# • Notes Set 16: Bipolar Noise analysis topics

- noise figure of BJT at low frequencies.
- resistive feedback amplifier noise figure

ECE ___ Notes Set 16

Consisting of:

- Noise figure relationships for Bipolars.
- A few final "circuits" noise problems/examples.

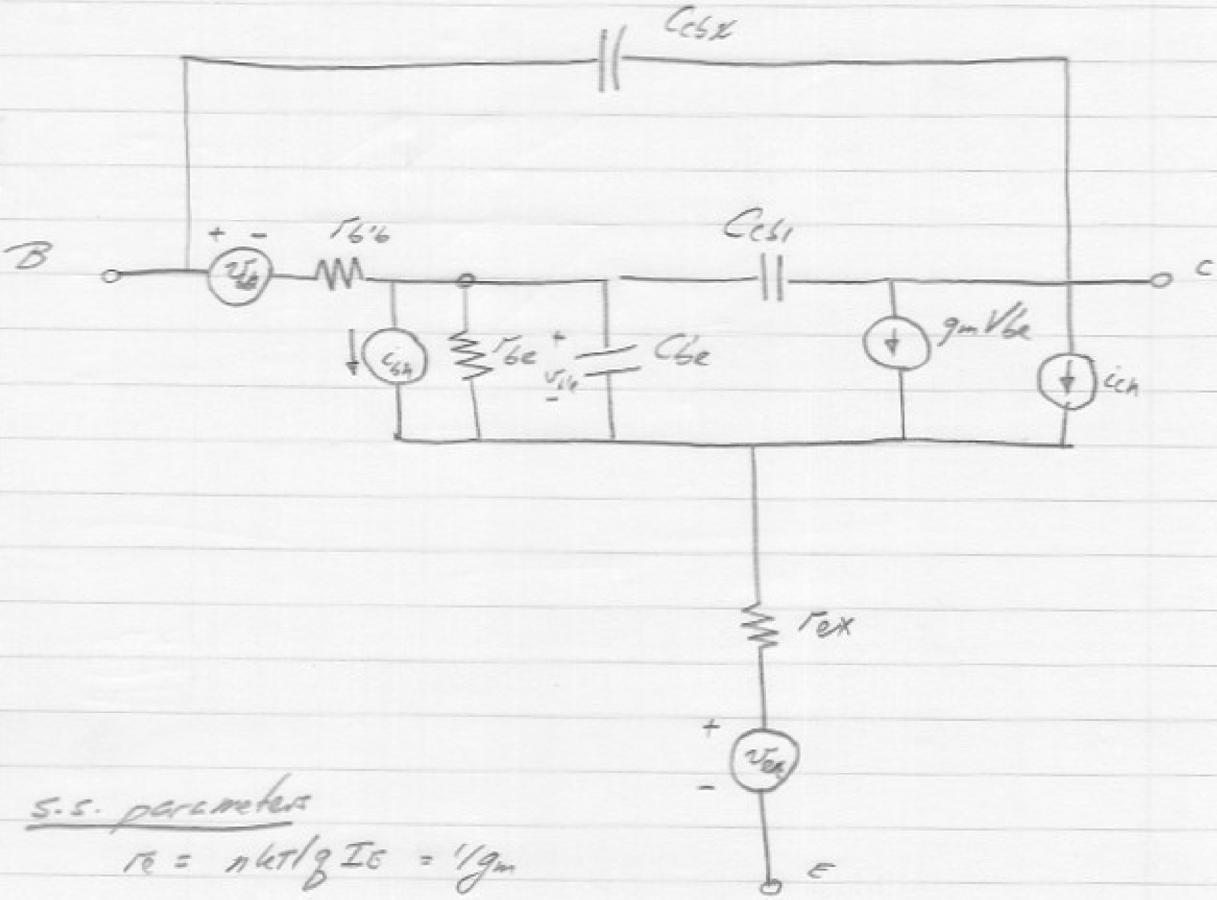
---

First:

Example for the purpose of showing method

Low-Frequency bipolar amplifier.

# Bipolar Transistor Noise Model




<u>s.s. parameters</u>

$r_e = nkT/q I_E = 1/g_m$

$r_{b'e} = \beta r_e$ $\qquad C_{b'e} = C_{be,depl} + g_m \tau_f$

<u>Noise terms:</u> All are statisticaly independant (uncorrellated)

$\frac{d}{df} \langle v_{bn}^2 \rangle = 4kT r_{b'b}$ — base thermal noise.

$\frac{d}{df} \langle i_{bn}^2 \rangle = 2q I_b$ — base shot noise.

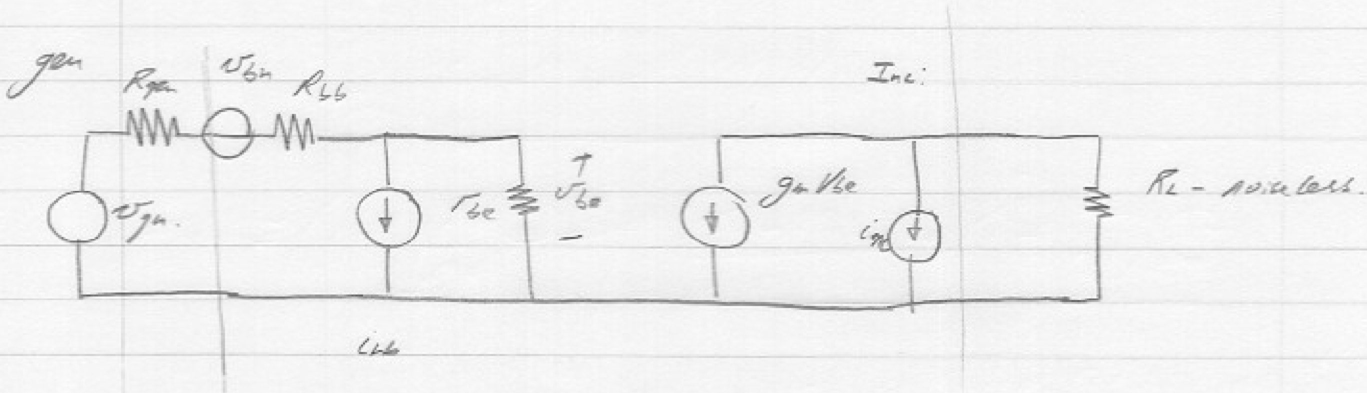
$\frac{d}{df} \langle i_{cn}^2 \rangle = 2q I_c$

$\frac{d}{df} \langle v_{en}^2 \rangle = 4kT r_{ex}$ — thermal noise of the emitter contact resistance.

The advantage of the hybrid-$\pi$ noise model is precisely that the noise terms are uncorrellated.

## Noise figure at low frequencies:

initially - omit the ~~base~~ emitter contact resistance:




$$F = \frac{\text{total noise power at output, with noisy device}}{\text{" " " " " noiseless device.}}$$

$v_{gn}$ = generator noise $\Rightarrow$ spectral density $\frac{d}{df}\langle v_{gn}^2\rangle = 4kT R_{gen}$

$v_{bn}$ = base thermal noise: $\rightarrow \frac{d}{df}\langle v_{bn}^2\rangle = 4kT R_{b'b}$

$i_{bn}$ = base shot noise: $\frac{d}{df}\langle i_{bn}^2\rangle = 2q I_b$

$i_{nc}$ = collector shot noise. $\frac{d}{df}\langle i_{nc}^2\rangle = 2q I_c$

output noise due to generator:

$$v_{out} = v_{gn} \cdot \frac{r_{be}}{r_{be} + r_{gen} + R_{bb}} \cdot g_m R_L$$

$$r_{be} = \beta r_e \qquad g_m = 1/r_e$$

$$\frac{d}{df}\left\langle \frac{v_{out}^2}{R_L} \right\rangle = \frac{dP_{out}}{df} = 4kT r_{gen} \cdot \left[ \frac{\beta r_e}{\beta r_e + R_{gen} + R_{bb}} \cdot \frac{1}{r_e} \right]^2 R_L .$$

---

output noise due to $R_{b'b}$ - same analysis as above.

$$\frac{dP_{out}}{df} = 4kT R_{b'b} \left[ \frac{\beta r_e}{\beta r_e + R_{gen} + R_{bb}} \cdot \frac{1}{r_e} \right]^2 R_L$$

clearly must make $r_{b'b} << r_{gen}$!

output noise due to $i_{nb}$.

$$v_{out} = i_{nb} \cdot \left[ [R_{gen} + R_{bb}] \,//\, r_{be} \right] \cdot \frac{R_L}{r_e}$$

$$\frac{dP_{out}}{df} = \frac{v_{out}^2}{R_L} = 2q I_b \left[ \frac{\beta r_e}{\beta r_e + R_{gen} + R_{bb}} \cdot \frac{1}{r_e} \cdot (R_{gen} + R_{bb}) \right]^2 R_L$$

$$= 2q \frac{I_c}{\beta} \left[ R_{gen} + R_{b'b} \right]^2 \cdot \left[ \frac{\beta r_e}{\beta r_e + R_{gen} + R_{bb}} \cdot \frac{1}{r_e} \right]^2 \cdot R_L$$

now use $I_c \cong I_E$ & $\frac{V_T}{I_E} = \frac{kT}{q I_E} = r_e$

$$= \frac{2kT}{\beta r_e} \cdot \left[ R_{gen} + R_{b'b} \right]^2 \left[ \frac{\beta r_e}{\beta r_e + R_{gen} + R_{bb}} \cdot \frac{1}{r_e} \right]^2 \cdot R_L$$

output noise due to $i_{nc}$ – collector shot noise

$$\frac{dP_{out}}{df} = 2qI_c \cdot R_L =$$

Now use $kT/qI_E = r_e$ & $I_C \cong I_E$

$$= \frac{2kT}{r_e} \cdot R_L$$

Noise figure, again, takes the ratio of these...

$$F = 1 + \frac{R_{b'b}}{R_{gen}} + \frac{2kT}{\beta r_e}\left[R_{gen} + R_{b'b}\right]^2 \frac{1}{4kT\,R_{gen}}$$

$$+ \frac{2kT/r_e}{4kT\,r_{gen}} \left[\frac{\beta r_e + R_{gen} + R_{b'b}}{\beta r_e} \cdot r_e\right]^2$$

$$\cong$$

$$F = 1 + \frac{R_{b'b}}{R_{gen}} + \frac{1}{2\beta}\,\frac{1}{r_e}\,\frac{\left[R_{gen} + R_{b'b}\right]^2}{R_{gen}}$$

$$+ \frac{1}{2}\,\frac{r_e}{r_{gen}} \left[\frac{\beta r_e + R_{gen} + R_{b'b}}{\beta r_e}\right]^2$$

noise figure, at low frequencies, with no emitter contact resistance:

$$F = 1 \qquad \text{generator term!}$$

$$+ \frac{1}{2\beta}\frac{1}{r_e}\frac{[R_{gen}+R_{b'b}]^2}{R_{gen}} \qquad \text{base shot noise}$$

$$+ \frac{R_{b'b}}{R_{gen}} \qquad \text{base thermal noise}$$

$$+ \frac{1}{2}\frac{r_e}{r_{gen}}\left[\frac{\beta r_e + R_{gen} + R_{b'b}}{\beta r_e}\right]^2 \qquad \text{collector shot noise.}$$

now clearly at low frequencies one uses a sufficiently large device to ensure $R_{b'b} << R_{gen}$. Examine also the other terms:

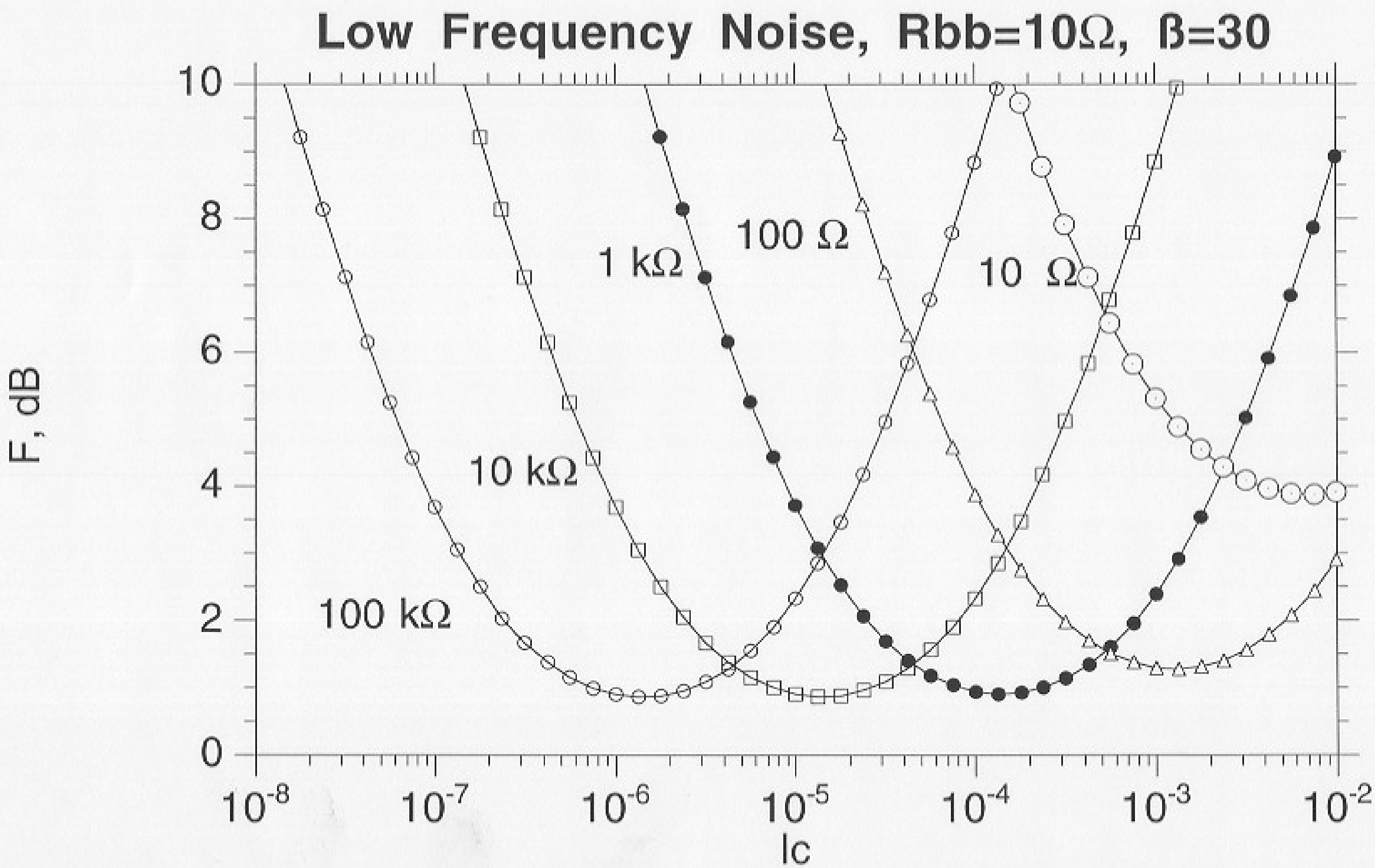
$$A = \frac{1}{2\beta r_e}\frac{[R_{gen}+R_{b'b}]^2}{R_{gen}} = k_1/r_e$$

$$B = \frac{1}{2}\frac{r_e}{r_{gen}}\left[\frac{\beta r_e + R_{gen} + R_{b'b}}{\beta r_e}\right]^2 \cong \frac{1}{2}\frac{r_e}{r_{gen}} \quad \text{if } \beta r_e >> R_{gen}+R_{b'b}.$$

$$= k_2 r_e$$

The sum $A+B$ is minimized by choosing $k_1/r_e = k_2 r_e$, e.g.

$$r_e = \sqrt{k_1/k_2} = \ldots$$



Low Frequency Noise, Rbb=10Ω, ß=30
F, dB
Ic
0
2
4
6
8
10
$10^{-8}$
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
100 kΩ
10 kΩ
1 kΩ
100 Ω
10 Ω

## optimum bias - low frequency, idealized

$$r_{e,opt} = \sqrt{\frac{1}{\cancel{2}\beta}\,\frac{\left[R_{gen} + R_{bb}\right]^2}{\cancel{R_{gen}}}\cdot \cancel{2}\cancel{R_{gen}}} = \frac{\left[R_{gen} + R_{bb}\right]}{\sqrt{\beta}}$$

$$\boxed{r_{e,opt} = \frac{R_{gen} + R_{bb}}{\sqrt{\beta}}}$$

---

$$F_{min} = 1 + \frac{R_{b'b}}{R_{gen}} + \frac{1}{2\sqrt{\beta}}\,\frac{R_{gen} + R_{b'b}}{R_{gen}}$$

$$+ \frac{1}{2\sqrt{\beta}}\,\frac{R_{gen} + R_{bb}}{R_{gen}}\left[\frac{\beta r_e + R_{gen} + R_{bb}}{\beta r_e}\right]^2$$

Now, note that the last term is problematic (messy).

However, when we plug into the expression for $r_{e,opt}$, the fraction becomes $\left[\frac{\sqrt{\beta} + 1}{\sqrt{\beta}}\right]^2$

$$F_{min} = 1 + \frac{R_{b'b}}{R_{gen}} + \left(\frac{R_{gen} + R_{b'b}}{R_{gen}}\right)\frac{1}{\sqrt{\beta}} \cdot \left(1 + \frac{\left(\frac{\sqrt{\beta}+1}{\sqrt{\beta}}\right)^2}{2}\right)$$

so for $\beta$ reasonably large, this is

$$F_{min} \cong 1 + \frac{R_{b'b}}{R_{gen}} + \frac{R_{gen} + R_{b'b}}{R_{gen}} \cdot \frac{1}{\sqrt{\beta}}$$

$$\boxed{F_{min} \cong \left(1 + \frac{1}{\sqrt{\beta}}\right)\left(1 + \frac{R_{b'b}}{R_{gen}}\right)} \quad \boxed{r_{s,opt} \cong \frac{R_{gen} + R_{bb}}{\sqrt{\beta}}}$$

again this assumes:

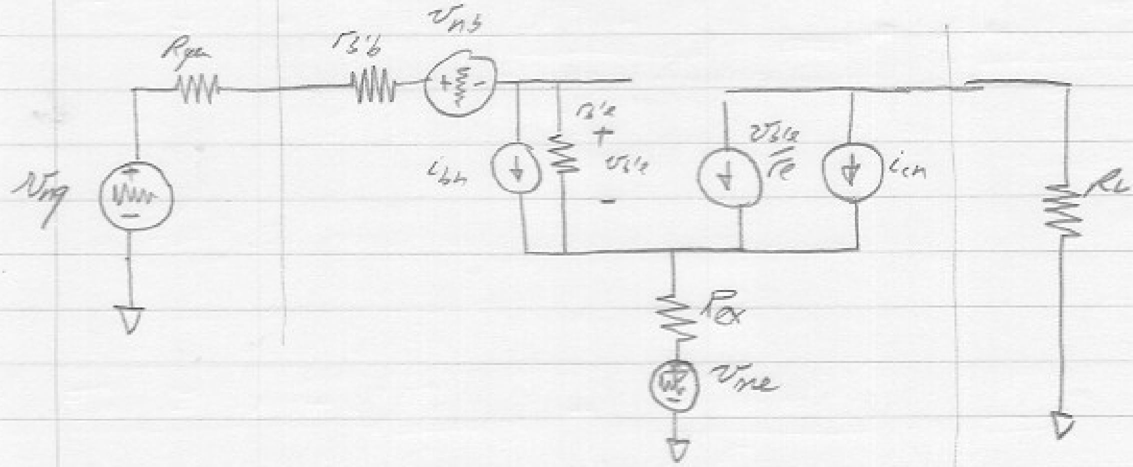
- no emitter resistance
- low frequency model

conclusions for the mid-band analysis are the usual ones:

- Need $R_{b'b} \ll R_{gen}$
- Need $\beta \gg 1$

At low frequencies, if we choose a big device, we should be able to attain $F \cong 0.7\,dB$

Now let's look at the effect of emitter resistance:




I will immediately start using degenerated gain expressions, or the analysis will be intractable.

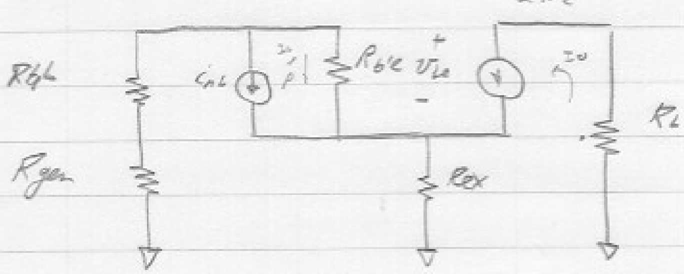
<u>output power due to generator:</u>

$$\frac{dP_{out}}{df} = 4kTR_{gen} \cdot \left[ \frac{\beta(\cancel{r_e + R_{ex}})}{\beta(r_e + R_{ex}) + R_{gen} + R_{bb}} \cdot \frac{1}{\cancel{r_e + R_{ex}}} \right]^2 \cdot R_L$$

<u>output power due to base thermal noise</u>

$$\frac{dP_{out}}{df} = 4kT R_{bb} \left[ \frac{\beta(\cancel{r_e + R_{ex}})}{\beta(r_e + R_{ex}) + R_{gen} + R_{bb}} \cdot \frac{1}{\cancel{r_e + R_{ex}}} \right]^2 \cdot R_L$$

output power due to $i_{nb}$:




TRICKY!

$$v_o \cong i_{nb} \cdot \left(R_{bb}+R_{gen}\right) \| \left(\beta R_{ex}+\beta r_e\right) \frac{R_L}{r_e+R_{ex}}$$

$$+ \frac{R_{ex}}{r_e+R_{ex}+\frac{R_{bb}+R_{gen}}{\beta}} \cdot R_L \cdot i_{nb}$$

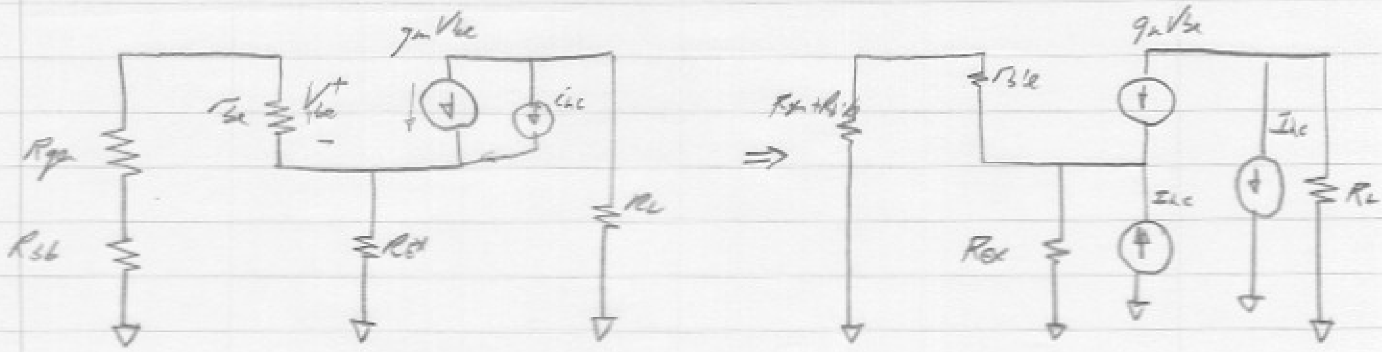
$$\frac{1}{R_L}\frac{v_o}{i_{nb}} = \left[\frac{\beta(R_{ex}+r_e)}{R_{gen}+R_{bb}+\beta(R_{ex}+r_e)} \cdot \frac{1}{r_e+R_{ex}} \cdot (R_{bb}+R_{gen})\right] \cdot$$

$$+ \frac{\beta R_{ex}}{R_{gen}+R_{b'b}+\beta(r_e+R_{ex})}$$

$$= \frac{\beta(R_{ex}+R_{bb}+R_{gen})}{R_{gen}+R_{bb}+\beta(r_e+R_{ex})}$$

$$\frac{dP_{out}}{df} = 2q I_b \cdot \left[\frac{\beta R_{ex}+\beta R_{bb}+\beta R_{gen}}{R_{gen}+R_{bb}+\beta r_e+\beta R_{ex}}\right]^2 \cdot R_L$$

## output power due to $I_{nc}$




$$\frac{d}{df}\langle I_{nc}^2\rangle = 2qI_c$$

$$\frac{I_{out}}{I_{nc}} = 1 - \frac{R_{EX}}{R_{EX} + r_e + \frac{R_{gen}+R_{b'b}}{\beta}}$$

$$\frac{dP_{out}}{df} = \left[\frac{r_e + \frac{R_{gen}+R_{b'b}}{\beta}}{r_e + R_{EX} + \frac{R_{gen}+R_{b'b}}{\beta}}\right]^2 \cdot R_L \cdot 2qI_c$$

$$= \left[\frac{\beta r_e + R_{gen} + R_{b'b}}{\beta(r_e + R_{EX}) + R_{gen} + R_{b'b}}\right]^2 \cdot R_L \cdot 2qI_c$$

## output power due to $I_{ne}$

$$\frac{I_{out}}{I_{ne}} = \frac{R_{ex}}{R_{ex} + r_e + \frac{R_{gen} + R_{b'b}}{\beta}} \quad ; \frac{d\langle I_{ne}^2\rangle}{df} = \frac{4kT}{R_{ex}}$$

$$\frac{dP_{out}}{df} = \frac{4kT}{R_{ex}} \left[ \frac{R_{ex}}{R_{ex} + r_e + \frac{R_{gen} + R_{bb}}{\beta}} \right]^2 \cdot R_L$$

---

Noise figure

$$F = 1 + \frac{R_{b'b}}{R_{gen}} +$$

$$+ \frac{2qI_b}{4kTR_{gen}} \left[ \frac{\beta r_e + \cancel{\beta R_{ex}} + R_{gen} + R_{bb}}{\beta r_e + \cancel{\beta R_{ex}} + R_{gen} + R_{bb}} \cdot \frac{\cancel{\beta} R_{ex} + \cancel{\beta} R_{bb} + \cancel{\beta} R_{gen}}{\cancel{\beta}} \right]^2$$

$$+ \frac{2qI_c}{4kTR_{gen}} \left[ \frac{\beta r_e + R_{gen} + R_{b'b}}{\beta} \right]^2$$

$$+ \frac{4kT/R_{ex}}{4kTR_{gen}} \left[ \frac{\beta^{*} R_{ex}^{*}}{\beta} \right]^2$$

**Noise Figure** Low frequency including external emitter resistance.

$$F = 1 + \frac{R_{b'b}}{R_{gen}}$$ generator & base thermal noise terms

$$+ \frac{1}{2\beta} \frac{1}{r_e} \frac{\left[ R_{bb} + R_{ex} + R_{gen} \right]^2}{R_{gen}}$$ base shot noise

$$+ \frac{1}{2} \frac{r_e}{r_{gen}} \left[ \frac{\beta r_e + R_{gen} + R_{b'b}}{\beta r_e} \right]^2$$ collector shot noise

$$+ \frac{1}{2} \frac{R_{ex}}{R_{gen}}$$ external emitter resistance noise!

This is a nice simple expression, but it does not reduce the answer into the form which I was looking for! Lets see what happens if we try to lump the emitter resistance noise together with the collector shot noise.

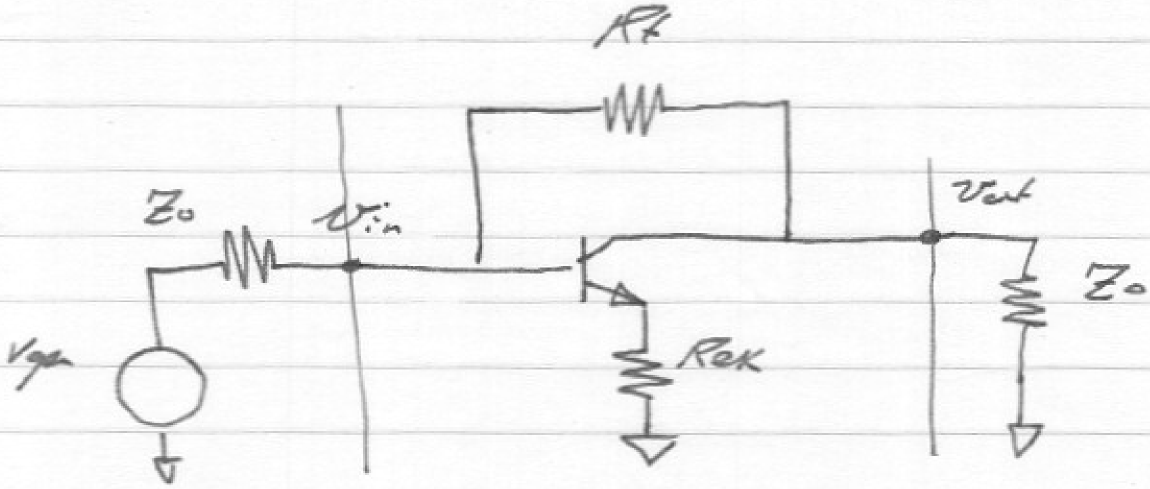
external: $4kTR_{ex}$

internal $2kTr_e$

[ I also include an
example problem for mid-band noise analysis.

- This will not be worked in lecture -

- Circuit is Resistive Feedback amplifier




defining: $\tilde{g}_m = \dfrac{1}{kT/qI_E + R_{ek}}$

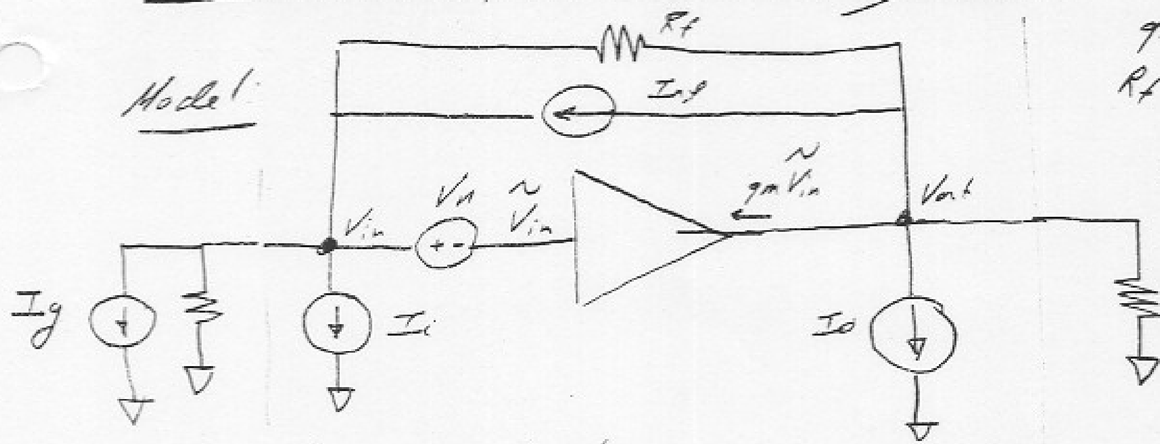
then if we desire $Z_{in} = Z_{out} = Z_o$
and $\dfrac{v_{out}}{v_{in}} = -A.$

$\tilde{g}_m = \dfrac{1-A}{Z_o}$ and $R_f = Z_o(1-A)$

For $\beta$ large.

# Feedback Amplifier noise analysis:

Model:




$g_m = \frac{1+A}{Z_o}$

$R_f = (1+A) Z_o$

A is used as positive

Work by superPosition:

$\equiv$ $I_g$: generator noise:

$$V_o = +A \frac{Z_o}{2} I_g \, ; \quad d\langle I_g^2 \rangle / df = 4kT/Z_o$$

$$\frac{d\langle P_o \rangle}{df} = A^2 \frac{\cancel{Z_o^2}}{\cancel{2^2}} \frac{1}{\cancel{Z_o}} \frac{4kT}{\cancel{Z_o}} = A^2 kT \quad \text{of course.}$$

$I_i$ - input noise current of amplifier:

$$V_o = (+AZ_o/2)\, I_i$$

$I_o$: output noise current:

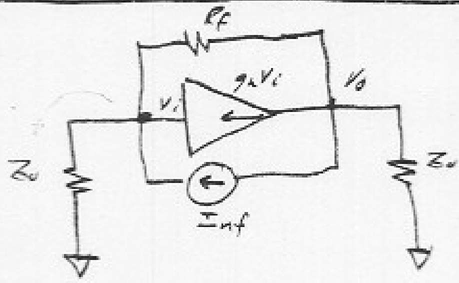
$$V_o = -I_o Z_o/2$$

$V_n$: input noise voltage

$V_n$ on input can be treated as a current $I_n = V_n g_m$ on the output, so output noise voltage is

$$V_o = +V_n \left[ g_m Z_o/2 \right] = V_n \left[ \frac{1+A}{2} \right]$$

Rf - feedback resistor noise:




$$V_i\left[1/Z_0 + \frac{1}{Z_0}\,\frac{1}{1+A}\right] + V_o\left[\frac{-1}{Z_0}\,\frac{1}{1+A}\right] = I_{nf}$$

$$V_i\left[\frac{1+A}{Z_0} - \frac{1}{Z_0}\,\frac{1}{1+A}\right] + V_o\left[1/Z_0 + \frac{1}{Z_0}\,\frac{1}{1+A}\right] = -I_{nf}$$

$$\frac{-V_o}{Z_0 I_{nf}} = \frac{\left(1 + \frac{1}{1+A}\right) + \left(1+A - \frac{1}{1+A}\right)}{\left(1 + \frac{1}{1+A}\right)^2 - \left(1+A - \frac{1}{1+A}\right)\left(-\frac{1}{1+A}\right)}$$

$$= \frac{(1+A+1) + (\cancel{1} + A^2 + 2A - \cancel{1})}{(1+A+1)^2 + (1+A^2+2A-1)} \cdot (1+A)$$

$$= \frac{2 + 3A + A^2}{A^2 + 4A + 4 + A^2 + 2A}\,(1+A)$$

$$= \frac{(A+1)^2(A+2)}{2A^2 + 6A + 4} = \frac{(A+1)^2(A+2)}{2\left[A^2 + 3A + 2\right]}$$

$$= \frac{1}{2}\,\frac{(A+1)^2\cancel{(A+2)}}{(A+1)\,(A+2)} = \frac{1}{2}(A+1) \quad !$$

$$\Rightarrow \boxed{V_o = -(1+A)\cdot\frac{Z_0}{2}\cdot I_{nF}}$$

$$\frac{d\langle P_0\rangle}{df} = \frac{1}{Z_0}\,\frac{d\langle V_0^2\rangle}{df} = \frac{1}{Z_0}\left[(1+A)^2\,\frac{Z_0^2}{4}\right]\left[4kT/R_f\right]$$

$$= \frac{1}{\cancel{Z_0}}\left[1+A\right]^{\cancel{2}}\,\frac{Z_0^2}{\cancel{4}}\,\frac{4kT}{\cancel{Z_0}}\,\frac{1}{\cancel{1+A}} = kT\left[1+A\right]$$

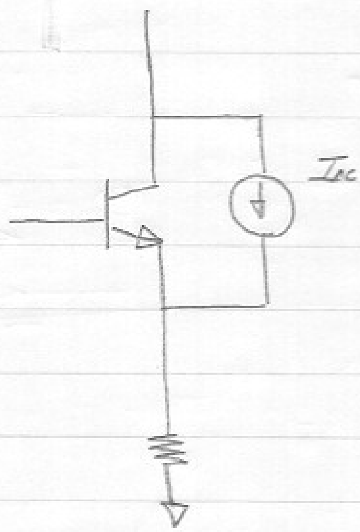
$$\boxed{\frac{d\langle P_0\rangle}{df} = kT\left[1+A\right]}$$

due to $R_f$

Note particularly that since $A$ is the voltage gain, not the power gain, that the relative noise contribution of $R_f$ becomes minor for high amplifier gains.

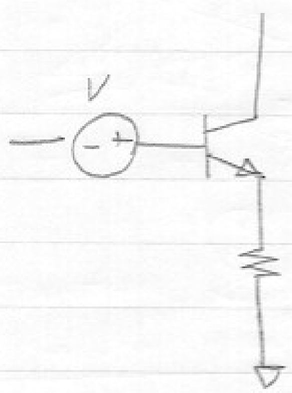
Mid-Band Noise Model – Transistor with emitter degeneration

Effect of Collector shot noise:



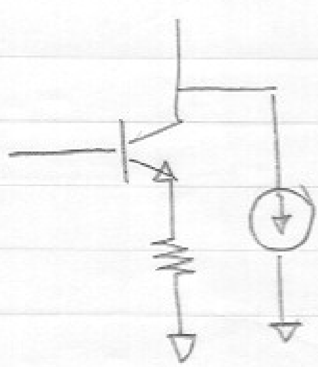

$$\frac{d}{df}\langle I_{nc}^2\rangle = 2qI_c = 2kT/r_e$$

$\Rightarrow$




$$V = I_{nc}\cdot r_e$$

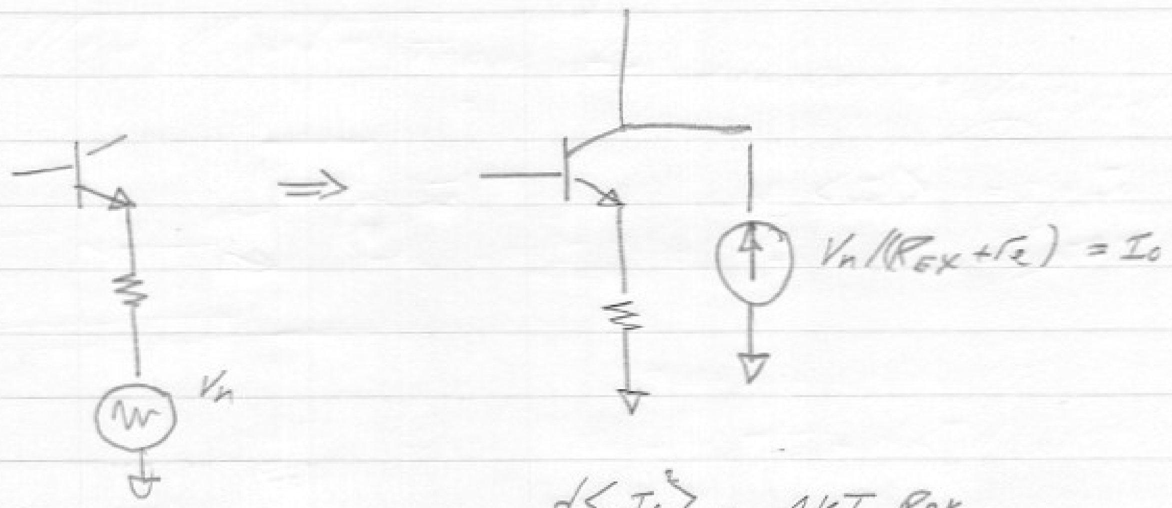
$\Rightarrow$



$$\tilde{I} = \frac{V}{r_e + R_{ex}} = I_{nc}\cdot\frac{r_e}{r_e + R_{ex}}$$

$$\frac{d\langle\tilde{I}_2\rangle}{df} = 2KT\,\frac{r_e}{(r_e + R_{ex})^2}$$

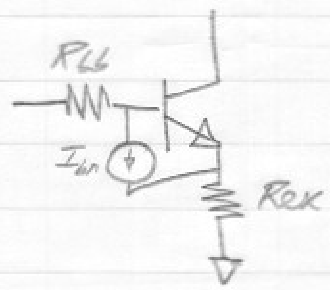
## Effect of Rex thermal noise:

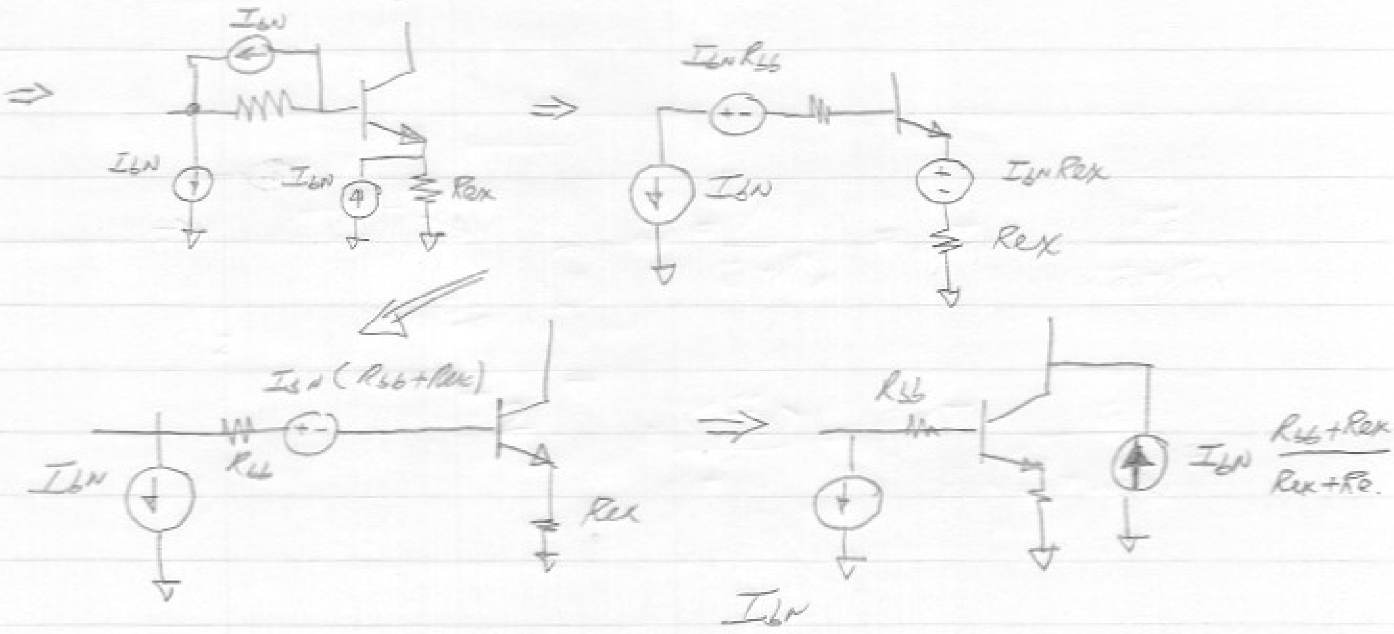



$$\frac{d\langle I_0^2\rangle}{df} = \frac{4kT\,R_{ex}}{(R_{ex}+r_e)^2}$$

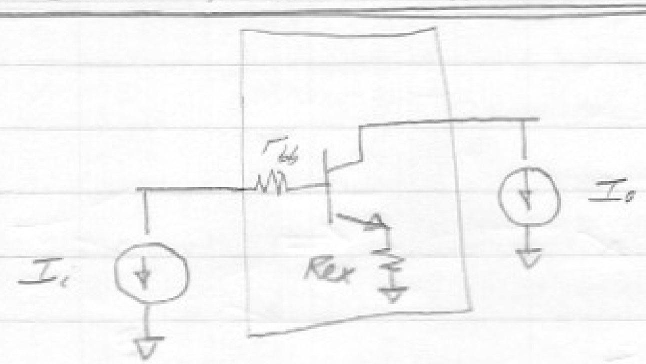
---

Base shot noise?
this will give correlated
input & output noise terms.



## So Mid-band transistor noise Model is:




$$\frac{d\langle I_i^2\rangle}{df} = 2q\,I_b = \frac{2kT}{\beta r_e}$$

$$\frac{d}{df}\langle I_o^2\rangle = \frac{4kT[R_{bb}+R_{ex}] + 2kT\,r_e}{(R_{ex}+r_e)^2} + \underbrace{2q\,I_b}_{=\,2kT/\beta r_e}\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)^2$$

$$\frac{d}{df}\langle I_i I_o\rangle = -\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)\cdot 2q\,I_b$$

So now substitute these back into the amplifier.

output noise terms:

generator: $\frac{d}{df}\langle P_{out}\rangle = A^2 kT$ | feedback resistor $\frac{d}{df}\langle P_o\rangle = kT[1+A]$

amplifier input noise current:

$$\frac{d}{df}\langle P_{out}\rangle = A^2 \frac{Z_o^2}{4} \cdot \frac{2kT}{\beta r_e}\,\frac{1}{Z_o} = A^2\,\frac{Z_o}{r_e}\,\frac{2kT}{4\beta}$$

but $g_m Z_o = Z_o/r_e = 1+A$

$$= A^2\,\frac{Z_o}{4}\cdot\frac{2kT}{\beta r_e} = \frac{A^2 Z_o}{4}\,2qI_b$$

amplifier output noise current

$$\frac{d}{df}\langle P_{out}\rangle = \left[\frac{4kT\,Z_o\,(R_{bb}+R_{ex}) + 2kTr_e}{(R_{ex}+r_e)^2} + \frac{2kT}{\beta r_e}\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)^2\right] \cdot Z_o/4$$

$$\frac{d}{df}\langle P_{out}\rangle = kT\,\frac{Z_o}{R_{ex}+r_e}\left[\frac{R_{bb}+R_{ex}+r_e/2}{R_{ex}+r_e} + \frac{2qI_b}{4}\,\frac{(R_{bb}+R_{ex})^2}{R_{ex}+r_e}\right]$$

Correllation term

$$\frac{d}{df}\langle P_{out}\rangle = 2\frac{d}{df}\langle I_i I_n\rangle \cdot \frac{AZ_o}{2}\cdot\left(-\frac{Z_o}{2}\right)\frac{1}{Z_o}$$

$$= +\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)\frac{A\,Z_o}{\cancel{2}}$$

Total output noise power:

$$\frac{d}{df}\langle P_o\rangle = A^2kT + [1+A]\,kT$$ generator, Rf

$$+ 2qI_b\left[\frac{A^2Z_0}{4} + \frac{Z_0}{4}\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)^2 + \frac{AZ_0}{2}\left(\frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right)\right]$$

base shot noise

base thermal

$$+ kT\,\frac{Z_0}{R_{ex}+r_e}\left[\frac{R_{bb}+R_{ex}+r_e/2}{R_{ex}+r_e}\right]$$

Rex

collector shot

$$\frac{d}{df}\langle P_o\rangle = A^2kT + [1+A]\,kT$$ generator, Rf

$$+ 2qI_b Z_0 \frac{1}{4}\left[A + \frac{R_{bb}+R_{ex}}{R_{ex}+r_e}\right]^2$$ (of course) Base shot.

$$+ kT\,(1+A)\left[\frac{R_{bb}+R_{ex}+r_e/2}{R_{ex}+r_e}\right]$$ base, Rex thermal collector shot.

To get noise figure, divide by $A^2kT$ ...

## Noise figure

$$F = 1 + \left[\frac{1+A}{A^2}\right] \qquad \text{generator, } R_f$$

$$+ \frac{2q I_b Z_0/4}{kT}\left[1 + \frac{1}{A}\,\frac{R_{bb} + R_{ex}}{R_{ex} + r_e}\right]^2 \qquad \text{Base shot}$$

$$+ \left[\frac{1+A}{A^2}\right]\left[\frac{R_{bb} + R_{ex} + r_e/2}{R_{ex} + r_e}\right] \qquad \text{base thermal, } R_{ex} \text{ thermal, collector shot.}$$

The above is written in physical form. To get the result in terms of $r_e$, use $2qI_b = 2kT/\beta r_e$

$$F = 1 + \left[\frac{1+A}{A^2}\right] + \frac{1}{2\beta}\,\frac{Z_0}{r_e}\left[1 + \frac{1}{A}\,\frac{R_{bb} + R_{ex}}{R_{ex} + r_e}\right]^2$$

$$+ \left[\frac{1+A}{A^2}\right]\left[\frac{R_{bb} + R_{ex} + r_e/2}{R_{ex} + r_e}\right]$$

we also end up with more "useful" expressions if we use

$(R_{ex}+r_e)^{-1} Z_o = 1+A \Rightarrow (R_{ex}+r_e)^{-1} = Z_o^{-1}(1+A)$

$$F = 1 + \left[\frac{1+A}{A^2}\right] \qquad \text{generator, } R_f$$

$$+ \frac{2q I_b Z_o/4}{kT}\left[1 + \frac{1+A}{A}\frac{R_{bb}+R_{ex}}{Z_o}\right]^2 \qquad \text{base shot}$$

$$+ \left[\frac{1+A}{A}\right]^2 \frac{R_{bb}+R_{ex}+r_e/2}{Z_o} \qquad \text{base thermal, } R_{ex} \text{ thermal, collector shot.}$$

Again use $2qI_b = 2kT/\beta r_e$ to get another useful form:

$$F = 1 + \left[\frac{1+A}{A^2}\right] + \frac{1}{2\beta}\frac{Z_o}{r_e}\left[1 + \frac{1+A}{A}\frac{R_{bb}+R_{ex}}{Z_o}\right]^2$$

$$+ \left[\frac{1+A}{A}\right]^2 \frac{R_{bb}+R_{ex}+r_e/2}{Z_o}$$

finally, start approximating:

If Rex is larger than re, then Rex + re/2 $\cong$ re + Rex.

we then get:

$$F \cong 1 + \left[\frac{1+A}{A^2}\right] + \frac{1}{2\beta}\,\frac{Z_0}{r_e}\left[1 + \frac{1+A}{A}\,\frac{R_{bb}+R_{ex}}{Z_0}\right]^2$$

$$+ \underbrace{\left[\frac{1+A}{A}\right]^2 \frac{R_{bb}}{Z_0}}_{R_{bb}\ \text{thermal.}} + \left[\frac{1+A}{A^2}\right]$$

Rex thermal + collector shot

This is

$$F \cong 1 + 2\left[\frac{1+A}{A^2}\right] + \frac{1}{2\beta}\,\frac{Z_0}{r_e}\left[1 + \frac{1+A}{A}\,\frac{R_{bb}+R_{ex}}{Z_0}\right]^2$$

$$+ \left[\frac{1+A}{A}\right]^2 \frac{R_{bb}}{Z_0}$$

If we can approximate that $A+1 \cong A$, we have:

$$F \approx 1 + \frac{2}{A} + \frac{Z_0}{2\beta r_e}\left[1 + \frac{R_{bb}+R_{ex}}{Z_0}\right]^2 + \frac{R_{bb}}{Z_0}$$

now, the findings here can be quite profound!