  dBm,  $P_{in1} = 6$  dBm (11 dB gain; 1 dB compression) and PAE=20%

@  $P_{out1} = 20$  dBm,  $P_{in1} = 10$  dBm (10 dB gain; 2 dB compression) and PAE=PAE<sub>MAX</sub>=30%

**Design for maximum overall PAE:**

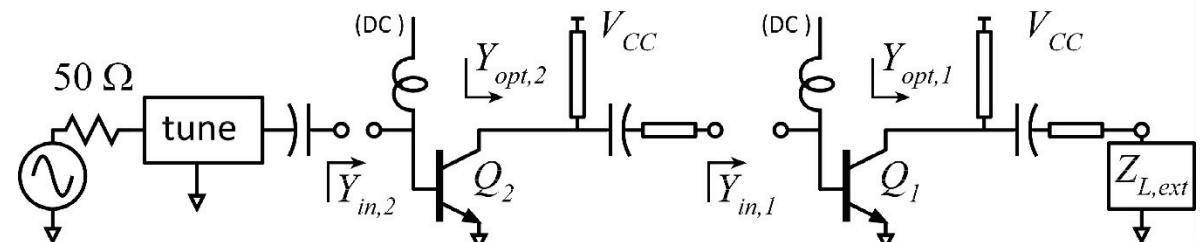
both stages must simultaneously operate at peak PAE.

→  $Q_2$  must reach peak PAE with  $P_{out2} = 10$  dBm.

→  $A_{E2} = A_E / 10$

→ @  $P_{out2} = 10$  dBm,  $P_{in2} = 0$  dBm (10 dB gain) and PAE=PAE<sub>MAX</sub>=30%

→ @  $P_{out2} = 0$  dBm (small signal),  $P_{in2} = -12$  dBm (12 dB gain) and PAE is low.



2-stage amplifier:

@  $P_{out1} = 20$  dBm,  $P_{in2} = 0$  dBm (20 dB total gain); 30% PAE for each stage; 30% overall PAE

@  $P_{out1} = 0$  dBm (small signal),  $P_{in2} = -24$  dBm (24 dB gain).

4 dB gain compression between small-signal and peak PAE

# Multi-stage power amplifiers: driver stage sizing

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Assume for  $Q_1$ :

- @  $P_{out1} = 0 \text{ dBm}$  (small signal),  $P_{in1} = -12 \text{ dBm}$  (12 dB gain) and PAE is low.
- @  $P_{out1} = 17 \text{ dBm}$ ,  $P_{in1} = 6 \text{ dBm}$  (11 dB gain; 1 dB compression) and PAE=20%
- @  $P_{out1} = 20 \text{ dBm}$ ,  $P_{in1} = 10 \text{ dBm}$  (10 dB gain; 2 dB compression) and PAE=PAE<sub>MAX</sub>=30%

Design for less gain compression:

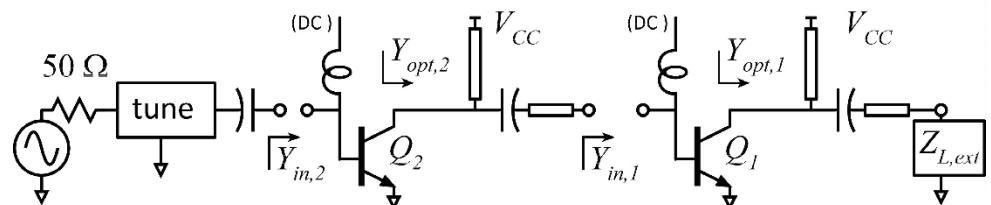
size  $Q_2$  such that  $Q_2$  has 1 dB compression when  $Q_1$  has 2 dB compression.

$\rightarrow Q_2$  must reach 1 dB gain compression with  $P_{out2} = 10 \text{ dBm}$ .

$\rightarrow A_{E2} = A_E / 5$

$\rightarrow @P_{out2} = 10 \text{ dBm}$ ,  $P_{in2} = -1 \text{ dBm}$  (11 dB gain) and PAE=20%

$\rightarrow @P_{out2} = 0 \text{ dBm}$  (small signal),  $P_{in2} = -12 \text{ dBm}$  (12 dB gain) and PAE is low.



2-stage amplifier:

@  $P_{out1} = 20 \text{ dBm}$ ,  $P_{in2} = -1 \text{ dBm}$  (21 dB total gain); PAE<sub>1</sub> = 30%; PAE<sub>2</sub> = 20%  $\rightarrow$  PAE<sub>overall</sub> =  $(P_{out1} - P_{in2}) / (P_{DC1} + P_{DC2}) = 28.7\%$

@  $P_{out1} = 0 \text{ dBm}$  (small signal),  $P_{in2} = -24 \text{ dBm}$  (24 dB gain).

3 dB gain compression between small-signal and peak PAE

# Over-sized driver stages

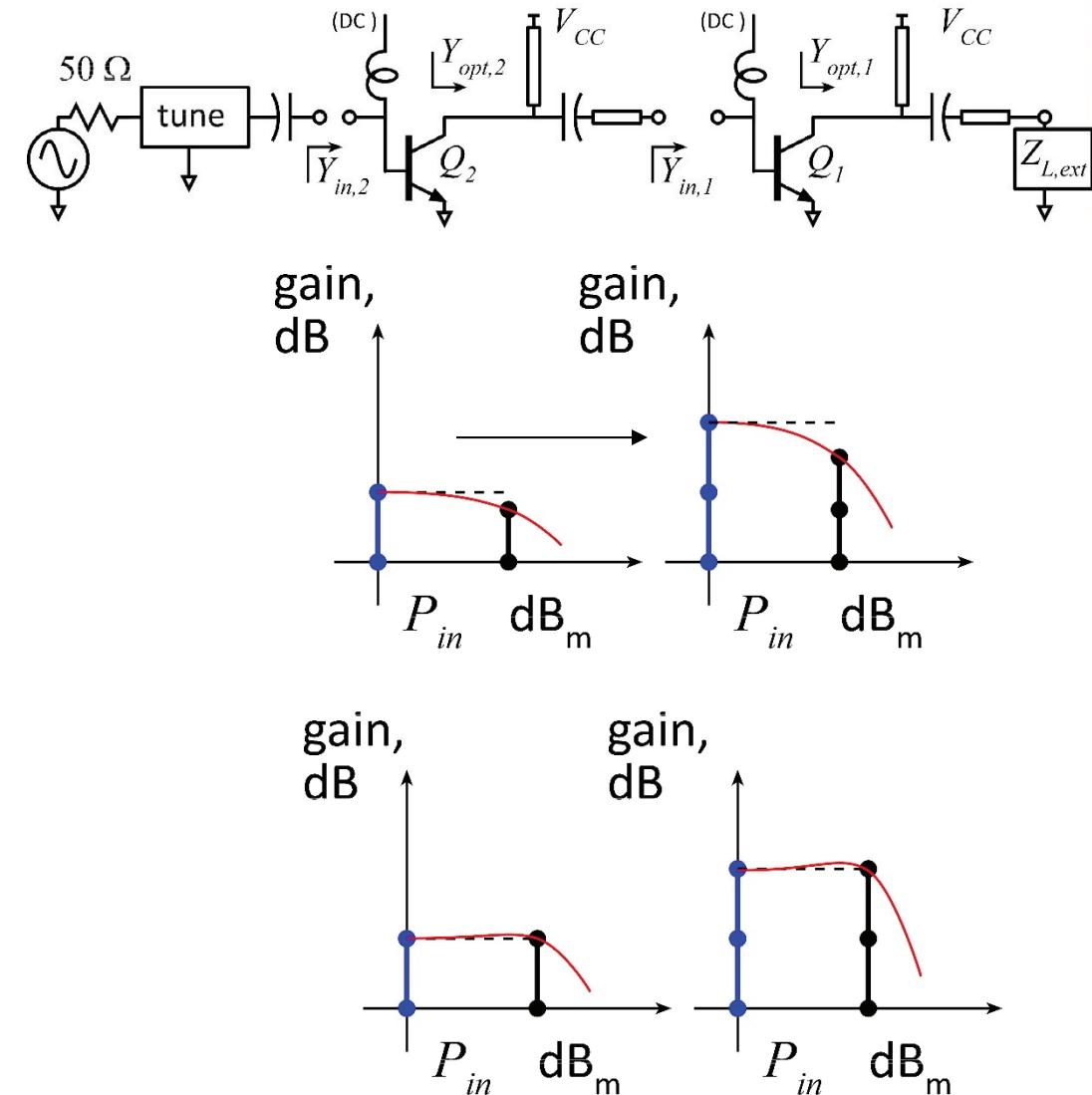
In a multi-stage amplifier, if driver stages are sized for peak PAE, then gain compression at PAE will increase rapidly as the number of stages is increased to increase the overall gain.

Over-sizing the driver stages will reduce the accumulated gain compression, but will reduce the overall PAE.

Adaptive bias circuits can be used in some systems to flatten the  $\text{dB}(P_{out})$  vs.  $\text{dB}(P_{in})$  characteristics.

Overall digital pre-distortion can also be applied.

These are advanced topics, but important ones.



# Multi-stage PAs: $I_{\max}$ , $Z_{\text{load}}$ , transistor size problems

Consider this 2-stage amplifier:

$$P_{out1} = 1000 \text{ mW}; P_{in1} = 100 \text{ mW};$$

$$P_{out2} = 100 \text{ mW}; P_{in2} = 10 \text{ mW}$$

Assume  $(V_{\max} - V_{\min}) = 4$  Volts and recollect that

$$Z_L = 1/Y_L = R_L + j0\Omega = (V_{\max} - V_{\min})/I_{\max}$$

$$I_{DC} = I_{\max}/2$$

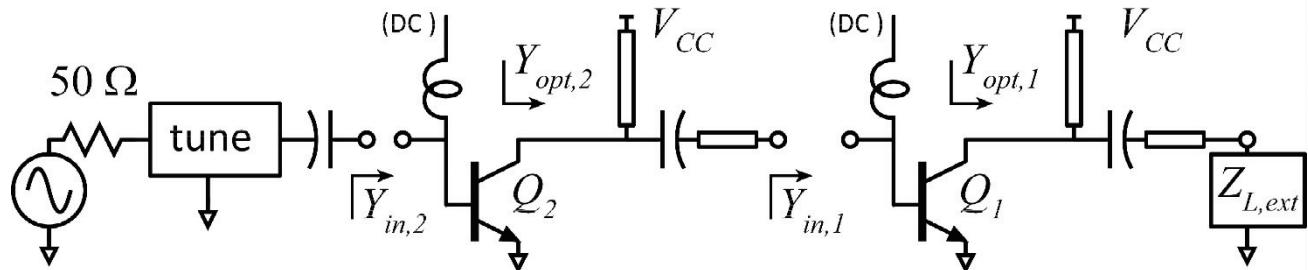
$$P_{RF,\max} = (V_{\max} - V_{\min})I_{\max}/8 = (V_{\max} - V_{\min})^2/8R_L$$

BJTs or FETs with  $J_{\max} = 2 \text{ mA}/\mu\text{m}$  current density

Then

Stage 1:  $I_{DC} = 1 \text{ A}$ ;  $R_L = 2 \Omega$ ;  $N_G W_G$  or  $N_E L_E = 1000 \mu\text{m}$

Stage 2:  $I_{DC} = 100 \text{ mA}$ ;  $R_L = 20 \Omega$ ;  $N_G W_G$  or  $N_E L_E = 100 \mu\text{m}$



Extremely low load impedances, extremely high currents, extremely large transistor sizes.

This is the microwave power-combining problem