• Notes Set 21: Sensitive Experiments: Noise in the Physics Lab

- How to measure TINY things. integration bandwidth. minimum detectable signal.
- Example: search for little green men from alpha centari (e.g. CETI)
- Observation period and equivalent noise bandwidth. Averaging as a form of filtering. 1/f noise and observation times.
- Example: electrooptic sampling
- Example: atomic force microscopy.

ECE ____ Notes Set 21

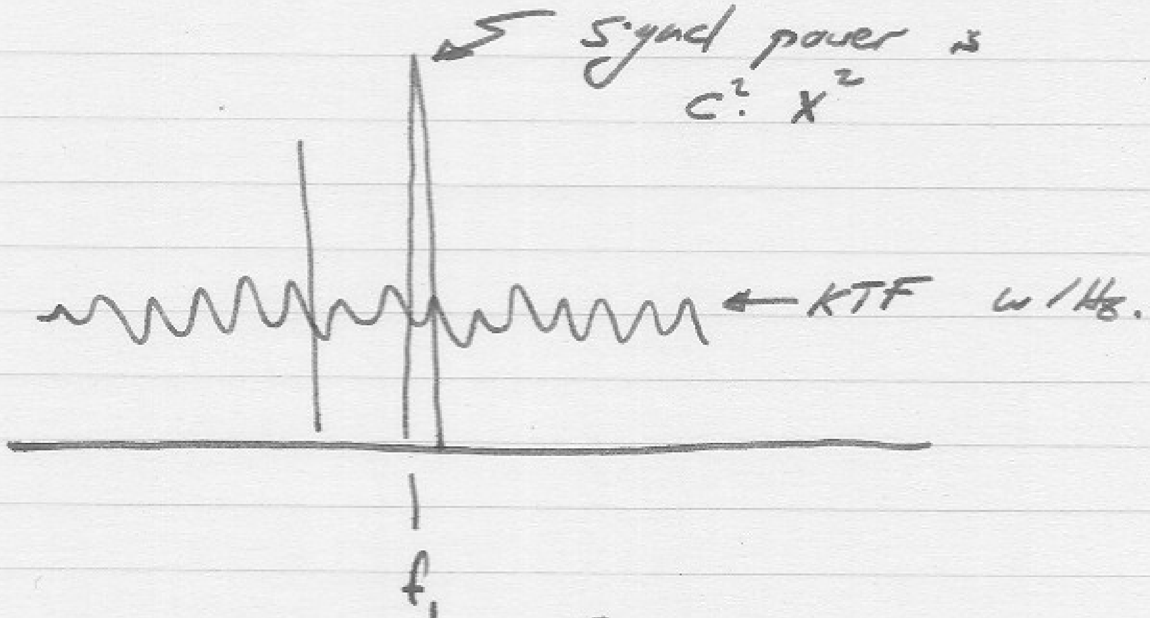
Sensitive Measurements

* How to measure *tiny* things

* How to measure a 10^{-10} meter vibration, a 10^{-5} radian optical phase shift

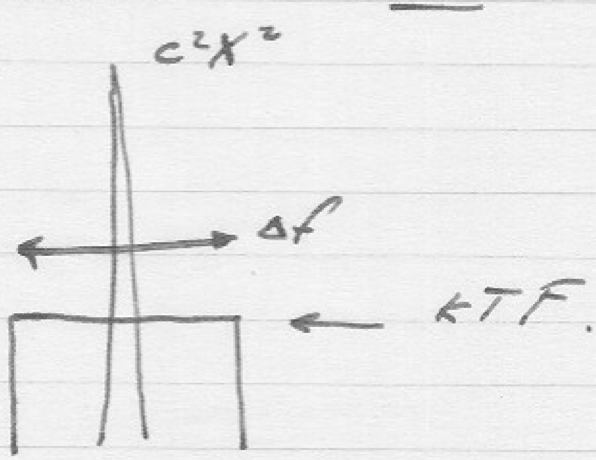
Is this hard? remember that we have already learned how to listen to music with a 10^{-14} W signal!

Key idea: Detect signal in noise background KTF, or the like ...



/// Signal is sinewave at f_1
amplitude is $C \cdot X$
X = parameter under measurement.

Filter Signal in BandWidth $\underline{\Delta f}$



Signal power : $c^2 X^2$

Noise power : $KTF(\Delta f)$

$$S/N = c^2 X^2 / KTF(\Delta f)$$

<u>If</u> we can calibrate the experiment to determine c, then:

$$X_{measured} = X_{actual} + X_{noise}$$

where X_{noise} : $\sigma^2_{x_{noise}} = \dfrac{KTF\Delta f}{c^2}$

$$\sigma_{x_{noise}} = \frac{1}{c}\sqrt{KTF_{a}} \cdot \sqrt{\Delta f}$$

σ_x noise is often called the "<u>Minimum Detectable Signal</u>"

e.g. the signal at unit S/N ratio,

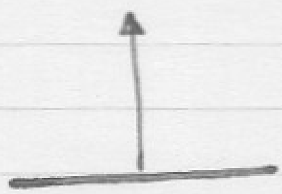
or, equivalently, the standard deviation of the measurement.

$$\boxed{\begin{aligned} \sigma_{x\,noise} &= \text{Minimum detectable signal} \\ &= \text{Standard deviation of Measurement error} \\ &= \frac{1}{C} \cdot \sqrt{KTF} \cdot \sqrt{\Delta f} \end{aligned}}$$

The hidden assumption here is

signal has discrete power spectrum:

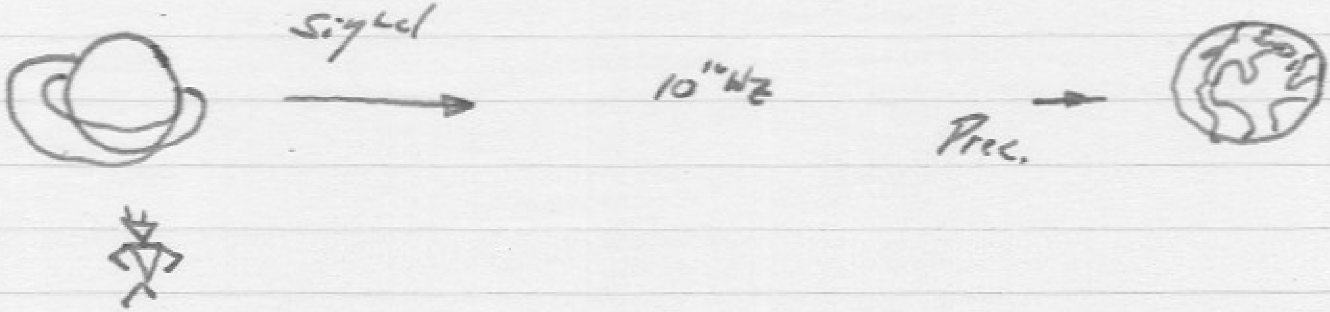
$$S_{signal}(f) = c^2 x^2 \cdot \delta(f - f_1)$$



* In this case we can measure to arbitrarily small precision by making Δf small.

* This is real! small signals can be measured.

<u>Simplified example</u> [Search for Extraterrestrial Intelligence.]



little green men.

SETI

Lets work in terms of antenna power.

Assume $K T_{rad} + (F-1) k\, T_{planet\,eart} = K \cdot 100^\circ$ Kelvin

e.g. $T_{eq} = 100^\circ K$.

Suppose my receiver is a DSP spectrum analyzer with 10^{-3} Hz bandwidth.

$$\underbrace{K}_{1.38(10^{-23})\ J = 1.38(10^{-23})\ w/Hz.}$$

$$P_{min} = K \cdot 100°K \cdot 1 \cdot 10^{-3}\ Hz$$

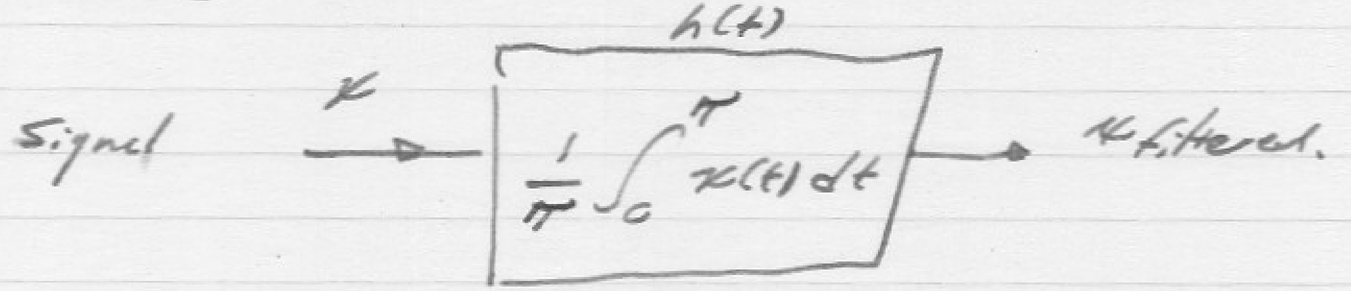
$$\boxed{P_{min} = 1.4(10^{-24})\ \text{watts}\ !}$$

Pretty remarkable:

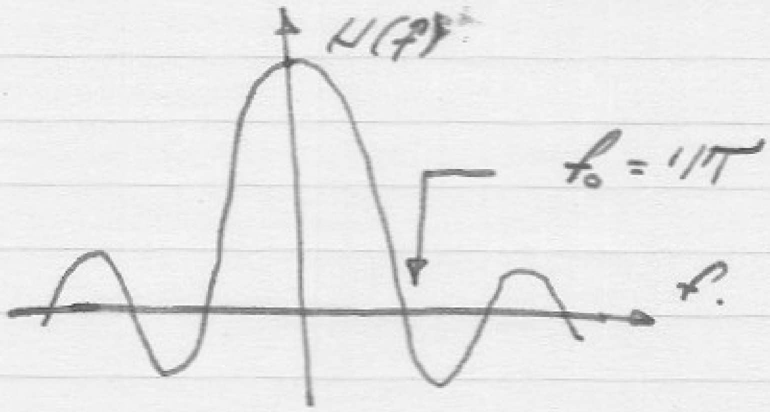
But we have a problem:

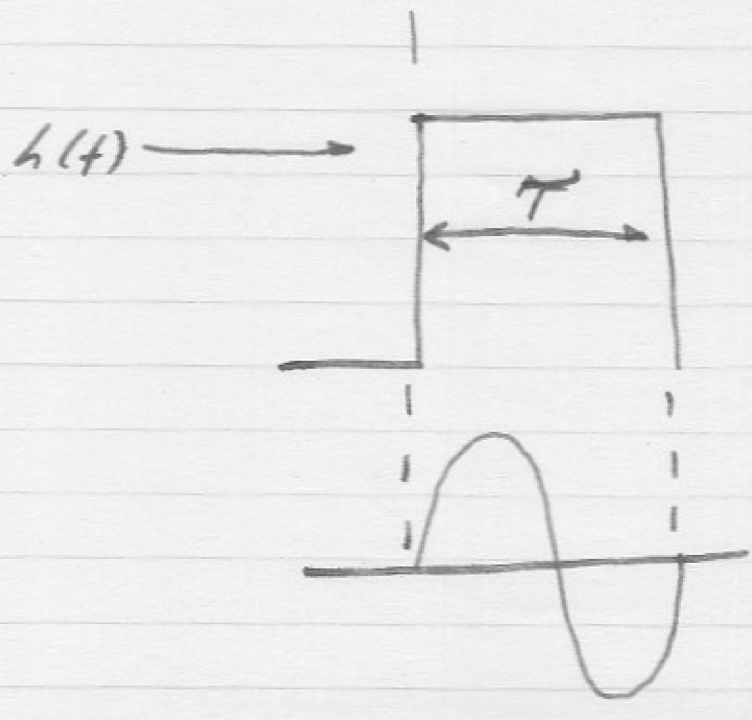
1 spectral bin $= 10^{-3}$ Hz.

How Long Did the measurement take?



Filter $H(\omega) = \dfrac{\sin \omega T/2}{\omega T/2} = \dfrac{\sin 2\pi f T/2}{2\pi f T/2}$





first null
in freq response.

Equivalent Filter Bandwidth:

$$\Delta\omega_{eq} = \int_0^\infty H(\omega)\, d\omega$$

$$\Delta f_{eq} = \int_0^\infty H(f)\, df = \int_0^\infty \frac{\sin 2\pi f T/2}{2\pi f T/2}\, df$$

$$\boxed{\Delta f_{eq} = \frac{1}{T}}$$

Equivalent Receiver bandwidth $= \frac{1}{T}$

where T is time of observation

This is of Major philosphical importance:

* spectral density of noise = KTF w/Hz.

* observed for time T

* Gives Noise Power (variance)

$$\boxed{\sigma_{noise} = KTF/T} \quad W$$

"Good things come to

those who wait"

parenthetical note:

Noise power = KTF/T

duration of observation = T

Noise Energy = Noise power · Duration of observation

$$\boxed{\text{Noise Energy} = KTF}$$

* This we will use in communications analysis

Back to our little green Men:

If we wait 1000 seconds $\longrightarrow \tau = 1000$ sec

$P_{minimum} = k\,T_{eq} / \tau = k(100°K)/1000$ sec

$P_{minimum} = 1.4 \cdot 10^{-24}$ Watts   1000 sec observation

or a better way of looking at it:

$E_{minimum} = k\,T_{eq} = 1.4\,(10^{-21})$ Joules.

Our minimum detectable signal (σ) is
1.4 (10^{-21}) Joules. in Energy

The signal Energy = $E_{signal} = P_{signal} \cdot \tau$

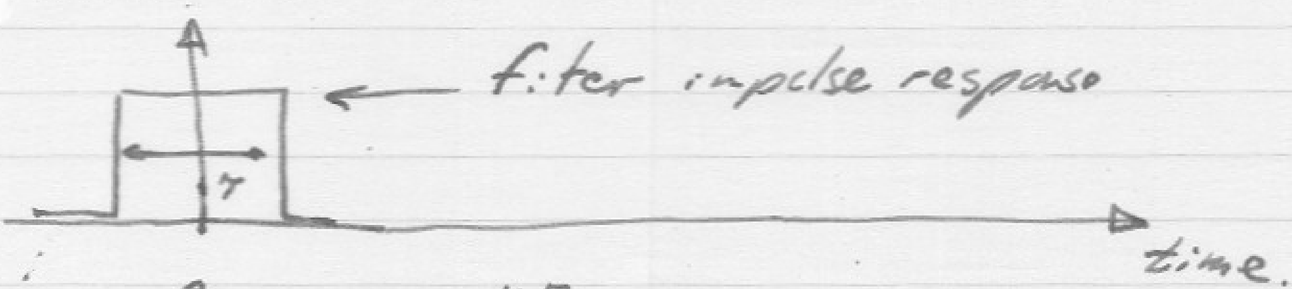
S/N increases with τ because we are capturing
more Energy

Note that I can average 10 measurements to improve ~~P~~minimum (sum of $\perp\!\!\!\perp$ random variables → signal grows 100 times, noise grows 10 times)

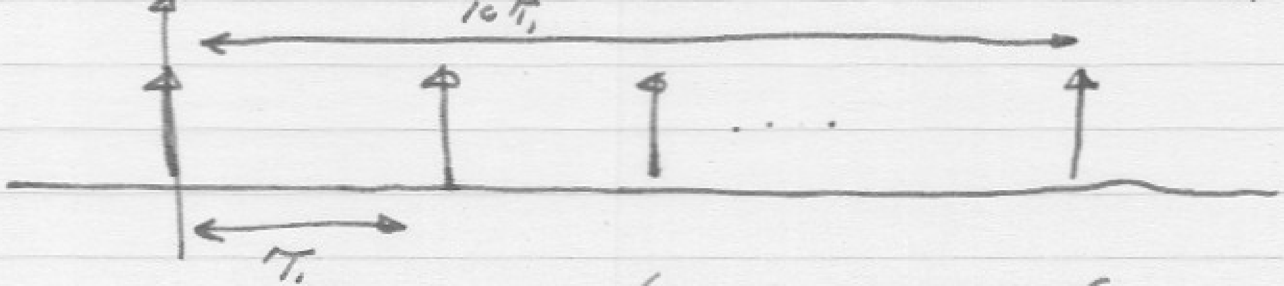
but this is just the same as observing the signals ~~to~~ 10 times longer.

$\boxed{\text{– Conclusion: Signal averaging \& signal filtering are the same thing.}}$

Pictures:

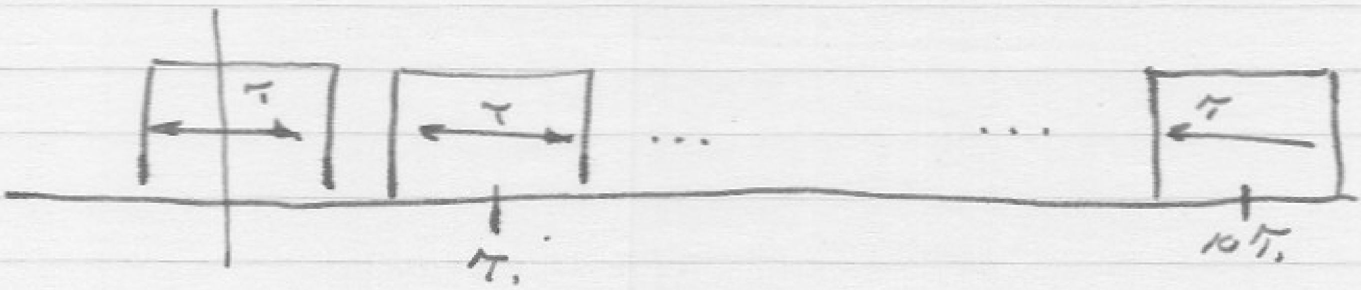


← filter impulse response



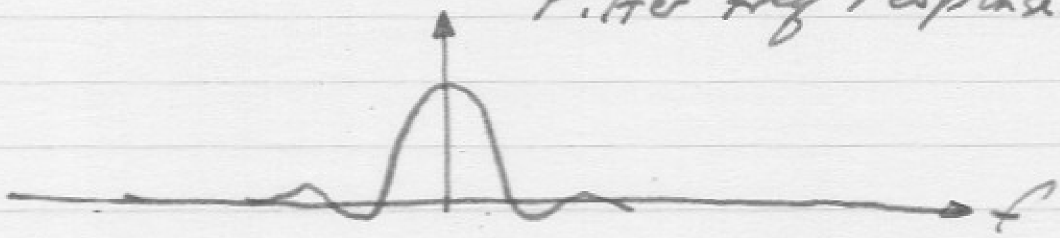
impulse response of averaging

10 averages taken at increments τ₁

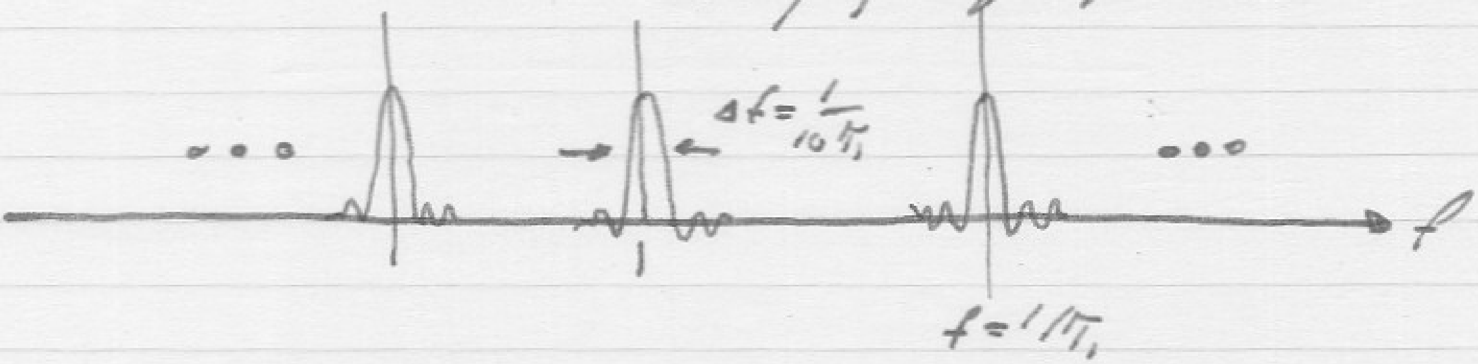


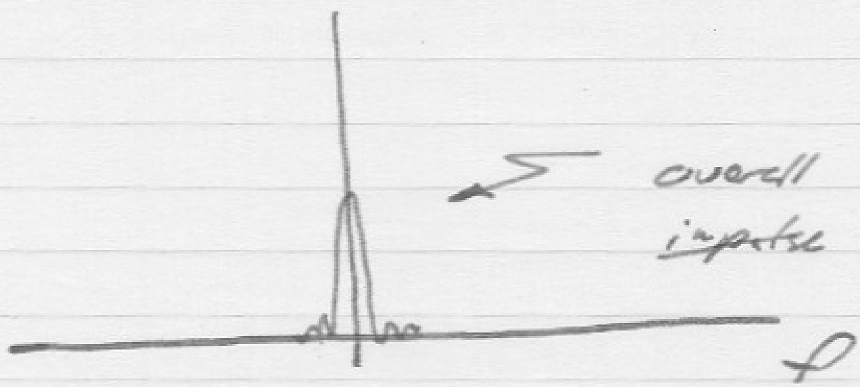
overall filter impulse response.

Filter freq response.



Averaging freq response:



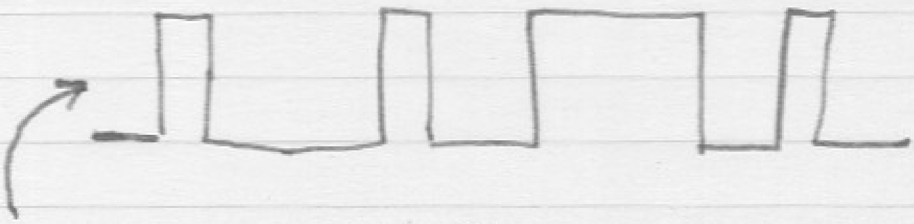


overall filter freq. ~~impulse~~ response.

Overall, the point is

— averaging & filtering are the same —

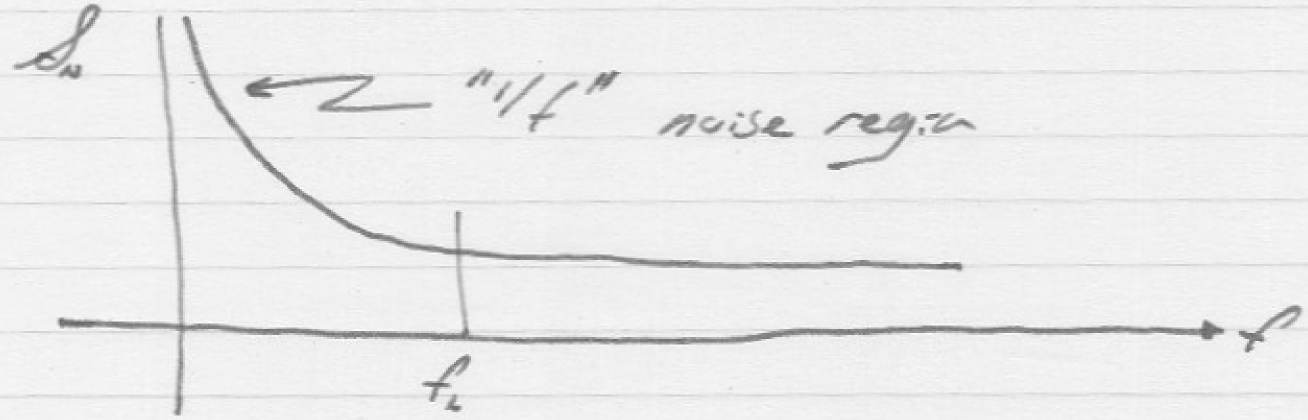
we have a net noise Energy = $\underline{KTF}$



~~noise~~ Signal Energy = Signal Power ×

how long we listen

1/f noise & modulation



* Most systems have some 1/f noise corner f_L

* Signal/Noise greatly degraded if detection done below f_L, because (noise figure) & noise background much much higher.

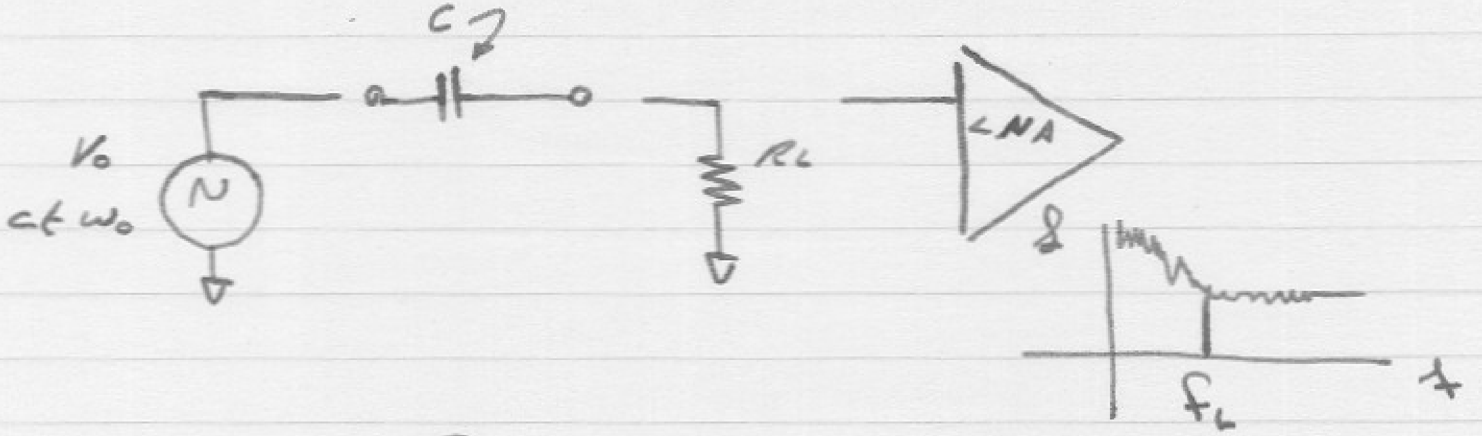
- Solution involves modulation of signal so that it is upconverted in frequency -

Chopping, Frequency or Amplitude Modulation and "lock-in" (heterodyne) detection.

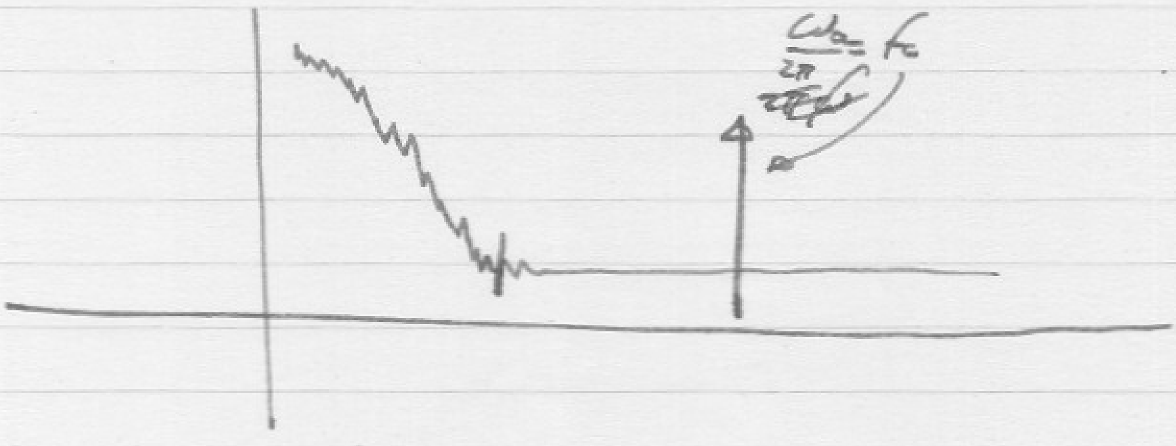
* Modulation of signal under observation so that it has a spectral line at a frequency above the 1/f noise corner.

This cannot always be done, particularly ~~that~~ as we cannot modulate the things which have 1/f noise, or the 1/f noise will be upconverted too.

Example Attofarad Meter

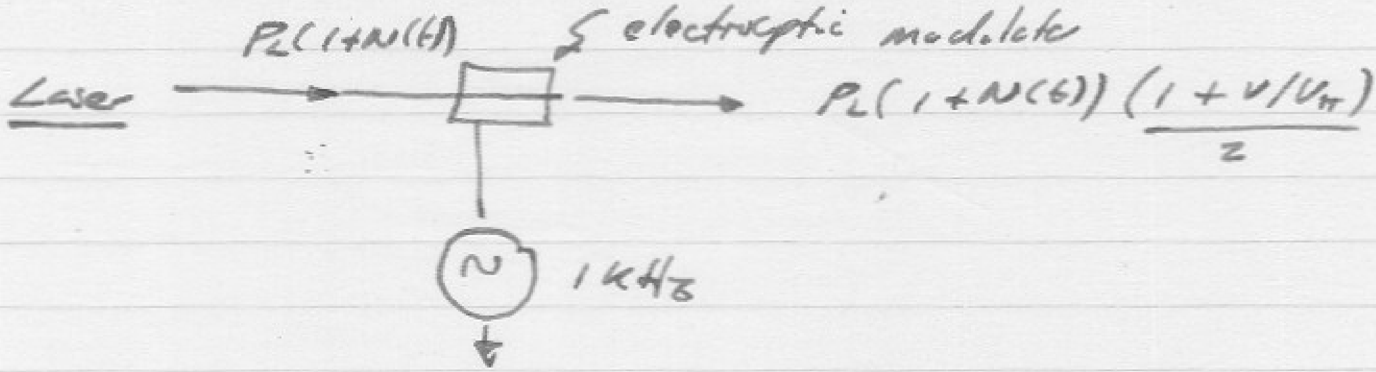


- apply V_o @ ω_o,
  current $I = \omega C V_o$ flows ($\omega RC \ll 1$)
  measure $V_o = \omega RC V_o$ with LNA.

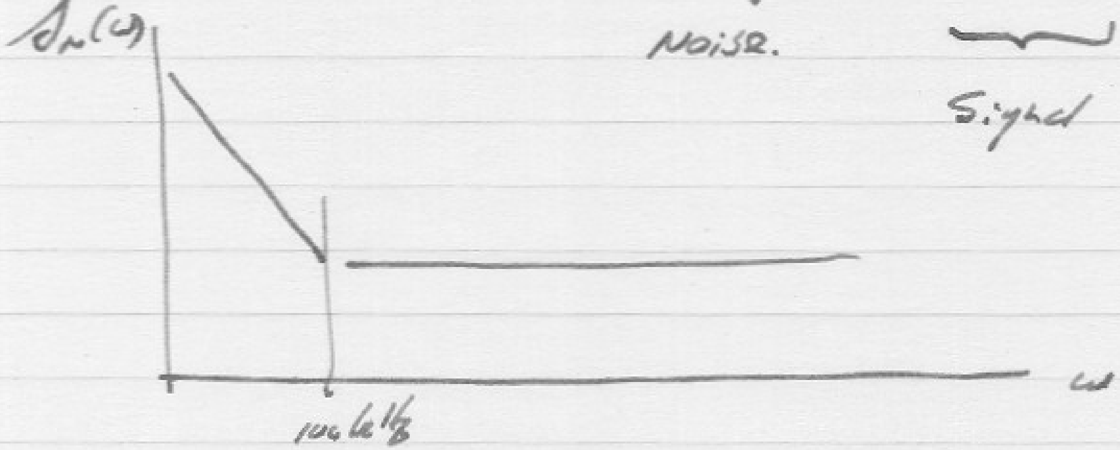


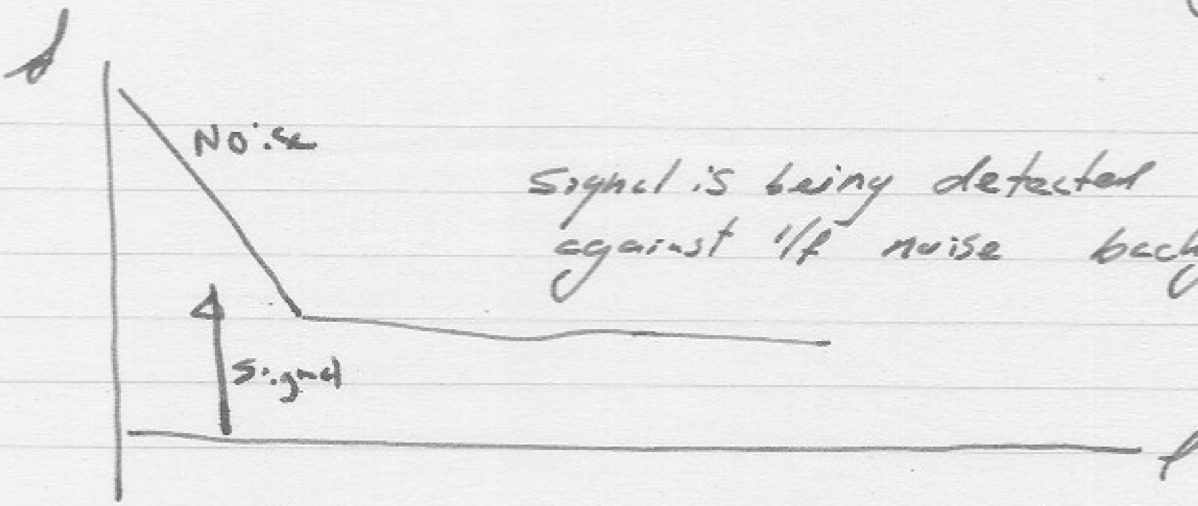
- choose $f_o \gg f_L$.

Example: Electrooptic Sampling



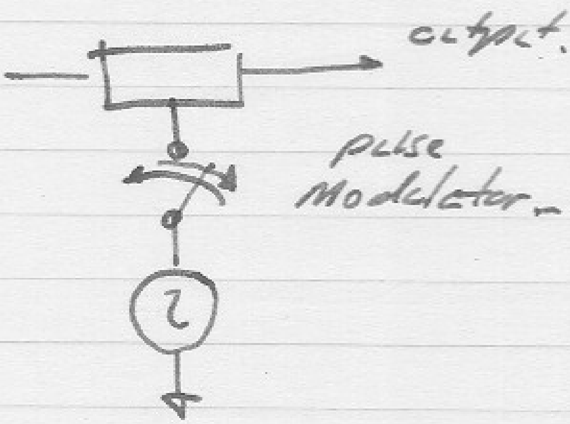
$$\text{output power} \propto P_L + \underbrace{P_L N(t)}_{\text{Noise}} + \underbrace{P_L \frac{V}{V_\pi}}_{\text{Signal}}$$





Signal is being detected against 1/f noise background.

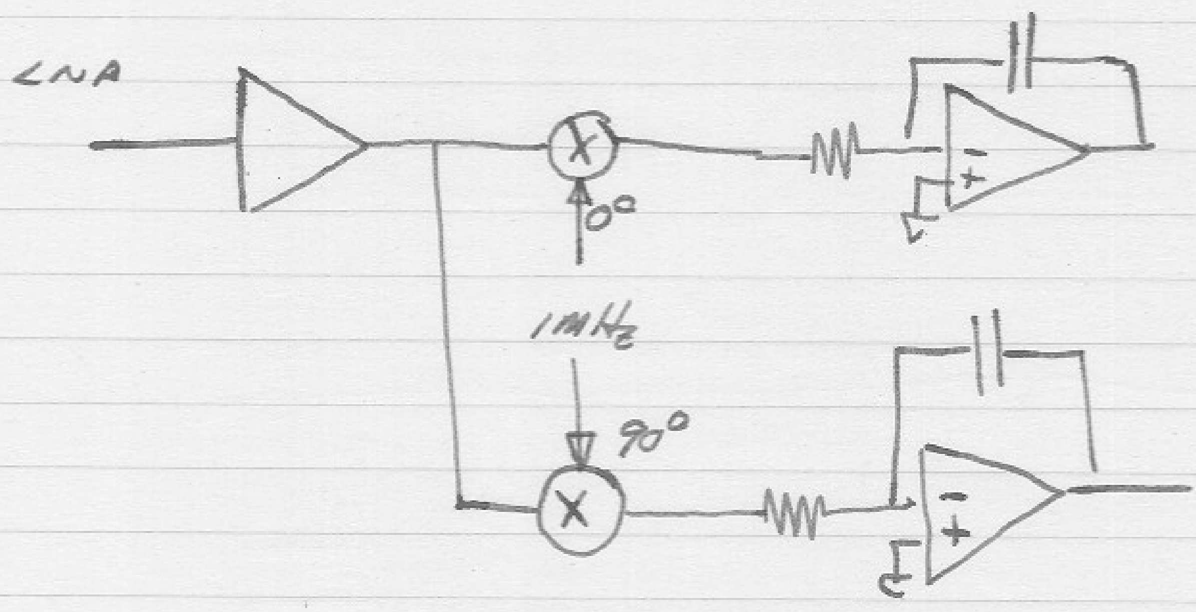
Laser



pulse modulator – chopper @ 1 MHz.

Now Signal @ 1 MHz

We now recover signal with a phase-sensitive detector:



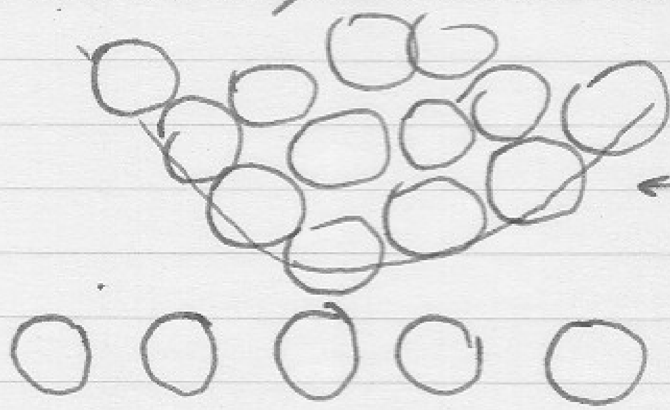
The above, a DSB receiver,

is known in the business as a

"Lock in Amplifier"

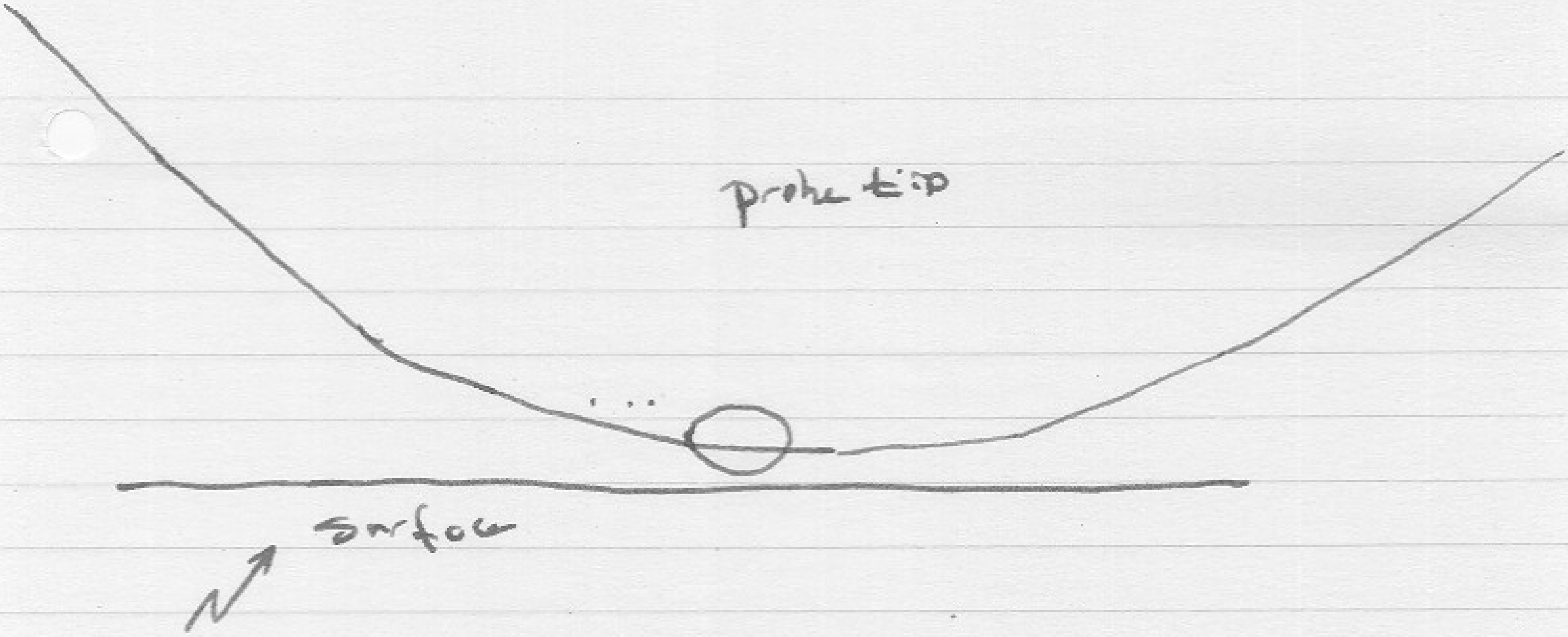
<u>Example</u> Atomic Force Microscope:

– one possible implementation –



electron cloud of atoms in probe tip.

electron "clouds" of atoms in surface..

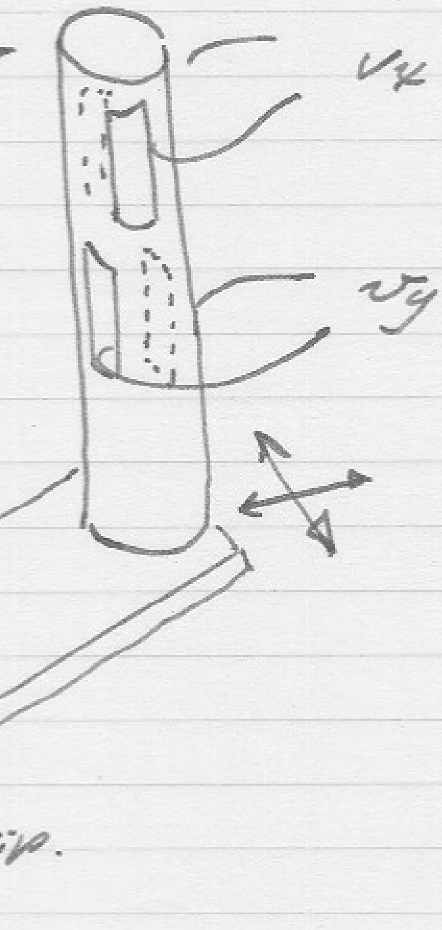


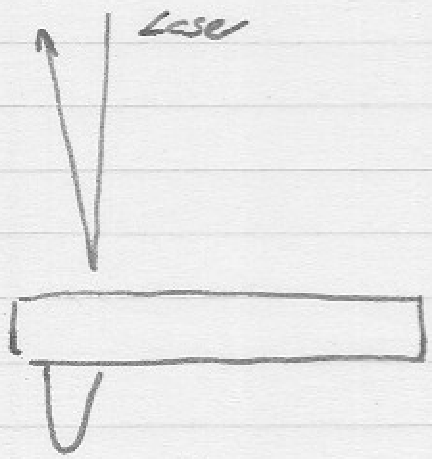
The point: one tip atom sticks out farthest & forms the probe!

Piezo bar

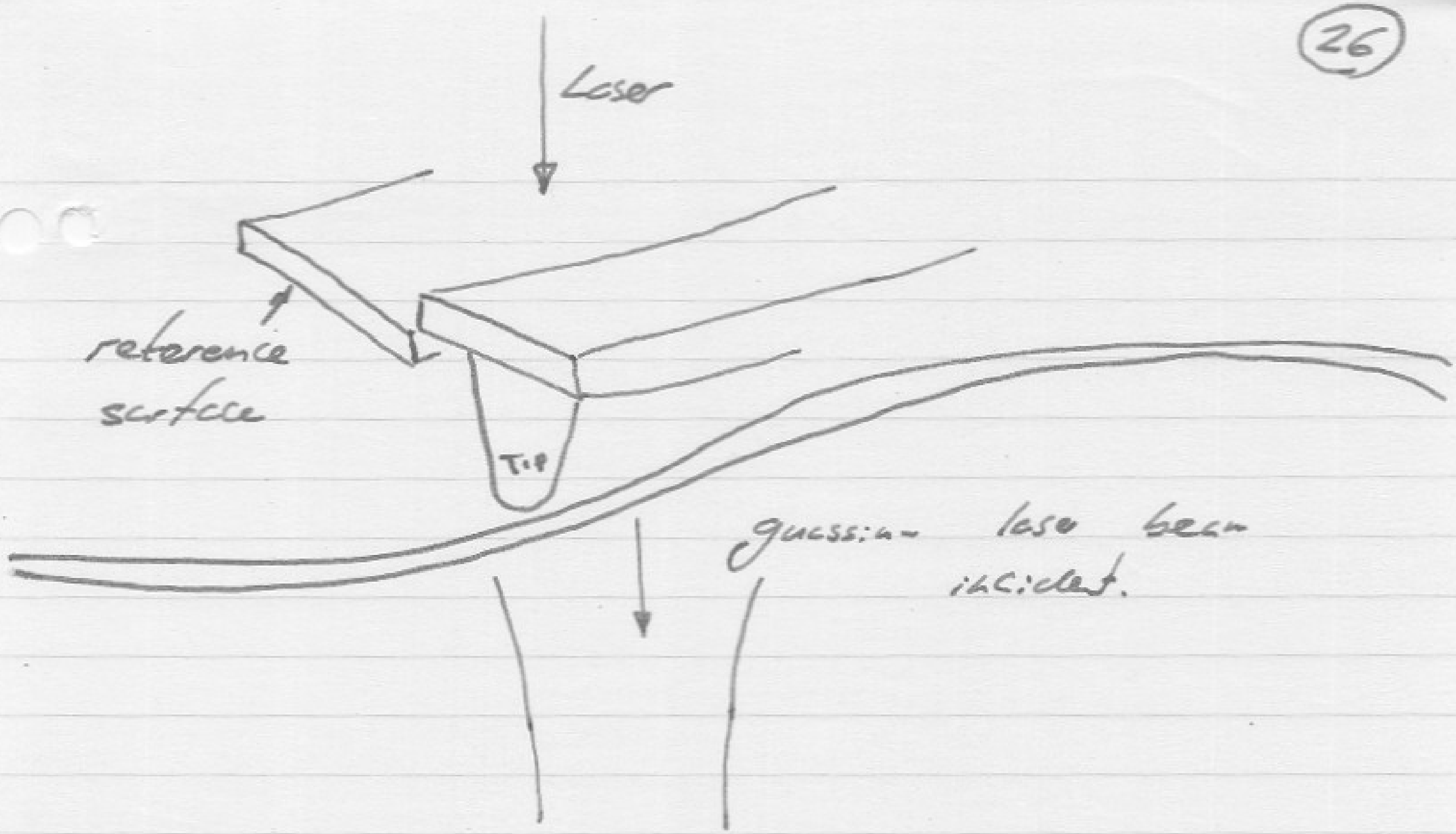
used to raster scan

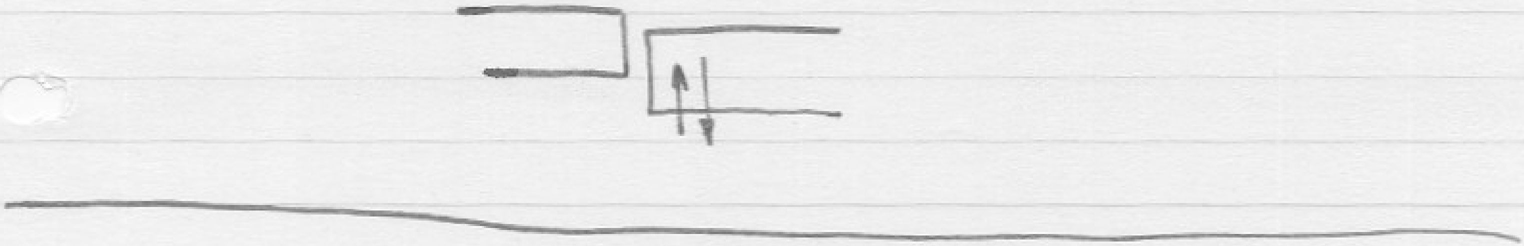
tip over surface.

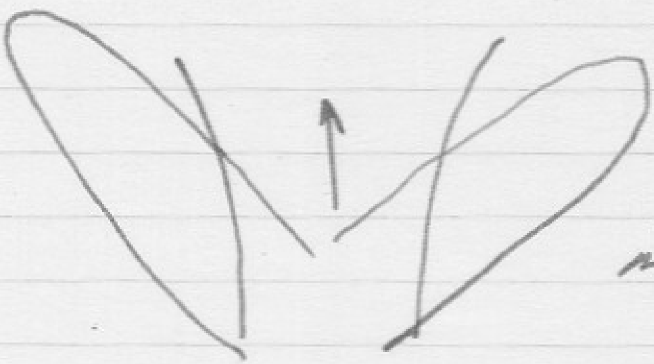




& vertical displacement measured by laser...



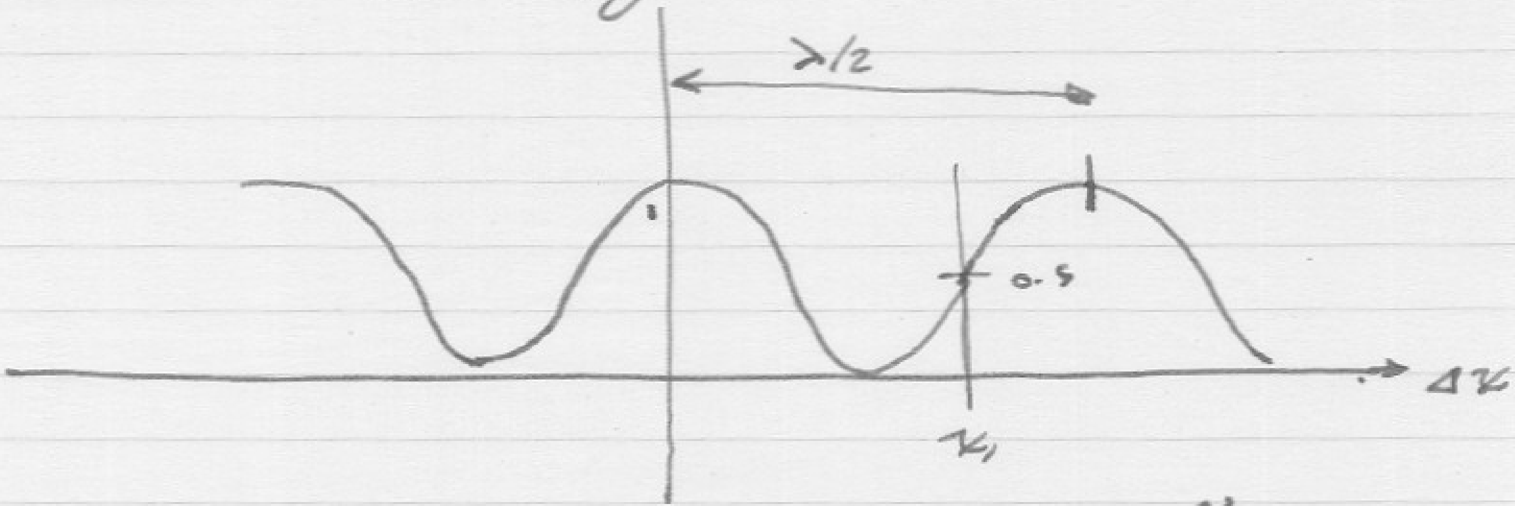




Gaussian &

first antisymmetric mode reflected back.

Preflected, gaussian:



$$P_{ref} = 1/2\,(1 - \cos(2\pi\,\Delta x/(\lambda/2))\;\tilde{P}_0$$

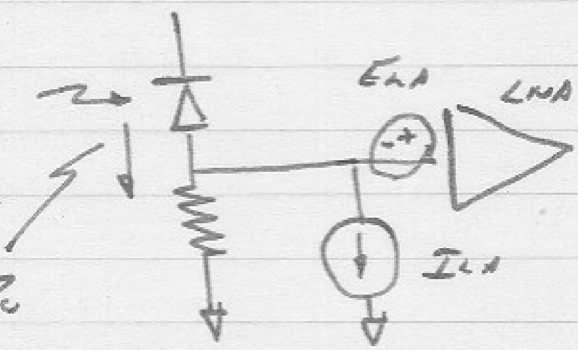
at x_1: $$\delta P_{ref} = \frac{\tilde{P}_0}{2} + \frac{\tilde{P}_0}{2}\,\frac{4\pi}{\lambda}\cdot\delta x$$

$$\boxed{P_{out} = P_0 + P_0\cdot\frac{4\pi}{\lambda}\cdot\delta x}$$

The laser beam has shot noise

(Poisson process of attenuation)

with spectral density $\rightarrow 2h\nu P_0$



$I_0 = \frac{\eta q}{h\nu} P_0$

P_0 is the power absorbed by the photodiode.

$$I_{ph} = I_0 + I_0 \frac{4\pi}{\lambda} \delta x + I_n$$

$$S_{I_n} = \frac{4kT}{R} + S_{I_{LA}} + \frac{S_{E_{LA}}}{R^2} + 2qI_0$$

$\cong 2qI_0$ if well designed.

$$I_{ph} = \overset{\overset{DC}{\uparrow}}{I_0} + I_0 \frac{4\pi}{\lambda} \cdot \delta x + I_n$$

$$= I_0 + I_0 \left(\frac{4\pi}{\lambda}\right)\left(\delta x + x_{noise}\right)$$

$$S_{x\,noise} = \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{2q}{I_0}\right) \qquad \frac{\text{meters}^2}{\text{Hz.}}$$

Suppose we observe for period τ

$$\sigma^2_{x\,noise} = \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{2q}{I_0}\right)\left(\frac{1}{\tau}\right)$$

$$\sigma^2_{x\,noise} = \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{2q}{I_0 \tau}\right)$$

Depends upon how many electrons $I_0\tau/q$ we collect from photodiode.

lets choose $\lambda \cong 600\ nm$ (red HeNe laser)

$$I_0 = 1\ mA$$

$$\tau = 100\ \mu s$$

$$\boxed{\sigma_{x\,noise} = 8.5 \cdot 10^{-18} \text{ meters!}}$$

* this assumes we have translated signal out of 1/f region - scanning can do this..

* We have neglected the acoustic noise (kT) of the cantilever. Fix this by making the cantilever very high mechanical Q, or cooling it to Liquid helium temperatures.

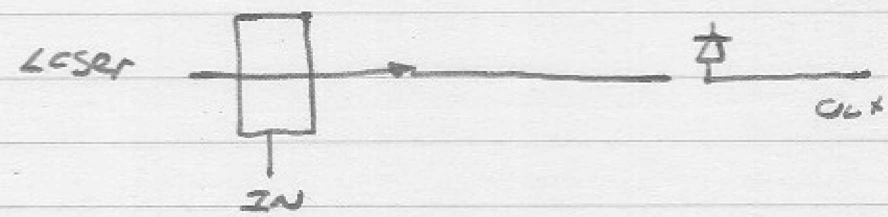
Acoustic Noise is in fact dominant!

Measuring Dinos' Photodetectors:

problem: hard to tell photodiode from

modulator transfer function in link measurement.

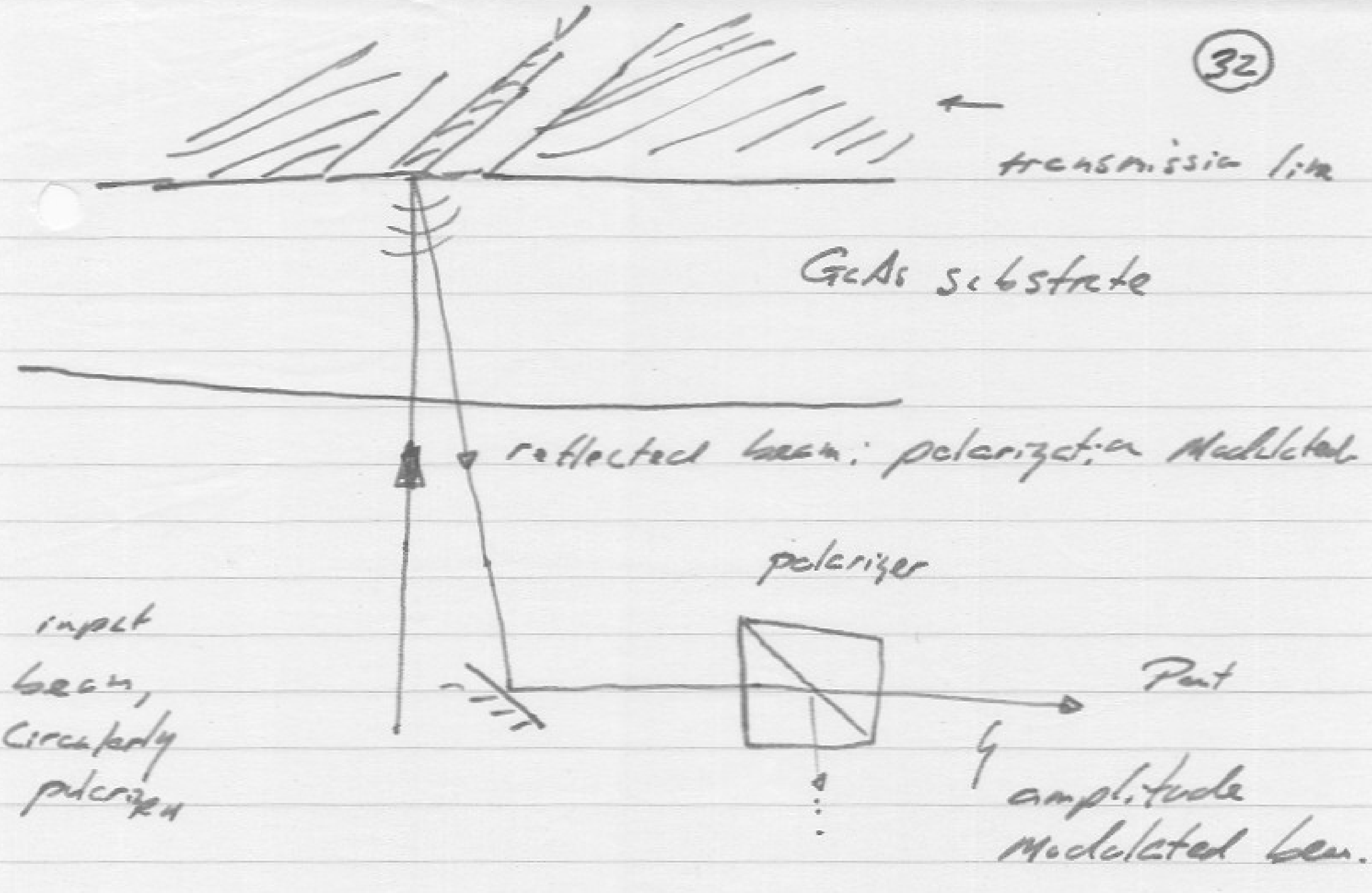
modulator: $H_m(f)$ photodiode: $H_p(f)$



Measured IN-out: $H_{meas} = H_m H_p$

$H_p = ?$

Solution: use bulk electrooptic effect modulator

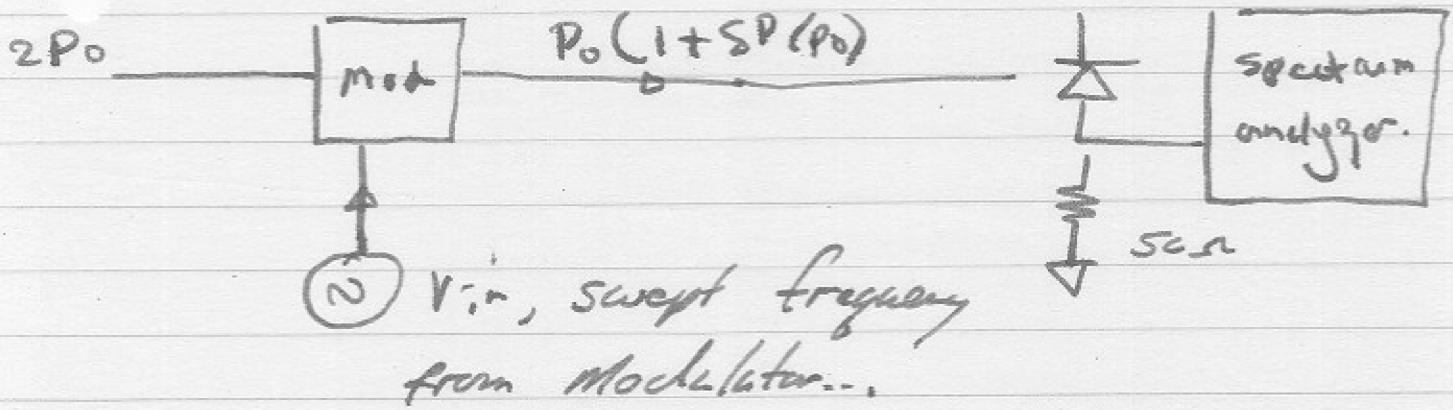


$$\boxed{\frac{\delta P_{out}}{P_{out}} = \frac{\delta V}{5\,kV}}$$

Modulator has wide bandwidth ~ 500 GHz, limited by transit time of beam through substrate.

But signal is <u>small</u>.

photodiode:



photodiode signal is very small

$$I_{out} = I_0 + I_0 \frac{\delta V}{(5kV)} + I_{noise}$$

$$S_{I_{noise}} = 2qI_0 + \frac{4kT}{50\Omega} \cdot F_{SA}$$

$$\cong \frac{4kT}{50\Omega} \cdot F_{SA} \quad \text{if} \quad F_{SA} \simeq 40dB.$$

<u>Can we detect the signal?</u>

Set spectrum analyzer @ $\Delta f = 10\text{ Hz}$.
$I_0 = 1\text{ mA}$.

Minimum detectable voltage =

$$V_{min} = \sqrt{\left(\frac{4kT}{50\Omega} \cdot F_{SA}\right)\Delta f} \cdot \frac{5k\Omega}{I_0}$$

$$V_{min} = \sqrt{\frac{4kTF_{SA} \cdot \Delta f}{50\Omega}} \cdot \frac{5k\Omega}{I_0}$$

$$\boxed{V_{in} = 29\text{ mV}}$$

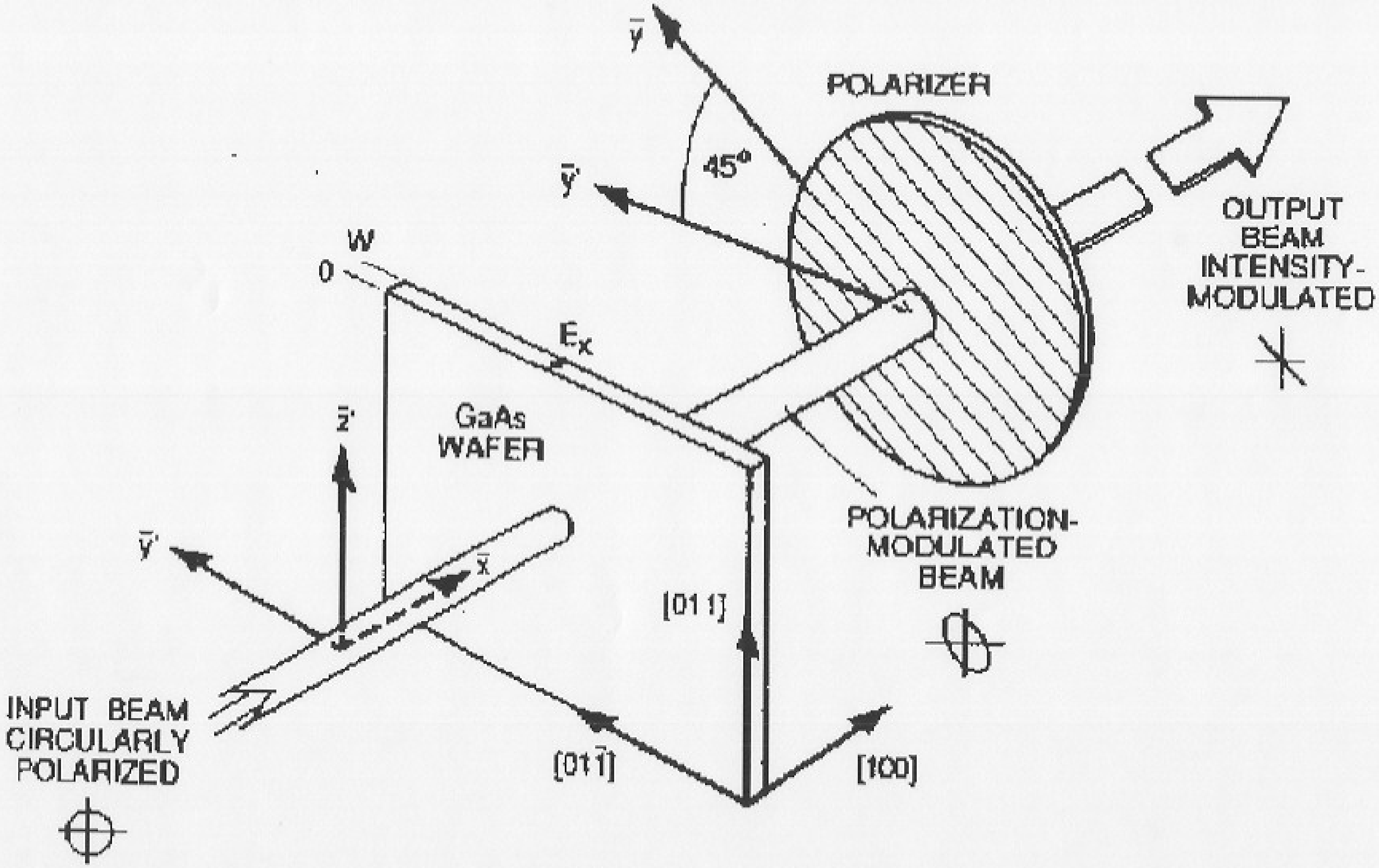
So, If we can apply a 1 Volt signal

$\Rightarrow S/N = \frac{1V}{29mV}$

$$\boxed{S/N = 30.8\text{ dB}}$$

We can measure the signal comfortably...

Gallium Arsenide electrooptic intensity modulator

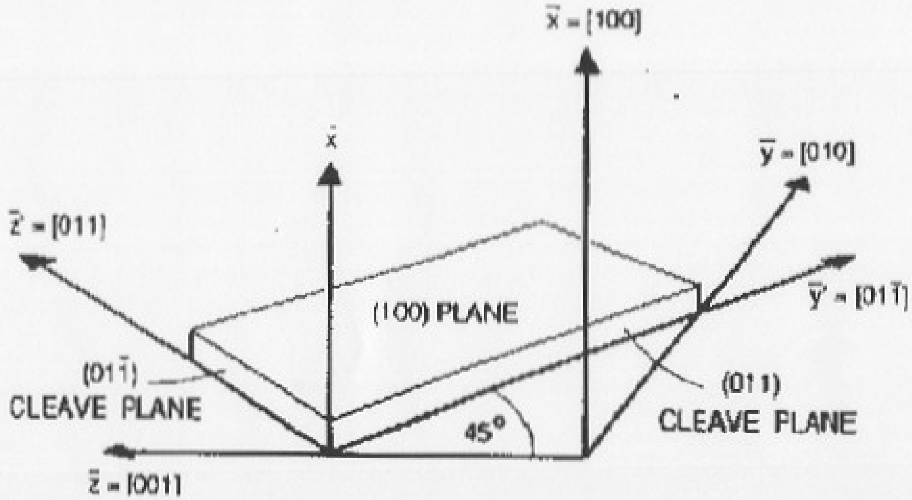


Electrooptic Probing

Field-Induced birefringence in [100]-cut GaAs:

$$N_{[01\bar{1}]} - N_{[011]} = N_0^3 r_{41} E_{[110]}$$

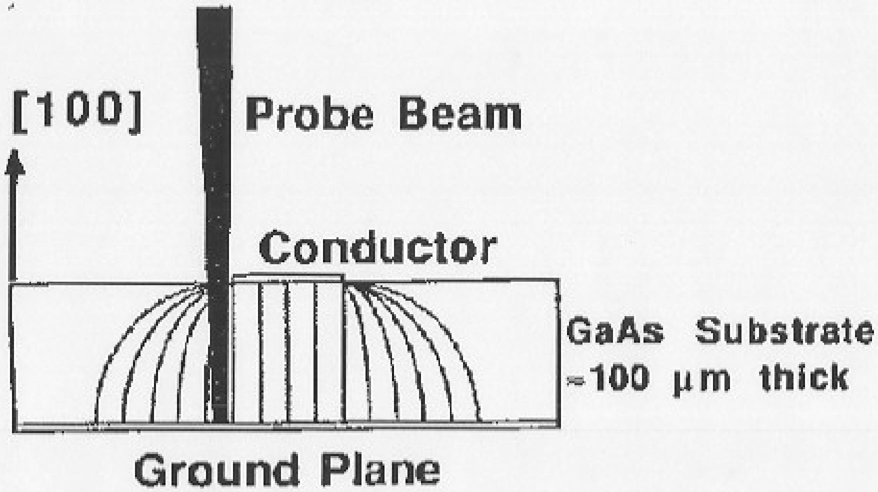
A sub-bandgap probe beam is passed in the [100] direction through the substrate, and the birefringence measured with a polarization interferomenter. The interferometer output is proportional to the potential difference across the wafer.



Principal axes and cleave planes in (100)-cut Gallium Arsenide.

Electrooptic: Frontside/backside probing

Frontside probing



Backside probing