• Notes Set 20: Noise in Signal Sources (Phase noise)

- handwaving re 1/f noise. Small-angle PM. Spectral descriptions of PM and AM
- Amplitude noise and Phase noise sidebands. Energy relationships to find timing deviations. Phase noise in oscillators. Van Der Pol Model.
- Derivation of oscillator phase noise spectrum, 1/f and 1/f^3 regions
- Example: Phase lock loop for subpicosecond laser timing stabilization including: phase detector noise, AM-PM conversion, etc.
- Comments on PLL design generally

ECE_Notes set 18.2U

$\boxed{\text{Noise in Signal sources}}$

In other words

AM noise and FM noise

Phase noise

Timing Noise

Timing Jitter, etc...

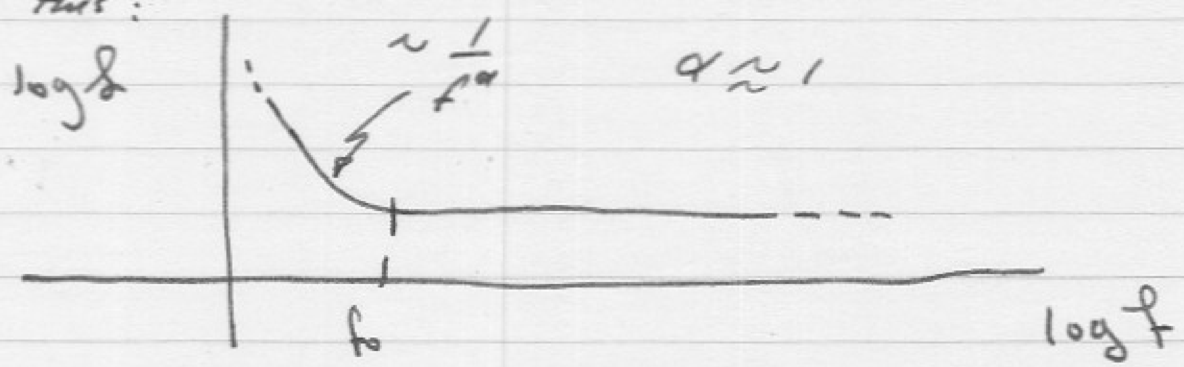
Once upon a time I spent
considerable time learning how to
measure & quantify (phase noise)
"timing jitter" & how to
suppress it in a laser ~~done~~ through
use of a -phase-lock-loop-

Please: try not to use
the term "jitter" - very
imprecise & unscientific language.

"1/f noise"

Most devices produce noise spectral densities

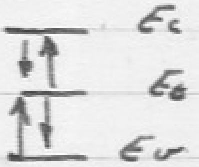
like this:



Noise power increases rapidly for frequencies

below a "1/f" noise corner frequency

<u>Why?</u>

1) 

} random processes involving population and depopulations of traps involve <u>long</u> storage times.

These can be found using the principle of detailed balance, similar to the Shockley-Read-Hall method for generation/recombination <u>analysis</u>.

If we assume a distribution of traps over energy, a leakage current power spectrum can be found, and has a "$1/f^{\alpha}$" variation over a wide range of frequencies. <u>Surfaces</u> of semiconductors are great concentrations of <u>traps</u>.

devices with high surface/volume ratios (e.g. fets) have high surface effects and high 1/f noise. BJT's generally are better. Our bit's, with their high surface/volume ratios will resemble Fets...

2) Microphonics & mechanical vibrations, particularly in big flashlamp-pumped lasers, etc.

1/f noise is often very important, particulary with phase noise...

Caution - regarding notation

we have often used:

$\frac{\partial \langle e_L e_L^* \rangle}{\partial f}$, where $\int_{f_1}^{f_2} \frac{\partial \langle e_L e_L^* \rangle}{\partial f} df$

is the power between f_1 & f_2. This is called a single-sided spectral density.

Here we shall use

$\mathcal{S}_=(\omega)$: $$P = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathcal{S}(\omega)\, d\omega$$

$$= \int_{-\infty}^{+\infty} \mathcal{S}\left(f = \frac{\omega}{2\pi}\right) df$$

... or double sided spectral densities:

Small Angle Phase Modulation

Consider a signal

$$v(t) = v_0 e^{j\omega_0 t}$$

now give it some "timing jitter"

$$v(t) = v_0 e^{j\omega_0 (t + J(t))} = v_0 e^{j(\omega_0 t + \theta(t))}$$

suppose $J(t) = T_M \cos \omega_M t$:

use $e^x \sim 1 + x$ for $x \ll 1$

$$v(t) \sim v_0 e^{j\omega_0 t} \left(1 + j\omega_0 J(t)\right)$$

$$= \underbrace{v_0 e^{j\omega_0 t}}_{\text{Signal}} + \underbrace{v_0 e^{j\omega_0 t} \left(j\omega_0 J(t)\right)}_{\text{Modulation.}}$$

$$\boxed{v(t) = v_0 e^{j\omega_0 t} + \left(v_0 e^{j\omega_0 t}\right) \cdot j\omega\, T(t)}$$

write a cosinusoidal $T(t) = T_m \cos \omega_m t$

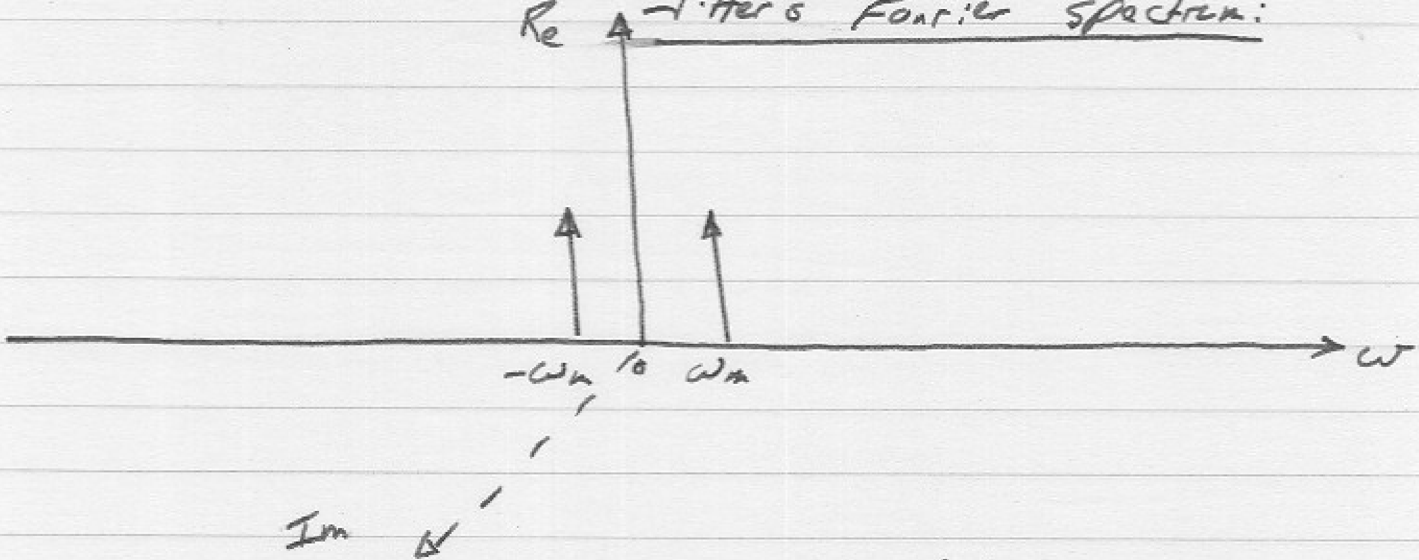
$$= \frac{T_m}{2}\left[e^{j\omega_m t} + e^{-j\omega_m t} \right]$$

so:

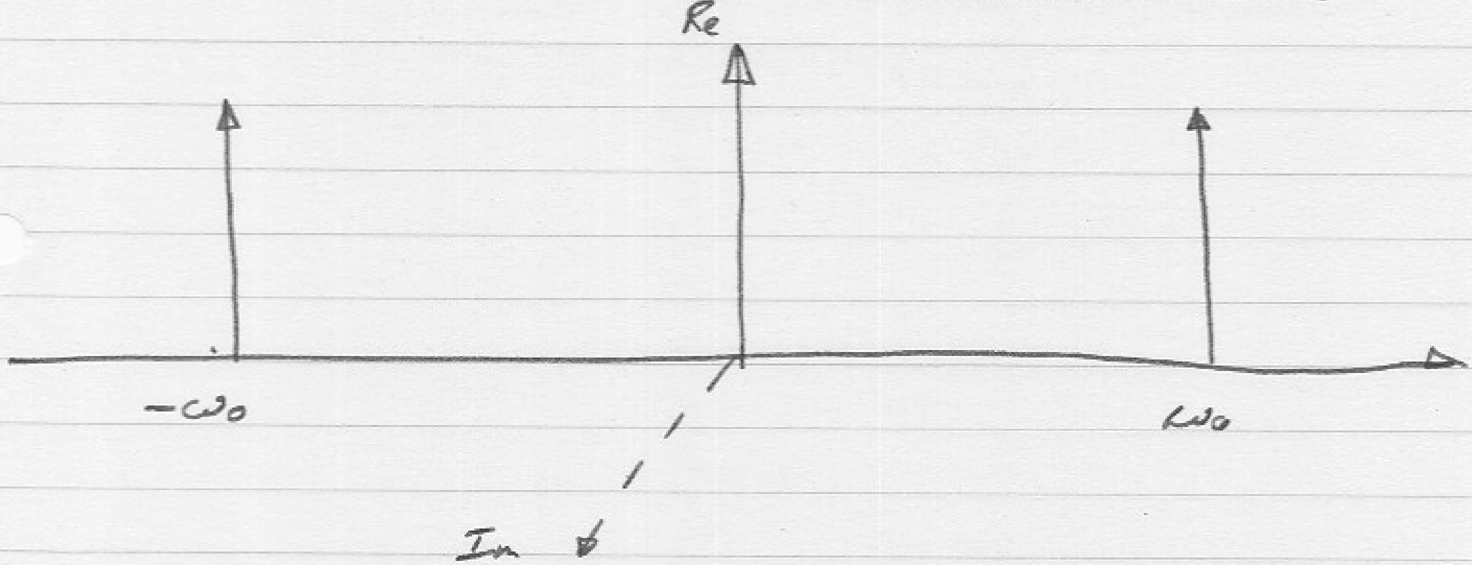
$$v(t) = v_0 e^{j\omega_0 t} + \frac{1}{2}\, j\omega_0 v_0 T_m \left[e^{j(\omega_0+\omega_m)t} + e^{j(\omega_0+\omega_m)t} \right]$$

$\boxed{J(t) = T_m \cos \omega_m t}$

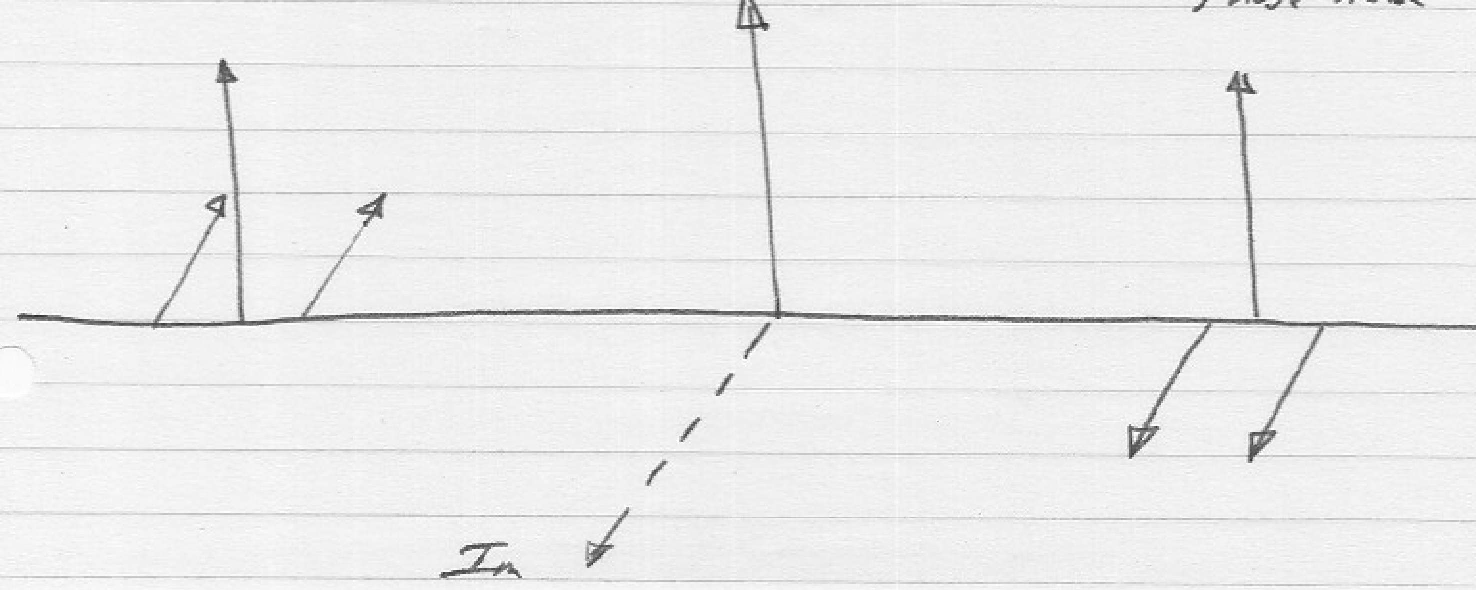
Jitter's Fourier spectrum:



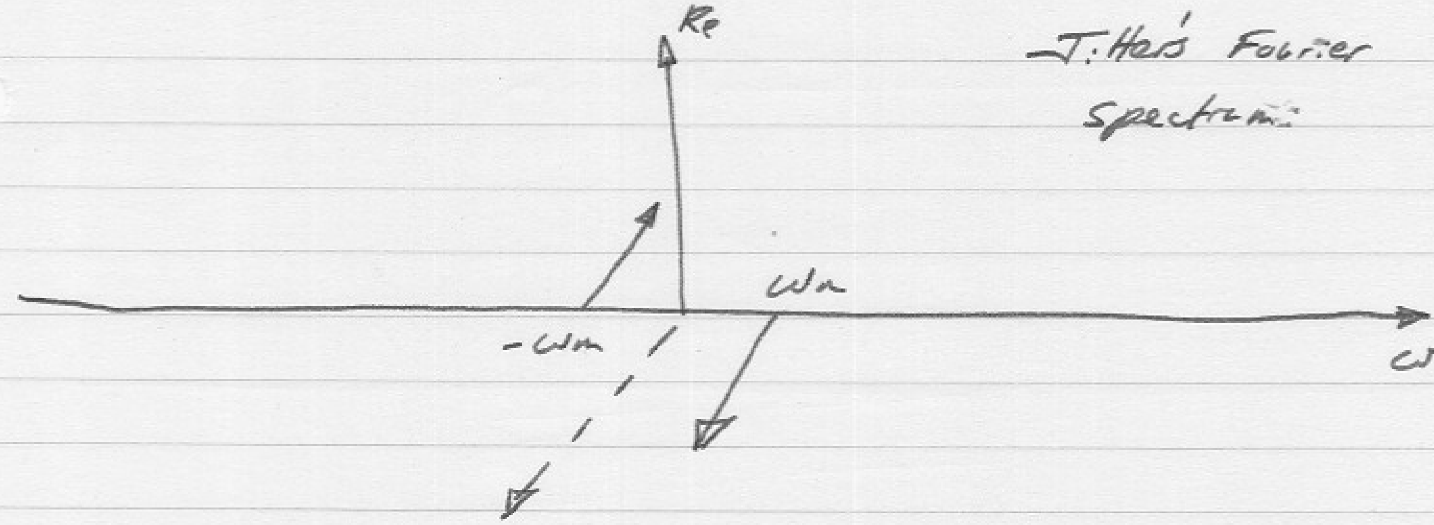
unmodulated carrier



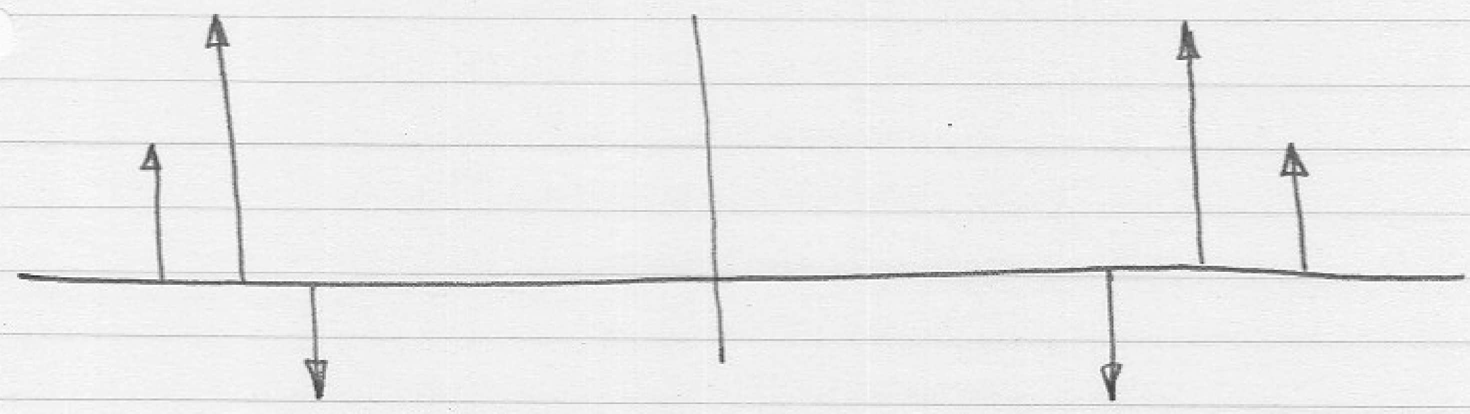
Carrier with ~~jitter~~ phase noise



Jitter's Fourier spectrum:



Carrier with phase noise



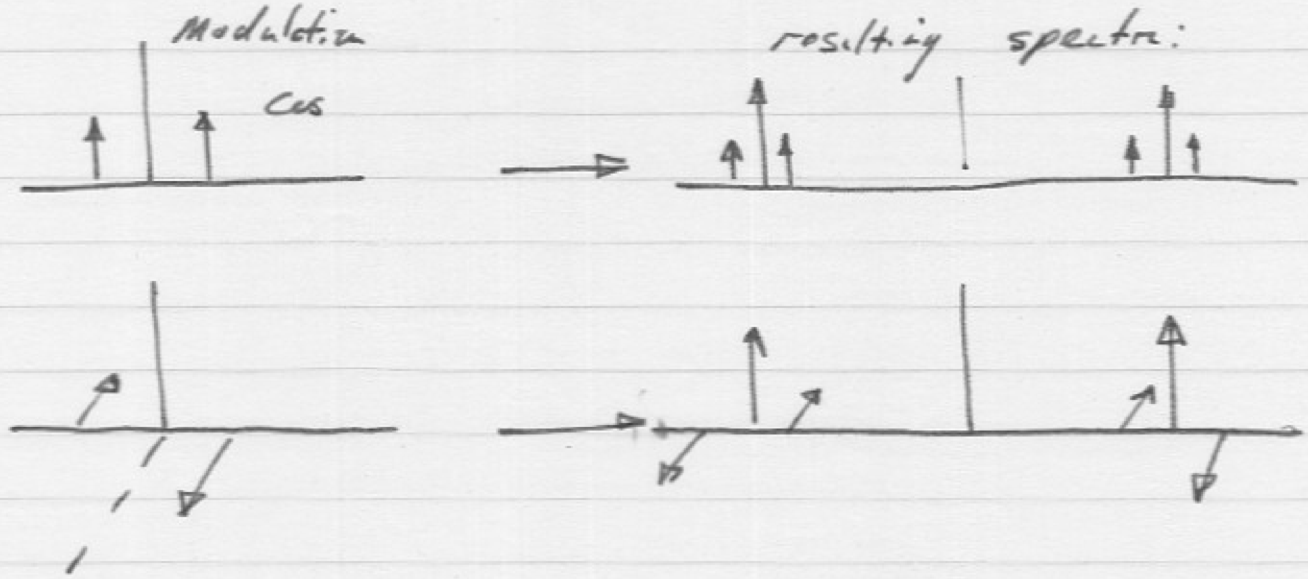
Key points, small phase angles

* Phase modulation results in sidebands whose amplitude $\left(\frac{\omega_0 T_m}{2}\right)$ is the phase deviation

* Phase modulation sidebands are

Skew Hermetian

as opposed to am sidebands, which are Hermetian



Major Comment

There is lots of math here, if we try. Time is now running short. I will include a paper to give the gory details...

IF we have a sinusoidal/cosinusoidal signal with Deterministic Phase Modulation

$$\underline{v(t) =} \; v_0 \cos[\omega_0(t + J(t))] = v_0 \cos[\omega_0 t + \Theta(t)]$$

$J(t)$ is the timing modulation in seconds

$\Theta(t)$ is the phase modulation in radians

$\Theta(t) = \omega_0 J(t)$

Then the signal spectrum is:

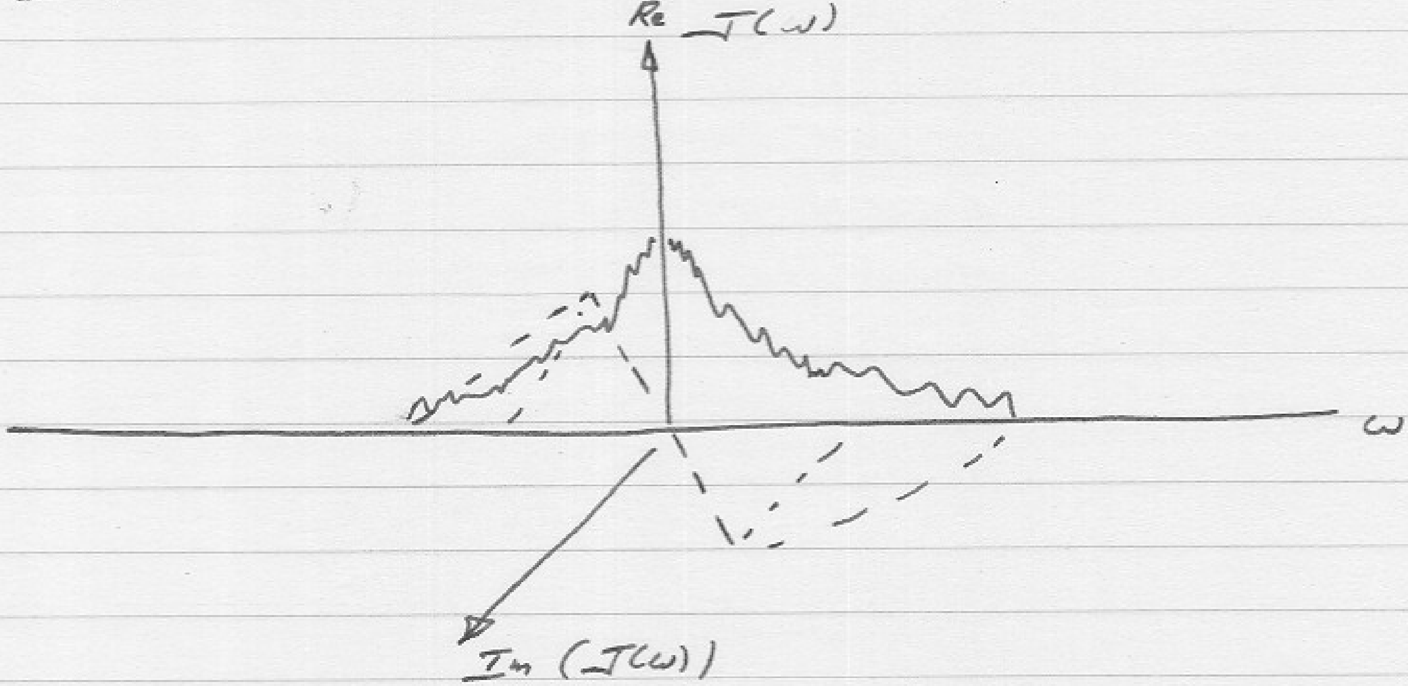
$$V(\omega) = \frac{v_0}{2}\,\delta(\omega - \omega_0)$$

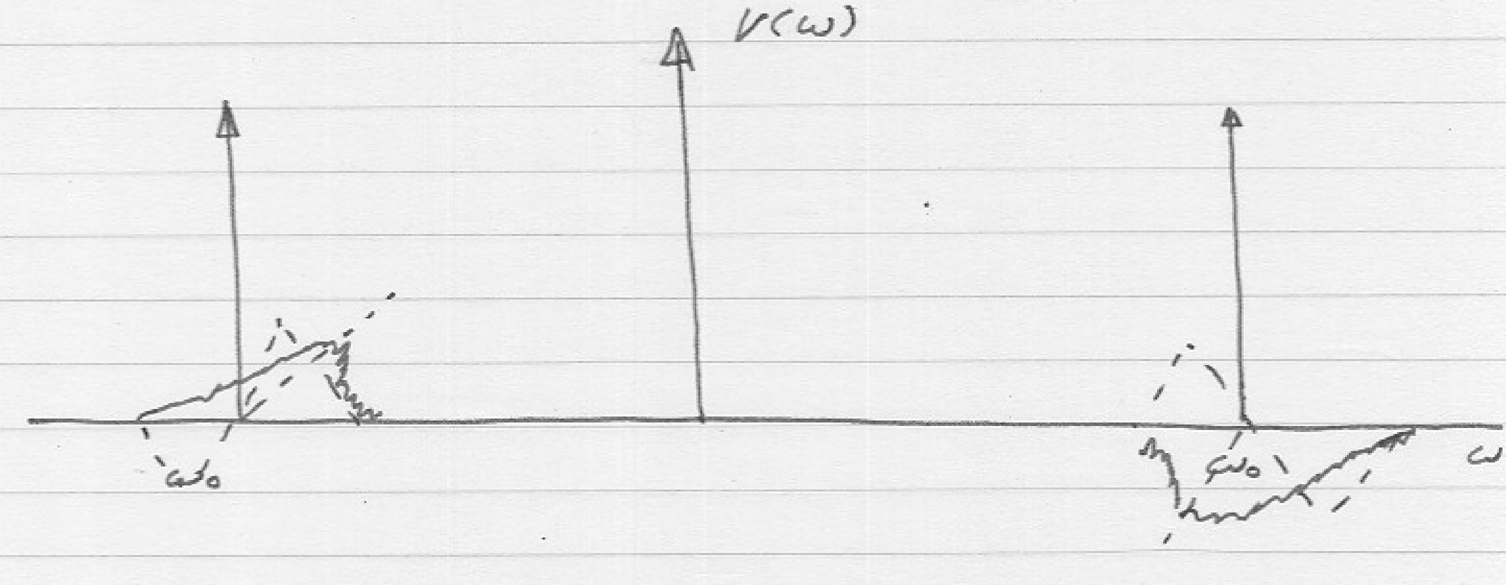
$$+ \frac{v_0}{2}(j\omega_0)\,J(\omega - \omega_0)$$

$$+ \frac{v_0}{2}\,\delta(\omega + \omega_0)$$

$$+ \frac{v_0}{2}(j\omega_0)\,J(\omega + \omega_0)$$

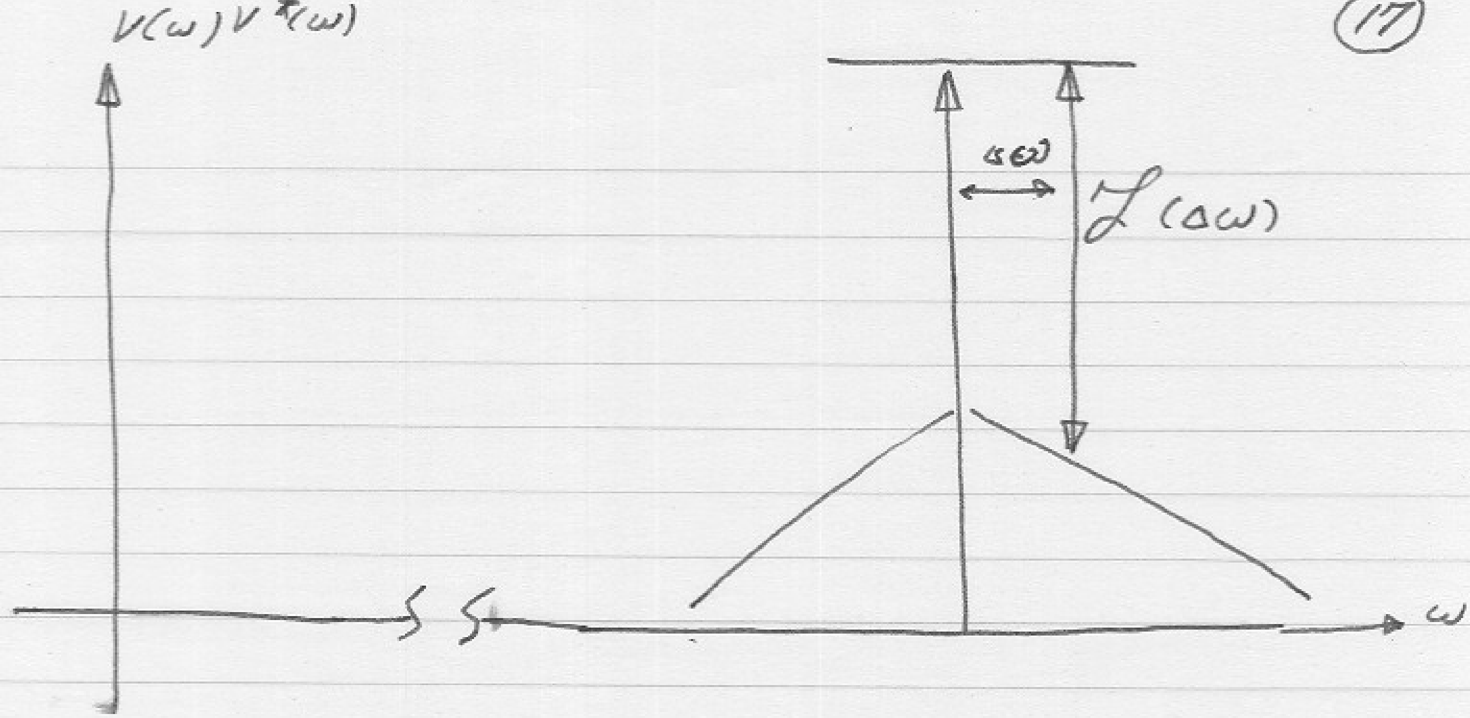
A picture will help:





These are the fourier spectra V(ω)

lets look at the power spectra $V(\omega)V^*(\omega)$



To find Timing / Phase fluctuations:

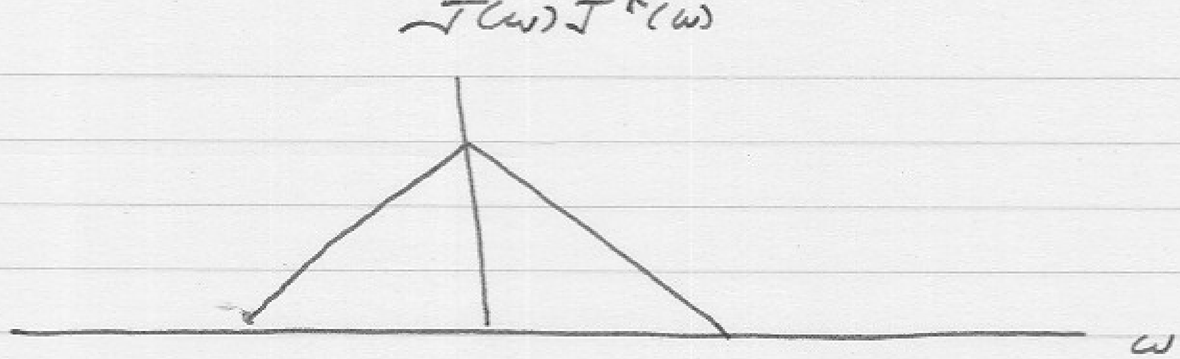
☆ $\mathcal{L}(\omega)$, defined above, is the single sideband phase noise spectral density: relative power in carrier vs harmonics.

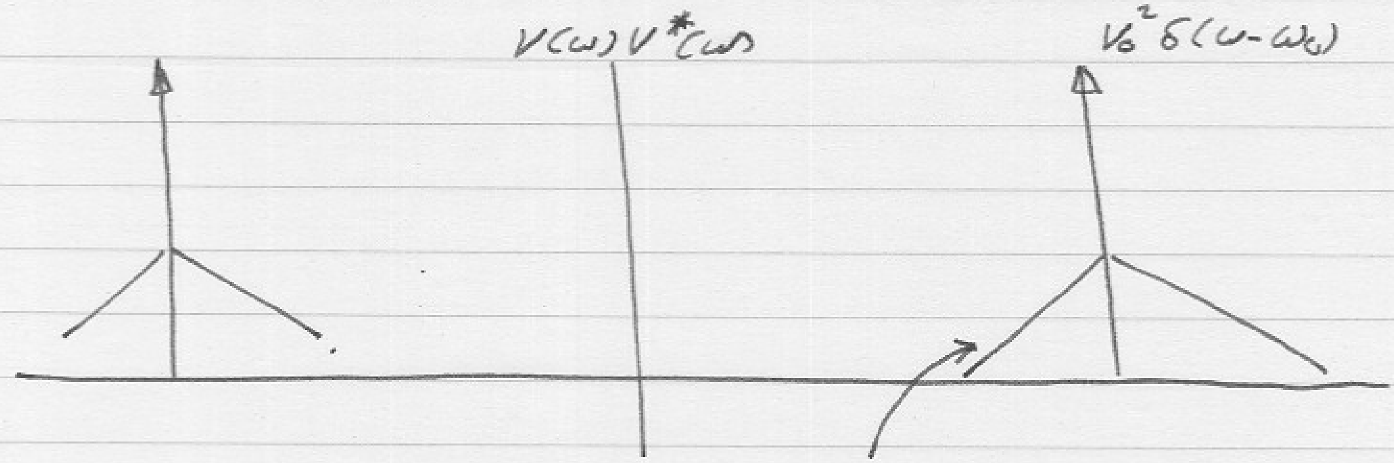
☆ for our assumption: $\Theta(\omega) << 2\pi$ - small angles -

$$\mathcal{L}(\omega) = \Theta(\omega)\,\Theta^*(\omega) = \mathcal{S}_\Theta(\omega)$$

"phase noise Spectral density" of

spectral density of the phase deviations.

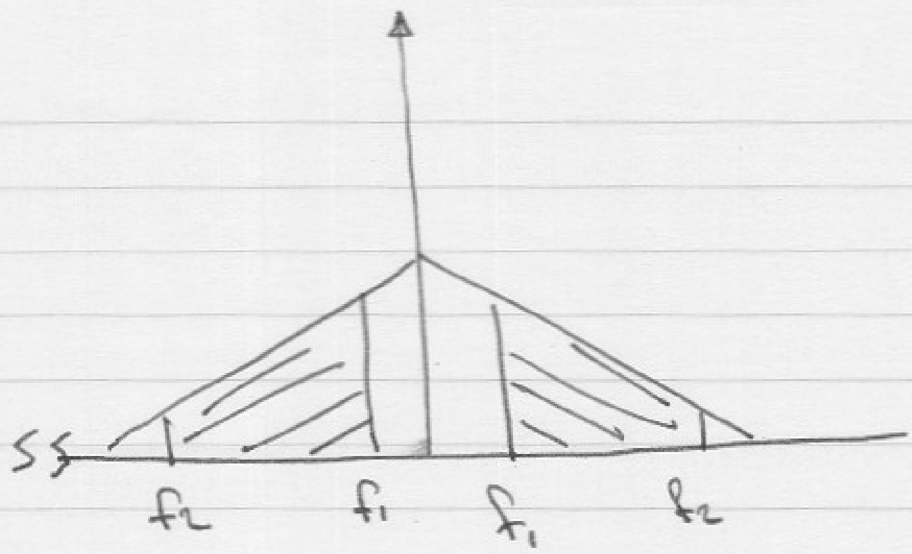




$= V_0^2 \cdot \omega_0^2 \cdot J(\omega) J^*(\omega)$

$\updownarrow$

$= V_0^2 \, \Theta(\omega) \, \Theta^*(\omega)$



$$\frac{\text{Energy in the sidebands}}{\text{Energy in the carrier}} = \langle \theta^2 \rangle$$

= mean-squared phase deviation

$\langle J(t)^2 \rangle$ = mean-squared timing deviation

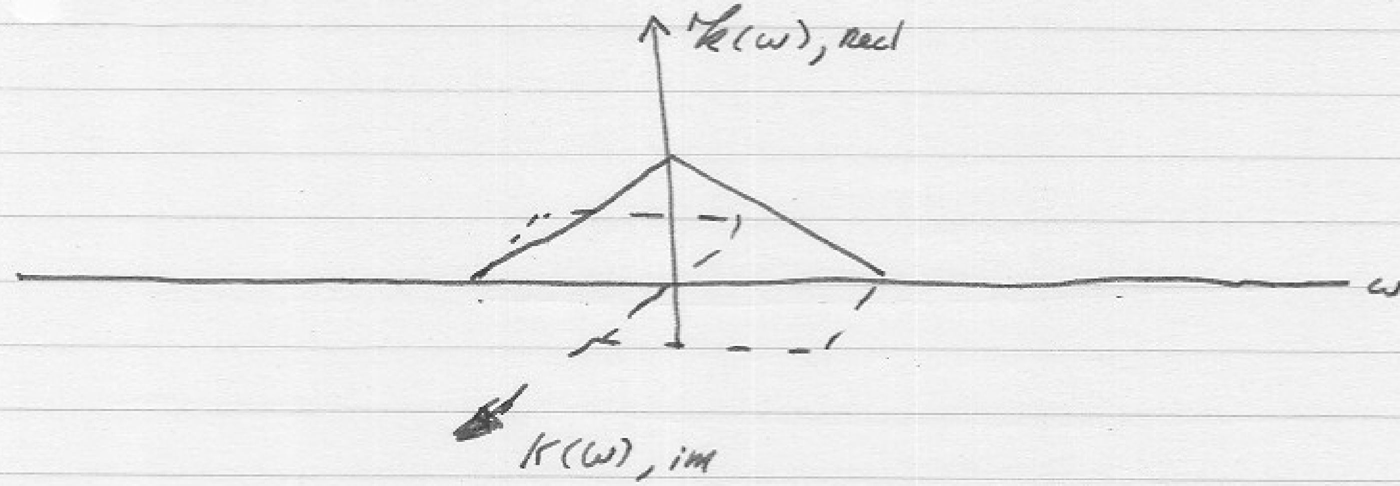
$$= \frac{1}{\omega_0^2} \langle \theta^2 \rangle$$

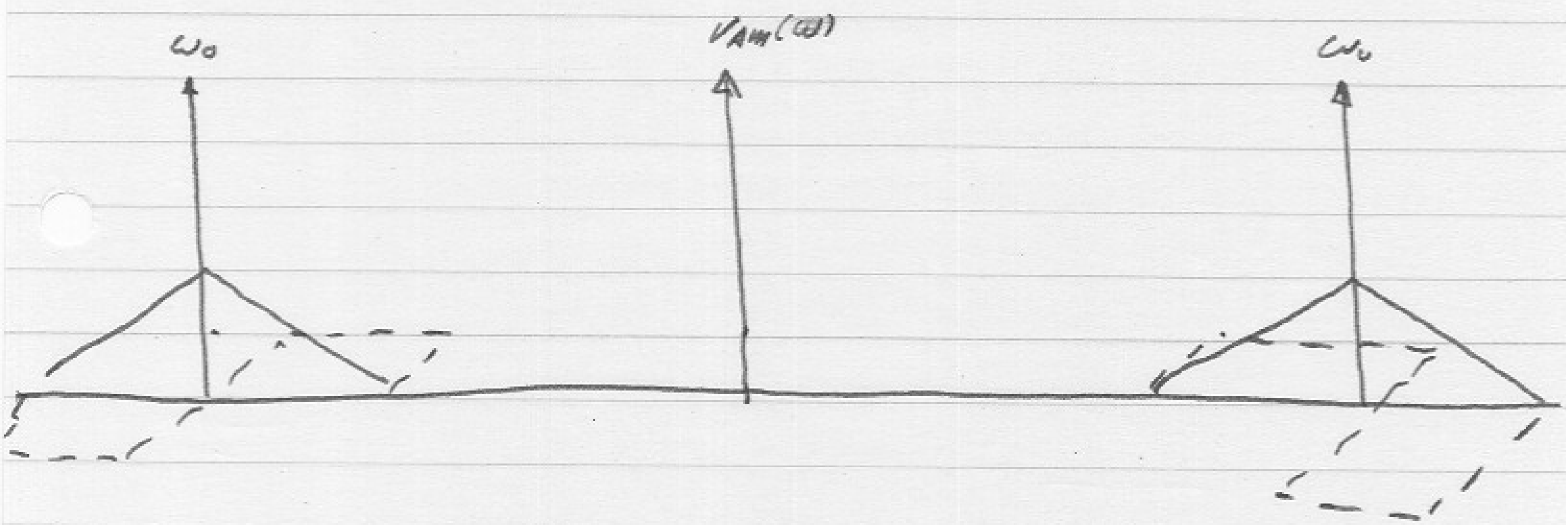
— Now you too can measure phase noise & "timing jitter" —

Modulation

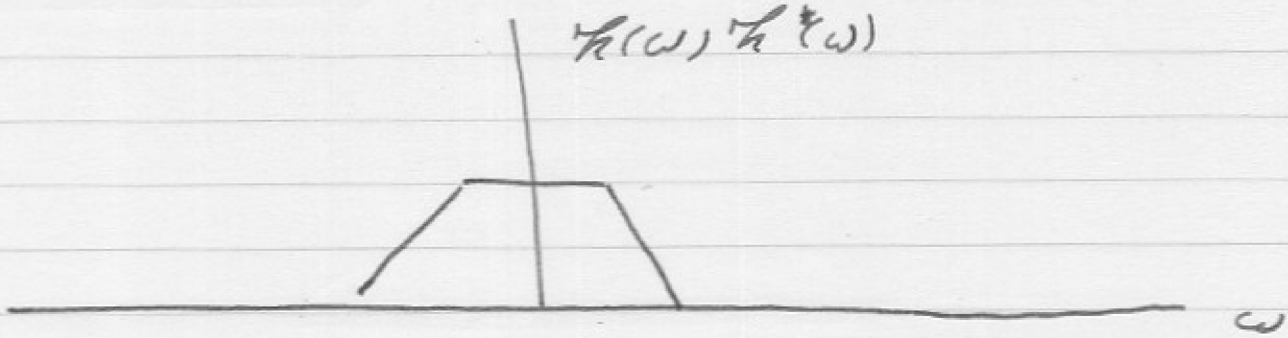
What about AM?

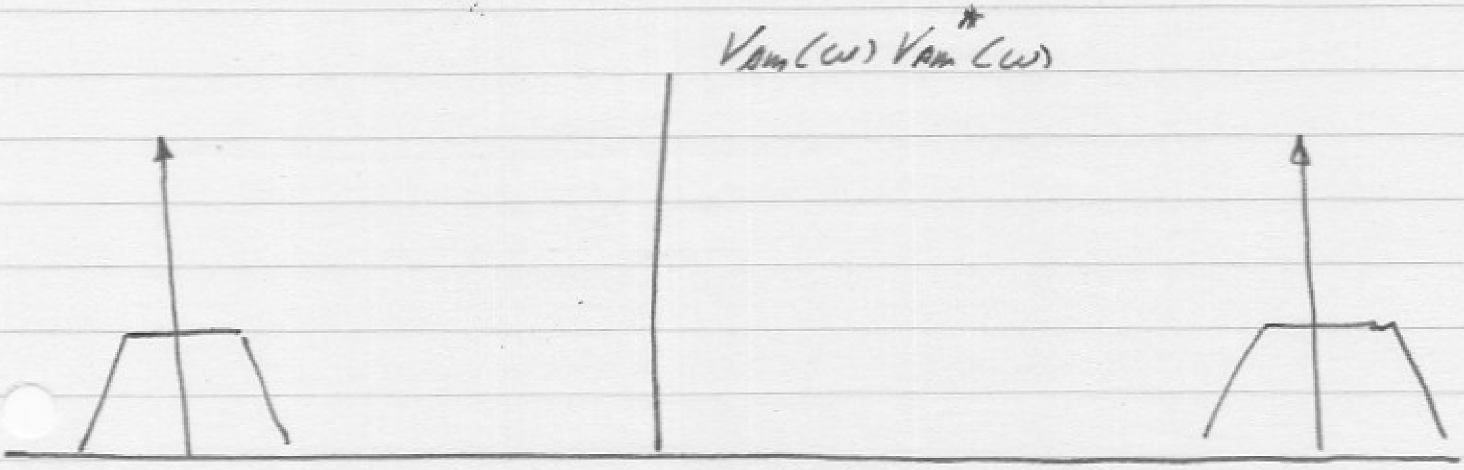
$$V_{AM}(t) = v_0\left[1 + k(t)\right]\cos(\omega_0 t)$$





Now, once we look at power spectra





Note the following:

1) Mean-Squared AM Modulation is found by the ratio of sideband to carrier power.

2) How do we tell AM from PM?

- signals are distinct $v(t) = V_0 \cos(\omega_0 t + \Theta(t))$

vs

$v(t) = V_0 (1 + M(t)) \cos(\omega_0 t)$

- Fourier spectra are distinct:

skew Hermetian (PM)

vs

Hermetian (AM) sidebands.

- Power spectra are not distinct.

So we can tell AM from PM using a good* (*we'll define later) phase detector.

= We cannot directly tell AM from PM using a spectrum analyzer.

= Fortunately, for many electronic signal sources, circuit physics makes PM >> AM, so sidebands are predominantly PM, and a spectrum analyzer can be used to measure PM.

Comment on Math of Random AM / PM:

* Consider $v(t) = v_0 \cos(\omega_0 t + \theta(t))$

  where $\theta(t)$ is a stationary, ergodic, gaussian R.P.

  with spectral density $S_\theta(\omega)$

* $v(t)$ is not stationary, of course.

* To calculate spectra, we must make $v(t)$ at least

  first-order stationary by setting instead:

  $v(t) = v_0 \cos(\omega_0 t + \theta(t) + \theta_{dc})$

  where θ_{dc} is a random (not time-varying)

  initial phase distributed uniformly over $[-\pi, \pi]$

We must be careful with the consequences of this:

* using a spectrum analyzer, θ_{dc} is clearly real because we have no d.c. phase synchronization between the signal and the spectrum analyzer.

* But in dealing with signals with harmonics, e.g. a pulse train:

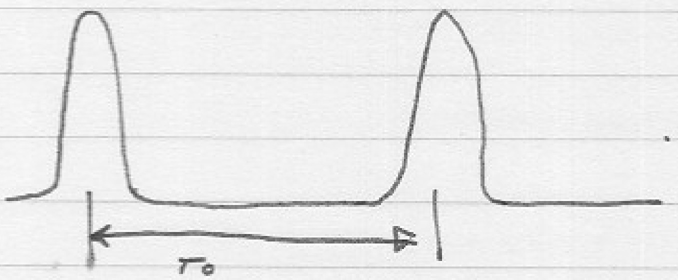
$$v(t) = v_0 \sum_n \delta(t - nT - J(t))$$

$$= v_0 \sum_m a_m \exp\left[j \underset{\substack{\uparrow \\ \frac{2\pi}{T}}}{\omega_0} m (t + J(t))\right]$$

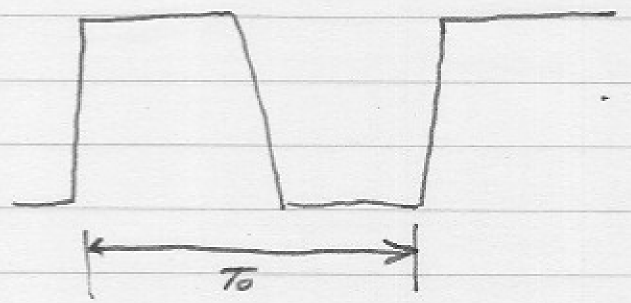
the phase noise sidebands between successive harmonics are 100% correlated; yet to calculate a "stationary" power spectrum of this highly nonstationary process, we have to assume that each harmonic $e^{j m \omega_0 T}$ has a separate random starting phase $\theta_{dc,m}$.

This assumption, clearly nonphysical, loses the correlation between sidebands of successive harmonics...

Subject to this caution, a pulse train



... from a laser



... or from a logic gate...

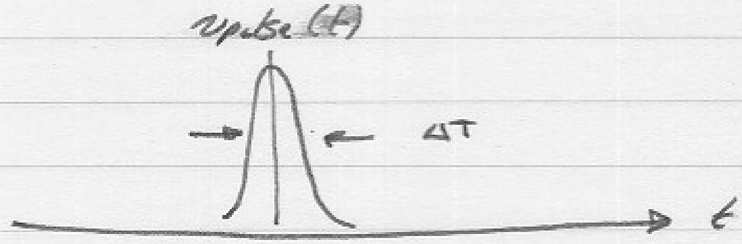
... which is written

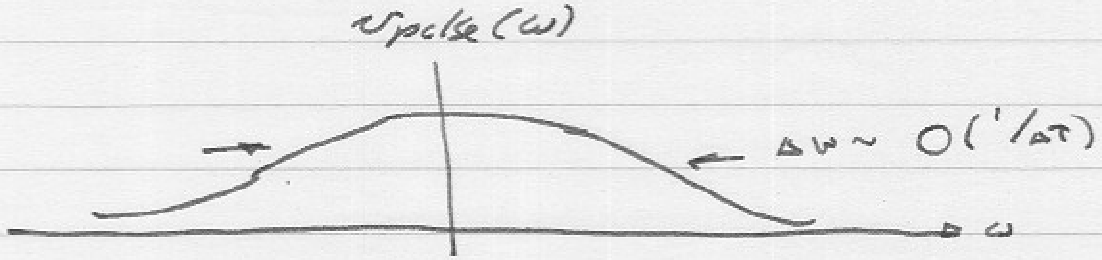
$$v(t) = \sum_{n=-\infty}^{+\infty} v_{pulse}(t - nT_0 - J(t))$$

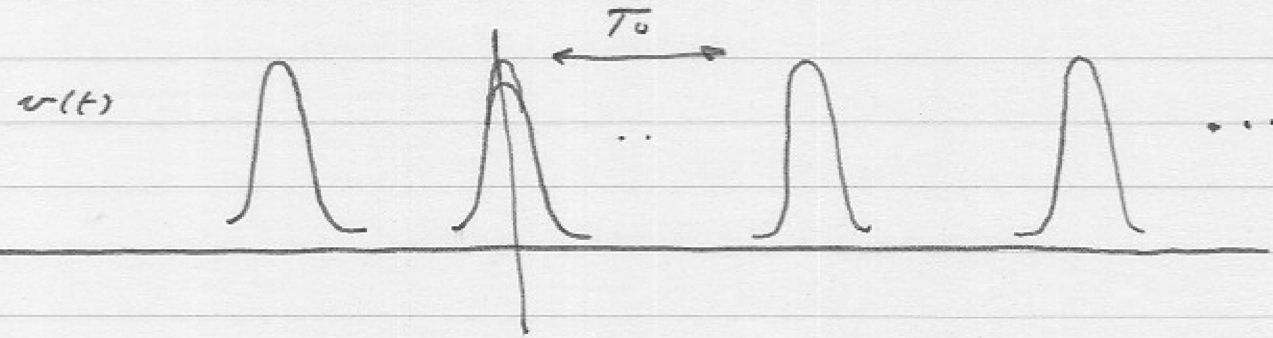
$$\boxed{2\pi / \omega_0 = T_0}$$

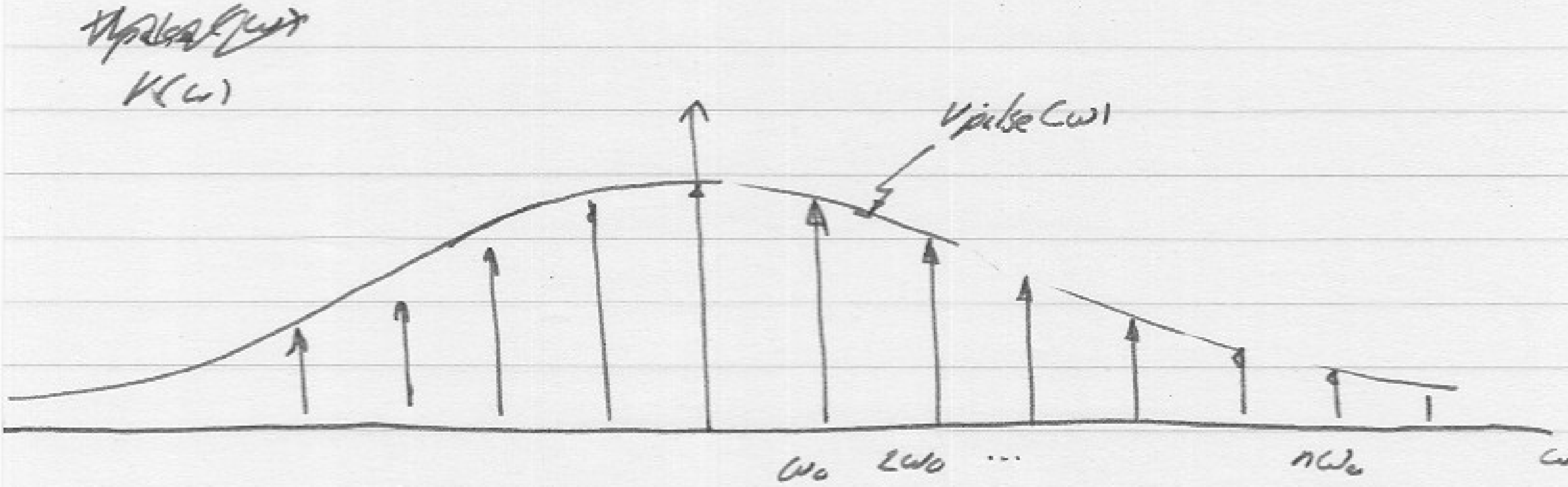
$$= \sum_{m=-\infty}^{+\infty} V_{pulse}(\omega) e^{jm\omega_0 (t - J(t))}$$

No phase Modulation



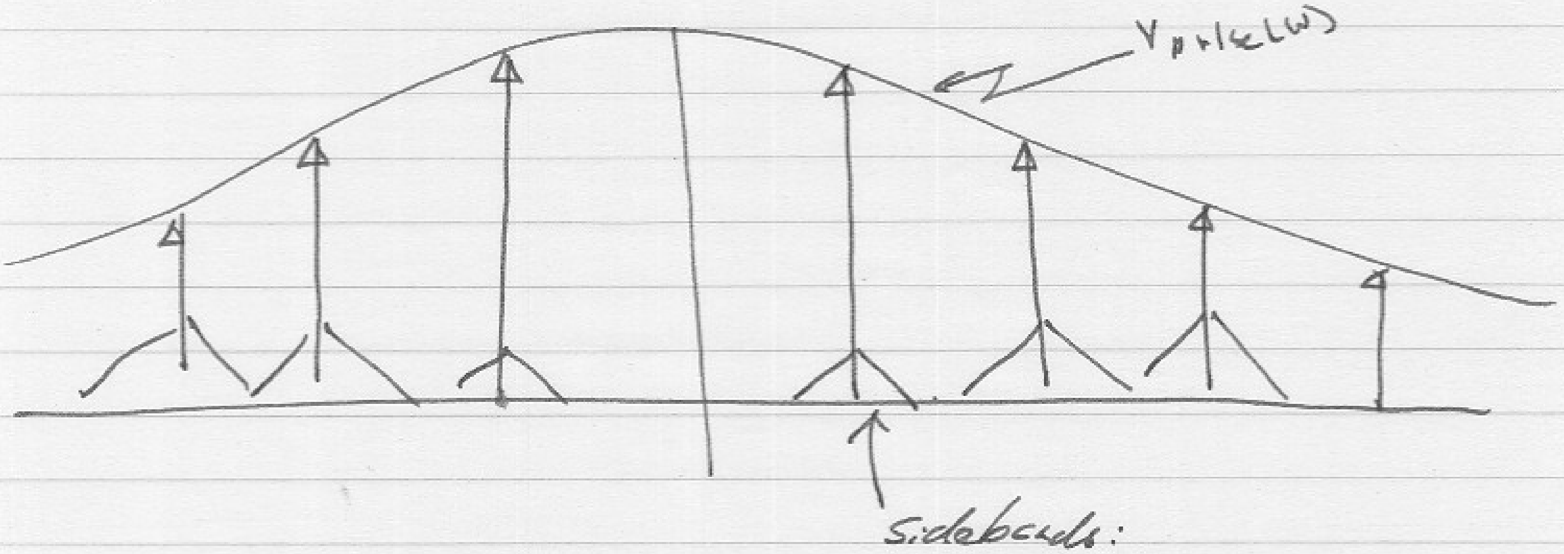






If we add "timing ~~jitter~~" - no random

timing deviations, we get:



for AM: $\frac{\text{Sideband power}}{\text{Carrier power}}$ = mean-squared amplitude modulation

... so this is the same on all harmonics.

for PM: $\frac{\text{sideband power}}{\text{Carrier power}}$ = mean squared phase deviation

$$= n^2 \omega_0^2 \langle J(t)^2 \rangle$$

... so PM sideband power grows as the

square of the harmonic order.

Note that

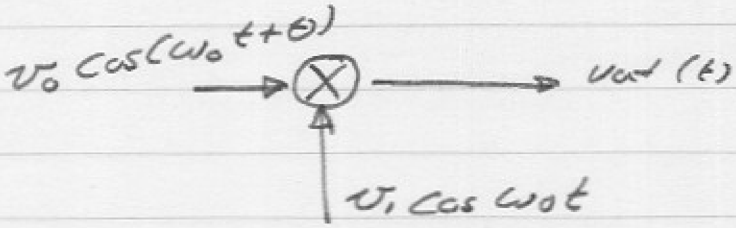
* This can be used to conveniently <u>measure</u> phase noise with a spectrum analyzer.

* At very high harmonics other spectral terms will crop up. These include pulse-width-noise spectral terms, etc.

* One implication of the <u>above</u>: Frequency multiplication in a nonlinear element <u>increases</u> phase noise.

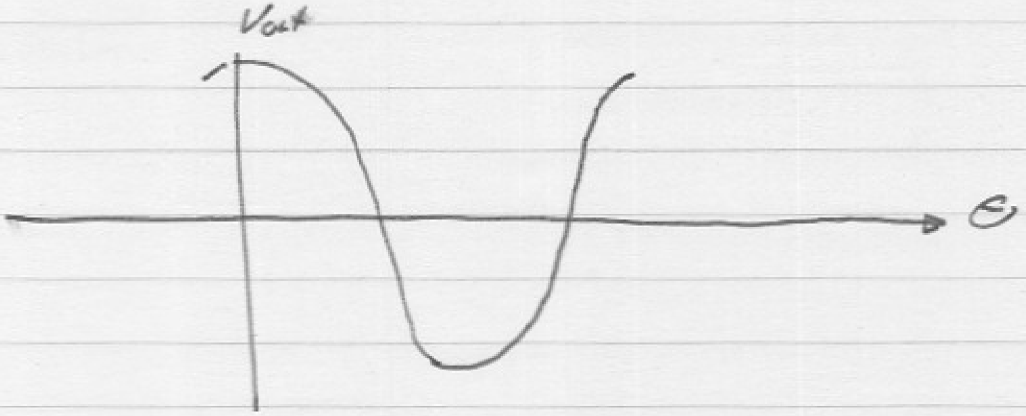
Why? - Because the output <u>timing</u> tracks the input <u>timing</u>, hence in N:1 multiplication the output phase deviation is N:1 bigger than the input phase deviation.

Things to know about phase detectors:



$$V_{out}(t) = \frac{v_1 v_0}{v_{mix}} \cos(\omega_0 t + \theta)\cos(\omega_0 t) + \overset{\text{dc offset}}{V_{os}}$$

$$= \frac{v_1 v_0}{v_{mix}} \cos\theta + V_{os} + (\text{term at } 2\omega_0\text{, filtered out})$$



this looks wonderful:

mixers are phase detectors.

But beware

If: $v_0(1+N(t))\cos(\omega_0 t+\Theta(t))$ is the input

$$\underset{\uparrow \text{ AM}}{N(t)} \qquad \underset{\uparrow \text{ PM.}}{\Theta(t)}$$

Then

$$V_{out} = \frac{v_1 v_0 (1+N(t))}{v_{mix}} \cos\Theta + V_{os}$$

is the output. Write $\Delta\Theta = -\Theta + \pi/2$

$$V_{out} = \frac{v_1 v_0}{v_{mix}} \Delta\Theta \cdot (1+N(t)) + V_{os}$$

$$V_{out} \simeq \underbrace{\frac{v_1 v_0}{v_{mix}} \Delta\Theta}_{\text{desired}} + \underbrace{\frac{v_1 v_0}{v_{mix}} \Delta\Theta \cdot N(t)}_{\text{corruption from AM.}} + V_{os}$$

Corruption from AM may look second-order,

but watch out. unless $\Delta\Theta \sim 0$, it is not

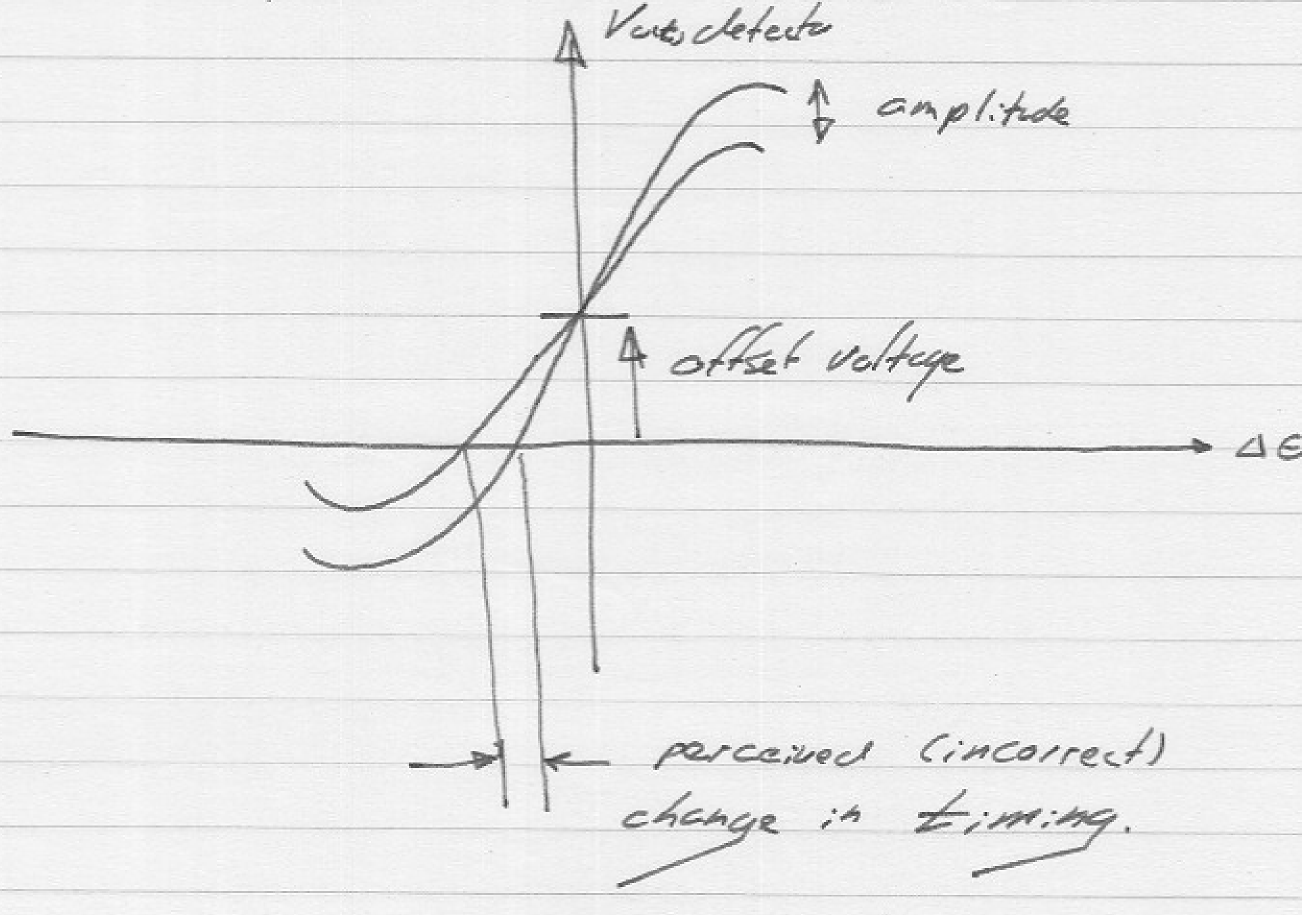
We might set the phase of the input to obtain

$V_{out} \sim 0\,V$. Then $\Delta\Theta \neq 0$

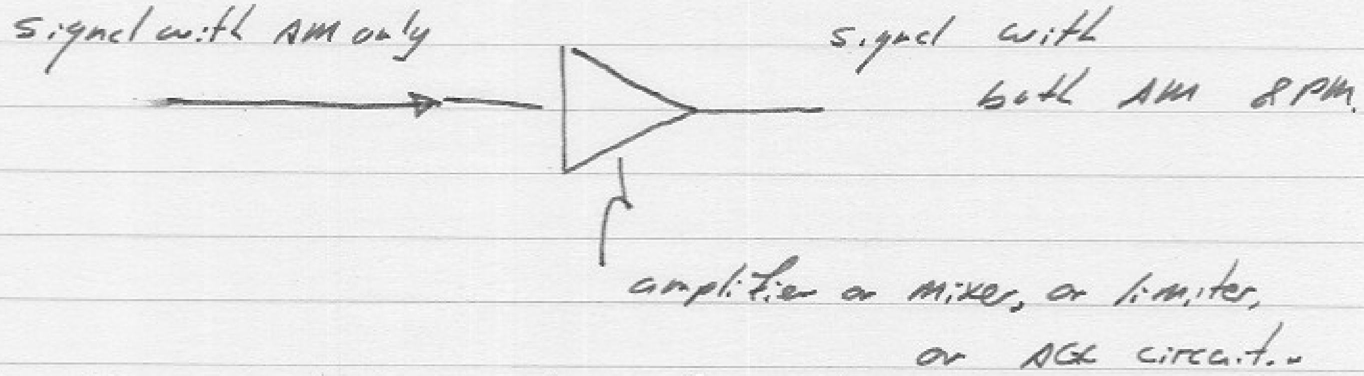
$$\Delta\Theta = v_{os} \cdot \left(\frac{v_{mix}}{v_1 v_0} \right) \neq 0$$

e.g. mixer offset voltage ends up indirectly giving

us AM corruption.



The reader is also cautioned that nonlinear elements can & will convert AM into PM.



the effect is due to three-frequency mixing, and is strongly related to third-order intercept. Read my paper.

$$S_\Theta(\omega) \le 9 \left(\frac{P_{in}}{P_{3oi}}\right)^2 S_{AM}(\omega)$$

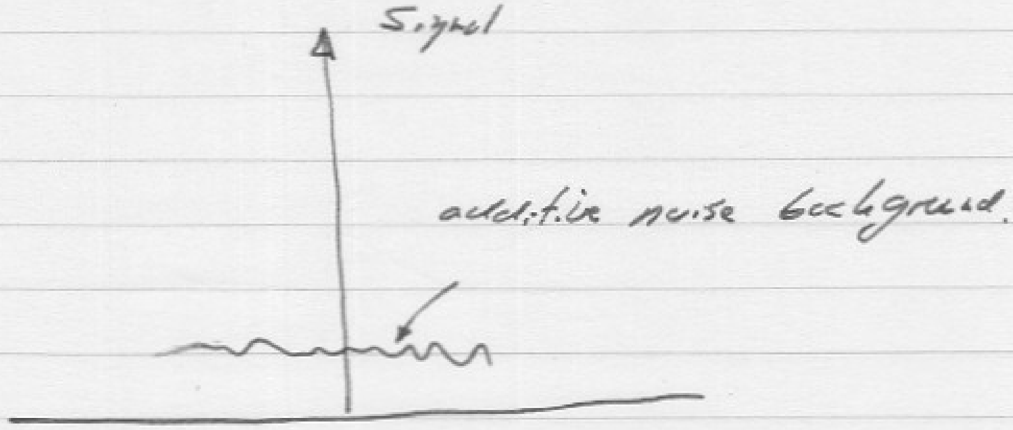
input AM

output phase modulation

P_{in} - input power to device

P_{3oi} - input-referred third-order intercept.

If we $\boxed{\text{add}}$ noise



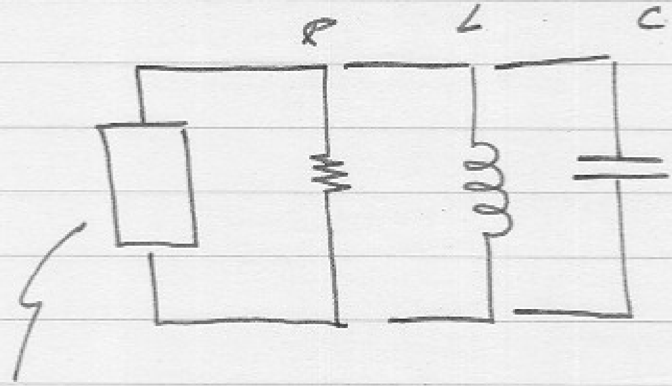
- Phase of noise is random

↳ 50% of noise power is AM, 50% is PM.

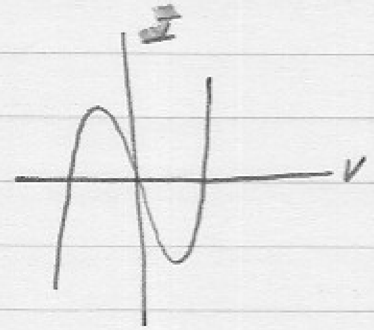
↳ Additive noise must be watched in Phase lock-loops — !

Phase noise in Oscillators:

Oscillator model:

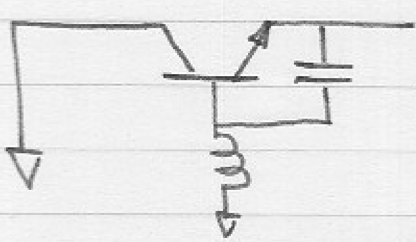


saturating negative resistance.



← cubic? fit??
need quadratic term.

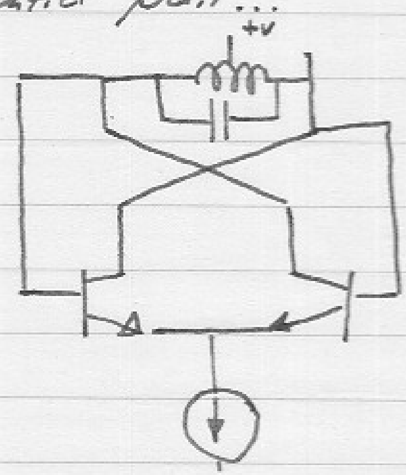
Examples of negative resistance:



← R_{out} is negative at some ω

or 2-terminal: RTD / Gunn / Impatt, etc.

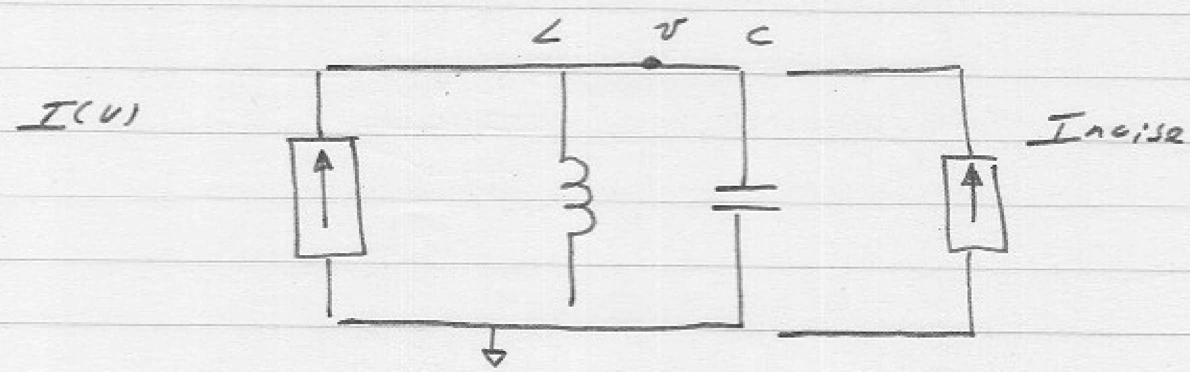
Or differential pair...



In all of these we model the <u>combination</u> of

the & R as

$$I(v) = \alpha_1 v - \beta_1 v^2 - \gamma_1 v^3 + I_{noise}.$$



$$C\frac{dv}{\partial t} + \frac{1}{L}\int v\,dt = I(v) + I_{noise}$$

[write $I_{noise} = I_n \cos \omega_n t$, summed over a range of ω_n.]

$$C\frac{\partial^2 v}{\partial t^2} + \frac{v}{L} + \frac{d}{dt}\left[-\alpha_1 v + \beta_1 v^2 + \gamma_1 v^3\right] = \omega_n I_n \sin \omega_n t$$

simplify:

$\alpha = \alpha_1 / C = 1/R'C$

$\beta = \beta_1 / c$

$\gamma = \gamma_1 / c$ & $\frac{I_n}{C} =$

$\omega_0 = 1/\sqrt{LC}$

$$\boxed{\frac{\partial^2 v}{\partial t^2} + \omega_0^2 v - \frac{\partial}{\partial t}\left[\alpha v - \beta v^2 - \gamma v^3\right] = \frac{\omega_n^2}{\omega_n C} I_n \sin \omega_n t}$$

This, without the β term,

is the Van Der Pol oscillator equation.

First set $\beta \to 0$ & noise $I_n \to 0$

$$\frac{\partial^2 v}{\partial t^2} + \omega_0^2 v - \frac{\partial}{\partial t}\left[\alpha v - \gamma v^3\right] = 0$$

$$\longrightarrow v(t) = a_0 \cos(\omega_0 t + \theta)$$

$$a_0 = \sqrt{\frac{4\alpha}{3\gamma}}$$

The picture here:

over one cycle of the sinewave,

the average negative resistance is zero:

$$0 = \int_{\text{one cycle}} v(t)\left[\alpha_1 v - \gamma_1 v^2\right] dt \qquad \text{where } v = a_0 \cos(\omega_0 t)$$

$$\longrightarrow a_0 = \sqrt{4\alpha/3\gamma}$$

Now with $\beta = 0$ but very small $I_n \neq 0$

$$\frac{\partial^2 v}{\partial t^2} + \omega_0 v - \frac{\partial}{\partial t}\left[\alpha v - \gamma v^3\right] = \omega_n^2 I_n \sin \omega_n t$$

→ oscillation @ ω_0, amplitude $a_0 \sim a$
oscillation @ ω_n, amplitude b

$$b^2 = \frac{I_n^2}{C^2} \frac{1}{\alpha^2 + 4(\omega_n - \omega_0)^2}$$

$$a_0^2 \sim a^2 = \frac{4\alpha}{3\gamma} = \frac{4\alpha_1}{3\gamma_1}$$

we have also written $\tilde{R} = 1/\alpha_1$ = net small signal negative resistance.

$$b^2 = I_n^2 \tilde{R}^2 \cdot \frac{1}{1 + 4(\omega_n - \omega_0)^2 \tilde{R}^2 C^2}$$

now $\frac{1}{\tilde{R} C} = \Delta\omega_{res}$ is the linewidth of the resonator, so

$$\boxed{b^2 = I_n^{\,2}\, \tilde{R}^2 \frac{1}{1 + 4\left(\frac{\omega_n - \omega_0}{\Delta\omega_{res}}\right)^2}}$$

so If I_n has spectral density $\left(\frac{4KT}{\tilde{R}} \cdot F\right)$ then

$$\boxed{\mathcal{L}(\Delta\omega) = \frac{4KTF}{a^2} \cdot \tilde{R} \frac{1}{1 + 4\left(\frac{\Delta\omega}{\Delta\omega_{res}}\right)^2}}$$

where $a = \sqrt{\frac{4\alpha}{3\gamma}}$ is the oscillation voltage

$\tilde{R} = 1/\alpha$ is the net negative resistance.

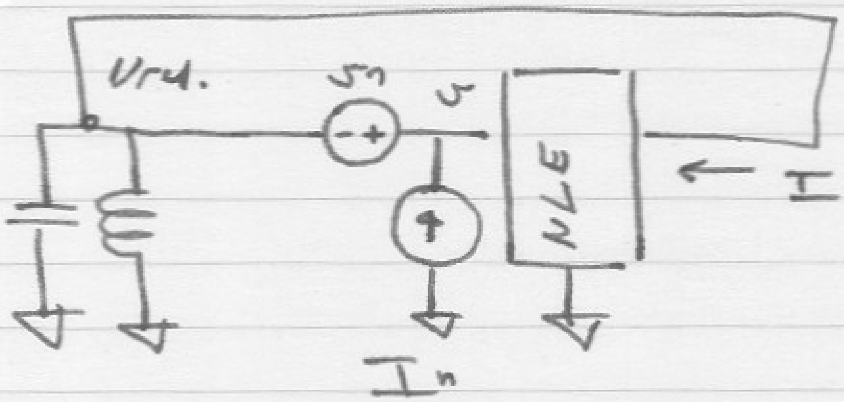
$\Delta\omega_{res} = \frac{1}{\tilde{R} C}$ is the resonator bandwidth.

≡ The above simply predicts that the noise current is filtered by the bandpass of the resonator.

We might have been tempted to skip the nonlinear analysis, but remember that the problem is highly nonlinear.

Now lets quantitatively look at the effect of β_2 ...

Look at βv^2 term & "1/f" noise:



$I = \alpha_1 v + \beta_1 v^2 + \beta\gamma_1 v^3$

We consider now only low-freq. noise of the Non-linear element...

Note first that as drawn, I_n has no effect.

Input to NLE:

$$v = v_{res} + v_n \; ; \; v_{res} \sim a_0 \cos\omega_0 t + \dots$$

output of NLE:

$$I = \beta(a_0 \cos\omega_0 t + v_n)^2 \quad + \text{ terms in } \omega_0 \text{ only.}$$

From which:

$$\delta I = 2\beta_1 (a_0 \cos\omega_0 t)\, v_n$$

+ terms in $a_0 \cos\omega_0 t$ only

+ terms in v_n^2 (order ϵ^2)

If $v_n = v_{n0} \cos\omega_n t$, then

$$\delta I = \frac{2\beta_1 a_0 v_{n0}}{2} \left\{ \cos[(\omega_n + \omega_0)t] + \cos[(\omega_n - \omega_0)t] \right]$$

... e.g. the noise voltage $v_{n0} \cos\omega_n t$

... produces noise currents $\frac{\cancel{2}\beta_1 a_0 v_{n0}}{\cancel{2}} \cdot \cos((\omega_0 \pm \omega_n)t]$

$$\delta I = \beta_1 a_0 v_{n0} \cos((\omega_0 \pm \omega_n)t)$$

If v_{no} has a low-frequency spectral density

of $4kT\,F(\omega_n) \cdot \tilde{R}$

Then δI has a spectral density of

$$4kT\,F(\omega_n)\,\beta_1^2\,a_0^2\,\tilde{R} = \frac{4kT\,F(\omega_n)}{\tilde{R}} \cdot \left(\frac{4\beta_1^2}{3\alpha_1\gamma_1}\right)$$

so from our previous analysis:

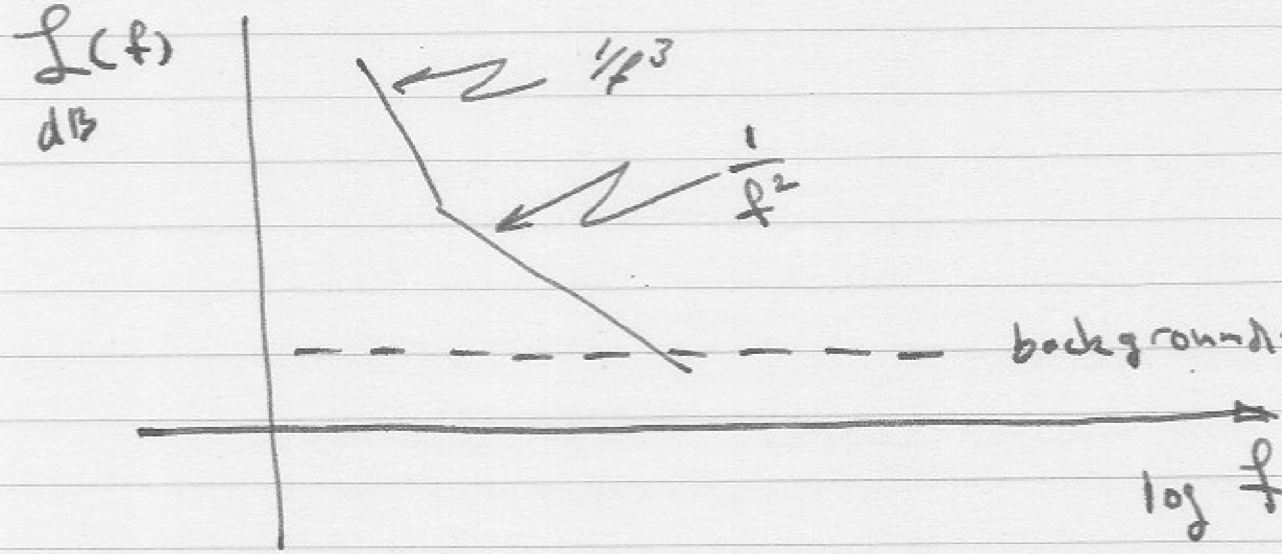
$$\mathcal{L}(\Delta\omega) = \frac{4kT\,F(\Delta\omega)}{a^2} \cdot \frac{4\beta_1^2}{3\alpha_1\gamma_1}\left[\frac{1}{1+4\left(\frac{\Delta\omega}{\Delta\omega_{res}}\right)^2}\right]$$

this is upconversion of

the transistors' $1/f$ noise $\left(F(\Delta\omega) \sim 1/\Delta\omega^{\alpha}\right)$

to around carrier.

we observe:



$\frac{1}{f^2}$ is filtered high-frequency noise

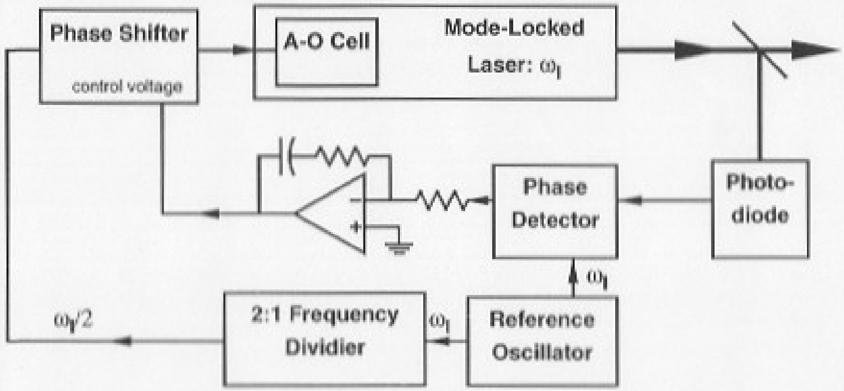
$\frac{1}{f^3}$ is upconverted & filtered 1/f noise.

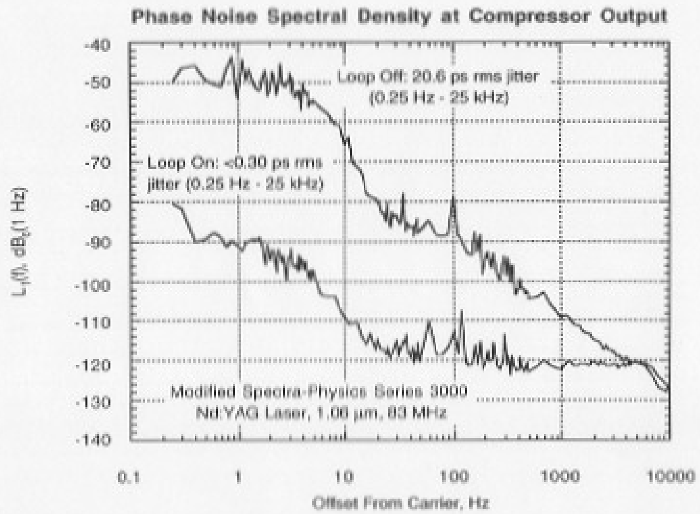
We note

1) βv^2 can be eliminated (greatly reduced) by appropriate circuit design.

2) Source impedance at low frequencies will change effect of $1/f$ noise!

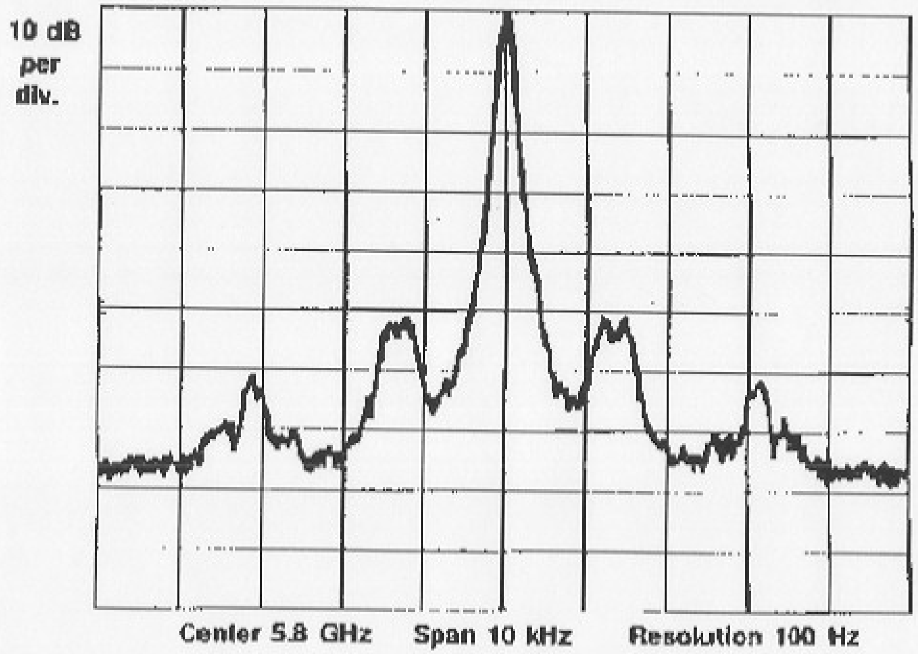
Laser Timing Stabilization





This method is applicable to any actively mode-locked laser. Passively mode-locked lasers can be phase-locked by introducing an electrically-controlled cavity tuning element

phase noise integration



$$\left\| J^2(t) \right\| = \sigma_J^2 = \frac{1}{\pi} \int_{\omega_{low}}^{+\omega_l/2} S_J(\omega)\, d\omega$$

ω_{low} is the low - frequency limit of integration

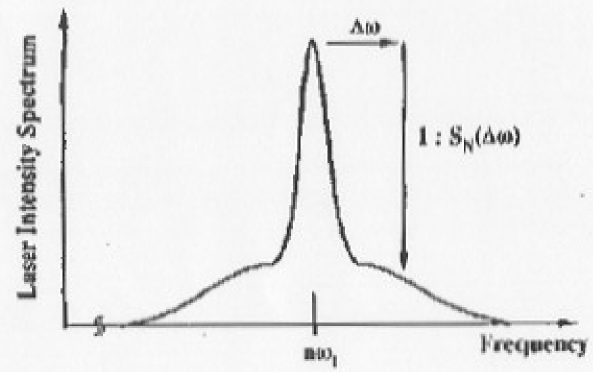
$\approx \pi / T$, where T is the period of observation

In words: Power in sidebands divided by power in carrier is mean-squared phase deviation in radians. Divide by the radian frequency to obtain the timing deviation

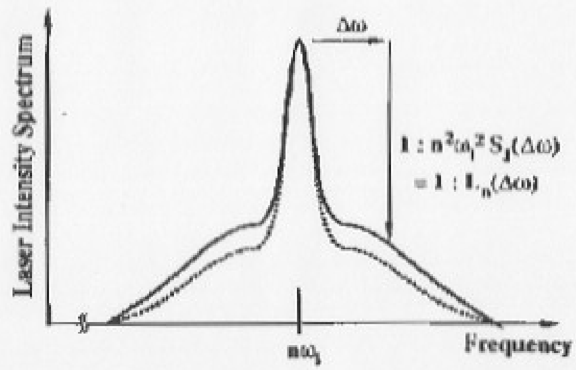
phase noise theory and measurement

I(t) laser intensity
P=average intensity
T=pulse repetition period
σ_t =RMS pulse width
N(t)=intensity fluctuations
J(t) timing fluctuations

Amplitude-Noise Sidebands: Low Harmonics



Phase-Noise Sidebands: High Harmonics



phase noise theory and measurement

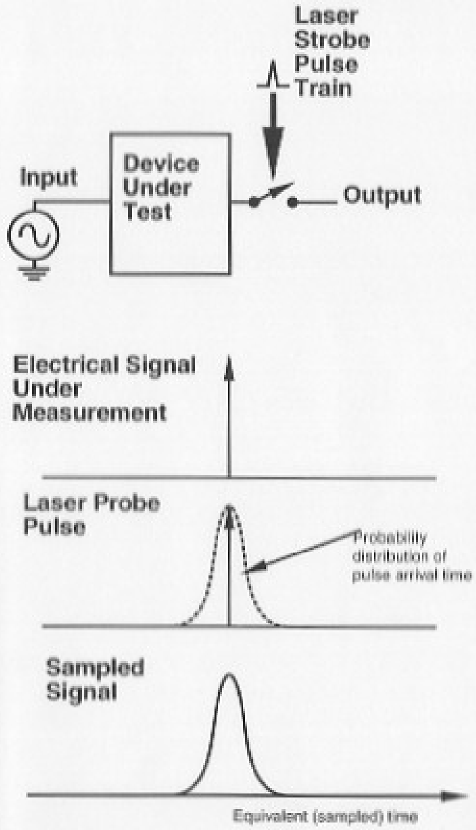
Laser timing and amplitude fluctuations:

$$I(t) \cong \overline{P}T(1+N(t))\sum_{-\infty}^{+\infty}\frac{1}{\sigma_t\sqrt{2\pi}}\exp\left[-(t-nT-J(t))^2/2\sigma_t^2\right]$$

Laser Spectrum

$$S_I(\omega) \approx \overline{P}^2\exp\left(-\omega^2\sigma_t^2\right)\sum_{-\infty}^{+\infty}\begin{bmatrix} 2\pi\delta(\omega-\omega_0) & \text{...laser harmonics} \\ +S_N(\omega-\omega_0) & \text{...AM sidebands} \\ +n^2\omega_l^2 S_J(\omega-\omega_0) & \text{...FM sidebands} \end{bmatrix}$$

Laser Sampling and Timing Jitter



Mode-locked lasers derive their pulse repetition rate from the cavity round-trip time. This resonator, *in terms of the laser intensity modulation intensity envelope*, has relatively poor Q (poor finesse) and pulsed lasers have substantial pulse timing fluctuations.

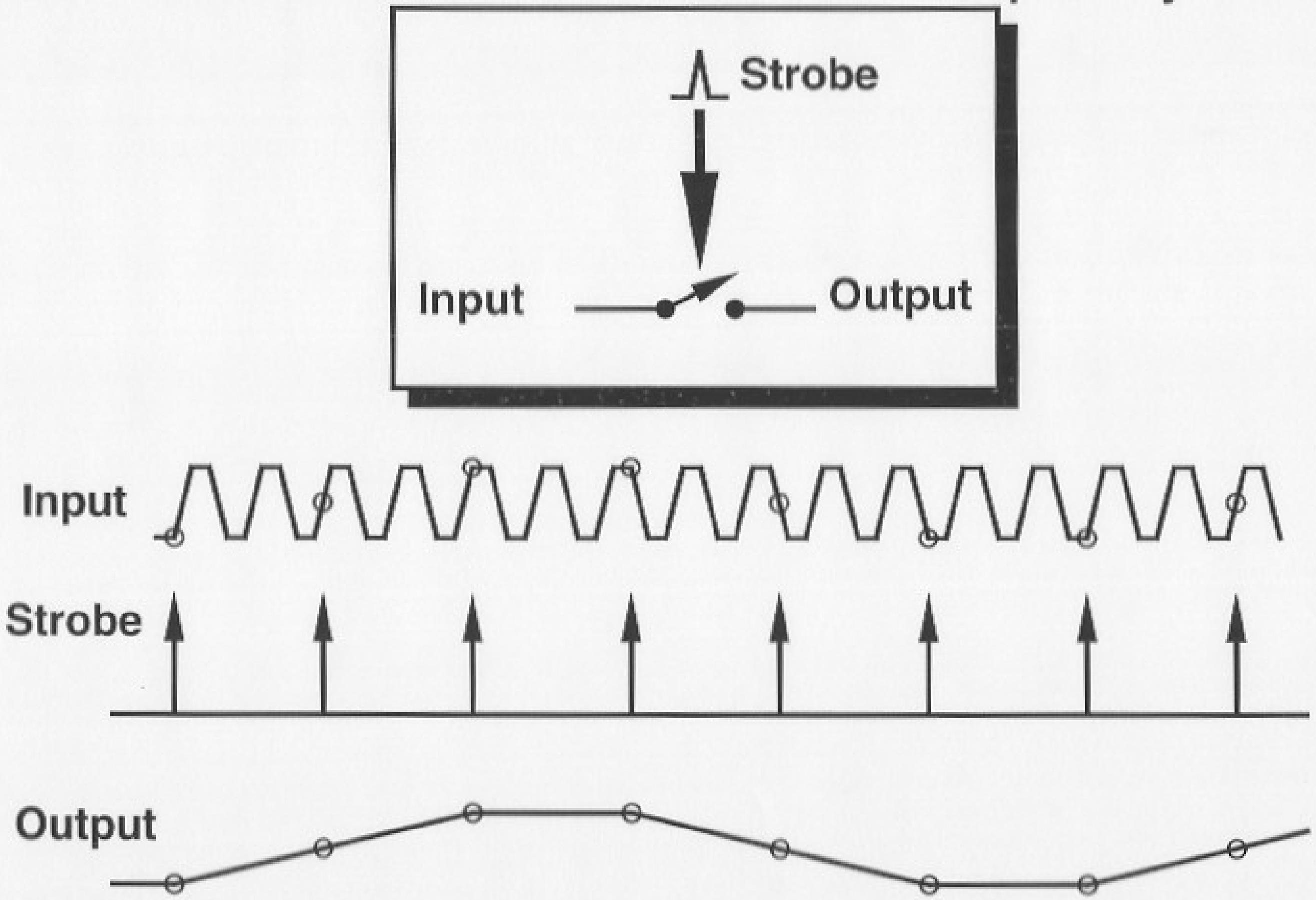
Relative timing fluctuations of the laser and electrical signal source degrade the system time resolution.

Good microwave synthesizer: ≈0.2 ps rms jitter

Mode-locked YAG laser: ≈3-10 ps rms
(0.3 ps if phase-locked)
CPM laser: ≈5 ps rms jitter

Sampling

Reducing the repetition frequency (bandwidth) of a signal so that it can be measured with low-frequency instruments



If the strobe signal has repetition frequency f_0 and the input signal has repetition frequency $nf_0+\Delta f$, the sampled output will be at frequency Δf.

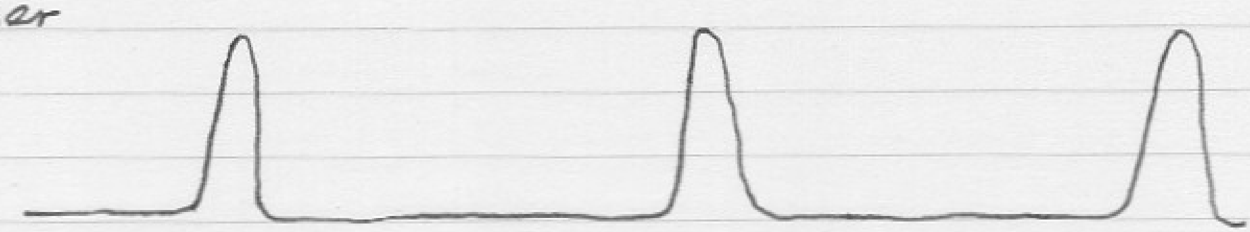
<u>2 specific examples:</u>

/// Normal Phase Locked Loop

/// PLL for laser timing stabilizer.

... Let's do the laser problem first...

Laser



$f_L = 100\,\text{MHz}$ nominal pulse rep. rate

- 1 ps pulse after compression.
- substantial timing & amplitude fluctuations.

Laser rep frequency $\omega_L = 2\pi(100\,\text{MHz})$

$$= \frac{2\pi}{T_0} = \frac{2\pi}{10\,\text{ns}}$$

Timing Fluctuations: $J(t)$

Amplitude: $a(1+N(t))$, $N(t)$ is the normalized amplitude fluctuation.

Spectra :

$$I_d(t) \longrightarrow S_I(\omega)$$
$$N(t) \longrightarrow S_N(\omega)$$

double sided radian based.

$$S|_{component} = ? = 4kTF$$

$$= No!$$

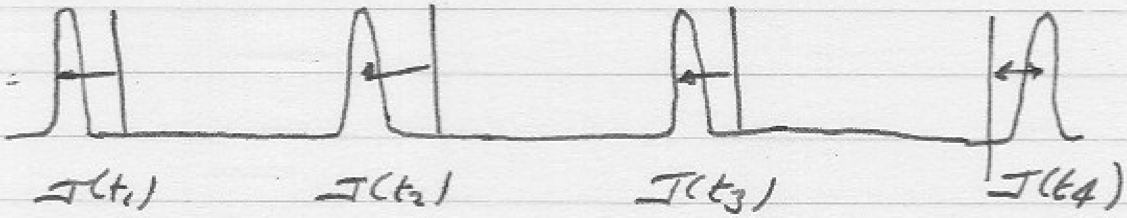
$$= 4kTF/2$$

$\equiv$ because we are using 2-sided spectral densities.

Shot noise: $2qI_{dc}$? No! $\rightarrow$ qI_{dc}

Resistor thermal noise: $4kTR$? No! $\rightarrow$ $2kTR$

Bandwidth of Noise



The continuous-time process $I(t)$ is being **sampled** by the pulse train at $f_c = \omega_c / 2\pi$

Nyquist Criterion: Noise limited to $\pm f_c/2$!

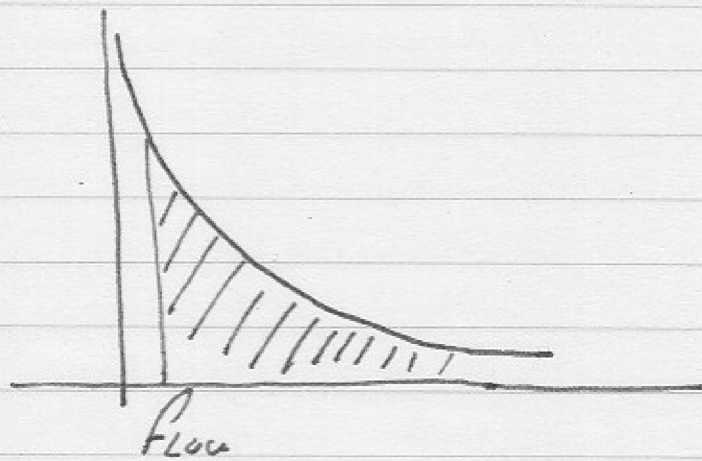
On the lower limit of integration, some process $x(t)$ may vary as $1/f^{\alpha} = f^{-\alpha}$ in spectral density

$$\langle x x \rangle = ? = 2 \cdot \int_{f_{low}}^{+f_{high}} k f^{-\alpha} \, df$$

$$= 2k \left[\frac{f^{1-\alpha}}{1-\alpha} \right]_{f_{low}}^{f_{high}}$$

$$= 2k \left(\left[-\frac{f_{low}^{1-\alpha}}{1-\alpha} \right] + \left(\frac{f_{high}^{1-\alpha}}{1-\alpha} \right) \right]$$

This is large or unbounded for $\alpha \simeq 1$ as $f_{low} \to 0$:



does it make sense to integrate from $f_{low} \cong 0$.

Example:

$f_{low} = 1$ millihertz $= \dfrac{1}{1000\ sec}$

$= \dfrac{1}{16\ \text{minutes}}$

has the instrument been on for 16 minutes?

General Comment: Real processes ain't stationary.

$4kT = S$, but T, the room temperature, fluctuates..

Assuming $S \sim f^{-\alpha}$ with $\alpha \sim 1$ & assuming very low f_{low} forces higher & higher demands on the stationarity of the physical process involved.

<u>Specific Comment</u> we generally care about the <u>change</u> in some physical parameter over some period of observation ΔT.

x = random process

- $x(t + \Delta T) - x(t)$ = change over period ΔT

= a random variable

$\langle [x(t+\Delta T) - x(t)]^2 \rangle$ = its <u>variance</u>

$$\langle (\Delta x)^2 \rangle = \langle x(t+\Delta T) \cdot x(t+\Delta T) \rangle$$

$$+ \langle x(t) \cdot x(t) \rangle - 2 \langle x(t)\, x(t+\Delta T) \rangle$$

$$= R_{xx}(0) + R_{xx}(0) - 2\, R_{xx}(\Delta T)$$

$$= 2 \left[R_{xx}(0) - R_{xx}(\Delta T) \right]$$

$$= 2 \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega)\, d\omega - 2 \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega) e^{-j\omega\tau}\, d\omega$$

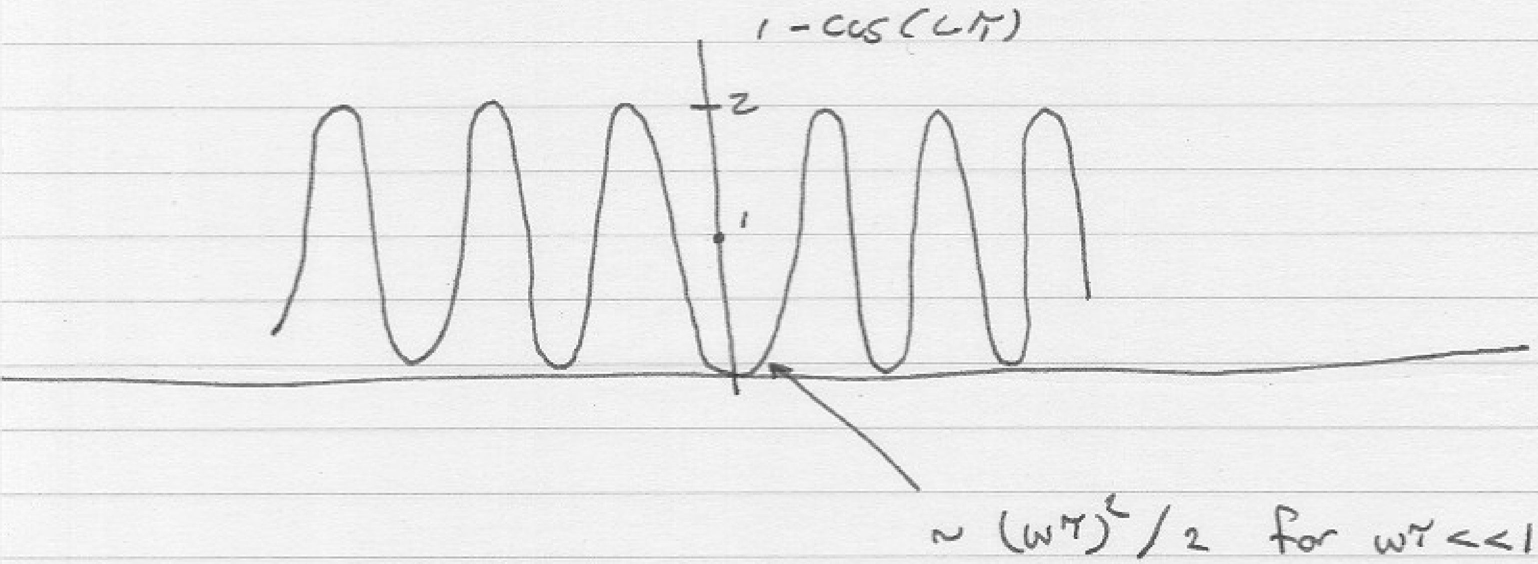
$$= 2 \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega) \left[1 - e^{-j\omega\tau} \right] d\omega$$

S_{xx} is real

$(1 - e^{-j\omega\tau})$ is hermatian so...

$$\boxed{\langle (\Delta x)^2 \rangle = 4 \cdot \frac{1}{2\pi} \int_{0}^{+\infty} S_{xx}(\omega) \cdot \left[1 - \cos(\omega\tau) \right] d\omega}$$

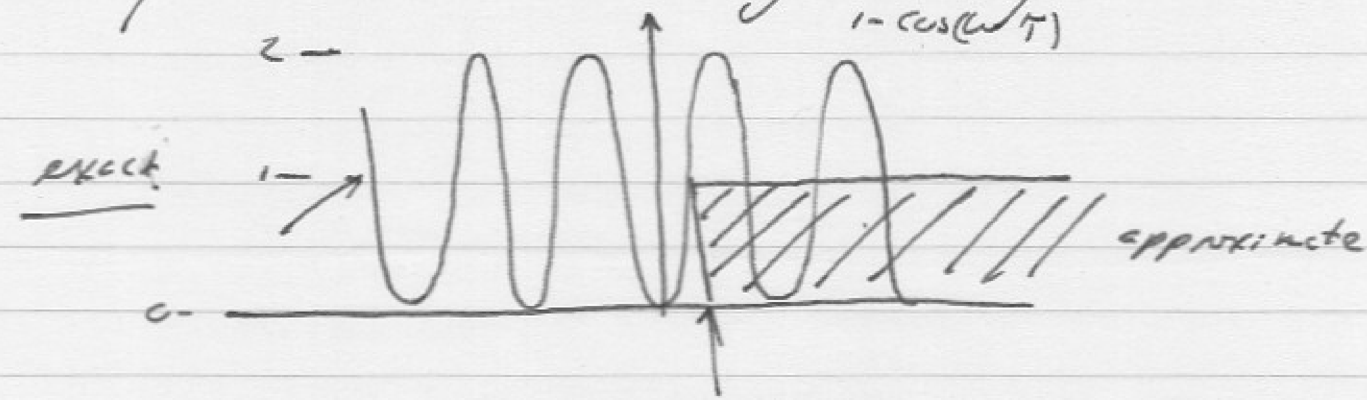
$$\langle (\Delta x)^2 \rangle = 2 \cdot \frac{2}{2\pi} \int_0^{\infty} S_{xx}(\omega) \cdot \left[1 - \cos(\omega\tau)\right] d\omega$$



$$\boxed{\begin{array}{l}\text{as long as } S_{xx}(\omega) < \dfrac{k}{f^3} \text{ for some } k \text{ at} \\ \text{low frequencies, } \langle (\Delta x)^2 \rangle \text{ is finite for } \underline{\text{any}}\ \Delta T\end{array}}$$

Pragmatic Approach:

Since we dont always no a priori what ΔT exactly we use on a given day, ...



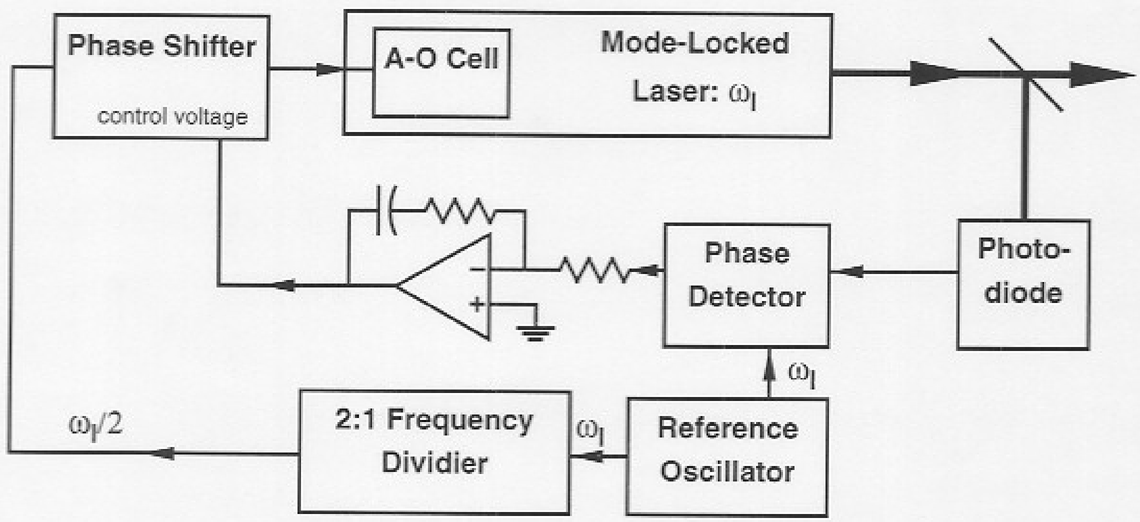
$$\omega_{low} = \frac{\pi}{2\Delta T}$$

$$f_{low} = \frac{1}{4\Delta T}$$

$$\boxed{\langle (\Delta x)^2 \rangle \simeq \int_{f_{low} = 1/4\Delta T}^{f_{high}} S_x(f)\, df}$$

where ΔT is the period of observation.

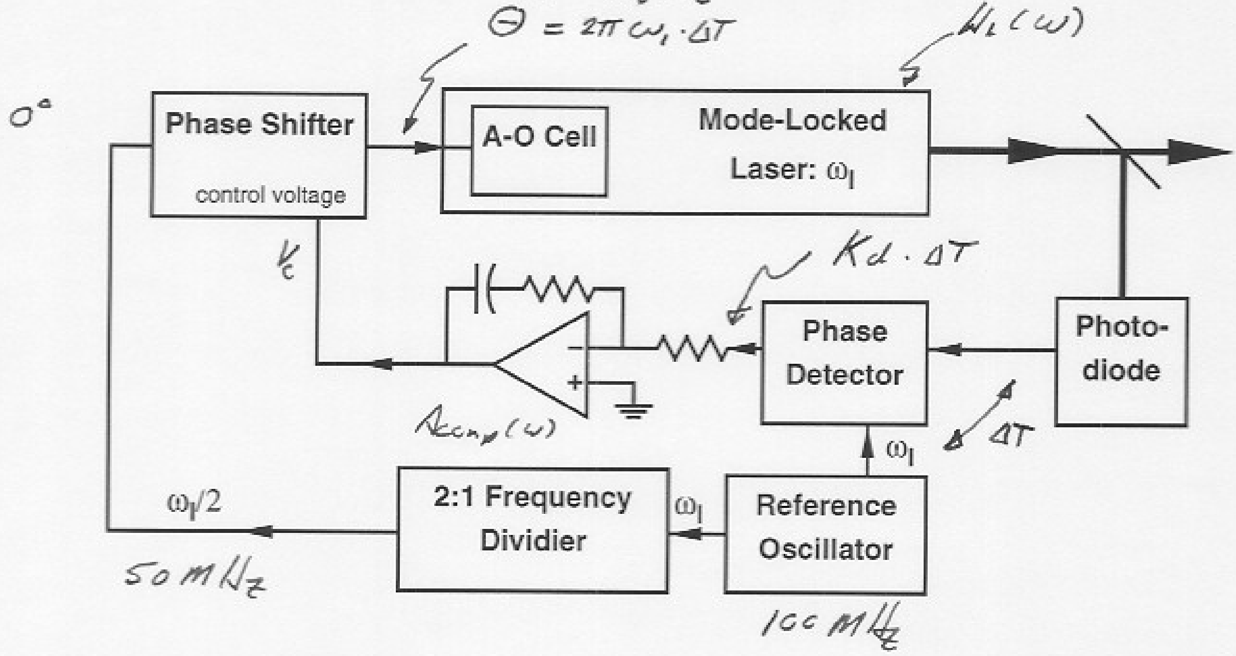
Laser Timing Stabilizer



Laser Timing Stabilizer

$\Delta T = K_s \cdot V_c$

$\Theta = 2\pi \omega_1 \cdot \Delta T$



Phase ~~detection~~ control loop transmission

$$G(\omega) = K_s \cdot K_d \cdot N_L(\omega) \cdot A_{comp}(\omega)$$

phase shifter constant

detector constant

Laser dynamic response

Amplifier gain vs frequency

... Like any control loop, the amplifier has lead / lag etc compensation to correct for the "plant" (laser) response.

design: $$G(\omega) = \frac{f_{loop}}{j f} = \frac{\omega_{loop}}{j \omega}$$

$$f_{loop} \simeq 4 - 5 \text{ KHz}.$$

suppression of phase noise of laser

basic feedback result: →

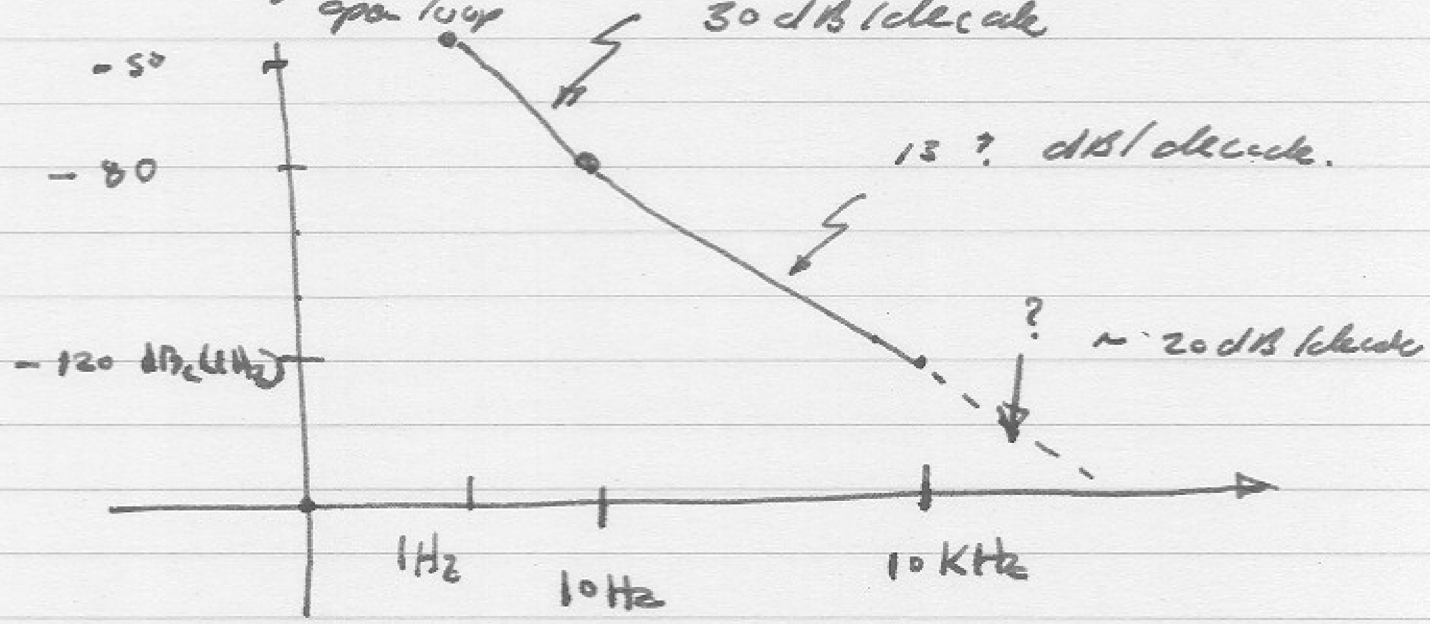
$$S_{I\,\text{closed loop}}(\omega) = \left\| \frac{1}{1+G(\omega)} \right\|^2 S_{I\,\text{open loop}}(\omega)$$

$$\cong \begin{cases} S_{\text{open loop}} & \omega \gg \omega_{\text{loop}} \\ S_{\text{open loop}} \cdot \dfrac{\omega^2}{\omega_{\text{loop}}^2} & \text{for } \omega \ll \omega_{\text{loop}}. \end{cases}$$

$$\mathcal{L}(\omega)_{\text{closed loop}} = \left\| \frac{1}{1+G(\omega)} \right\|^2 \cdot \mathcal{L}(\omega)_{\text{open loop.}}$$

$$\mathcal{L}(\omega) = S_\Theta(\omega) = \frac{1}{\omega_L^2} \cdot S_I(\omega)$$

Observed $\mathcal{J}(\omega)$:



... as best as my old notes show...

Objective: 0.3 ps rms timing "Jitter"
0.25 Hz – ? maximum observable.

$(0.3\ ps)^2 = \langle J(t)^2 \rangle$ over 1 sec. ΔT.

$\sigma_\Theta = \omega_L \cdot \sigma_J = 2\pi\,(100\ MHz) \cdot 0.3\ ps$

$\sigma_\Theta \cong 2 \cdot 10^{-4}$ radians !

$(\sigma_\Theta)^2 = 4 \cdot 10^{-8} = \frac{\text{Power in Sidebands}}{\text{Power in Carrier}}$

$\frac{P_{sideband}}{P_{carrier}} = \mathcal{L}(f) = 4 \cdot 10^{-8} = -74\ dBc$

$\downarrow$

$\mathcal{L}(f) = -74\ dBc$ ~~Average~~, integrated over phase noise bandwidth.

$$\mathcal{L}(f)_{\text{desired}} \sim -74 \text{ dBc} \quad (10 \text{ KHz})$$

$$= -74 \underset{(1\text{Hz})}{\text{dBc}} + 10 \log_{10}\left(\frac{1 \text{ Hz}}{10 \text{ KHz}}\right)$$

$$= -74 \text{ dBc } (1\text{Hz}) - 40 \text{ dB}$$

$$\boxed{\mathcal{L}(f)_{\text{desired}} = -114 \text{ dBc } (1\text{Hz})}$$

on the next sheet I will show the phase noise suppression for the 10 kHz loop.

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f, Hz	L(f)openloop	L(f) closedloop desired	Required Suppression	Actual loop suppression	Closed Loop L(f)
1	-50	-120	70	80	-130
10	-80	-120	40	60	-140
100	-93	-120	27	40	-133
1000	-106	-120	14	20	-126
10000	-120	-120	0	0	-120

Loop bandwidth 1.00E+04

All quantities Hz, dB or dBc (1 Hz) as appropriate.

... so the loop is adequate!

f, Hz	L(f)openloop dBc (1 Hz)	L(f) closedloop desired	N(f) dBc (1 Hz)	Maximum Permissible AM/PM conversion, dB
1	-50	-120	-65	-55
10	-80	-120	-75	-45
100	-93	-120	-85	-35
1000	-106	-120	-95	-25
10000	-120	-120	-100	-20

- Above is shown the laser Amplitude Noise, again in dBc (1Hz).

∴ Compare these to the desired -120 dBc (1Hz) to obtain a bound on the Maximum permissible level of AM-PM conversion.

AM-PM conversion by 3rd-order saturation:

$$S_{\substack{J \\ AM \to PM}}(\omega) \doteq \frac{9}{\omega_c^2} \cdot \left(\frac{P_{in}}{P_{3oi}}\right)^2 \cdot S_N(\omega)$$

equivalently,

$$S_{\substack{\theta \\ AM-PM}} = \mathcal{L}_{AM \to PM}(\omega) = 9\left(\frac{P_{in}}{P_{3oi}}\right)^2 S_N(\omega)$$

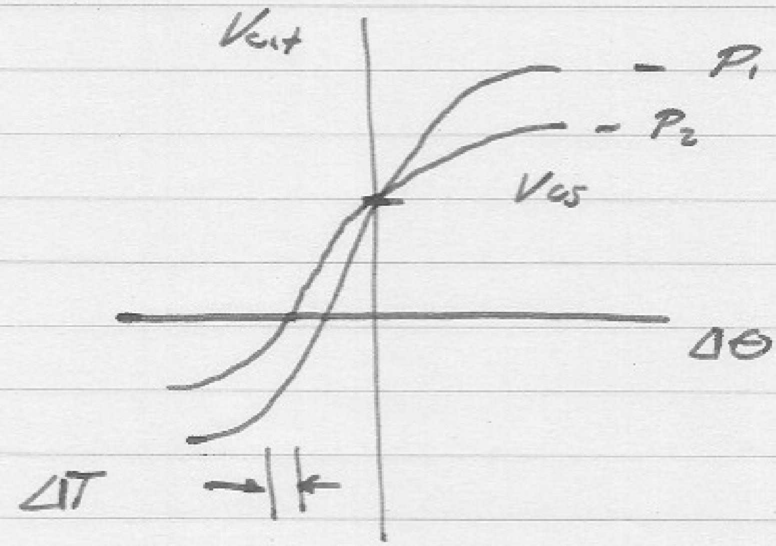
$$\text{AM-PM conversion} = 9\left(\frac{P_{in}}{P_{3oi}}\right)^2$$

$$= 10\log_{10} 9 + 20\log_{10}\left(\frac{P_{in}}{P_{3oi}}\right)$$

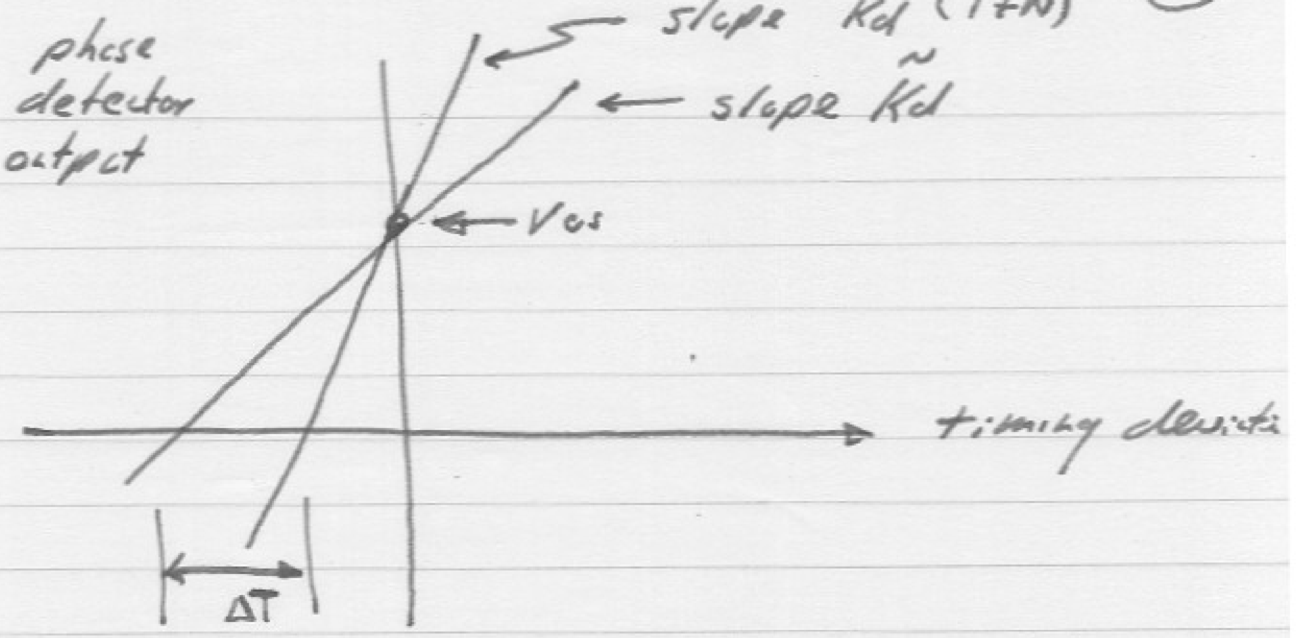
$$-55\,dB \geq 9.5\,dB + 2\cdot\left[P_{in}\Big|_{dB} - P_{3oi}\Big|_{dB}\right]$$

$$\boxed{P_{in,dBm} \leq P_{3oi,dBm} - 32\tfrac{1}{4}\ dB.}$$

All signal levels must be at least 32 dB ~~db~~ below the relevant third-order intercept points. Since there are several components, the design point was 35 dB below P_{3oi}



There are similar restrictions on the allowable AM-PM conversion arising indirectly from phase detector offset voltage.



$$\Delta T = N(t) \cdot \frac{V_{os}}{\tilde{K}d} \quad ; \quad \tilde{K}d \text{ in V/sec.}$$

$$\boxed{\Delta\Theta = N(t) \cdot \frac{V_{os}}{Kd} \quad ; \quad Kd \text{ in } \frac{\text{Volts}}{\text{radian}}}$$

$$\boxed{\underset{\substack{\text{Am-pm}\\ \text{detector offset}}}{\mathcal{L}(\omega)} = \left(\frac{V_{os}}{Kd}\right)^2 \cdot S_N(\omega)}$$

$$K_d = \left(\frac{\Delta V}{\Delta \Theta}\right)$$

$$\mathcal{L}(\omega) - S_N(\omega) = 20 \log_{10}\left(\frac{V_{os}}{K_d}\right)$$

$\uparrow$ $-120\,dBc$ (Hz) $\qquad$ $\uparrow$ $-65\,dBc$ $(1Hz)$

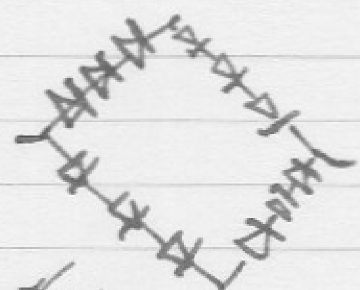
$$-55\,dB = 20 \log_{10}\left(\frac{V_{os}}{K_d}\right)$$

$$\boxed{\begin{array}{l} \dfrac{V_{os}}{K_d} \sim 2\,(10^{-3}) \\[2em] V_{os} \cdot \left(\dfrac{\Delta \Theta}{\Delta V}\right) \sim 2 \cdot 10^{-3} \text{ radians.} \end{array}}$$

In other words, the phase detector must be kept within 2 milliradians (0.1°) of quadrature in order to be sufficiently insensitive to AM noise!

- we need a very small v_{os} offset voltage
- or a very high K_d slope coefficient
- The high K_d can be obtained by increasing the input level to the mixer. But then third-order products kill us...
- High third-order ~~product~~ intercepts can be obtained by using <u>high</u> in<u>percept</u> mixers

For diodes, that is this...



but now I have increased the offset voltage...

- we have gone in circles & accomplished nothing.

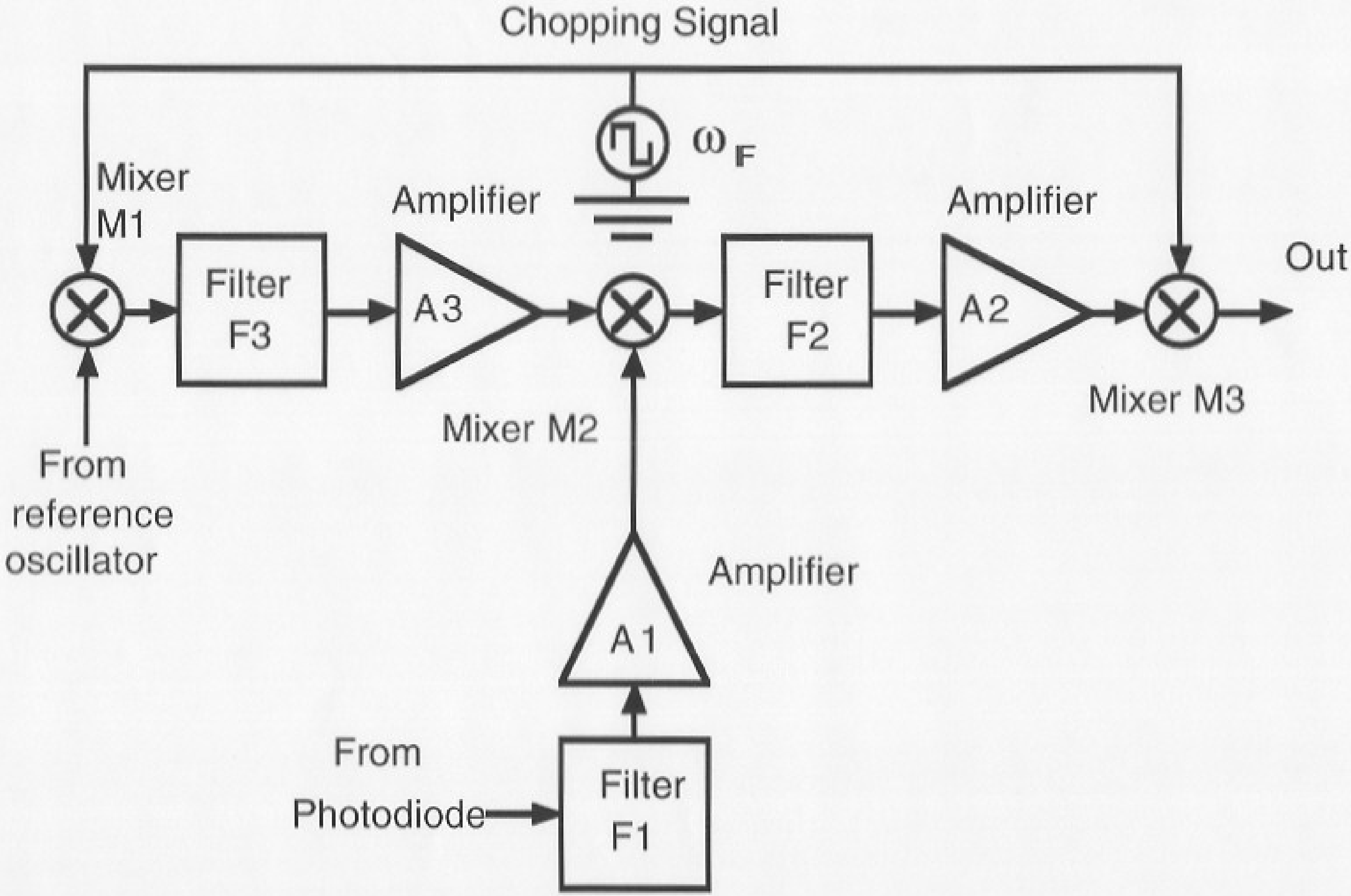
II The solution is to use <u>chopping</u> II

... More on this later ... in the context of sensitive measurements.

* We modulate the LO signal with a chopping signal. Say 1 MHz chopping

* The phase detector output is now a 1 MHz signal whose <u>amplitude</u> varies with the phase input. The 1 MHz <u>signal</u> is now spectrally separated from the DC offset voltage.

* We amplify this signal by 40 - 60 dB before (DSB) detecting it.



Chopper Stabilized Phase Detector

Noise The FRIIS formula for system Noise figure happily describes both the phase detection system and its subsequent baseband amplifier. Lets take a net noise figure of 10dB. 50% of the additive noise becomes phase noise...

so ~~take~~ ignoring the 50% factor...

$$\mathcal{L}_{\substack{min \\ \text{due to } kTF}}(f) = \frac{P_{in,\ \text{phase detector}}}{KTF}.$$

Ok, so lets put it all together.

Phase detector: $\begin{cases} IP_3 = +10\ dBm \\ F = 10\ dB \end{cases}$

→ 174 dB (1Hz) "dynamic" "range"

we must be 35 dB below IP_3

→ $P_{in} = -25\ dBm$

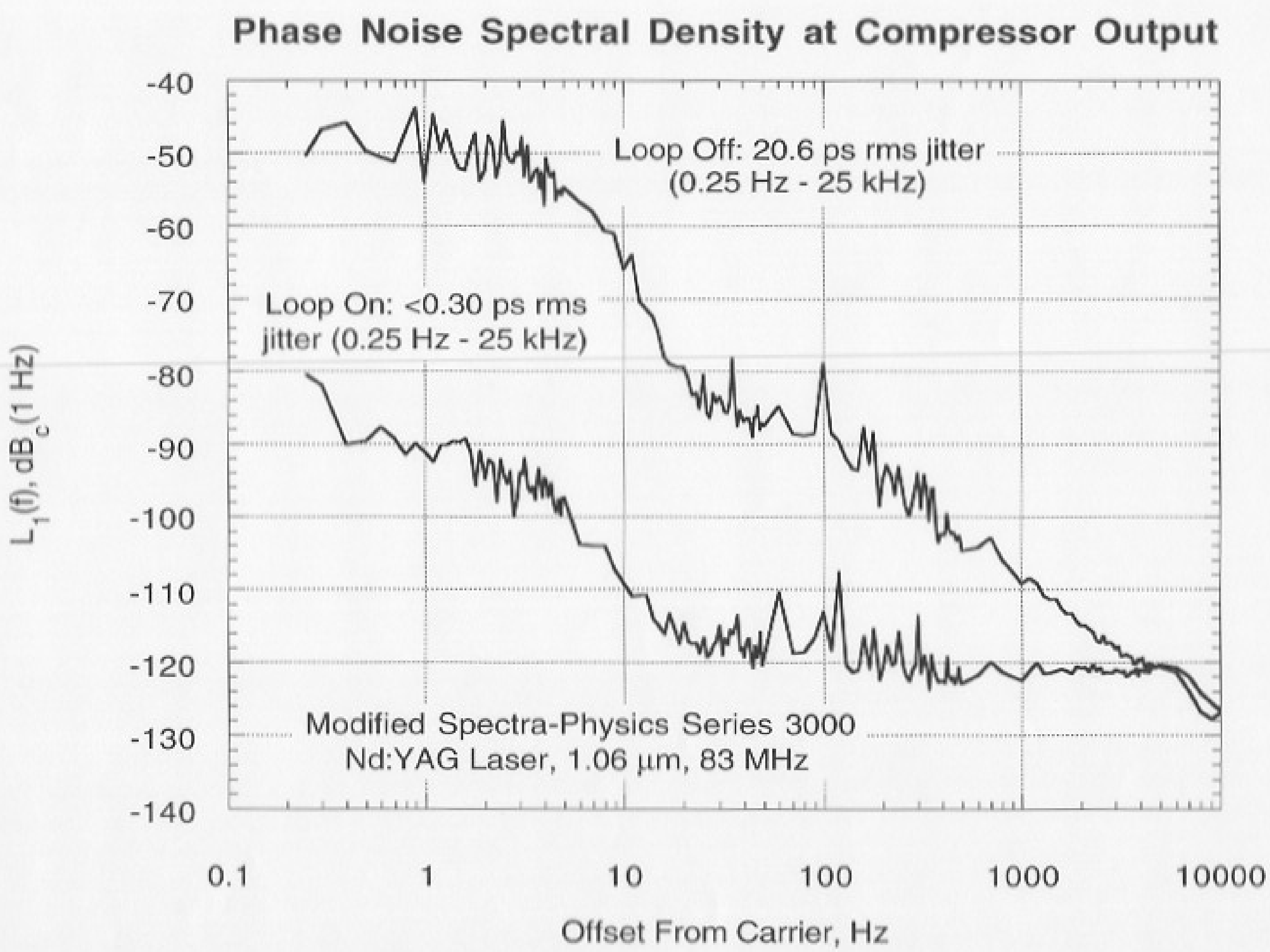
$$\frac{P_{in}}{kTF} = -25\ dBm + 173.8 - 10$$

$$S/N = -138.8 \cong -140\ dB\ (1Hz)$$

Given that we need $-120\ dBc\ (1Hz)$ $\mathcal{L}(f)$

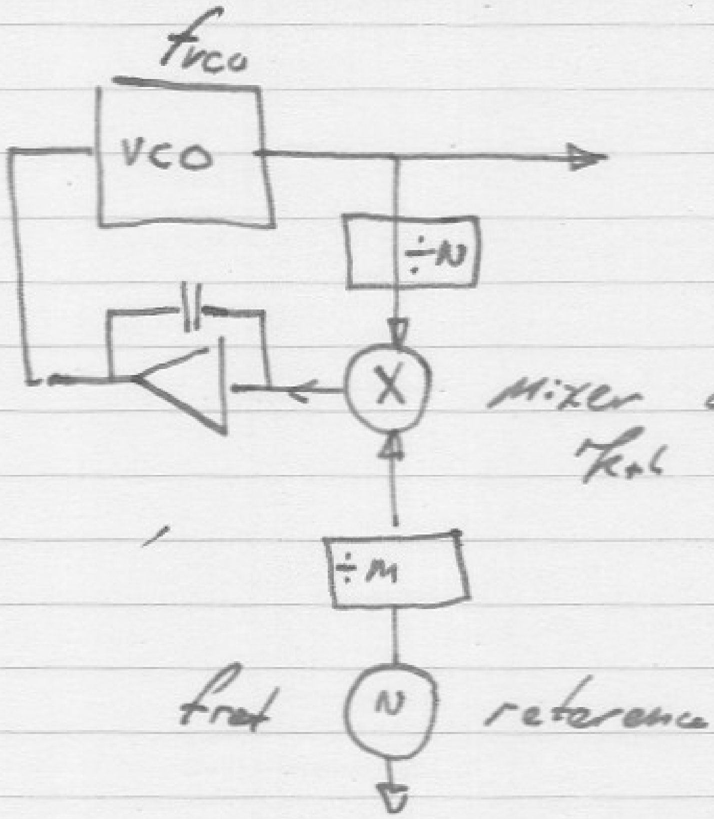
we are o.k.! 20 dB margin.

— End of design example —



Phase Noise Spectral Density at Compressor Output
$L_1(f)$, dB_c(1 Hz)
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
Loop Off: 20.6 ps rms jitter
(0.25 Hz - 25 kHz)
Loop On: <0.30 ps rms
jitter (0.25 Hz - 25 kHz)
Modified Spectra-Physics Series 3000
Nd:YAG Laser, 1.06 µm, 83 MHz
0.1
1
10
100
1000
10000
Offset From Carrier, Hz

Phase Lock- Loops Generally



sampler!

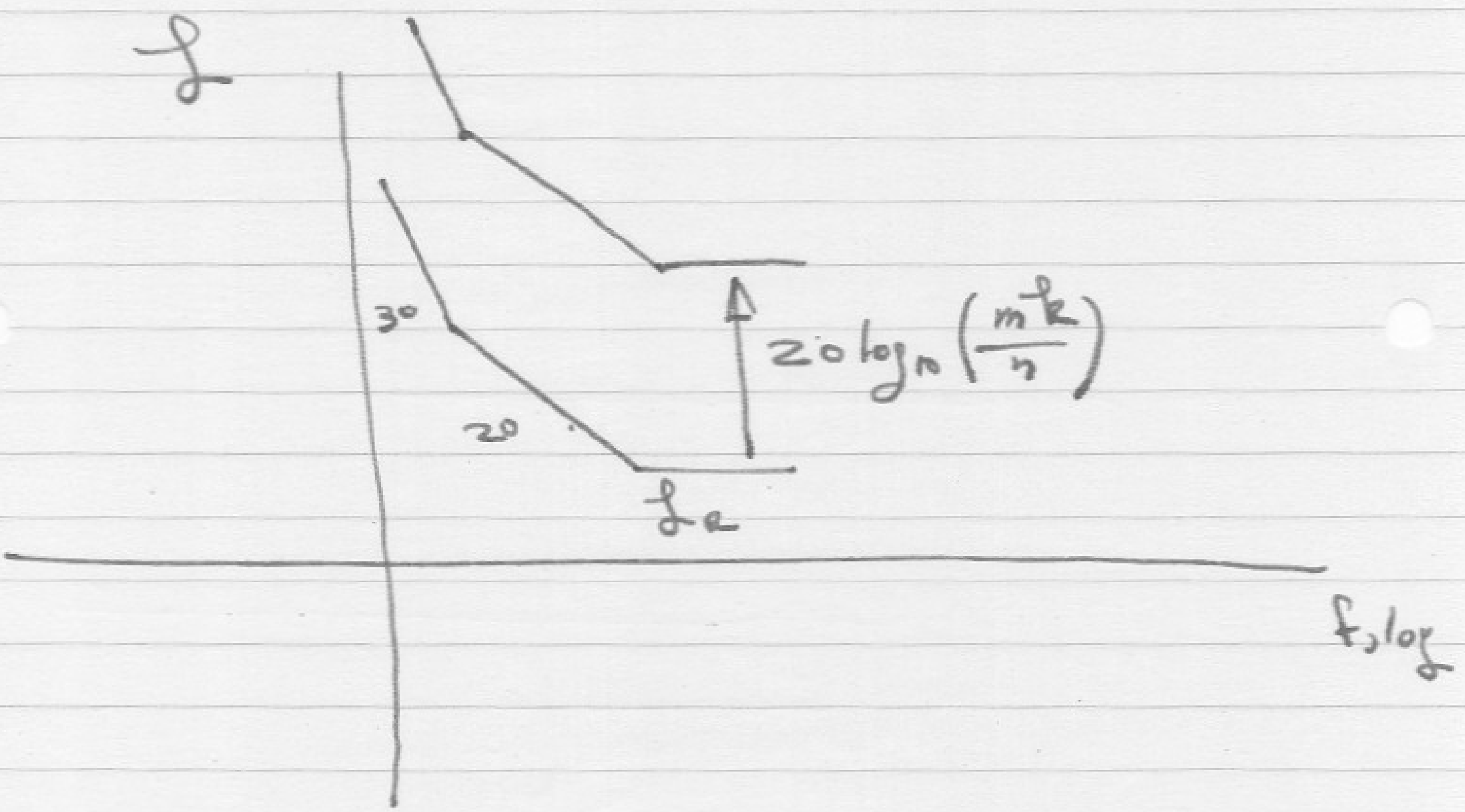
Mixer or harmonic mixer
kth harmonic

$$\boxed{f_{ref} \cdot \left(\frac{M}{N} \cdot k \right) = f_{vco}.}$$

This is how a synthesizer works.

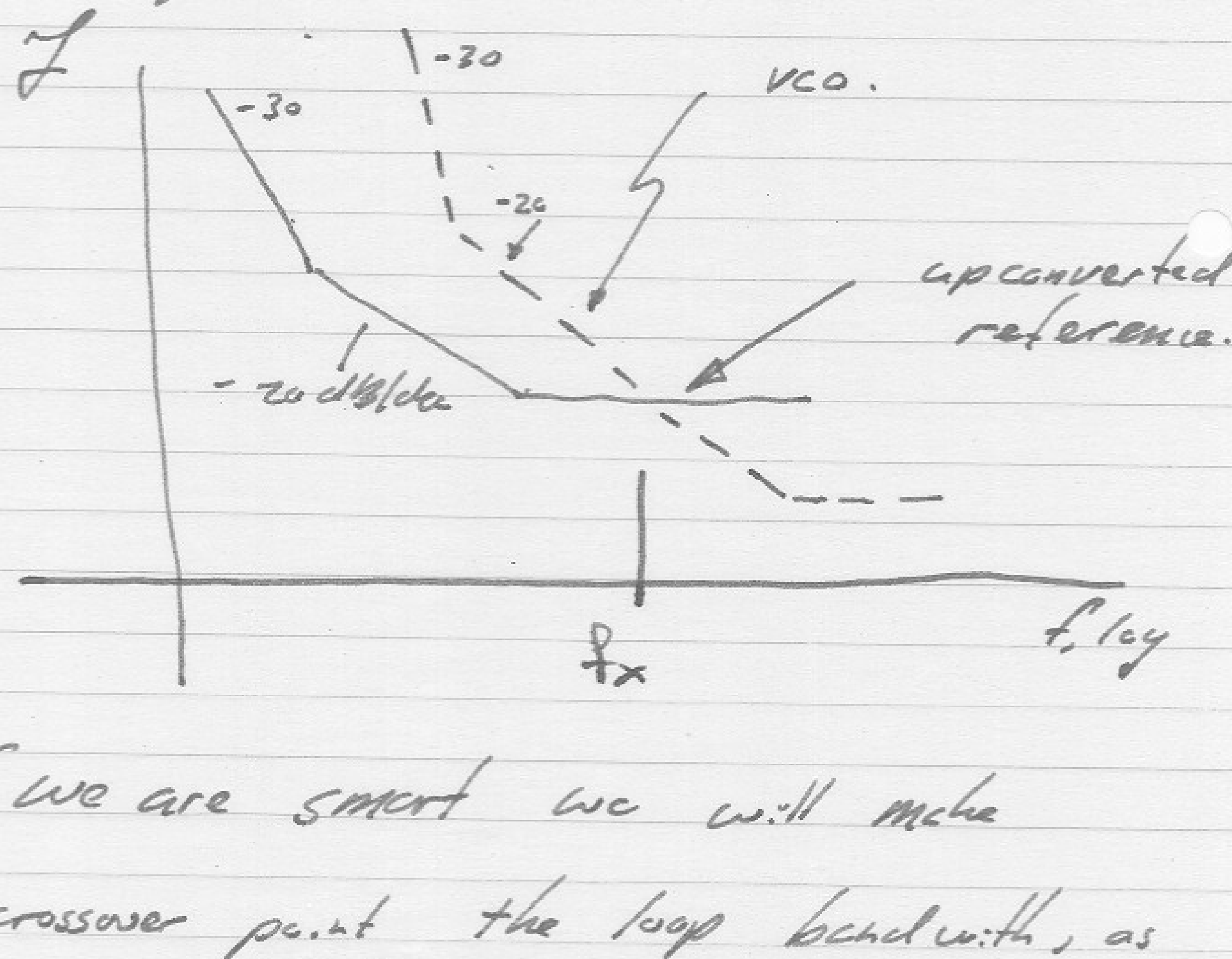
* Note that the reference is being upconverted by (m^k/n)

$$\mathcal{L}_{ref;\ upconverted} = \mathcal{L}_{ref} + 20\log_{10}\left(\frac{m^k}{n}\right)$$



The vco also has phase noise, & its resonator is probably poorer because

* It has to be tunable
  (but YTO's are great)
  YIG
* Better resonators generally at lower frequencies.



If we are smart we will make the crossover point the loop bandwidth, as

$$\mathcal{L}_{closed\ loop} = \begin{cases} \mathcal{L}_{vco} & \text{outside loop bandwidth} \\ \mathcal{L}_{Ref,\ mult} & \text{within loop bandwidth.} \end{cases}$$

Also/

1) The last example used a VCΦ, while PLL's usually use a VCO. The difference is an integration in the loop transfer function, plus a host of problems relating to loop locking behaviour

2) PLL's used for clock recovery in communication are driven by the randomly-fluctuating message. This is a big topic. Mixer/detector offset is of paramount concern.

Finally

The laser pll problem was hard because we were looking for 0.3ps fluctuations on a 100 MHz = 10ns signal. Much easier if the rep rates are higher, as the net phase angle gets bigger. Could have worked with not 100MHz but e.g. 10 GHz, the 100th laser harmonic. Dynamic range requirements then relaxed by 40dB, but the available hardware's ~~MHz~~ dynamic range may be 30dB (ok) to 50 dB (bad) poorer. Cost is also 40 dB higher...