• Notes Set 14: Circuit noise analysis methods

- Method of transposition of sources. Many examples worked
- En-In model of FET. Noise figure circles. Strong discrepancy between impedance and noise-matching.

ECE ______ Notes set 13 MJR

Circuit noise examples / Circuit noise Methods

First, an analytic technique which is

<u>very</u> <u>efficient</u> for some noise problems.

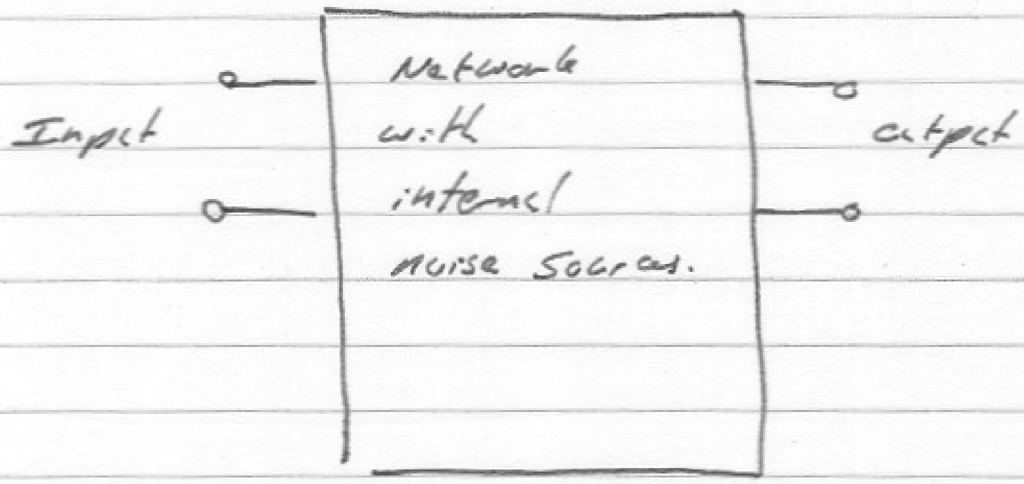
... and very inefficient for others...

Rodwells (unpublished) <u>method</u> of

<u>transposition of sources</u>

Rule:

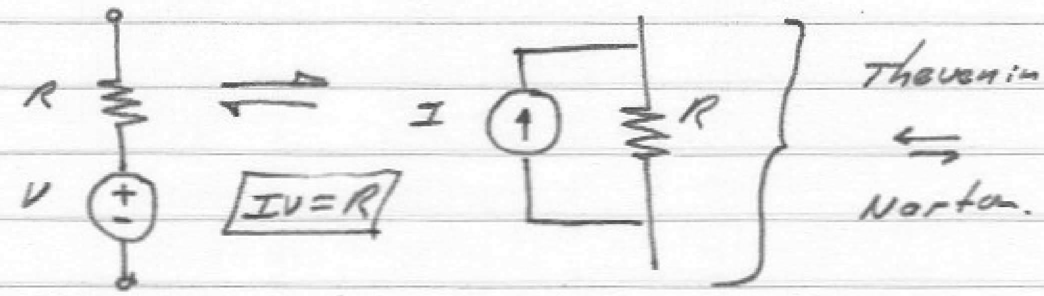
Separate problem into a generator (input) an output, and an amplifier.

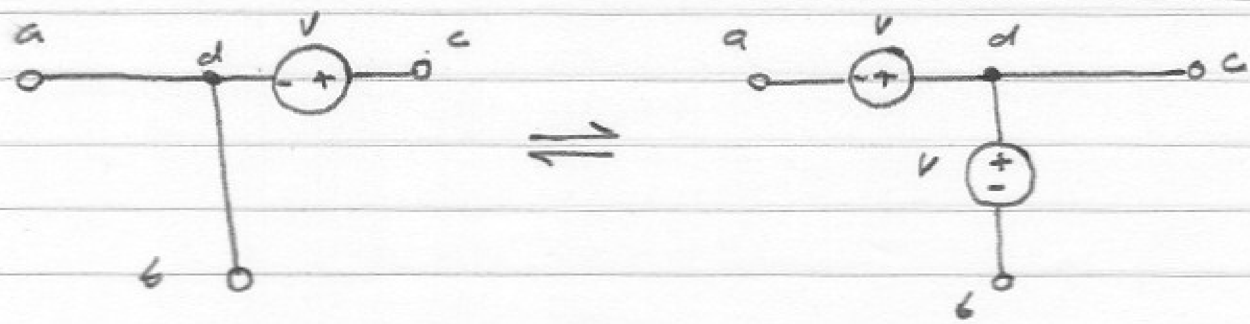


we desire to analyze the network to find the input-referred noise (voltage/current/power etc)

We can do this by a series of transformations to the internal network, transformations which may change many of the internal node voltages & branch currents, but which do not change the input or output node voltages or currents.

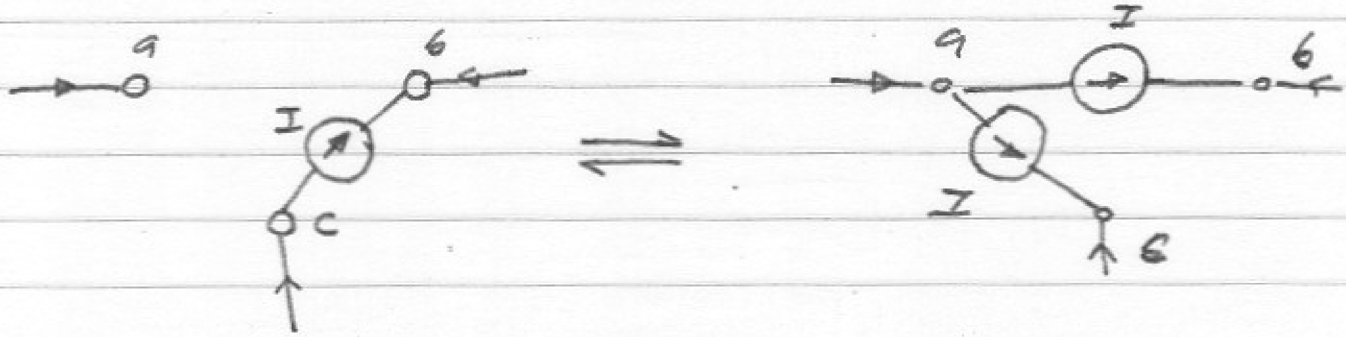
<u>allowed transformations:</u>



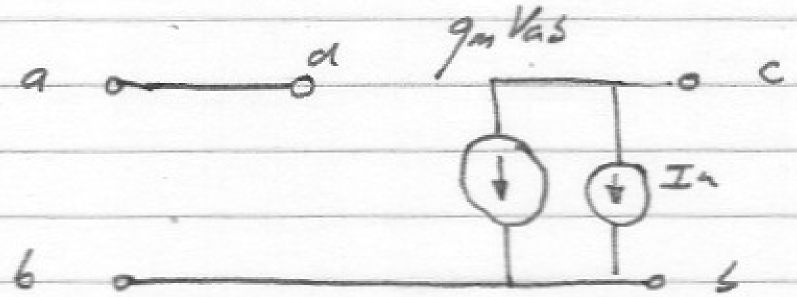


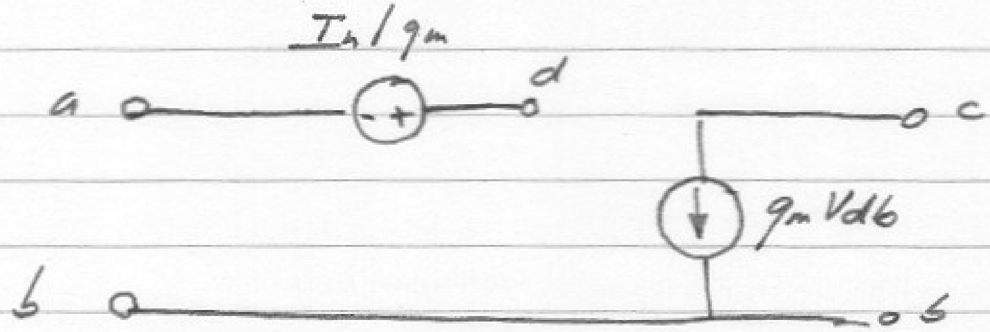
— pushing a voltage through a node —

only the node "d" voltage has changed; allowable unless "d" is the input or output node.



Pushing a current across a branch.





- Pushing a voltage through a transconductance - Only the node "d" (a newly-created node) voltage has changed

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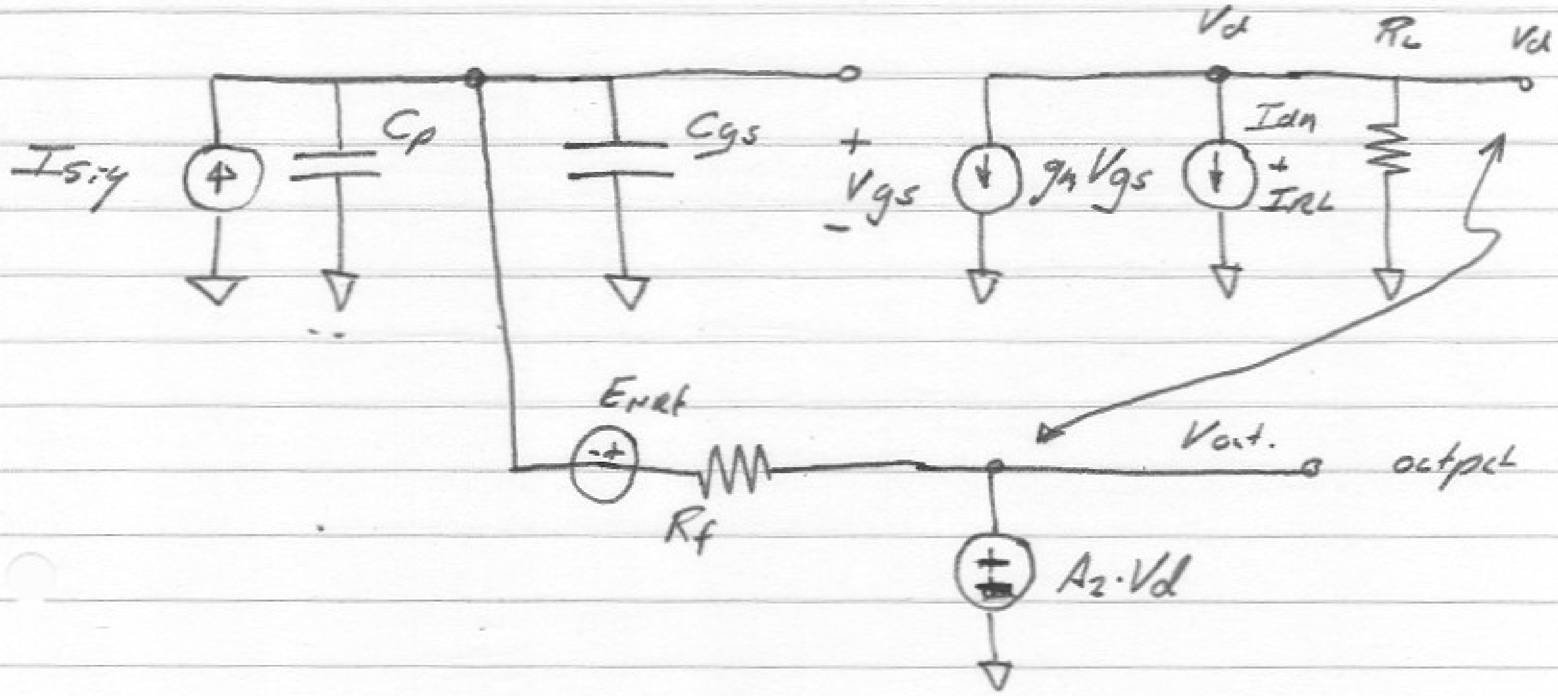
There are other transformations, but they should now be obvious.

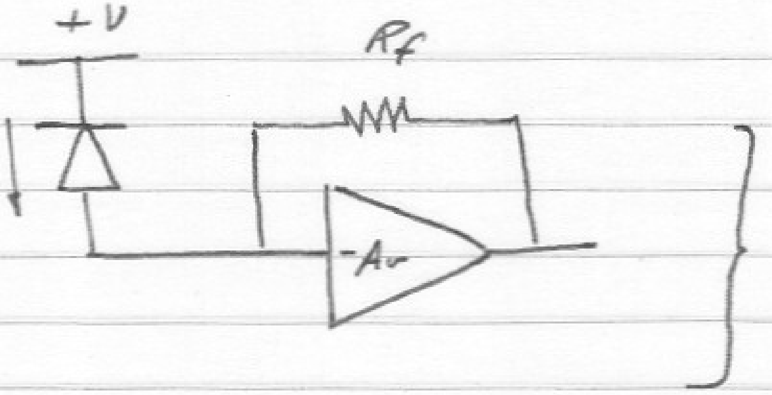
<u>Example</u> (only 1 given; method will appear repeatedly in circuit lectures)



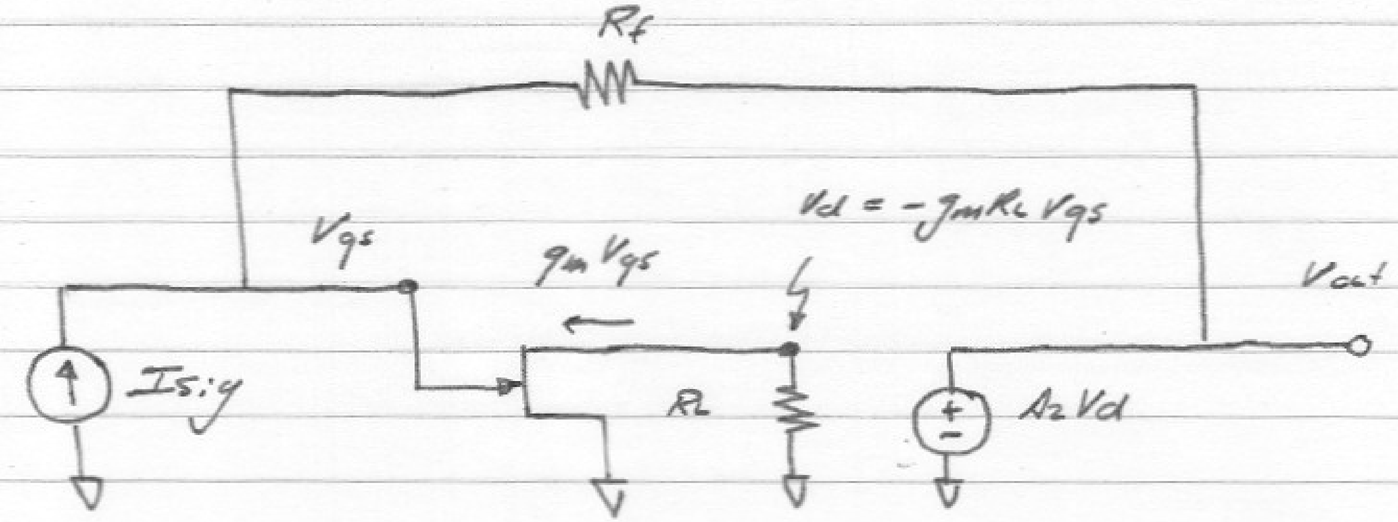
This is the model of a pin-fet optical receiver preamplifier.

I_{sig}

$= \frac{\eta q}{h\nu} P_{opt}$

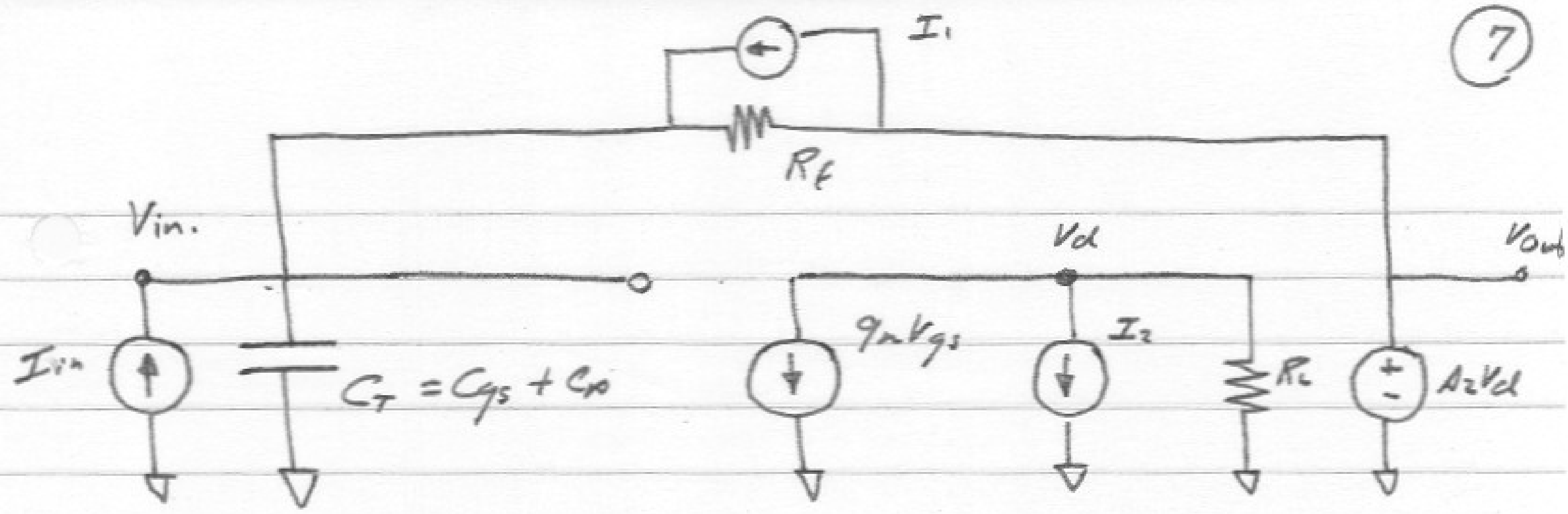


big picture



amplifier 1st stage detailed model | amplifier 2nd stage highly idealized.

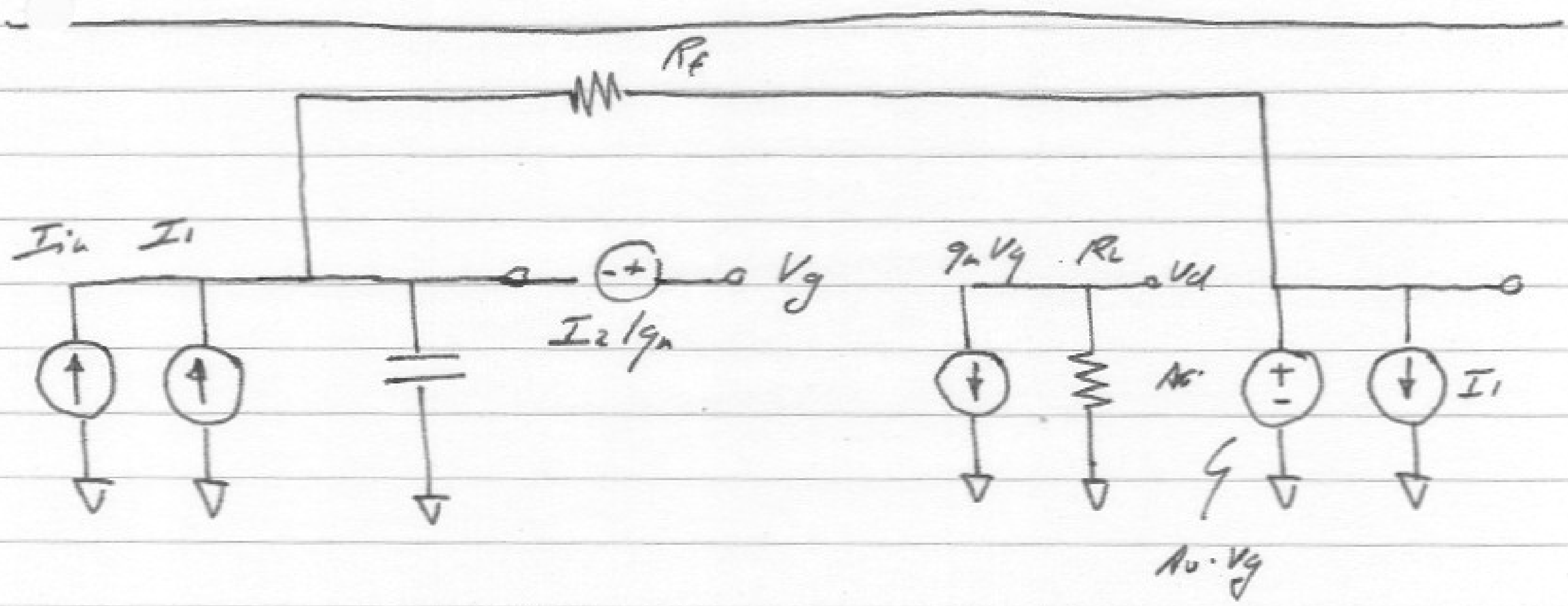
$$A_v = A_2 \cdot (g_m R_L)$$

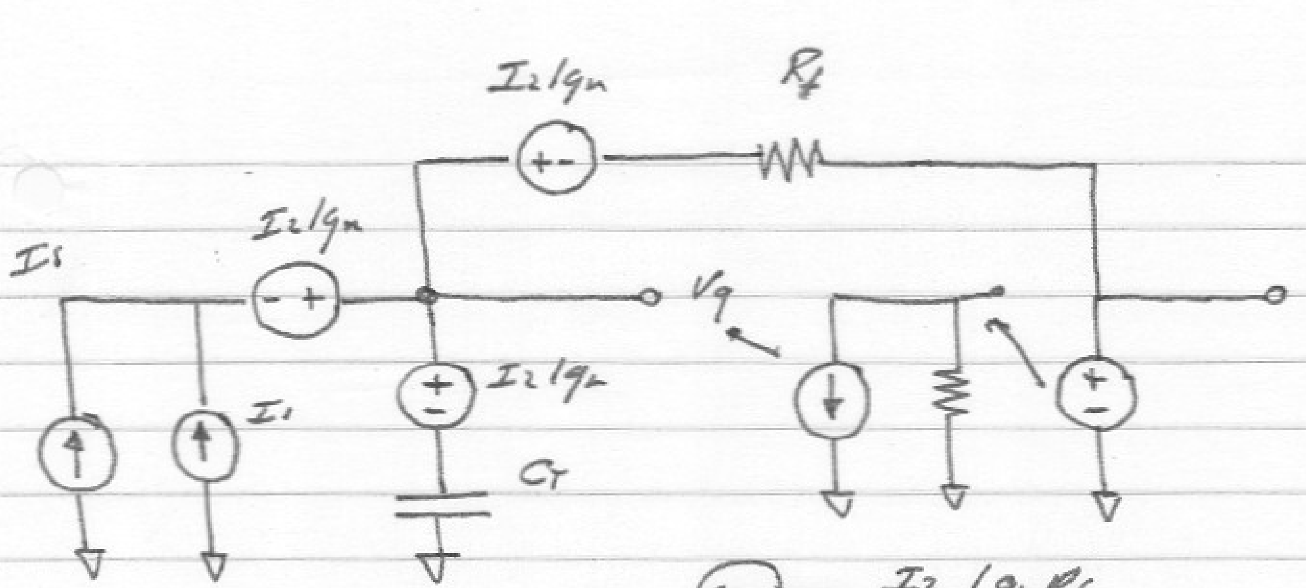


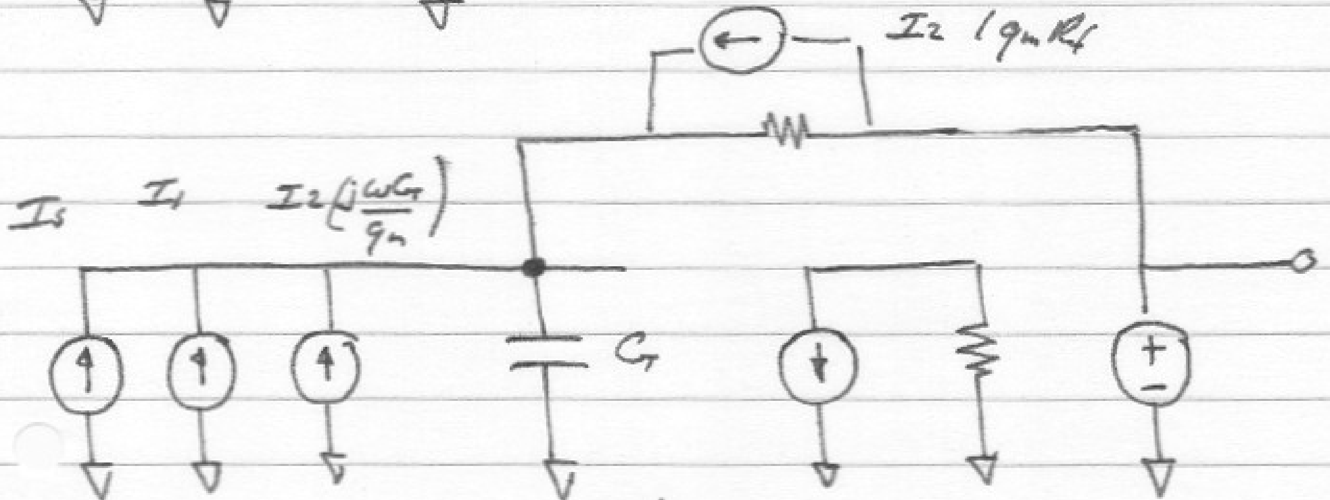
I_1: $S_{I_1 I_1} = 4kT / R_f$

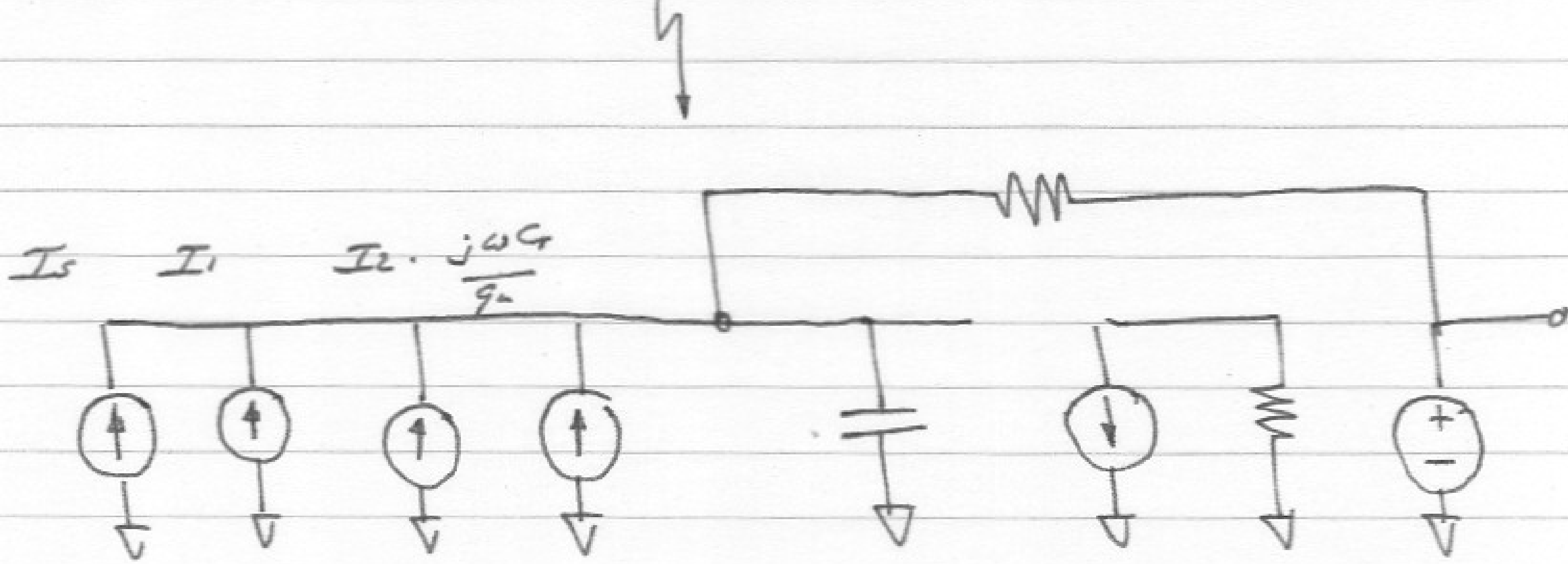
I_2: $S_{I_2 I_2} := 4kT \Gamma g_m + 4kT / R_L$

apply transformations which do not change V_{in} or V_{out}




I_2/g_m
R_f
I_s
I_2/g_m
V_g
I_1
I_2/g_m
C_T


$I_2/g_m R_f$
I_s
I_1
$I_2\left(\frac{j\omega C_T}{g_m}\right)$
C_T


I_s
I_1
$I_2 \cdot \frac{j\omega C_T}{g_m}$

$I_2 / g_m R_f$.

So the total input-referred noise-current spectral density is

$$\frac{d}{df}\langle I_T I_T^* \rangle = \frac{4KT}{R_f}$$

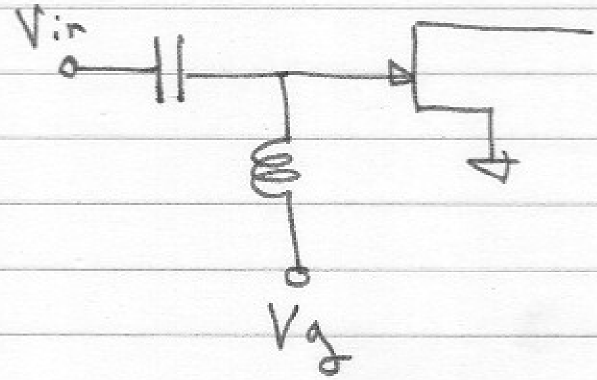
$$+ 4KT\Gamma g_m \left[\frac{\omega^2 C_T^2}{g_m^2} + \frac{1}{g_m^2 R_f^2} \right]$$

$$= \frac{4KT}{R_f} + \frac{4KT\Gamma}{g_m}\left[\omega^2 C_T^2 + \frac{1}{R_f^2} \right]$$

[... the method is faster than the lecture showed, as we can move generators on 1 drawing ...]

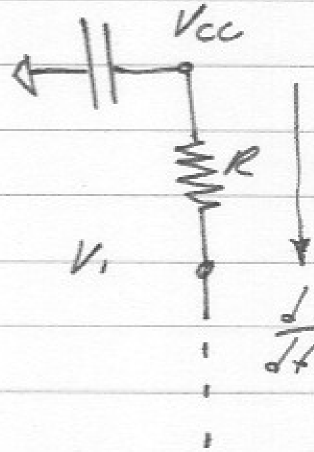
Cautionary Comment about biasing

bias tees are noiseless:



... but their low-frequency cutoff precludes their use in a number of applications.

If we need a bias current I:



$$I_{dc} = \frac{V_{cc} - V_1}{R} = \frac{\Delta V}{R}$$

$$\frac{d}{df}\langle I_{noise} I_N^* \rangle = 4kT/R = \frac{4kT I_{dc}}{\Delta V}$$

Make ΔV big to make the current source noiseless.

FET constant-current source

at low frequencies:

$$\frac{d}{df}\langle I_n I_n^* \rangle = 4kT\Gamma g_m$$

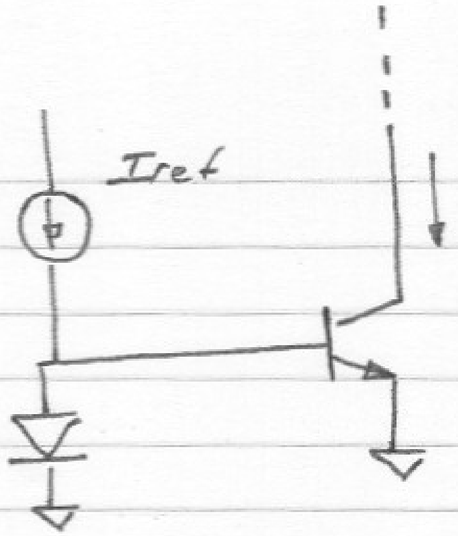
$$= 4kT\Gamma \cdot \left(\frac{g_m}{I_{dc}}\right) \cdot I_{dc}$$

since $(g_m / I_{dc})^{-1} \sim O(V_p)$, ...

$$\frac{d}{df}\langle I_n I_n^* \rangle \sim O\left[4kT\Gamma \ \frac{I_{dc}}{V_p}\right]$$

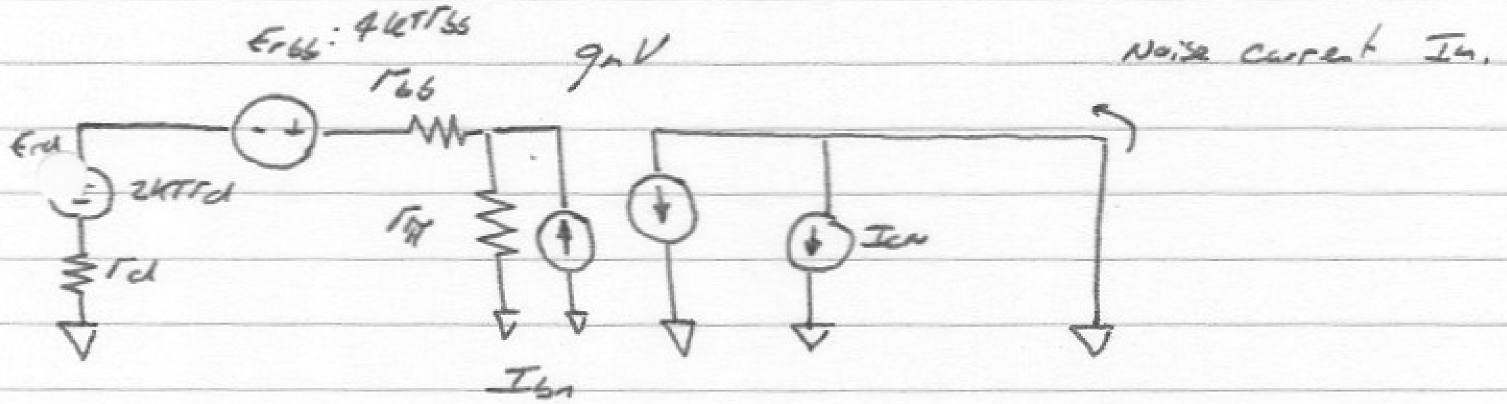
... this is noisier than the resistor by the ratio $\sim \left(\Delta V \cdot \frac{\Gamma}{V_p}\right)$

— Ouch — !



Bipolar current Mirror

Iref noiseless (?) → (really 4kT Iref(ΔV)



$$I_n = (E_{rd} + E_{rs}) \cdot \frac{r_\pi}{r_\pi + r_d + r_{bb}} \cdot g_m \cdot$$

$$+ (r_{bb} + r_d) \frac{r_\pi}{r_\pi + r_d + r_{bb}} \cdot I_{bn} \cdot g_m + I_{cn}$$

use $\beta / g_m = r_\pi$:

$$S_{I_n I_n}(f) = 4KT\left(r_{bb} + \frac{r_d}{2}\right)\left(\frac{\beta}{r_{bb} + r_d + \beta / g_m}\right)^2$$

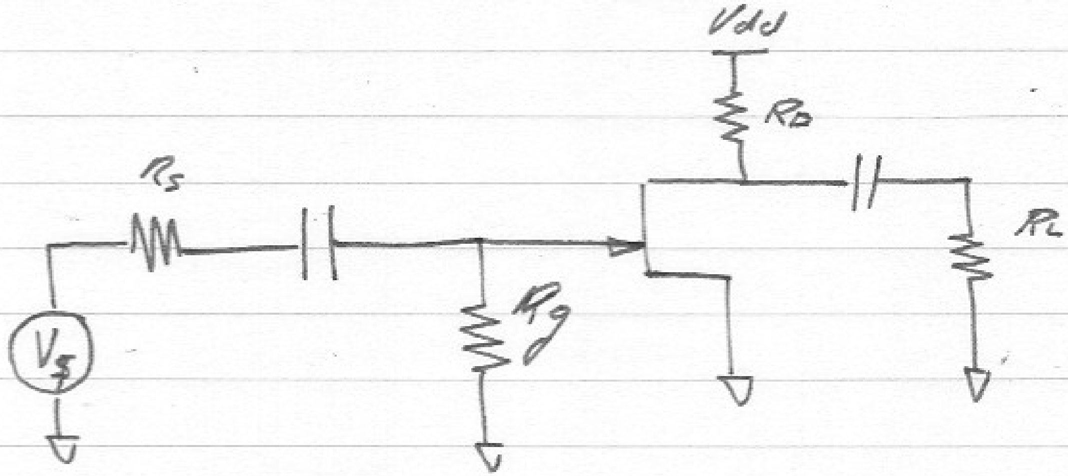
$$+ 2qI_b\left(\frac{\beta}{r_{bb} + r_d + \beta / g_m}\right)^2 (r_{bb} + r_d)^2$$

$$+ 2q I_{dc}$$

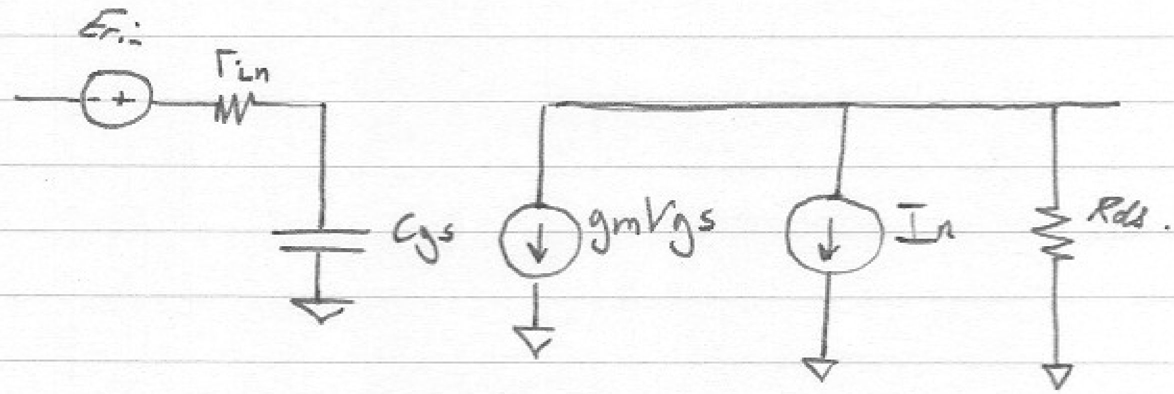
This gets complicated. The last term alone ($2qI_c$) is bigger than resistor current-source noise by the ratio $\left[\Delta V / 2(KT/q)\right]$

— ouch! —

Simple Common-source amplifier:



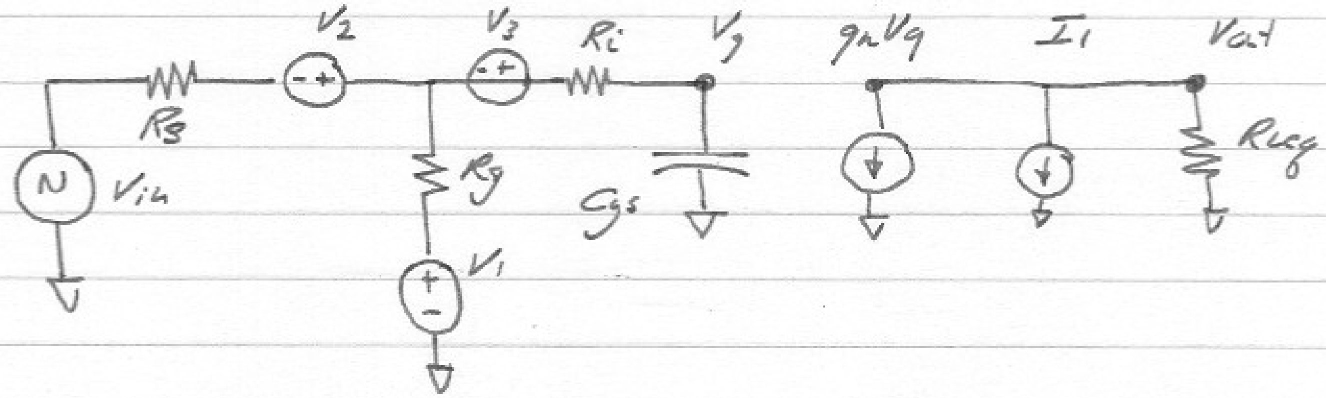
Since this is just an example, pick a simplified FET model



E_{rin}: $S = 4kT r_{in}$

I_n: $S = 4kT \Gamma g_m$

} zero cross-spectral density.



$V_1: \; 4kT R_g \qquad V_2: \; 4kT R_s \qquad V_3: \; 4kT R_i$

$I_1: \; 4kT \frac{2}{3} g_m + 4kT\left(1/R_d + 1/R_L \right) \; : \qquad R_{eq}^{-1} = R_{ds}^{-1} + R_d^{-1} + R_L^{-1}$

$$\frac{V_{out}}{V_{in}} = \frac{R_g}{R_g + R_s} \frac{1}{1 + j\omega C_{gs}(R_i + R_s \| R_g)} \cdot \left(-g_m R_{eq} \right)$$

Most efficient to work this by a mixture of methods.

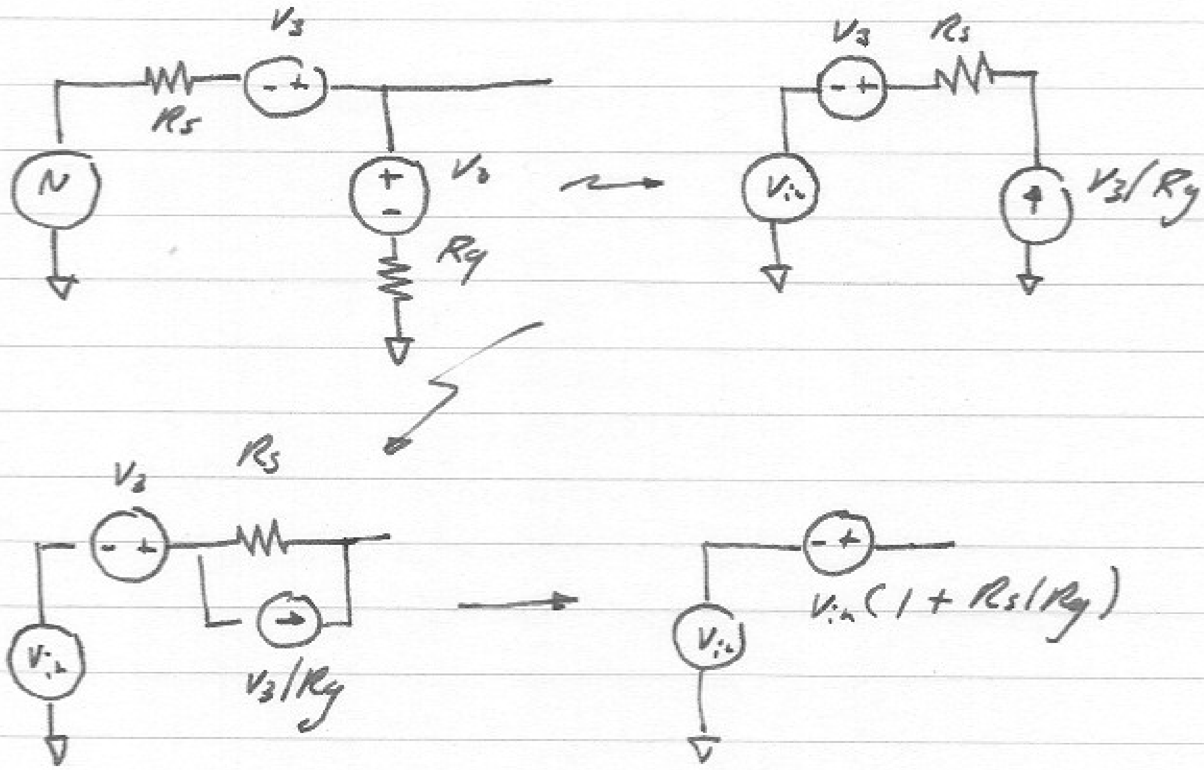
* V_2 already at input: leave it there...

$\frac{V_{out}}{I_1} = R_{eq},$

so, we can model I_1 by an input voltage of

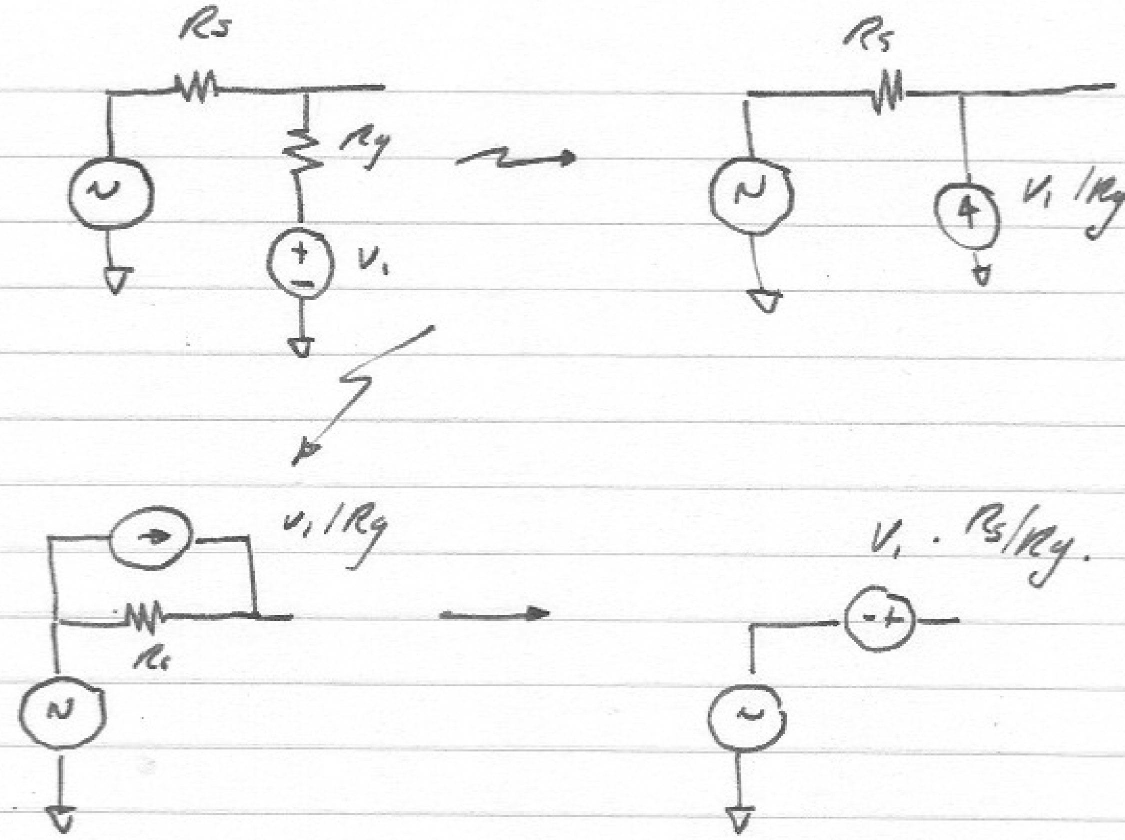
$$V_{eq}\Big|_{I_1} = \frac{-I_1}{g_m}\cdot\frac{R_g+R_s}{R_g}\left(1+j\omega C_{gs}(R_i+R_s\|R_g)\right)$$

V_3:



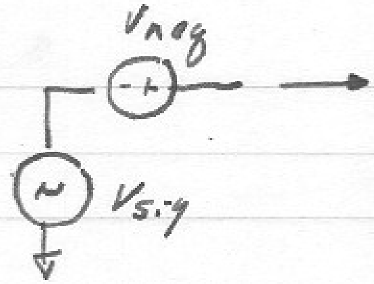
... easier to do than draw ...

V_1



So we can now gather terms & write the input-referred noise voltage ...

vn eq:



amplifier

This is a total noise voltage model

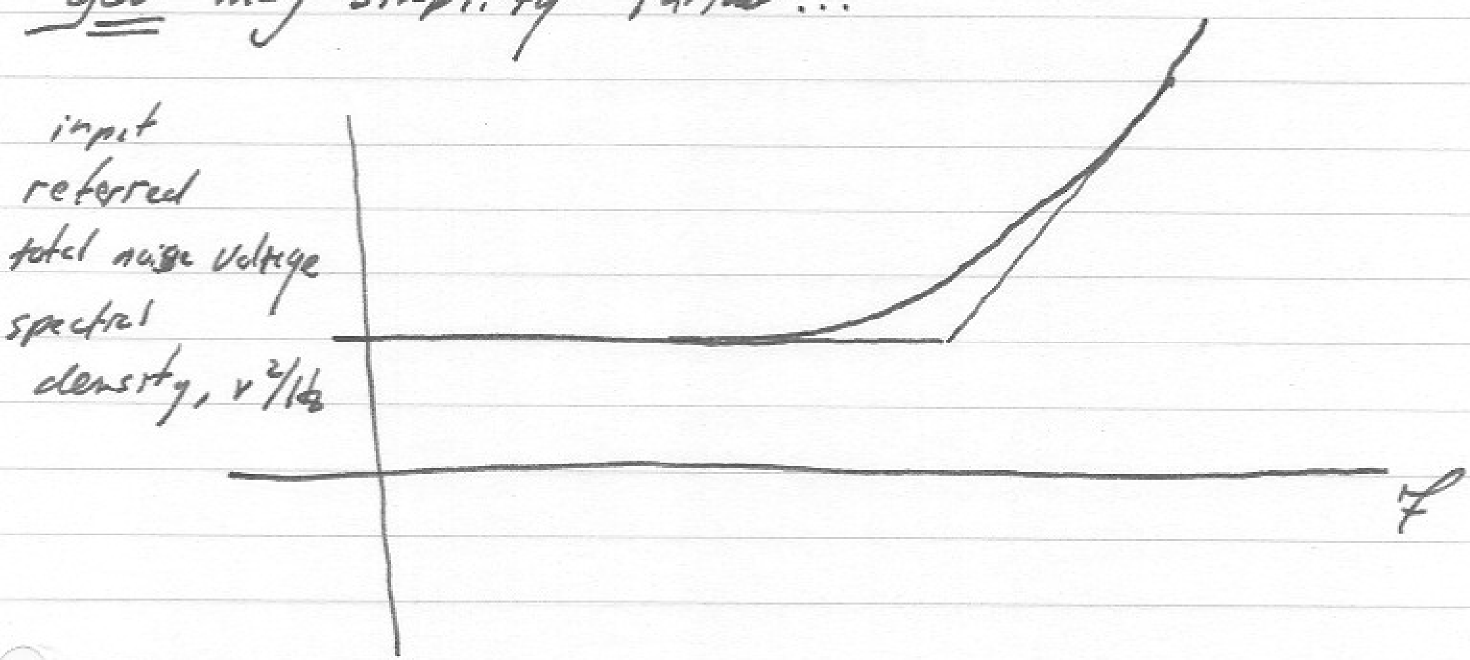
$$S_{v_{neq}}(f) = 4kT\Gamma g_m \cdot \left[\frac{R_g + R_s}{g_m R_g}\right]^2 \left[1 + \omega^2 C_{gs}^2 \cdot (R_i + R_s \| R_g)^2\right]$$

$$+ 4kTR_s$$

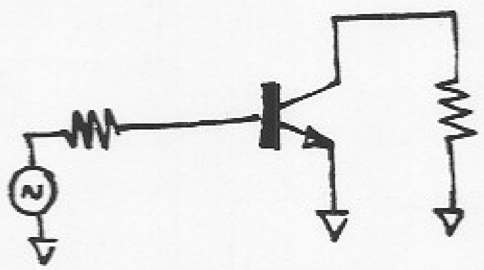
$$+ 4kTR_i \left[1 + R_s / R_g\right]^2$$

$$+ 4kTR_g \cdot \left[R_s / R_g\right]^2$$

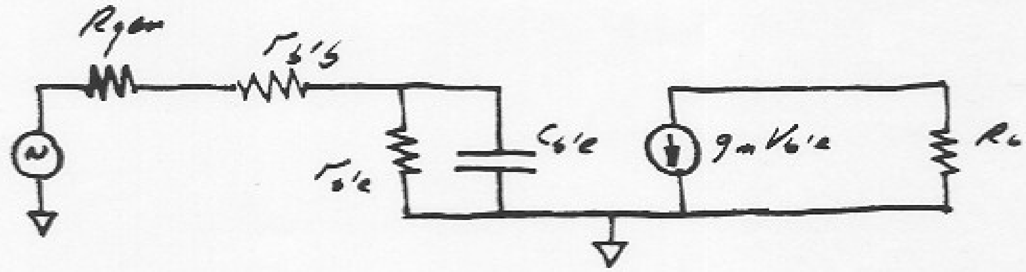
you may simplify further...



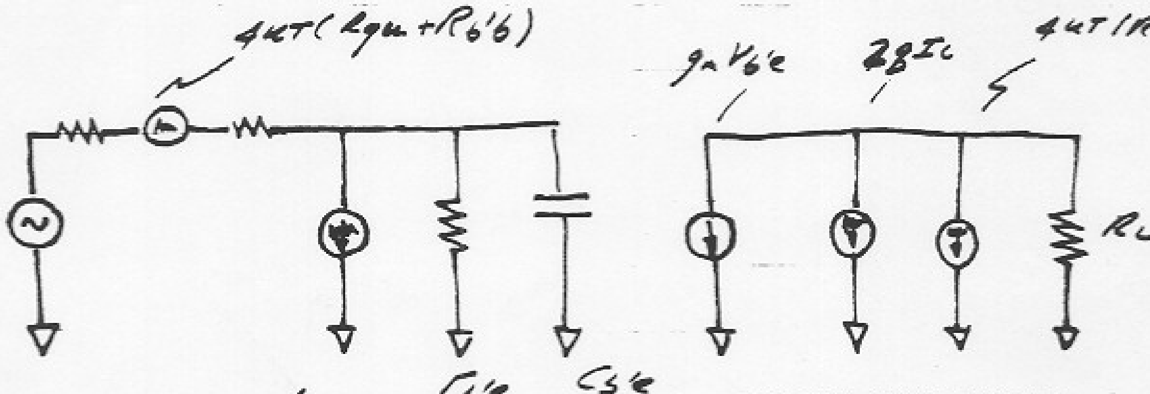
can do same analysis for bipolar c-e amplifier:



neglect R_{ce} (big)
neglect R_E & C_{cb} just to make the problem tractable



add noise sources:



$$\frac{d\langle i^2\rangle}{df} = 2q I_b = 2q\frac{I_c}{\beta}$$

Following similar steps, the input-referred noise voltage can be found:

$$\frac{d\langle E_{in}^2\rangle}{df} = 4kT\left(R_{gen} + R_{b'b}\right) + \frac{2qI_c}{\beta}\left(R_{gen} + R_{b'b}\right)^2$$

$$+ \frac{1}{g_m^2}\left(2qI_c + \frac{4kT}{R_L}\right)\left(1 + (2\pi f C_{b'e})^2\left[r_{b'e} \,\|\, (r_{b'b} + r_{gen})\right]^2\right)$$
$$\times\left(\frac{r_{b'e} + r_{b'b} + R_g}{r_{b'e}}\right)^2$$

<u>complex</u>: if β is reasonably large, so that $r_{b'e} \gg R_{gen} + r_{b'b}$

$$\boxed{\frac{d\langle E_{in}^2\rangle}{df} \cong 4kT\left(R_{gen} + R_{b'b}\right) + \frac{2qI_c}{\beta}\left(R_{gen} + R_{b'b}\right)^2 + \frac{1}{g_m^2}\left(2qI_c + 4kT/R_L\right)\left[1 + (2\pi f C_{b'e})^2(r_{b'b} + r_{gen})^2\right]}$$

optimization: note that $g_m = kT/qI_c$ & [illegible]

$$C_{b'e} = C_{b'e\,depl} + g_m(\tau_b + \tau_c)$$

base, collector transit times

by varying I_c, an optimum is found with lowest noise.

In general for bipolars:

<u>At lower frequencies</u>: low device noise depends on having <u>high β</u>, <u>low $r_{b'b}$</u>, and optimum bias.

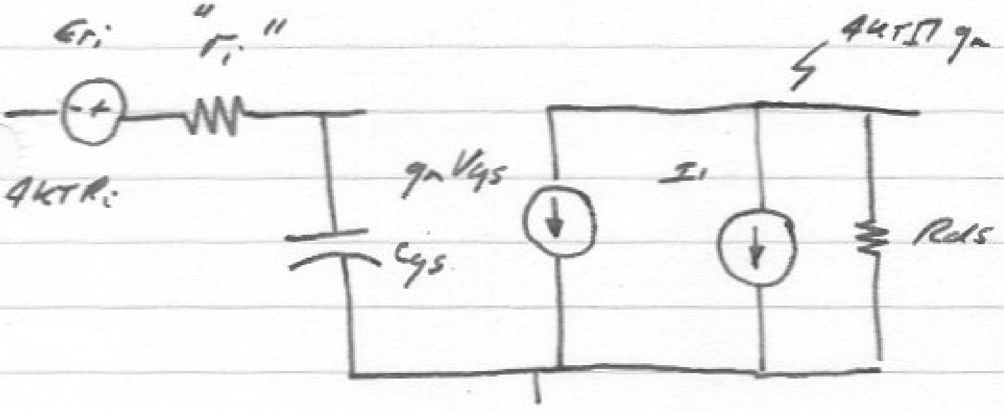
At very low frequencies (< 10kHz), 1/f noise processes become important.

<u>At high frequencies:</u> (high β), low $r_{b'b}$, low base and collector transit times, optimum bias

similar but not exactly the same parameters as for <u>power</u> gain.

<u>Another example:</u> Develop an $E_n - I_n$ Model of a <u>fet</u>

<u>Comment:</u> Again, I will simplify the FET noise Model. I am teaching <u>methods</u> here, so you can generalize as you wish.

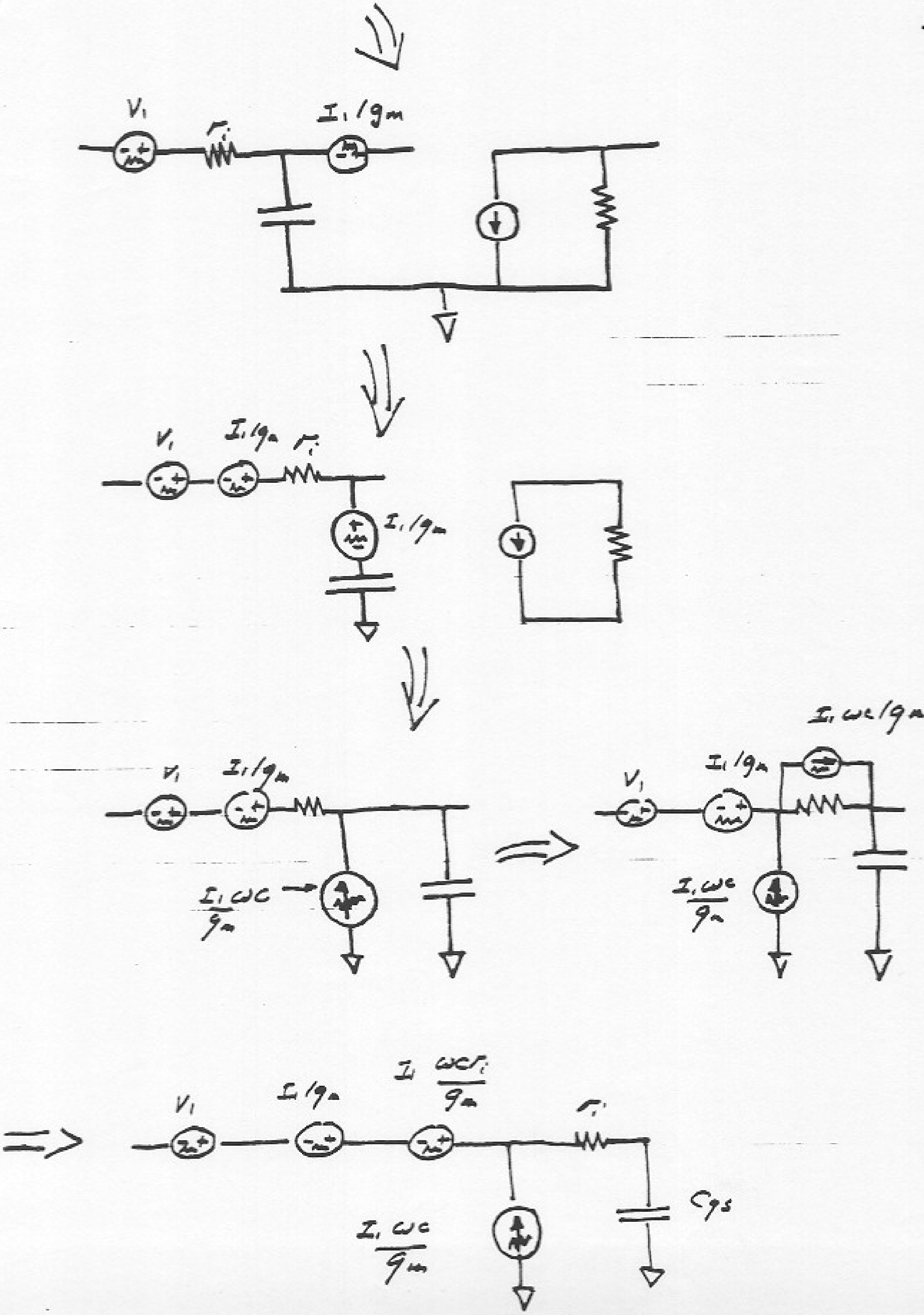


this model lumps $r_i + r_s + r_g \longrightarrow r_i$

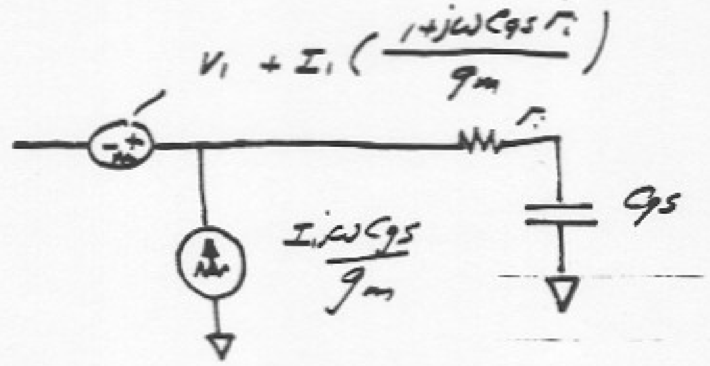
$$\frac{g_m}{1 + g_m R_s} \longrightarrow g_m$$

and small correction for Γ

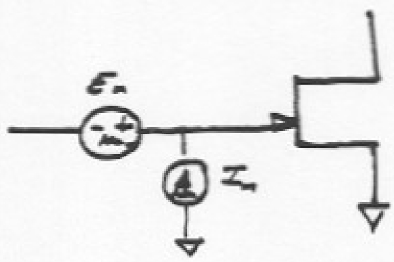
... in order to obtain the "hand analysis" fet noise Model.


V1
I1/gm
ri
V1
I1/gm
ri
I1/gm
V1
I1/gm
I1ωC/gm
I1ωC/gm
V1
I1/gm
I1ωC/gm
V1
I1/gm
I1 ωCri/gm
ri
Cgs
I1ωC/gm

$\Rightarrow$



$\Rightarrow$ Fet noise Model:



$$\frac{d\langle E_n^2\rangle}{df} = 4kTr_i + \frac{4kT\Gamma}{g_m}\left(1+(2\pi f C_{gs})^2 r_i^2\right)$$

$$\frac{d\langle I_n^2\rangle}{df} = \frac{4kT\Gamma}{g_m}(2\pi f C_{gs})^2$$

<u>and</u> $$\frac{d\langle E_n I_n^*\rangle}{df} = 4kT\Gamma g_m\left[\frac{1+j\omega C_{gs} r_i}{g_m}\right]\left[\frac{j\omega C_{gs}}{g_m}\right]^*$$

cross-spectral density

Making the substitution $r_i \to r_i + r_g + r_s$;

$$\frac{d}{df}\langle E_n E_n^*\rangle = 4kT(r_i + r_g + r_s) + \frac{4kT\Gamma}{g_m}\left[1 + \left(2\pi f C_{gs}(r_i + r_s + r_g)\right)^2\right]$$

$$\frac{d}{df}\langle I_n I_n^*\rangle = \frac{4kT\Gamma}{g_m}\left(2\pi f C_{gs}\right)^2$$

$$\frac{d}{df}\langle E_n I_n^*\rangle = \frac{4kT\Gamma}{g_m}\left[1 + j2\pi f C_{gs}(r_i + r_s + r_g)\right]\left[j\,2\pi f C_{gs}\right]^*$$

It is important to understand that this correllation is arising from circuit analysis, starting with a device model having uncorrellated physical random process.

Poor papers in the literature get confused regarding this point.

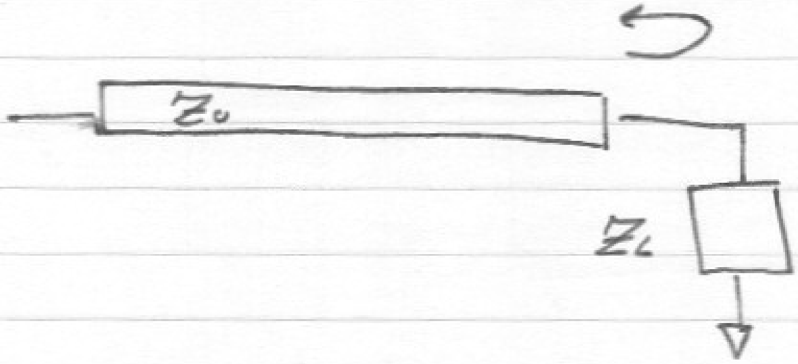
Noise figure Again

Earlier we had shown:

$$F = 1 + \frac{S_{vv}(f) + \|Z_g\|^2 S_{II}(f) + 2\,\mathrm{Re}\left[Z_g^* S_{vI^*}(f)\right]}{4KT \cdot \mathrm{Re}\left[Z_g\right]}$$

... and had found a minimum noise figure & an optimum generator impedance.

Microwave designers usually work with impedances indirectly, writing them as a function of reflection coefficients which would arise if that impedance were to terminate a transmission line:



$$\boxed{\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}}$$

From this the noise figure can be written in terms of the reflection coefficients, thus:

$$F = F_{min} + 4 \cdot \frac{R_n}{Z_0} \frac{\| \Gamma_g - \Gamma_{opt} \|^2}{(1 - \| \Gamma_g \|^2) \| 1 + \Gamma_{opt} \|^2}$$

where

F_{min} as before:

$$\Gamma_{opt} = \frac{Z_{opt}/Z_0 - 1}{Z_{opt}/Z_0 + 1} \qquad \Gamma_g = \frac{Z_g/Z_0 - 1}{Z_g/Z_0 + 1}$$

$$\frac{R_N}{Z_0} = \left[F(Z_g = Z_0) - F_{min} \right] \frac{|1 + \Gamma_{opt}|^2}{4 |\Gamma_{opt}|}$$

where $F(Z_g = Z_0)$ is the noise figure with $Z_g = Z_0$.

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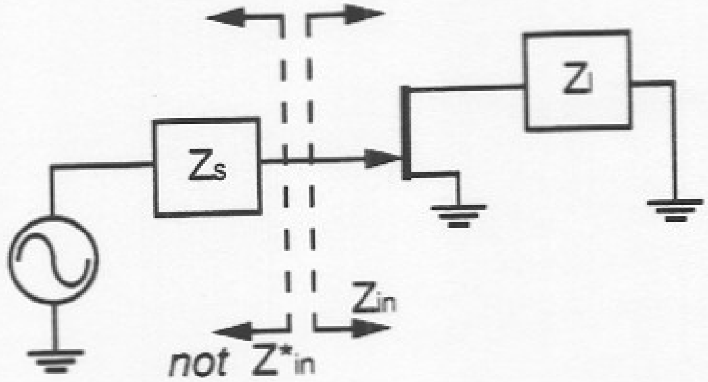
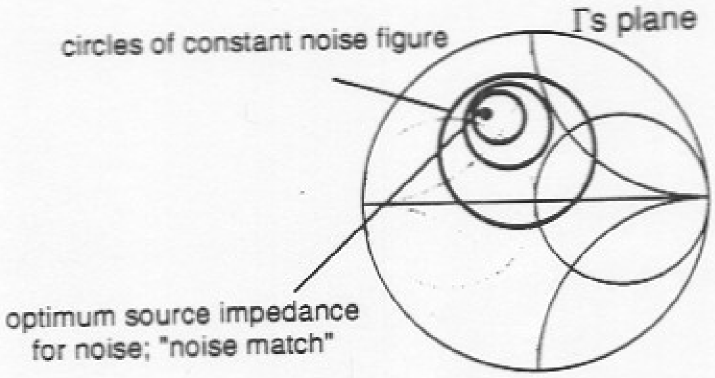
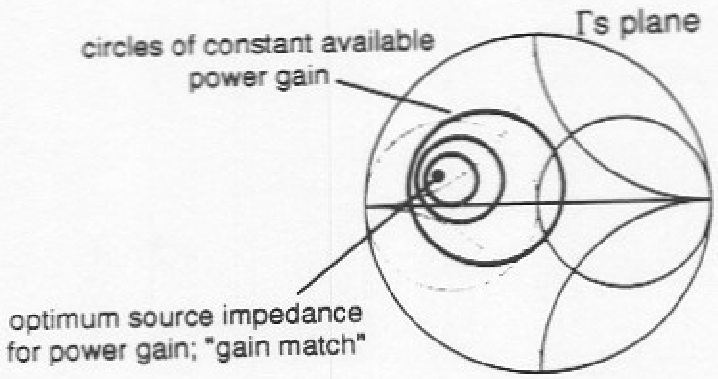
with some effort we can write Γ_{opt} & R_N directly in terms of the (E_n, I_n) spectral densities.

∴ don't bother, this math best left to a computer & there are no <u>new</u> <u>concepts</u> <u>involved</u>.





note that the match conditions are different!!! !!!

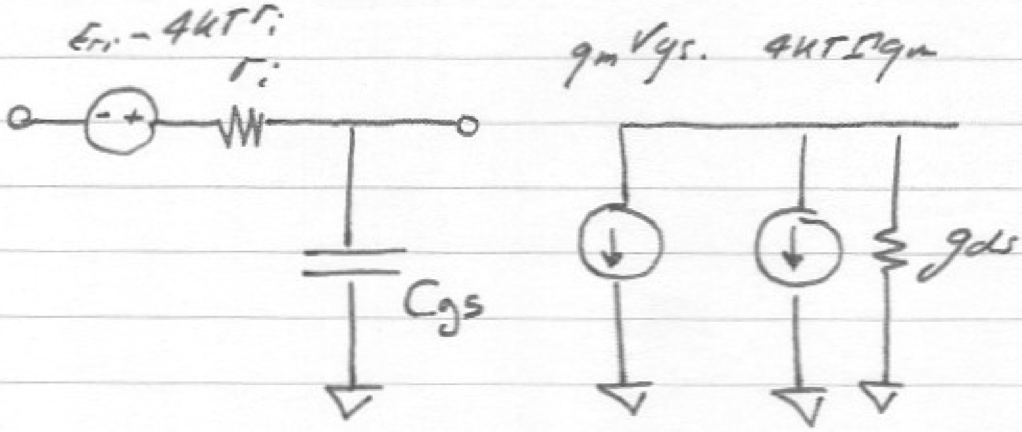


<u>Now note the following point:</u>

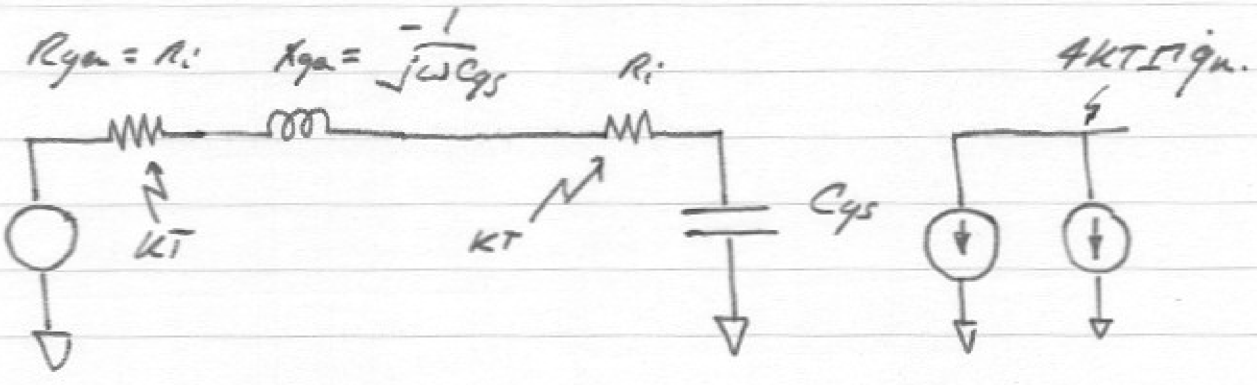
Z_s for optimum noise is not the (matched) Z_s for optimum gain.

<u>Many people are troubled by this; they should not be.</u>

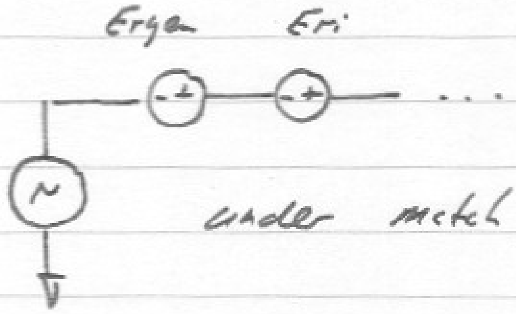
Simple example: FET



Lets impedance match on input:



Lets not even consider the drain noise $4kT\Gamma g_m$.



under match, $E_{ri} = E_{rgm}$ &

$F = 1 + 1 +$ terms due to $4kT\Gamma g_m$

noise due to R_i

Matching has made <u>F</u> <u>at</u> <u>least</u> <u>3dB</u> because R_i & R_{gm} contribute equally to the output. Low noise <u>clearly</u> requires a mismatch such that R_i contributes less strongly!!!