ECE 145C / 218C, notes set xx: filters (very quick summary)

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Filters (a very quick summary)

Filter design is an old but intellectually rich field.

What: "network synthesis": circuit diagram computed from transfer function.

When: ca. 1900 – 1940 (very roughly)

Who: Cauer, Foster, Butterworth, ...

Wikipedia has excellent summaries,
and excellent links into the archival literature.

Our purpose: rough understanding, and to be aware of the material

Butterworth Low-Pass Filter (1)

https://en.wikipedia.org/wiki/Butterworth_filter

Butterworth, S. (1930). "On the Theory of Filter Amplifiers"
Experimental Wireless and the Wireless Engineer. 7: 536–541

n^{th}-order (n-pole) Butterworth filter

$$H_n(s)H_n^*(s)=\frac{1}{1+(s/j\omega_0)^{2n}}$$

Poles lie on the complex circle of radius ω_0:

$$(s_p/j\omega_0)^{2N}=-1\Rightarrow s_p/\omega_0=j(-1)^{1/2n}=e^{j\pi/2}\left(e^{j\pi}e^{jm2\pi}\right)^{1/2n}$$

$$(s_p/j\omega_0)^{2N}=e^{j\pi/2}e^{j\pi/2n}e^{j\pi m/n}$$

.....where m is any integer.

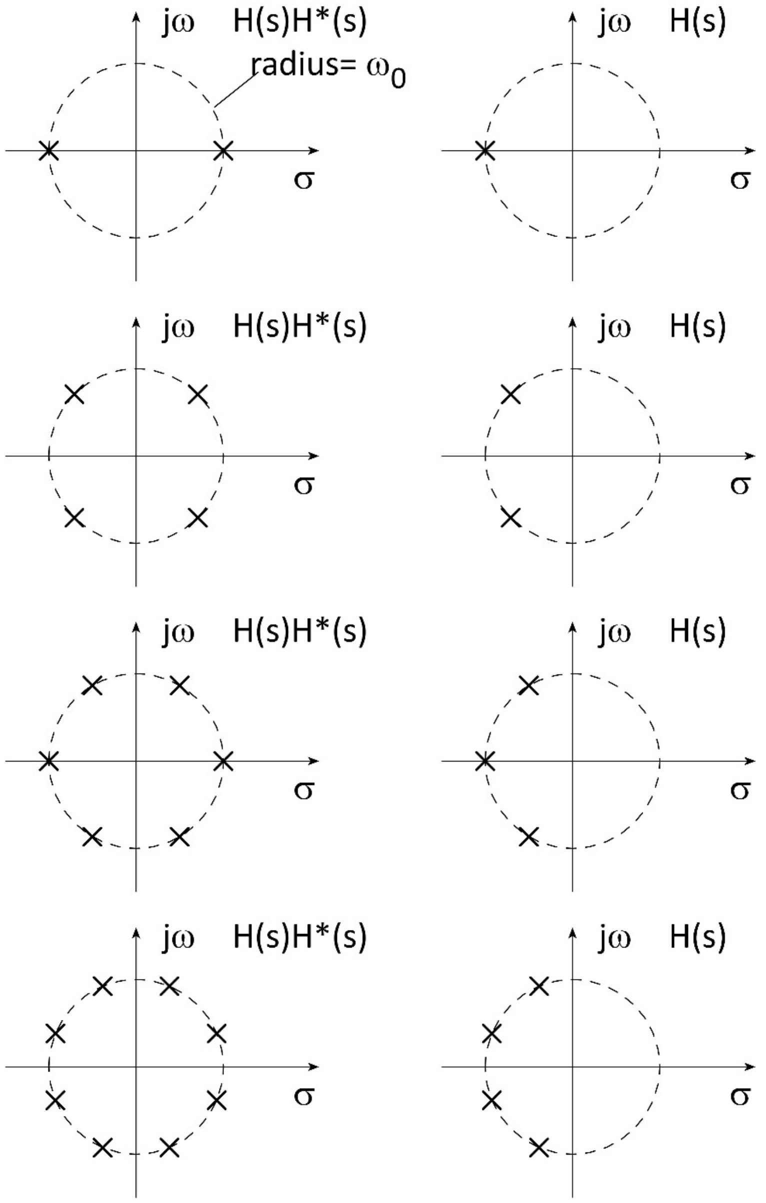
...angles are at $\pi/2+\pi/2n+2\pi m/2n$ radians, i.e.

...angles are at $90^0+180^0/2n+(180^0)m/2n$ degrees

There above gives $2n$ solutions for s_p,

but there are only N poles in H_n(s);

$\rightarrow$ pick the N solutions in the RHP (stable poles only).



Transfer function magnitudes

If $H(s)=\dfrac{(1+s\tau_{z1})(1+s\tau_{z2})(1+s\tau_{z3})...}{(1+s\tau_{p1})(1+s\tau_{p1})(1+s\tau_{p2})...}=\dfrac{\tau_{z1}\tau_{z2}\tau_{z3}...}{\tau_{p1}\tau_{p2}\tau_{p3}...}\dfrac{(s-s_{z1})(s-s_{z2})(s-s_{z3})...}{(s-s_{p1})(s-s_{p2})(s-s_{p3})....}$

where $s_{z1}=-1/\tau_{z1}$ etc.

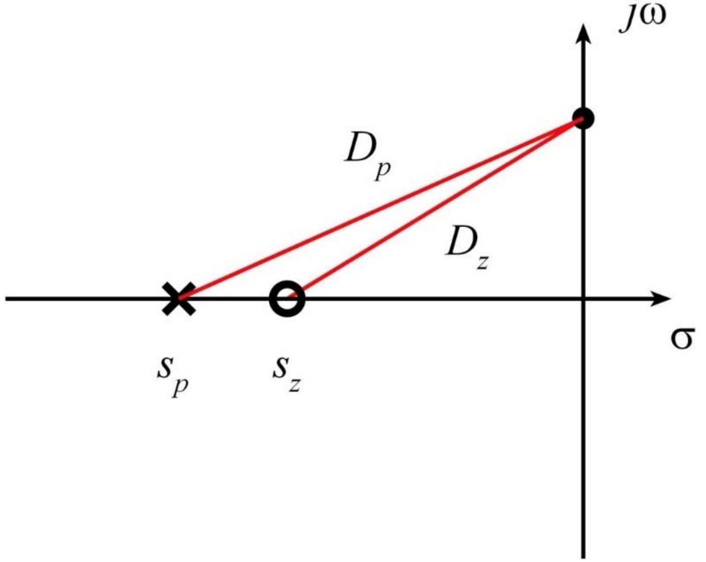
Then

$$\|H(s)\|=\frac{\tau_{z1}\tau_{z2}\tau_{z3}...}{\tau_{p1}\tau_{p2}\tau_{p3}...}\frac{\|s-s_{z1}\|\cdot\|s-s_{z2}\|\cdot\|s-s_{z3}\|...}{\|s-s_{p1}\|\cdot\|s-s_{p2}\|\cdot\|s-s_{p3}\|...}=\frac{\tau_{z1}\tau_{z2}\tau_{z3}...}{\tau_{p1}\tau_{p2}\tau_{p3}...}\frac{D_{z1}D_{z2}D_{z3}...}{D_{p1}D_{p2}D_{p3}}$$

where D_{zi} is the distance to s_{zi} and D_{pi} is the distance to s_{pi}.

Example with one pole and one zero

$$\|H(s)\|=\frac{\|1+s\tau_{zero}\|}{\|1+s\tau_{pole}\|}=\frac{\tau_z}{\tau_p}\frac{\|s-s_z\|}{\|s-s_p\|}=\frac{\tau_z}{\tau_p}\frac{D_z}{D_p}$$



Butterworth Low-Pass Filter (2)

https://en.wikipedia.org/wiki/Butterworth_filter

Butterworth, S. (1930). "On the Theory of Filter Amplifiers" Experimental Wireless and the Wireless Engineer. 7: 536–541

Transfer function is maximally flat,
meaning that for the n^{th}-order filter

$$\frac{d^{n-1} \| H(j\omega) \|}{d\omega^{N-1}} = 0$$

Transfer function cutoff becomes steeper
as the order N is increased,
but response is without ripples in passband.

$\| H(j2\pi f) \|$

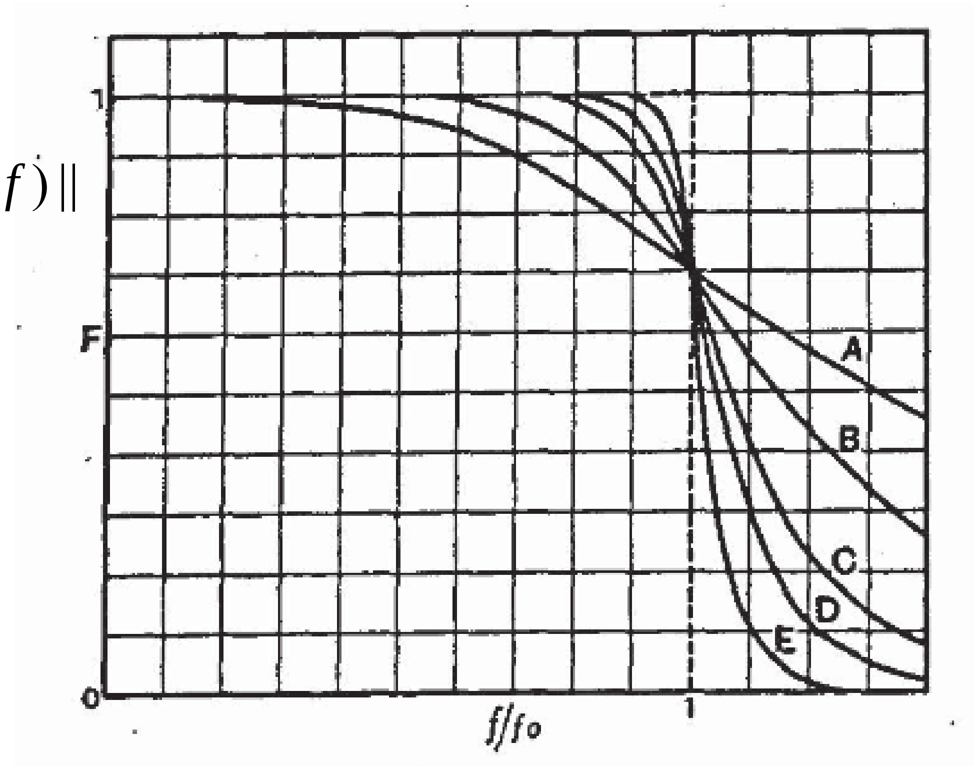


Figure from Butterworth, 1930

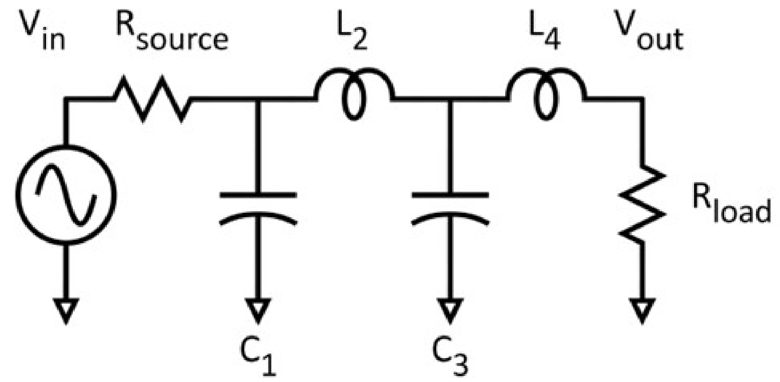
Butterworth filter values: Cauer synthesis

Given $R_{source} = R_{load} = R$

For an nth-order filter with $k = 1,...,n$

$$C_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\frac{1}{\omega_0 R} \text{ for } k = \text{odd}$$

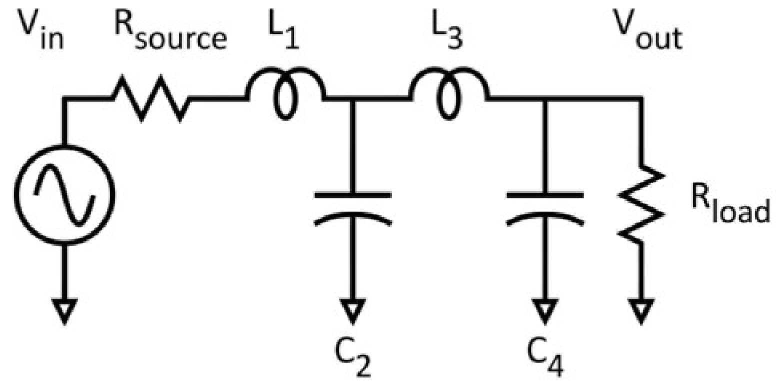
$$L_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\frac{R}{\omega_0} \text{ for } k = \text{even}$$



Given $R_{source} = R_{load} = R$

$$L_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\frac{1}{\omega_0 R} \text{ for } k = \text{odd}$$

$$C_k = 2\sin\left[\frac{2k-1}{2n}\pi\right]\frac{R}{\omega_0} \text{ for } k = \text{even}$$



https://en.wikipedia.org/wiki/Butterworth_filter#Cauer_topology

Filter forms with Rsource=0 Ohms are also possible, but the above forms are useful in doubly-terminated transmission-line environments.

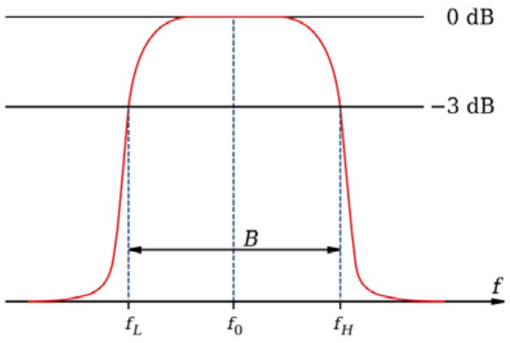
Band Pass filter transformed from Low-pass filter

Transformation:

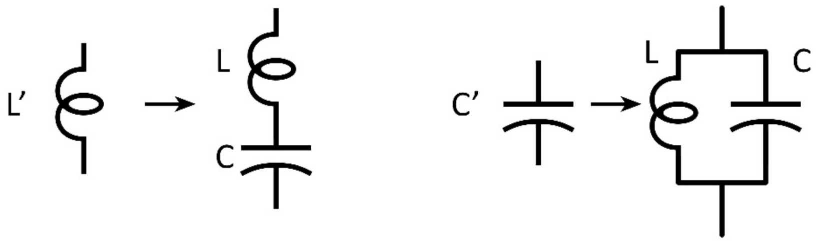
$$\frac{j\omega}{\omega'_c} \rightarrow Q\left(\frac{j\omega}{\omega_0}+\frac{\omega_0}{j\omega}\right)$$

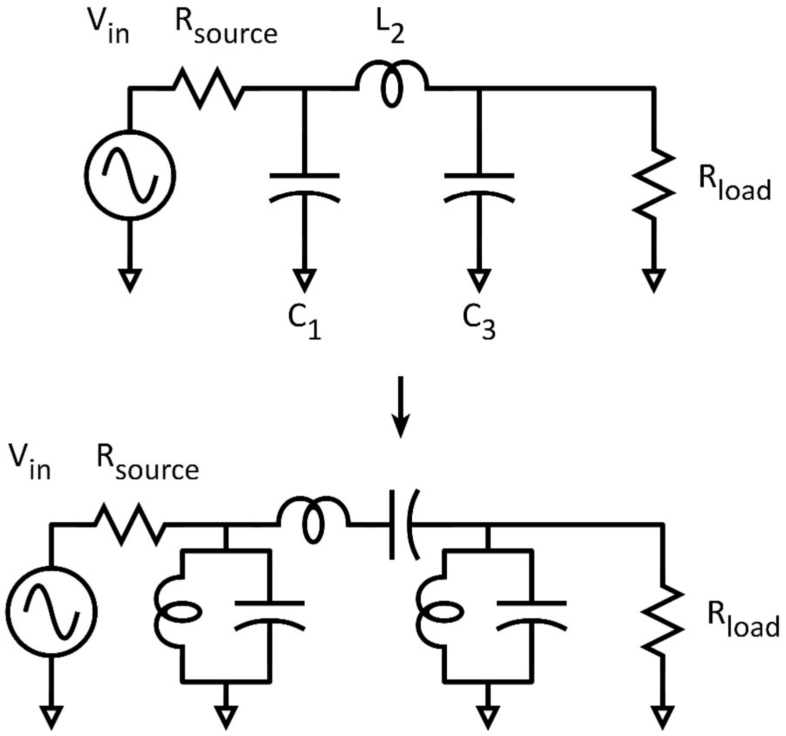
$$H_{new}\left(\frac{j\omega}{\omega'_c}\right)=H_{old}\left(Q\left(\frac{j\omega}{\omega_0}+\frac{\omega_0}{j\omega}\right)\right)$$

where ω'_c is the cutoff of the low-pass filter,
ω_0 is the band-pass center frequency and
$Q=\omega_0/\Delta\omega$, where $\Delta\omega$ is the band-pass filter 3-dB bandwidth.



From https://en.wikipedia.org/wiki/Prototype_filter





Inductors are transformed into series resonators with

$$L=\frac{\omega'_c Q}{\omega_0}L',\ C=\frac{1}{\omega'_c\,\omega_0 Q}\frac{1}{L'}$$

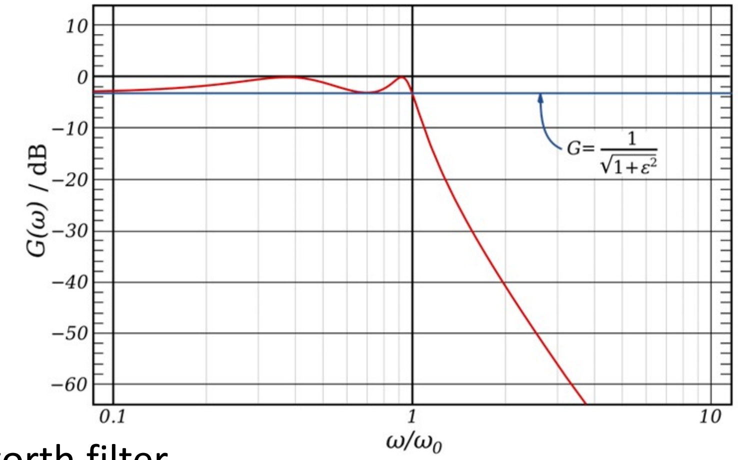
capacitors are transformed into parallel resonators with

$$C=\frac{\omega'_c Q}{\omega_0}C',\quad L=\frac{1}{\omega'_c\,\omega_0 Q}\frac{1}{C'}$$

Other Filter Transfer functions:

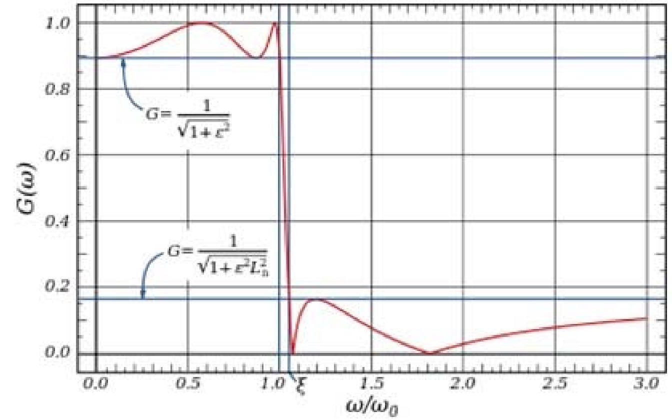
https://en.wikipedia.org/wiki/Chebyshev_filter#/media/File:Chebyshev_Type_I_Filter_Response_(4th_Order).svg

Chebyshev:
These provide steeper rolloff at the cutoff then a Butterworth filter
The disadvantage: more gain ripple in the passband.
Pole locations and element values can be found
in textbooks or on the web
see: https://en.wikipedia.org/wiki/Chebyshev_filter



Bessel-Thompson
these provide smaller group delay variation within the pass band than a Butterworth filter
The disadvantage: Less steep cut off than a Butterworth filter
Pole locations and element values can be found
in textbooks or on the web
see https://en.wikipedia.org/wiki/Bessel_filter

https://en.wikipedia.org/wiki/Elliptic_filter#/media/File:Elliptic_Filter_Response_(4th_Order).svg



Elliptic filters
These have zeros in the transfer function that provide steeper cut off
At the expense of ripple (peaks) In the transfer function in the stop band
see https://en.wikipedia.org/wiki/Elliptic_filter

What about Nyquist zero-ISI filters ?
Filters meeting Nyquist's vestigial sideband theore provide Zero intersymbol interference.
Neither Butterworth, Chebyshev, nor Bessel-Thompson filters precisely meet this criterion.
Appropriately selected Bessel-Thompson filters can come close…
… Or directly synthesize the desired filter from the desired root raise cosine transfer function using CAD tools.

Physical implementations

Physical inductors and capacitors, or transmission-lines and capacitors
common on IC filter implementations. Q is relatively limited.

Waveguide resonators
various design principles. Q's in the >1000 range
Large and heavy unless frequency is very high

Acoustics
Bulk acoustic waves
surface acoustic waves
very high Q's
operation to ~5-10 GHz.