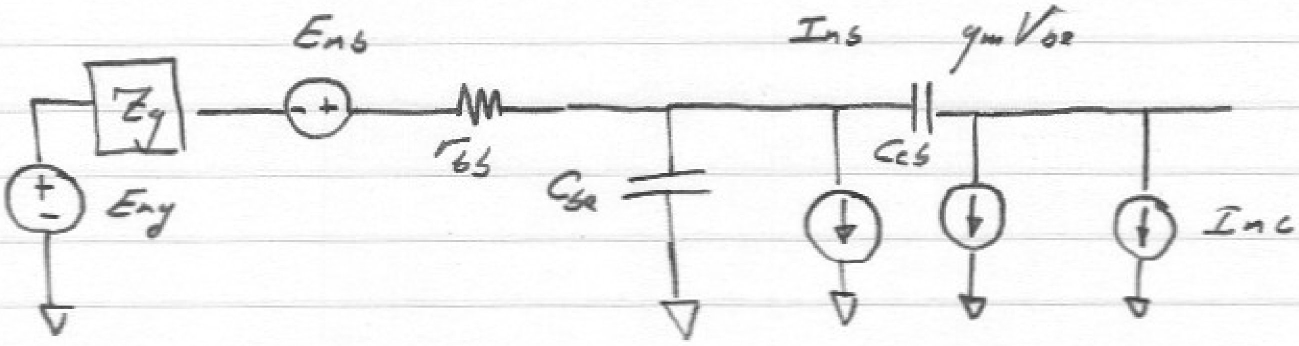
• Notes Set 17: Approximations to Bipolar minimum noise figure

- derivation of (long) expression for noise figure vs source impedance
- derivation of approximate expressions for MINIMUM noise figure

ECE ____ Notes set ~~14A~~ 17

derivations leading to F_{min} approximations for Bipolar Junction Transistors.

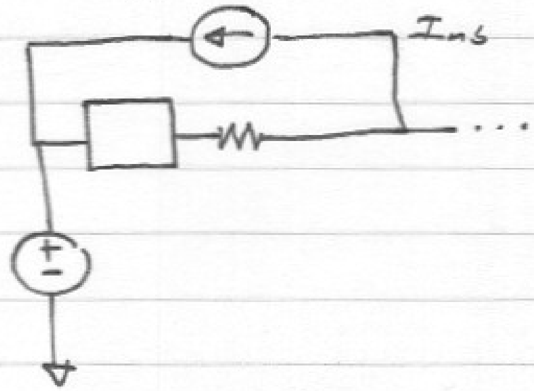


Assumptions: ~~[illegible]~~
$\omega C_{be} \gg 1/R_{be}$ e.g. $\omega \gg \omega_T/\beta$

Use transposition of sources:

= E_{nb} already at input (E_{ng})

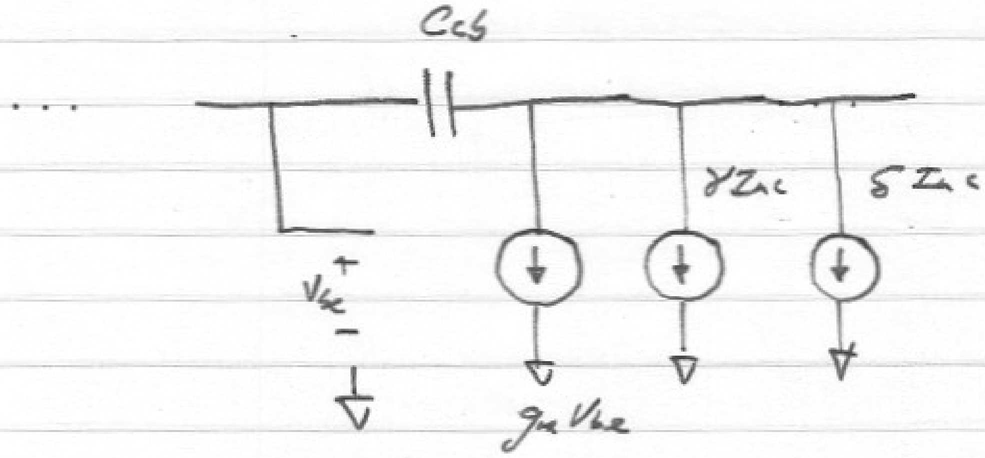
I_{nb}:



I_{nb} moves in one step to input:

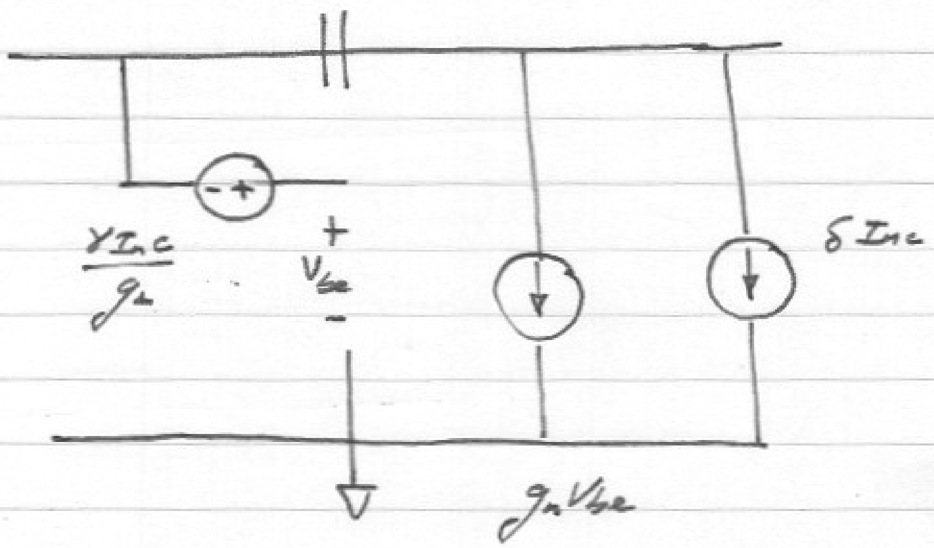
$$S_{vv}\Big|_{I_{nb}} = \left|(r_{bb} + R_{gen})^2 + X_{gen}^2\right| 2qI_b.$$

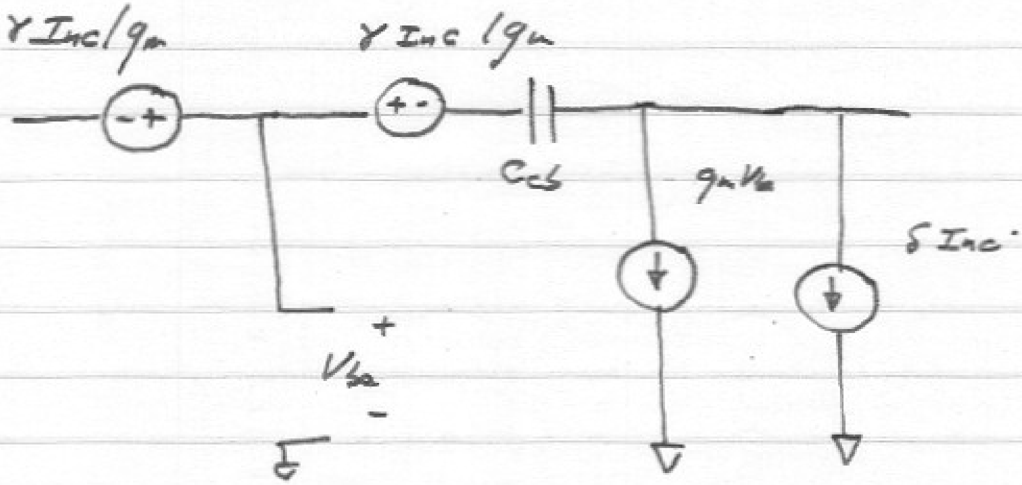
Lets work on I_{nc}:

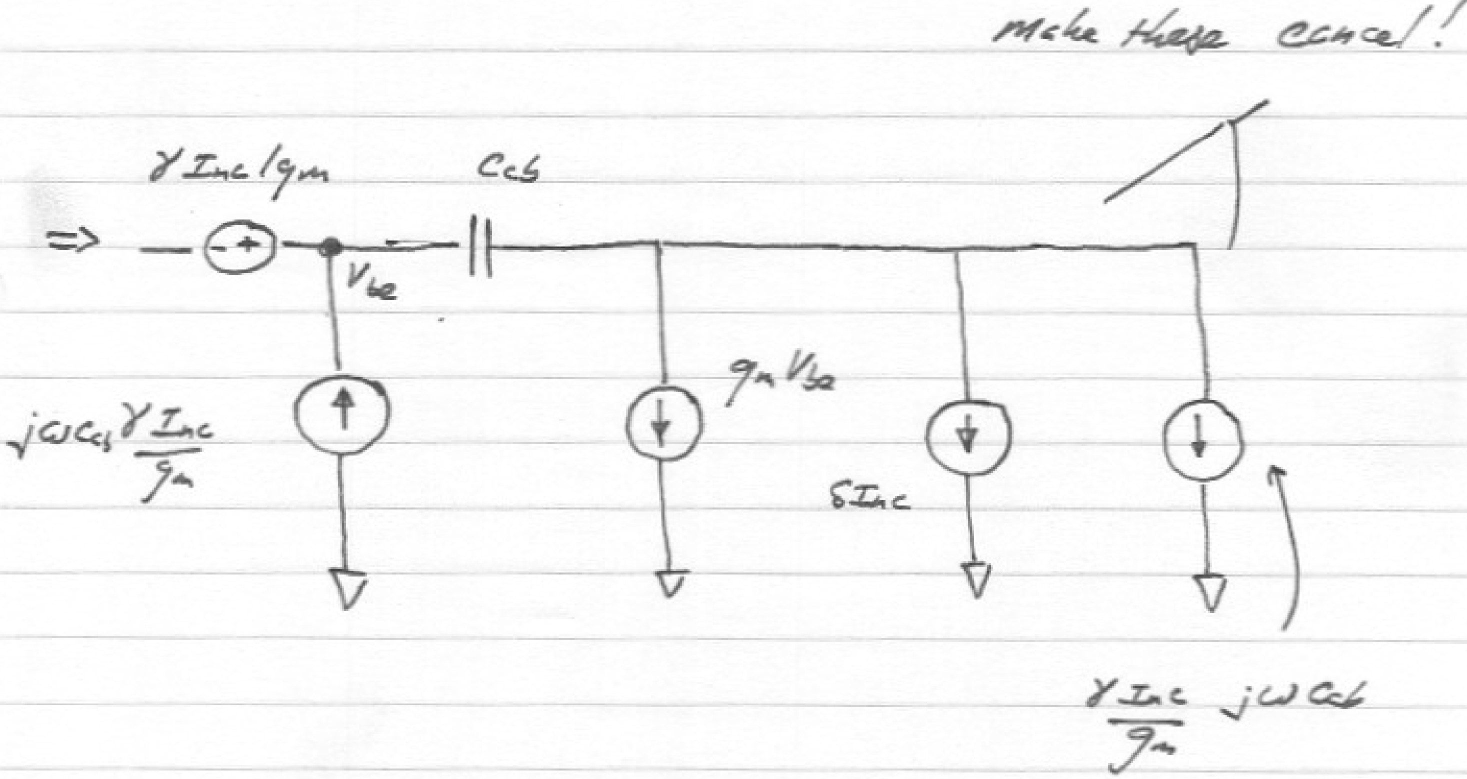


$\underline{\gamma + \delta = 1}$

$\Rightarrow$

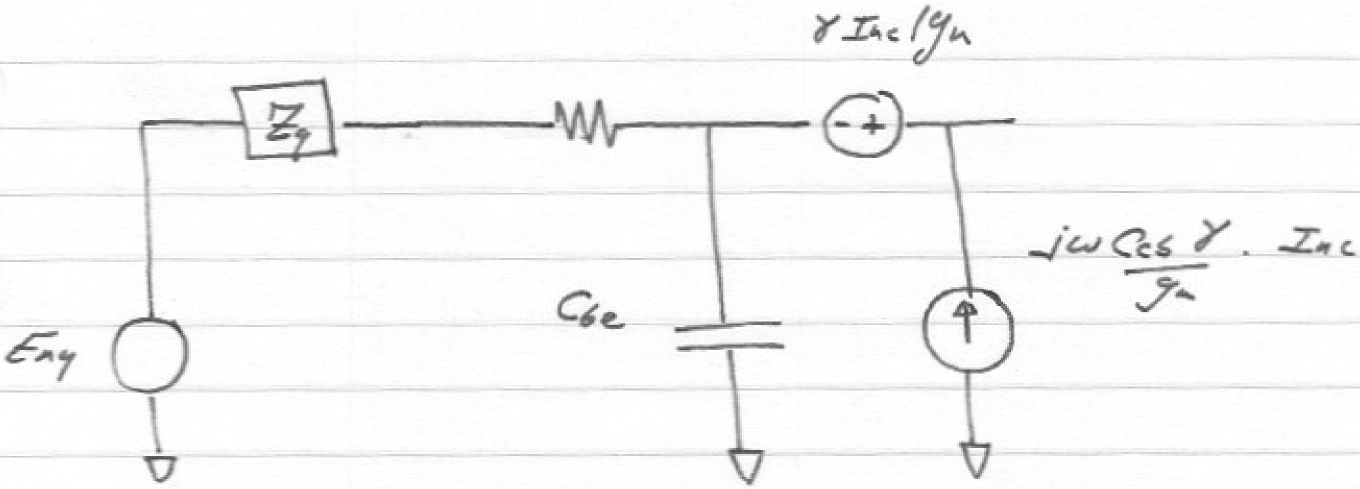




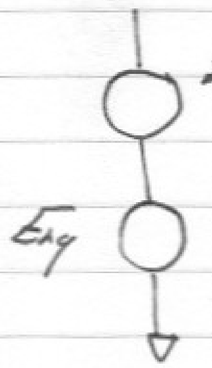


now choose $\underline{\delta} + \dfrac{\gamma}{g_m} \cdot \underline{j\omega C_{cb} = 0}$; $\gamma + \delta = 1$

$$\Rightarrow \boxed{\gamma = \frac{1}{1 - j\omega C_{cb}/g_m}}$$



by inspection



$$I_{nc} \cdot \left\{ \frac{\gamma}{g_m}\left(1 + j\omega C_{be}(r_{bb} + r_{gen} + jX_{gen})\right) + \frac{\gamma}{g_m}(j\omega C_{cb}) \cdot (r_{bb} + r_{gen} + jX_{gen}) \right\}$$

$$= I_{nc} \cdot \frac{\gamma}{g_m}\left(1 + j\omega(C_{be} + C_{cb})(r_{bb} + r_{gen} + jX_{gen})\right)$$

Input referred noise voltage due to I_{nc}:

$$E = \frac{I_{nc}}{g_m} \frac{1}{1 - j\omega C_{cb}/g_m} \left[1 + j\omega (C_{be} + C_{cb}) \cdot (r_{bb} + r_{gen} + jX_{gen}) \right]$$

$$= \frac{I_{nc}}{g_m} \frac{1}{1 - j\omega C_{cb}/g_m} \cdot \left[(1 - \omega (C_{be} + C_{cb}) X_{gen}) + j\omega (C_{be} + C_{cb})(r_{bb} + r_{gen}) \right]$$

$$EE^* = \frac{I_{nc} I_{nc}^*}{g_m^2} \frac{1}{1 + \omega^2 C_{cb}^2 / g_m^2} \left[\left\{ 1 - \omega X_{gen} (C_{be} + C_{cb}) \right\}^2 + \omega^2 (C_{be} + C_{cb})^2 (r_{bb} + r_{gen})^2 \right]$$

Total Input noise voltage:

$$S_{vv}(f) = 4kT R_{gen} \qquad \text{generator}$$

$$+ 4kT R_{bb} \qquad \text{base resistance}$$

$$+ \frac{2qI_c}{\beta}\left[(r_{bb} + R_{gen})^2 + X_{gen}^2\right] \qquad \text{base "shot"}$$

$$+ \frac{2qI_c}{g_m^2} \cdot \frac{1}{1 + \omega^2 C_{cb}^2 / g_m^2} \left\{ \begin{array}{l} \left\{1 - \omega X_{gen}(C_{be} + C_{cb})\right\}^2 \\ + \omega^2 (C_{be} + C_{cb})^2 (r_{bb} + r_{gen})^2 \end{array} \right\}$$

collector "shot"

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now use: $g_m = \frac{I_c}{V_T} \Rightarrow \frac{1}{g_m^2} = \frac{V_T^2}{I_c^2}$

$$\Rightarrow \frac{2qI_c}{g_m^2} = \frac{2qV_T^2}{I_c} = 2q\, V_T \cdot \frac{V_T}{I_c} = \underline{2kT r_e}$$

and: $2qI_c/\beta = \frac{qI_c}{kT}\,\frac{2kT}{\beta} = \frac{2kT}{\beta r_e}$

and: $C_{be} = g_m \tau_f + C_{je} = C_{je} + \tau_f / r_e$

$$S_{vv}(f) = 4kT R_{gen} + 4kT R_{bb}$$

$$+ \frac{2kT}{\beta r_e} \left[(r_{bb} + r_{gen})^2 + X_{gen}^2 \right]$$

$$+ \frac{2kT r_e}{1 + \omega^2 C_{cs}^2 / g_m^2} \left\{ \left[1 - X_{gen} \cdot \bar{\omega} (C_{je} + C_{cs} + \tau_f / r_e) \right]^2 + \omega^2 (C_{je} + C_{cs} + \cancel{g_m} \tau_f / r_e)^2 \cdot (r_{bs} + r_{gen})^2 \right\}$$

now we can write the noise figure...

$F = 1$ source

$+ R_{bb} / R_{gen}$ R_{bb}

$$+ \frac{1}{2 \beta r_e R_{gen}} \cdot \left[(r_{bb} + R_{gen})^2 + X_{gen}^2 \right]$$ base shot

$$+ \frac{r_e / 2 R_{gen}}{1 + \omega^2 C_{cb}^2 r_e^2} \left\{ \left[1 - X_{gen} \cdot \omega \left(C_{je} + C_{cb} + \tau_f / r_e \right) \right]^2 + \omega^2 \left(C_{je} + C_{cb} + \tau_f / r_e \right)^2 \left(r_{bb} + r_{gen} \right)^2 \right\}$$

Collector shot

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the <u>only</u> approximation here is neglecting r_{ex}.

To a reasonable approximation, this just adds a term r_{ex} / R_{gen} to F.

To find F_{min}, we have to

a) find the optimum $Z_{gen} = R_{gen} + jX_{gen}$ which minimizes F

b) then vary bias (vary r_e) to find the optimum low-noise bias.

This is arduous, and I won't attempt it.

Note that BJT noise analysis is <u>not</u> harder than FET noise analysis. The BJT equations are more complex because the model is more complete, modelling the variation of all small-signal & noise parameters with bias. We don't even attempt to do this with FETs.

Lets simplify: - Perhaps dangerously -

a) $1 + \omega^2 C_{cb}^2 r_e^2 \cong 1 \leftarrow$ ok

b) Ignore the X_{gen}^2 term in the base shot noise.

then $X_{gen} = \dfrac{(-1)}{j\omega(C_{je} + C_{cb} + \tau_f / r_e)}$ for lowest noise, and.

$$F \cong 1 + \frac{R_{bb}}{R_{gen}} + \frac{(r_{bb} + R_{gen})^2}{2 \beta r_e R_{gen}}$$

$$+ \frac{r_e}{2 R_{gen}} \left[\omega^2 (\underbrace{C_{je} + C_{cb} + \tau_f / r_e}_{C_T})^2 (r_{bb} + R_{gen})^2 \right]$$

$$= 1 + \frac{R_{bb}}{R_{gen}} + \frac{(r_{bb} + R_{gen})^2}{r_e R_{gen} 2 \beta}$$

$$+ \frac{r_e}{2 R_{gen}} \left[\omega^2 C_T^2 (r_{bb} + R_{gen})^2 \right]$$

$$F = 1 + \frac{R_{bb}}{R_{gen}} + \frac{(r_{bb} + R_{gen})^2}{2\beta r_e R_{gen}} +$$

$$+ \frac{r_e}{2R_{gen}} \left[\omega^2 C_T^2 \right] (r_{bb} + R_{gen})^2$$

$$= 1 + \frac{R_{bb}}{R_{gen}} + \frac{R_{bb}^2}{2\beta r_e R_{gen}} + \frac{2 R_{bb} r_{gen}}{2\beta r_e R_{gen}} + \frac{R_{gen}^2}{2\beta r_e R_{gen}}$$

$$+ \frac{r_e \omega^2 C_T^2}{2 R_{gen}} r_{bb}^2 + \frac{r_e \omega^2 C_T^2}{2 R_{gen}} 2 r_{bb} R_{gen}$$

$$+ \frac{r_e \omega^2 C_T^2}{2 R_{gen}} \cdot R_{gen}^2$$

$$F = 1 + \frac{1}{R_{gen}} \left\{ R_{bb} + \frac{R_{bb}^2}{2\beta r_e} + \frac{r_e \omega^2 C_T^2 R_{bb}^2}{2} \right\}$$

$$+ R_{gen} \left\{ \frac{1}{2\beta r_e} + \frac{r_e \omega^2 C_T^2}{2} \right\}$$

$$+ \frac{R_{bb}}{\beta r_e} + r_e \omega^2 C_T^2 R_{bb}$$

$$=$$

<u>so</u>

$$R_{gen}\Big|_{opt} = \sqrt{\frac{R_{bb}}{\left(\frac{1}{2\beta r_e} + \frac{r_e \omega^2 C_T^2}{2}\right)} + R_{bb}^2}$$

<u>and</u>

$$F_{min} = 1 + 2\left(\frac{1}{2\beta r_e} + \frac{r_e \omega^2 C_T^2}{2}\right) \sqrt{\frac{R_{bb}}{\left(\frac{1}{2\beta r_e} + \frac{r_e \omega^2 C_T^2}{2}\right)} + R_{bb}^2}$$

$$+ \frac{R_{bb}}{\beta r_e} + r_e \omega^2 C_T^2 R_{bb}$$

$$\boxed{F_{min} = 1 + \frac{1}{r_e}\left(\frac{1}{\beta} + \omega^2 C_T^2 r_e^2\right)\sqrt{\frac{2R_{bb}r_e}{1/\beta + \omega^2 C_T^2 r_e^2} + R_{bb}^2} + \frac{R_{bb}}{r_e}\left(\frac{1}{\beta} + \omega^2 C_T^2 r_e^2\right)}$$

~~... the term $(1/\beta + \omega^2 C_T^2 r_e^2)^{-1}$ should be recognized as h_{21} ...~~

Now, as in the Fukui FET model, we approximate that the first term in the radical dominates...

$$\boxed{F_{min} \approx 1 + \sqrt{\frac{R_{bb}}{r_e}}\sqrt{\frac{1}{\beta} + \omega^2 C_T^2 r_e^2} + \frac{R_{bb}}{r_e}\left(\frac{1}{\beta} + \omega^2 C_T^2 r_e^2\right)}$$

$$= 1 + \frac{R_{bb}}{\beta r_e} + \omega C_T r_e \sqrt{\frac{R_{bb}}{r_e}} \cdot \sqrt{1 + \frac{1}{\beta \omega^2 C_T^2 r_e^2}}$$

$$+ \frac{R_{bb}}{r_e}\omega^2 C_T^2 r_e^2$$

now we can eliminate the $\frac{1}{\beta\omega^2 C_T^2 r_e^2}$ pretty safely...

$$F_{min} \approx 1 + \frac{R_{bb}}{\beta r_e} + \sqrt{\frac{R_{bb}}{r_e}}\,\omega \cdot C_T r_e$$

$$+ \frac{R_{bb}}{r_e} \cdot \omega^2 C_T^2 r_e^2$$

Recognizing that $C_\pi r_e = 1/\omega_T$, we can write:

$$\boxed{F_{min} \simeq 1 + \frac{R_{bb}}{\beta r_e} + \sqrt{\frac{R_{bb}}{r_e}\left(\frac{\omega}{\omega_T}\right) + \frac{R_{bb}}{r_e}\left(\frac{\omega}{\omega_T}\right)^2}}$$

This can be compared directly with the Fukui FET expression.

... But the expression is very approximate.

Note that F_{min} depends not at all upon f_{max}.

Why? Well, for $f \simeq F_{max}$, the gain associated with F_{min} will become tiny, and the noise measure will become large. We really should calculate M_{opt}, not F_{opt}, but the math is a nightmare...

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The Noise Performance of Microwave Transistors

H. FUKUI

Abstract—Expressions for the noise parameters of microwave transistors are derived. The theory is based on a small-signal common-emitter equivalent circuit which includes a new basic noise equivalent circuit and the dominanting header parasitics. The theory is verified experimentally in the L-band (1 to 2 Gc/s) frequency range using Ge and Si microwave transistors. It is found that the header parasitics have little influence on the minimum noise figure, but do have large effects on the equivalent noise resistance and the optimum source admittance in the frequency region above about one-half of the series-resonant frequency resulting from the parasitics in conjunction with wafer parameters. For a quick evaluation of the noise performance, new approximate expressions are also given for the noise figure and for the optimum current which produces the lowest value.

Principal Symbols

A, B, C, D = Noise parameters
a = Drift potential
B_0 = Optimum source susceptance
B_s = Source susceptance
C_{BE} = Base-emitter header stray capacitance
C_{CB} = Collector-base header stray capacitance
C_{CE} = Collector-emitter header stray capacitance
C_c = $C_{c_1} + C_{c_2}$
C_{c_1} = Inner collector-base capacitance = $C_{T_{c_1}} + C_{D_c}$
C_{c_2} = Outer collector-base capacitance = $C_{T_{c_2}}$
C_{D_c} = Collector diffusion capacitance
C_{D_e} = Emitter diffusion capacitance = $1/\omega_1 r_1$
C_E = $C_{T_e} + C_{BE}$
C_e = $C_{D_e} + C_{T_e}$
$C_{T_{c_1}}$ = Inner collector-base transition region capacitance
$C_{T_{c_2}}$ = Outer collector-base transition region capacitance
C_{T_e} = Emitter-base transition region capacitance
D_0 = Diffusion constant of the minority carrier in the base region
E = Built-in field strength in the base region
e_B = Thermal noise voltage of the base resistance
F = Noise figure
F_{min} = Minimum noise figure
$(F_{min})_{HF}$ = Approximate high-frequency minimum noise figure
Δf = Narrow frequency interval
G_0 = Optimum source conductance
G_s = Source conductance
g_e = Real part of y_e

Manuscript received August 11, 1965.
The author is with Bell Telephone Laboratories, Inc., Murray Hill, N. J.

Reprinted from *IEEE Trans. Electron Devices*, vol. ED-13, pp. 329-341, Mar. 1966.