• Notes Set 22: Digital communications systems

- Linear Algebra. Waveforms as vectors in N-space.
- delta functions and sinusoids as basis functions. The fourier transform
- modulation formats / channel codes. Optimum reciever decision rules
- correlation and (equivalent) matched filter receiver. Receiver sensitivity and bit error rate. On-off keying. Binary antipodal modulation.
- example: sending data steams from saturn to earth.

ECE ______ Notes Set 22Ø.

Digital Communications Systems

Here we will find that Geometric picture is of unbounded value.

First lets review linear algebra so that we can treat signals as vectors.

Comment: Method & ideas common to

- Fourier transforms
- Quantum Mechanics.
-

Linear Algebra

A linear algebra consists of

- a set of mathematically defined objects
- the set of complex #'s.
- and a metric.

\# often called a "Linear space"

objects called "vectors"

metric called inner product

... but this may cause you to treat the ideas as being restricted to Euclidean geometry, while the rules allow much wider scope.

- We need an addition operation on the vectors $\bar{v}_i$:

$$\bar{v}_i + \bar{v}_j = v_j + v_i$$

$$v_i + (v_j + v_k) = (v_i + v_j) + v_k$$

... which must be both commutative & associative

- We need the null vector $\bar{0}$

$$\bar{V}_i + \bar{0} = V_i$$

- ... and the additive inverse ...

$$\bar{v}_i + (-\bar{v}_i) = 0$$

* we need scalar multiples:

$$\alpha(\bar{V}_i + \bar{V}_j) = \alpha \bar{V}_i + \alpha \bar{V}_i$$

$$(\alpha + \beta)\bar{V}_i = \alpha \bar{V}_i + \beta \bar{V}_i$$

$$\alpha(\beta \bar{V}_i) = (\alpha\beta)\bar{V}_i$$

* and we need a metric:

$\langle \bar{V}_i, \bar{V}_j \rangle$ with the properties

1) $\langle \bar{V}_i, \bar{V}_i \rangle \geq 0$

$= 0$ iff $\bar{V}_i = \bar{0}$

2) $\langle \bar{V}_i, \bar{V}_j \rangle = \langle V_j, V_i \rangle^*$

3) $\langle \bar{V}_i, \alpha \bar{V}_j + \beta \bar{V}_k \rangle =$

$$\alpha \langle \bar{V}_i, \bar{V}_j \rangle + \beta \langle \bar{V}_i, \bar{V}_k \rangle$$

Basic Statement

Any set of math objects $\bar{v}_i$

with scalars α and Metric $\langle \bar{v}_i, \bar{v}_j \rangle$

satisfying the above axioms forms

a Linear Space and is

Isomorphic with Vector Algebra in

Euclidean N-space.

<u>specifically</u>.

~~An~~ The set of functions $f(t)$ on some interval $[-T/2, T/2]$ together with scalar products $\alpha f(t)$ and inner products $\langle f(t), g(t)\rangle = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,g(t)\,dt$

forms such a linear space.

<u>Consequently</u>: We can speck of $f(t)$ being orthoganal to $g(t)$, perform the vector projection of $f(t)$ onto $h(t)$, speck of the "lenght" of $f(t)$, etc.

¿A Confusing Note to make you lost?

* We have already used $\langle g \rangle$ to denote statistical expectation. Now we have used $\langle g(t), h(t) \rangle$ to denote $\frac{1}{T}\int_{-T/2}^{T/2} f(t)\, g(t)\, dt$.

* Use $E[g]$ or $\cancel{\bar{g}}$ to avoid confusion.

  $\bar{g}$ just gets us in further trouble.

* This notational problem is profound, not accidental, because

$$E[g(x)\, h^*(x)] = \int_{-\infty}^{+\infty} g(x)\, h^*(x) \cdot f_x(x)\, dx$$

... clearly is also a linear-space operation, an inner product, but a different space

hence the use of $\langle \ \rangle$, inner product notation to denote expectation ... ouch!

* unit vectors: $\langle \bar{u}_i, \bar{u}_i \rangle = 1$

* a basis is a set of ortho-normal vectors which span the space, e.g.

$$\langle \bar{u}_i, \bar{u}_j \rangle = \delta_{ij}$$

and

$$\bar{f} = \sum_i \underbrace{\langle \bar{f}, \bar{u}_i \rangle}_{\text{Vector component}} \underbrace{\bar{u}_i}_{\text{unit vector.}}$$

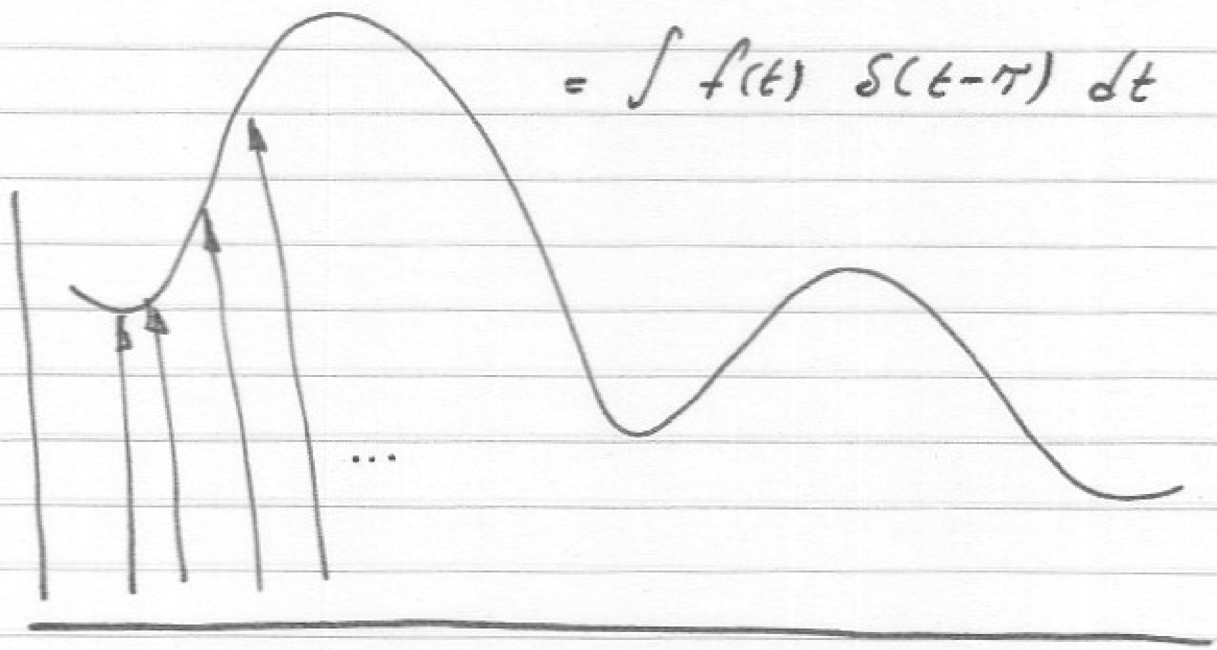
* We can write $f(x)$ as being made up of δ- functions:

$$\langle \delta(t_1), \delta(t_2) \rangle = \delta(t_1, t_2)$$
$$= 0 \quad \text{unless } t_1 = t_2$$

$$f(t) = \int_\tau \underbrace{\langle f(t), \delta(\tau) \rangle}_{\text{vector component}} \underbrace{\delta(\tau)}_{\text{basis vector}} \, d\tau$$

$$= \int f(t-\tau)\, \delta(\tau)\, d\tau$$

$$= \int f(t)\, \delta(t-\tau)\, dt$$



✱ Consider $e^{j\omega_i t}$; $\omega_i = i(2\pi/T)$

$$\langle e^{j\omega_i t}, e^{j\omega_j t} \rangle = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\omega_i t} e^{-j\omega_j t}\, dt$$

$$= \delta_{ij}$$

So $$\boxed{f(t) = \sum_i \langle f(t), e^{j\omega_i t} \rangle e^{j\omega_i t}}$$

This is the Fourier transform.

Note: If this looks like Quantum Mechanics, it should remember that it is called the

Dirac delta function.

[Ok I've just been very philosophical
now lets get very pragmatic.]

Remember:

"Nothing is as practical as a really good theory" - specifically the geometric picture makes optimum receiver design obvious.

Lets Continue with

general problem:

* transmitter wishes to communicate one of M messages m

Message:

m_1

$\vdots$

m_M

* transmitter sends (transmits) a unique vector $\bar{s}_i$ for each message m_i

Message	transmitted signal
m_1	$\rightarrow \bar{S}_1 = (S_{11}, S_{12}, \dots S_{1N})$
m_2	$\rightarrow \bar{S}_2 = (S_{21}, S_{22}, \dots S_{2N})$
$\vdots$	$\vdots$
m_i	$\rightarrow \bar{S}_i = (S_{i1}, S_{i2}, \dots, S_{iN})$
$\vdots$	$\vdots$
m_M	$\rightarrow \bar{S}_m = (S_{m1}, S_{m2}, S_{m3}, \dots S_{mN})$

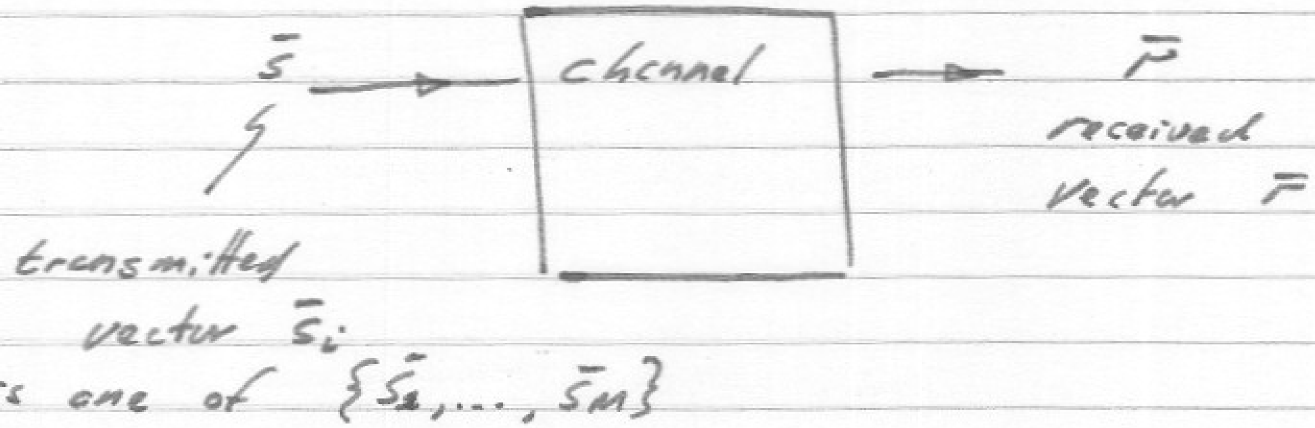
☆ this 1:1 mapping between a message and a symbol stream (vector) sent is called a <u>channel code</u>

or a <u>modulation format</u>

☆ The terms are equivalent

no distinction / boundary between codes & modulation methods.

For a moment lets take an arbitrary transmission channel

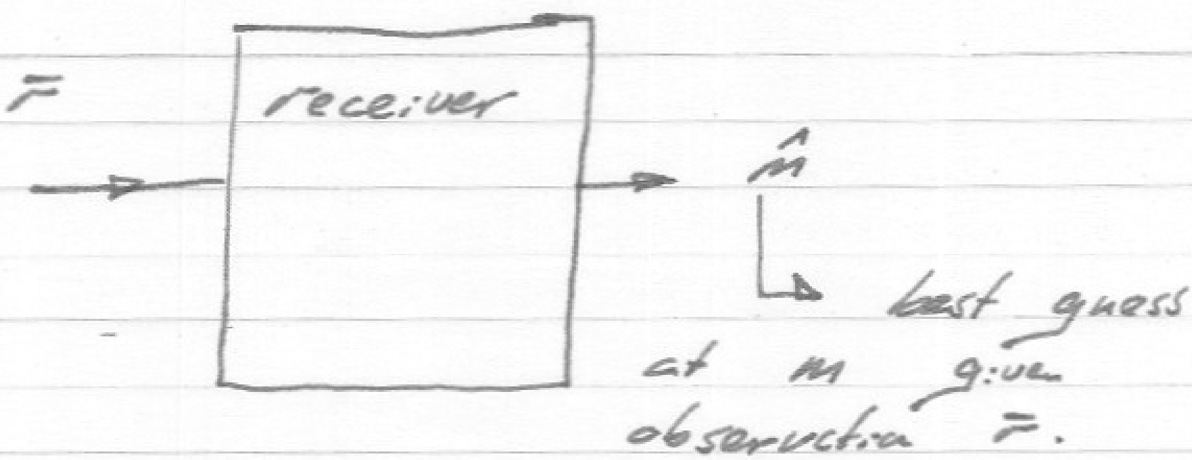


The channel is described by the conditional ~~den~~ probabilities (or probability densities) of receiving a vector $\bar{r}$ given that $\bar{s}_i$ was sent, e.g.

$$\left\{ P\{\bar{r}/\bar{s}_i\} \right\}_{\text{all } \bar{s}_i}$$

... describes the channel.

The receiver must make a decision



Our job is to determine the rule by which the receiver should best choose between the $\{m_1, \ldots, m_M\}$ possible messages, given that it has received the signal $\bar{r}$.

Comment

↳ We can't make a mathematical rule for decisions until we have decided on a cost function.

☆ Many "textbook" & "real" problems have vastly different costs for making errors of type I ("false alarm") vs type II ("alarm ignored")

examples: [radar: friend or foe?
NORAD: Missiles coming in?

☆ Concentrating on Generic data communication, we will choose to give all errors equal weight, hence will minimize simply the probability of error

Now we can define an

Optimum Receiver Rule

- assume r.v. $\bar{r}$ is a particular value $\bar{\rho}$
- Need to minimize P[Error]

$$P[E] = P[\text{Error}] = 1 - P[C] = 1 - P[\text{Correct}]$$

$$P[C] = \int_{\rho} P[C \mid \bar{r} = \bar{\rho}]\, p[\bar{\rho}]\, d\bar{\rho}$$

this is positive

So to minimize this

... we must minimize this

$\Downarrow$

<u>Obscure rule</u> given that $\bar{\rho}$ is received

choose $\hat{m}$ such that

$$P[C \mid \bar{r} = \bar{\rho}] = P[m = \hat{m} \mid \bar{r} = \bar{\rho}]$$

... is maximized ...

Oh: that means pick the most likely (highest probability) message given the received signal - not really suprising.

since $$P[m_i \mid r = \bar{\rho}] = \frac{P[m_i] P_r[\bar{\rho} \mid m_i]}{P_r[\rho]}$$

our rule is: simply: Pick the message m_i for which

$$P[m_i] \cdot P_r(\bar{\rho} \mid m_i)$$

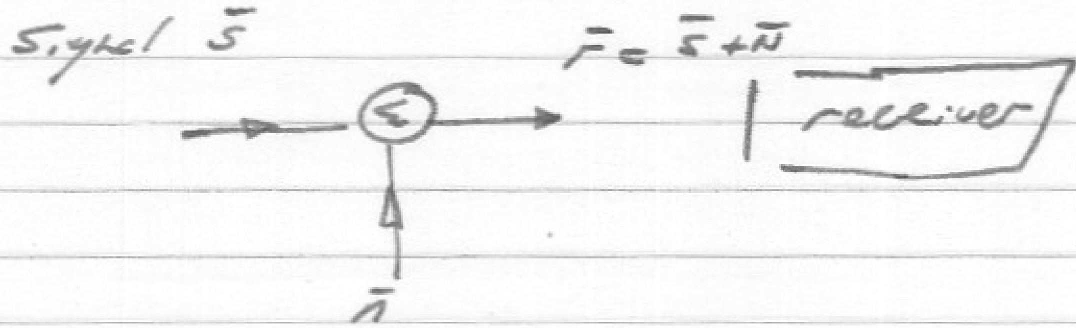
... is biggest

often we don't know what the message probabilities are, or they are all the same. Then our rule is just:

$$\boxed{\begin{array}{l}\text{Pick the message } m_i \\ \text{for which} \\ Pr(\bar{\rho} \mid m_i) \\ \text{is biggest}\end{array}}$$

Now Simplify

Make noise additive



* $\bar{N}$ is a random vector

$$\bar{N} = (n_1, n_2, n_3, \dots n_n)$$

with a covariance matrix $C_{\bar{N}\bar{N}^T} = \sigma^2 \cdot \begin{bmatrix} 1 & & & 0 & \\ & 1 & & & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & 1 \end{bmatrix}$

(* if N should have a different correlation matrix, we could come up with a change of basis for both $\bar{s}$ & $\bar{n}$ which gives the components of $\bar{n}$ a unit diagonal correllation matrix. this is called a whitening filter)

<u>So</u> $\bar{r} = \bar{s} + \bar{n}$

received, sent, white gaussian noise vector

Pick m_i such that we maximize

$$P_{\bar{r}}(\bar{\rho} / m_i) = P_{\bar{r}}(\bar{\rho} / \bar{s}_i)$$

$$= P_n(\bar{\rho} - \bar{s}_i / \bar{s}_i)$$

the noise & signal are independent, so

$$P_r(\bar{\rho} / m_i) = \underbrace{P_n(\bar{\rho} - \bar{s}_i)}$$

this is the probability of the noise taking

on the vector value $\bar{\rho} - \bar{s}_i$.

Since $\bar{n} = \bar{\rho} - \bar{s}_i$ is gaussian,

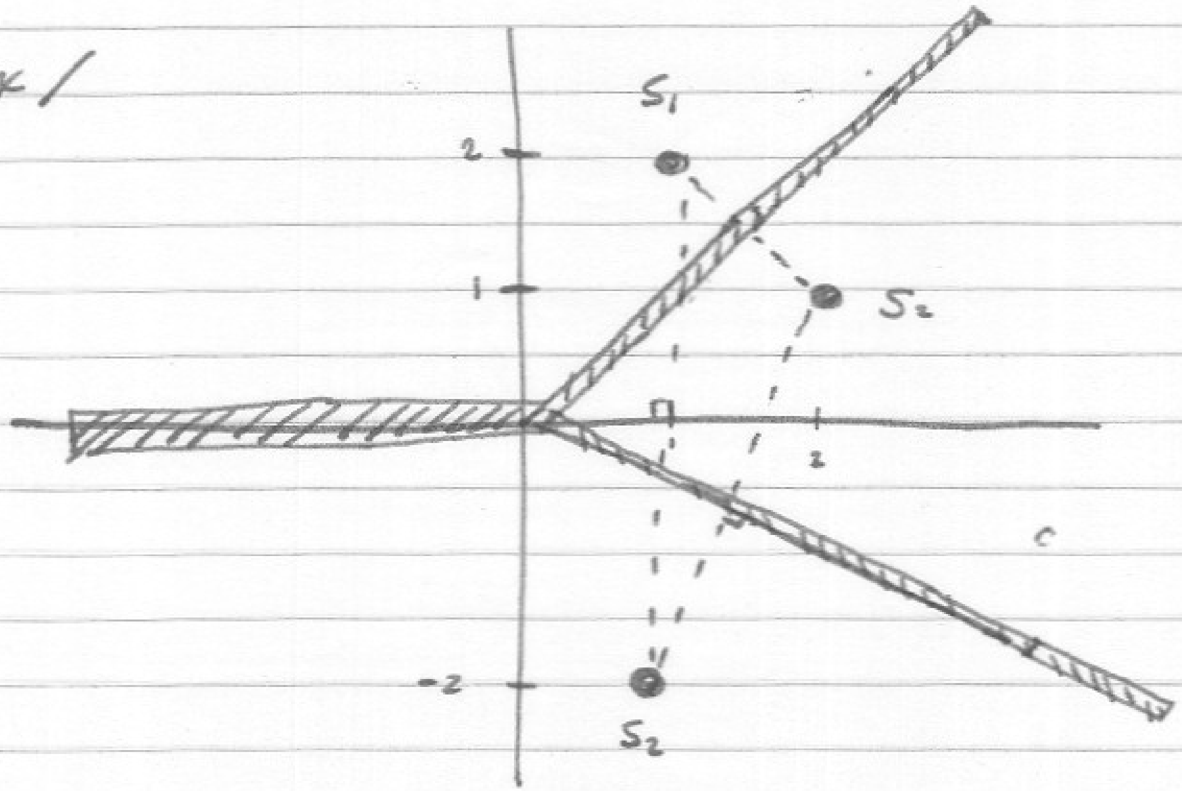
we minimize the probability of error

by picking the closest message vector $\bar{s}_i$

to the received vector $\bar{\rho}$:

Decision Rule:

Pick the message whose vector is closest to the received vector ex 1



☆ Decision boundaries found by perpendicular bisectors

☆ Pick a message if its the received signal falls in its decision region, e.g. is closer to it than others.

☆ Clearly it makes sense to pick evenly placed signal vectors. Information theory says it does not matter so much..

Irrelevant information

Ex: $s_1 = (1, 1, 0)$

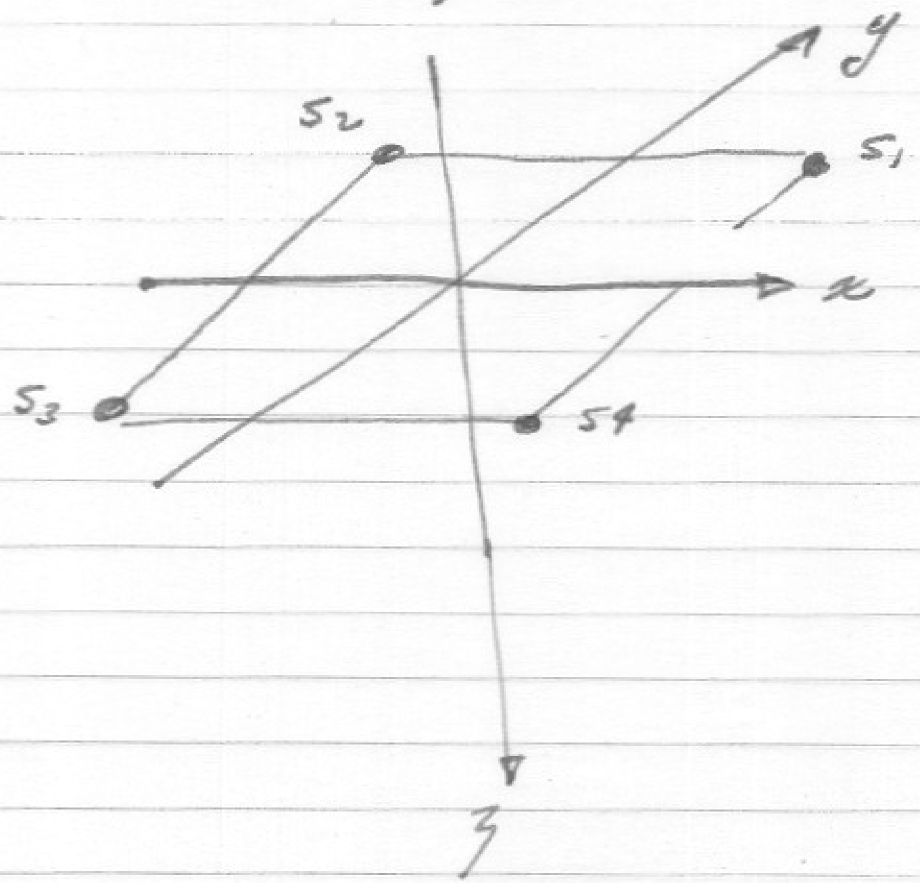
$s_2 = (-1, 1, 0)$

$s_3 = (-1, -1, 0)$

$s_4 = (1, -1, 0)$

x, y, z, components

the received signal has noise in x, y, & z components, but these are ⊥ of each other (if not, change vector basis so they are...)



* Clearly the z component will be irrelevant to this problem

* Developing the theorem of irrelevance mathematically will take more time than we have.

* But key point: once we are working in a vector space where noise has orthogonal (uncorrelated) components, we can consider only those vectors upon which the signal vector has an effect.

* We have worked digital communications with vectors

* but digital communications often uses waveforms

* but we have already shown that waveforms are vectors, given that the metric is $\langle f(t), g(t) \rangle$

$$\triangleq \int_{-T/2}^{T/2} f(t)\, g^*(t)\, dt.$$

Transmitter's message to send: | Signal sent

$$\left.\begin{array}{lcl} m_1 & \longrightarrow & s_1(t) \\ m_2 & \longrightarrow & s_2(t) \\ \vdots & & \\ m_m & \longrightarrow & s_m(t) \end{array}\right] \quad t \in [-T/2, T/2]$$

Recognize that this is a vector space.

Choose a set of orthonormal basis vectors $\{\varphi_i(t)\}$

e.g. $\langle \varphi_i, \varphi_j \rangle = \delta_{ij}$

[Note, once we get to physical problems we will set $\langle \varphi_i, \varphi_j \rangle = \delta_{ij} \cdot 1\,\text{Joule}$.

Number of Dimensions $\Longleftrightarrow$ Nyquist

I have 2 vectors s_1 & s_2 in a 3-dimensional space. I form other vectors $\alpha s_1 + \beta s_2$. Do I need 3 dimensions? No - all ~~dimensions~~ vectors $\alpha s_1 + \beta s_2$ fall in a 2-dimensional subspace.

$\Downarrow$

I have M vectors $\{s_1, \ldots s_M\}$ in an ∞-dimensional space. $\{s_1, \ldots s_M\}$ form at most an M-dimensional subspace. Signals don't fall outside that space, & neither do relevant components of the noise

So: of my messages

$$\left.\begin{matrix} m_1 \\ \vdots \\ m_M \end{matrix}\right\} \longrightarrow \left\{\begin{matrix} s_1(t) \\ \vdots \\ s_M(t) \end{matrix}\right.$$

there are $N \le M$ linear degrees of freedom

and hence, to be intelligent, we should:

* choose a set of basis vectors - orthonormal,

$$\{\varphi_i(t)\}$$

* ...with the first N of them

$$\varphi_1(t),\ \varphi_2(t),\ \ldots,\ \varphi_N(t)$$

...spanning the subspace of $s_1(t)$ through $s_M(t)$

* The remaining vector components & basis

vectors $\varphi_{N+1}(t), \ldots, \varphi_\infty(t)$

... are irrellevant, and we can ignore.

P.S: This is Nyquist's sampling theorem in a more general form, and an obvious one:

$$\boxed{\text{Given 10 unknown parameters, I need to make 10 linearly independent measurements. No need to make 1000}}$$

So we have solved the problem:

A) Find the vector components of the incoming signal.

B) See which signal vector it falls closest to!

wait?

How do we find a vector component?

relevant $\leftarrow$ | $\rightarrow$ irrelevant

$$r(t) = r_1\,\varphi_1(t) + r_2\,\varphi_2(t) + \dots + r_N\,\varphi_N(t) + r_{N+1}\,\varphi_{N+1}(t) + \dots$$

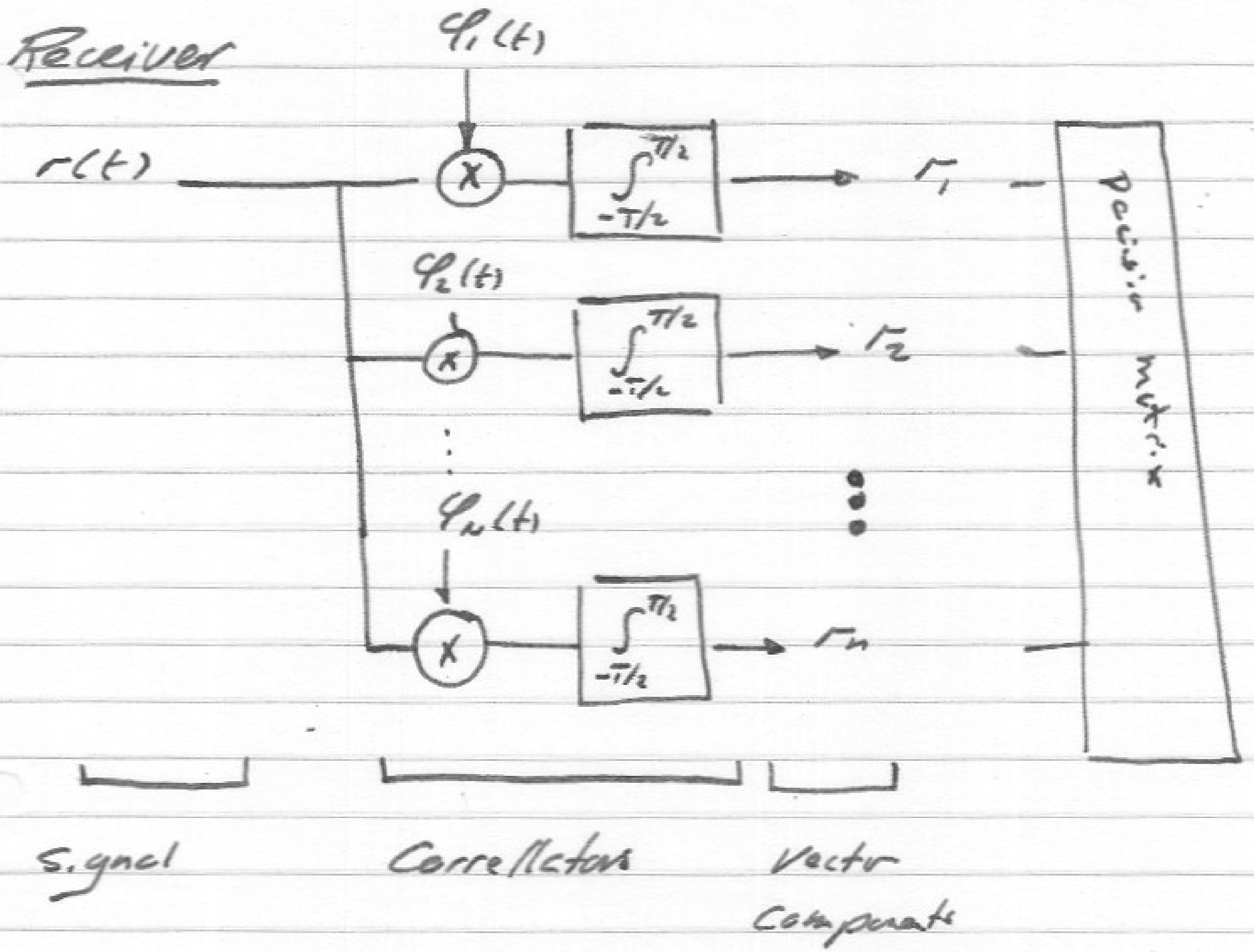
where:

$$r_1 = \langle r(t) \,|\, \varphi_1(t) \rangle$$

$$= \int_{-T/2}^{T/2} r(t)\,\varphi_1(t)\,dt$$

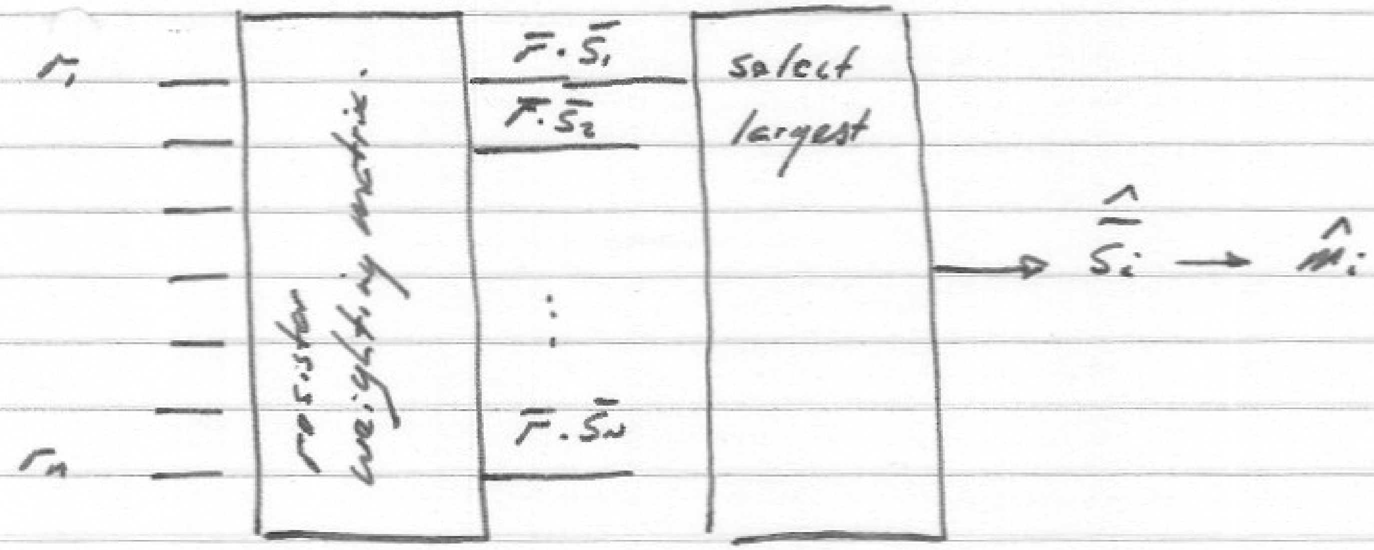
= The Correllation of $r(t)$ & $\varphi_1(t)$!

Receiver



The decision metrix looks at the vector $\bar{r}$ & determines which $\bar{s}_i$ is closest. It is an array of resistive weighting networks & comparators.

Here is the decision matrix:



we are trying to minimize

$$\| \bar{r} - \bar{s}_i \| = \| \bar{r} \|^2 + \| \bar{s}_i \|^2 - 2\, \bar{r}\cdot\bar{s}_i$$

so we can do this most easily by

maximizing $\bar{r}\cdot\bar{s}_i$

Note that

* $r_1 = \langle r(t), \varphi_1(t) \rangle = \int r(t)\, \varphi_1^*(t)\, dt$

is the projection of $r(t)$ onto $\varphi(t)$

* $\bar{r} \cdot \bar{s}_1 = r_1 s_1 + r_2 s_2 + \cdots + r_n s_n$

is the projection of r onto s_1

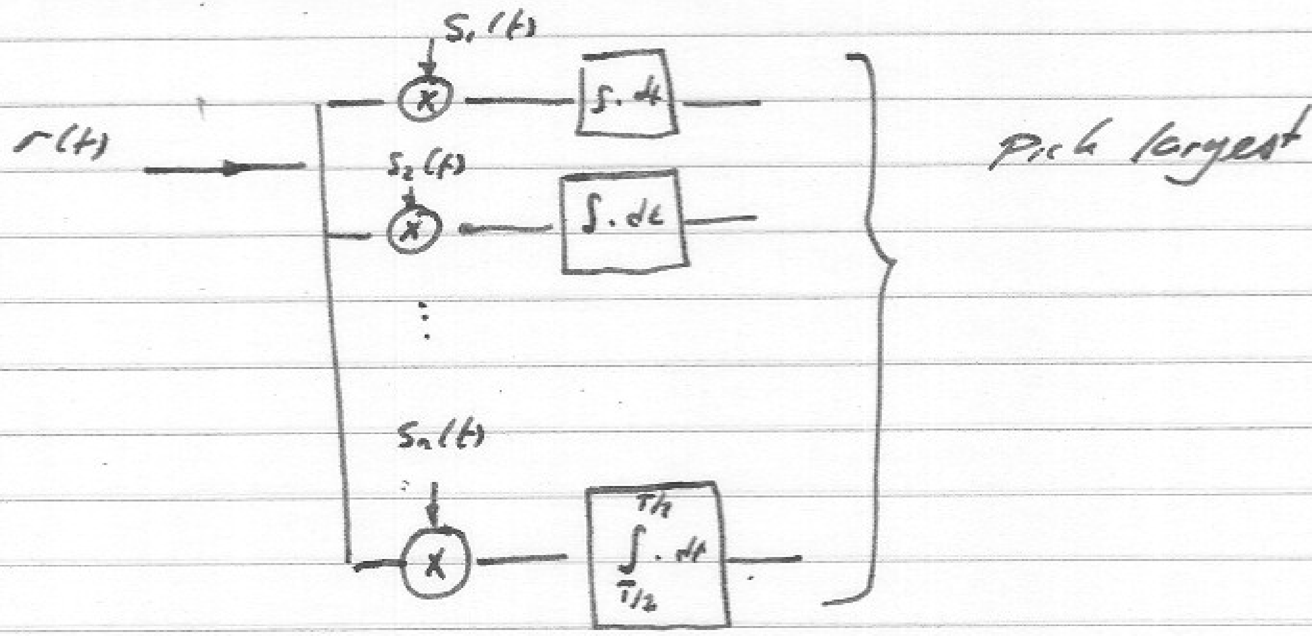
$\therefore$ we could have done both in a single step, although this might be either more or less expensive.

$$\bar{r} \cdot \bar{s}_1 = r_1 s_{11}^* + r_2 s_{12}^* + \cdots + r_n s_n^*$$

$$= \langle r(t), \varphi_1(t) \rangle \langle s_1(t), \varphi_1(t) \rangle^* + \cdots$$

$$= \langle r(t), s_1(t) \rangle$$

So another construction is:



This will be terribly more complex unless $s_1 \cdots s_N$ are linearly independent, which is quite unlikely.

"Matched filters"

$$r(t) \longrightarrow \boxed{h_1(t)} \longrightarrow u_1(t)$$

$$u_1(t) = \int_{-\infty}^{+\infty} r(\tau)\, h_1(t-\tau)\, d\tau \leftarrow \text{convolution}$$

set $\quad h_1(t) = \varphi_1(T-t) \leftarrow$ time reversal.

then

$$u_1(t) = \int_{-\infty}^{+\infty} r(\tau)\, \varphi_1(T-t+\tau)\, d\tau$$

$=$ filter output

If we look at filter output at $t=T$

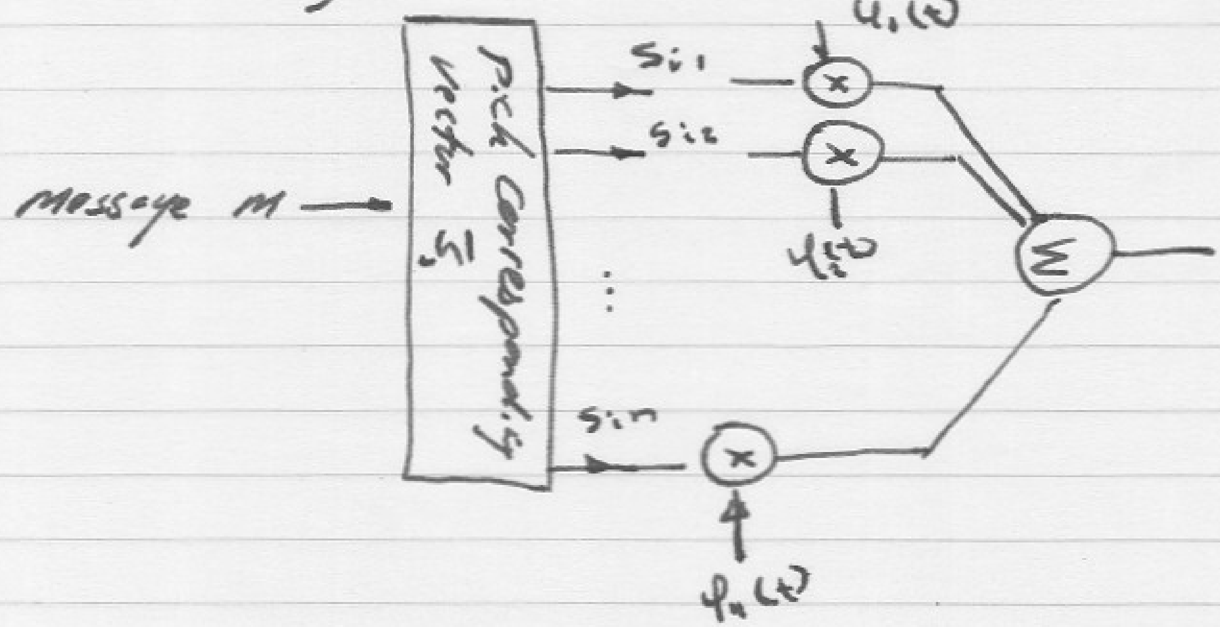
$$u_1(T) = \int_{-\infty}^{+\infty} r(\tau)\, \varphi_1(\tau)\, d\tau$$

$$= \langle r(t), \varphi_1(t) \rangle$$

* So, we can dispense with the correllators, if desired, by incorporating a filter whose impulse response is the time-reversal of the basis ~~of~~ function.

* If we choose to dispense with the weighting matrix, then the correlations must be performed against the transmitted signal set $\{s_i(t)\}$ & the filters have impulse responses $s_i(T-t)$.

by the way, the transmitter looks like so:



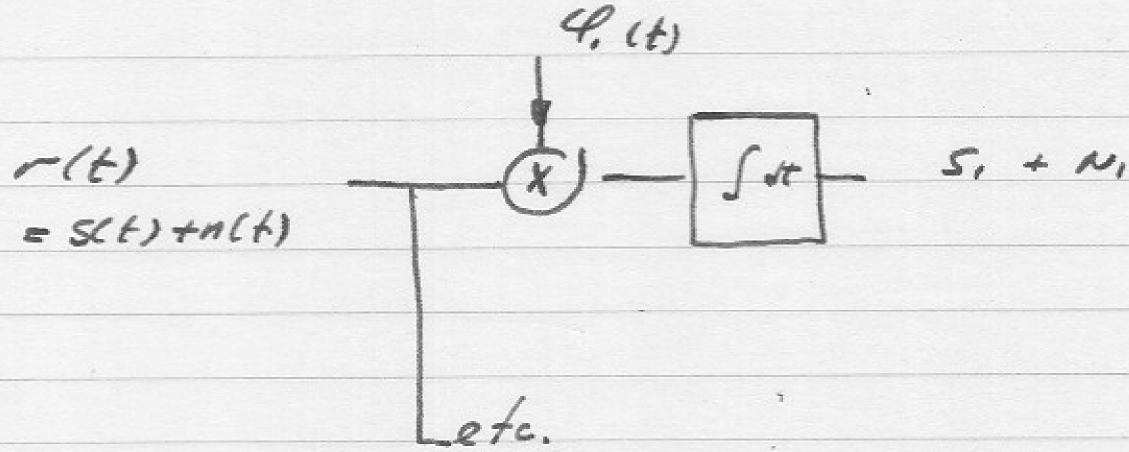
... this is the obvious way to do it but

1) there are other methods.

2) The above method is general.

The noise

noise waveform $n(t) \rightarrow$ noise vector $\bar{n}$



$$E[n_i, n_j] = E\left[\langle n(t), \varphi_i(t)\rangle \langle n(t), \varphi_j(t)\rangle\right]$$

$\cancel{\langle n_i, n_j\rangle}$

$E[n_i, n_j]$

$$E[n_i, n_j] = E\left[\int n(t)\,\varphi_i(t)\,dt \cdot \int n(\tau)\,\varphi_j(\tau)\,d\tau \right]$$

$$= E\left[\iint n(t)\,n(\tau)\,\varphi_i(t)\,\varphi_j(\tau)\,dt\,d\tau \right]$$

$$= \iint R_{nn}(t-\tau)\,\varphi_i(t)\,\varphi_j(\tau)\,dt\,d\tau$$

$$= \iint N_0\,\delta(t-\tau)\,\varphi_i(t)\,\varphi_j(\tau)\,dt\,d\tau$$

$$= \int N_0\,\varphi_i(\tau)\,\varphi_j(\tau)\,d\tau$$

$$= N_0 \langle \varphi_i, \varphi_j \rangle$$

$$E[n_i, n_j] = N_0\,\delta_{ij}$$

* Note that white noise gives orthogal (independent) projections onto orthogonal signals $\varphi_i(t)$, $\varphi_j(t)$

* My proof was ugly: can do ~~in a sheet~~ by vectors.

$$\langle n_i | n_j \rangle = \langle \varphi_j | n \rangle \langle n | \varphi_i \rangle$$

$$= \langle \varphi_j | \cdot \quad | n \rangle \langle n | \cdot | \varphi_i \rangle$$

$$= \langle \varphi_j | C_n | \varphi_i \rangle$$

$$= \langle \varphi_j | N_0 \delta_{i-j} | \varphi_i \rangle$$

$$= N_0 \langle \varphi_j | \varphi_i \rangle$$

$$= N_0 \delta_{ij}$$

So we send a vector $\bar{s}_i$ corresponding to m_i

add noise $\bar{n}$

and receive $\bar{r} = \bar{s}_i + \bar{n}$

= we find the vector components of $\bar{r}$

with a matched filter or correllation

receiver.

and pick the message m_i whose vector $\bar{s}$

is closest

Sensitivity:

note we have chosen

$$\langle \varphi_i(t), \varphi_j(t) \rangle = 1 \text{ Joule} \cdot \delta_{ij}$$

And we have found:

$$\langle n_i, n_j \rangle = N_0 \cdot \delta_{ij} = KTF \cdot \delta_{ij}$$

A signal $s(t)$ with energy $\mathcal{E}_{bit} = \mathcal{E}_b$

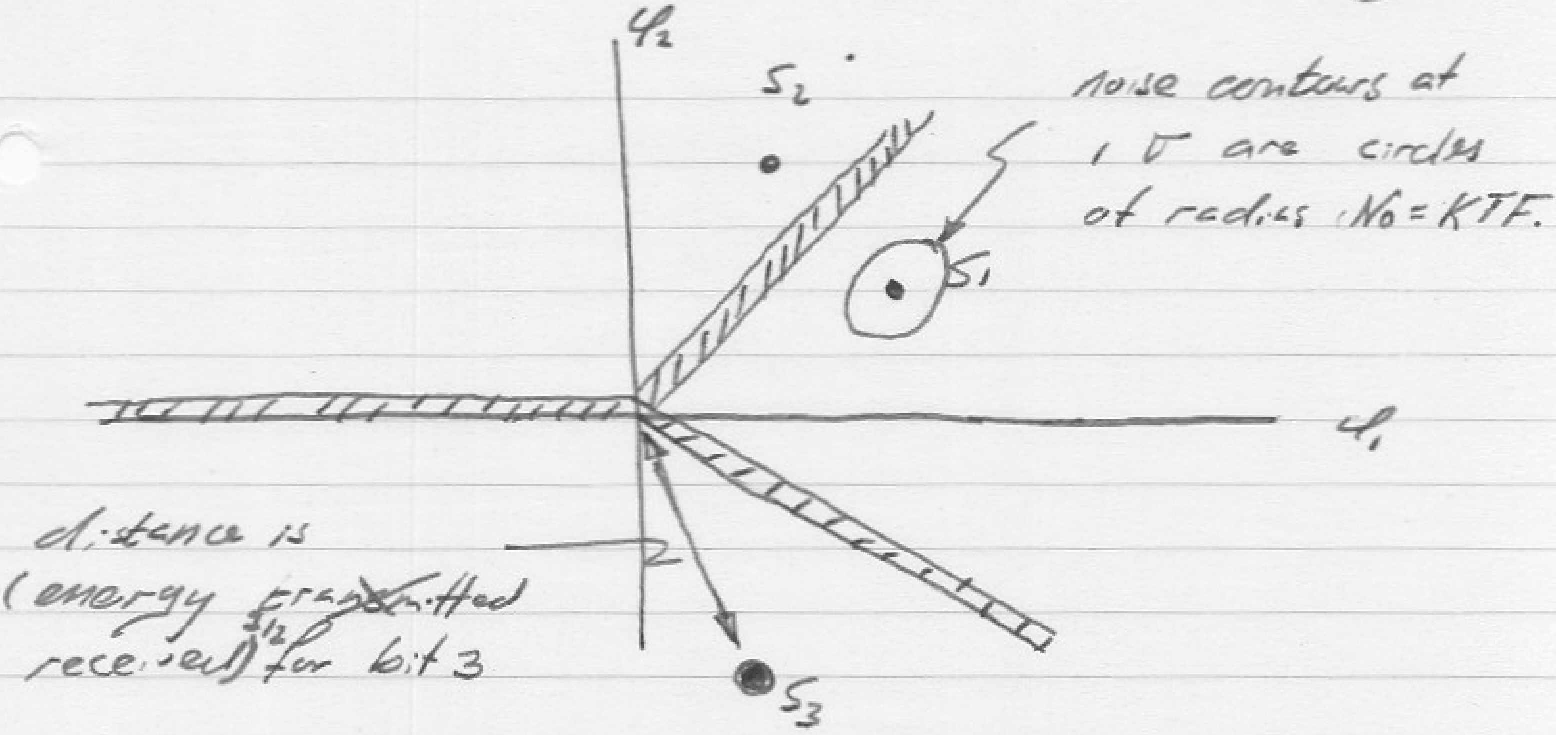
$$\mathcal{E}_{bit} = \int_{-\infty}^{+\infty} s(t)\, s^*(t)\, dt$$

has a vector $\bar{s}_i = s_{i1}\varphi_1 + s_{i2}\varphi_2 + \cdots s_{in}\varphi_n$.

whose length $\langle \bar{s}_i, \bar{s}_i \rangle = s_{i1} s_{i1}^* + \cdots + s_{in} s_{in}^*$

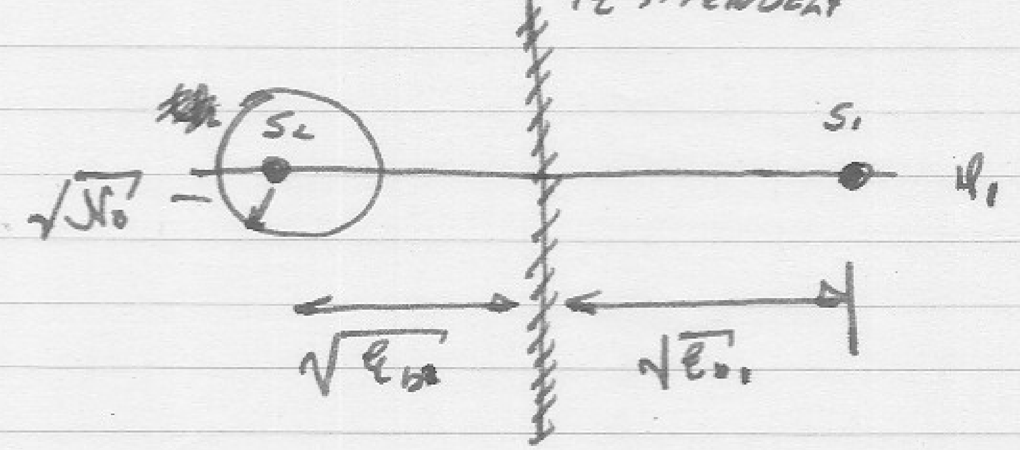
$$= \mathcal{E}_{bit,i}$$

$$\| \bar{s}_i^* \|^2 = \mathcal{E}_{bit\,i}$$

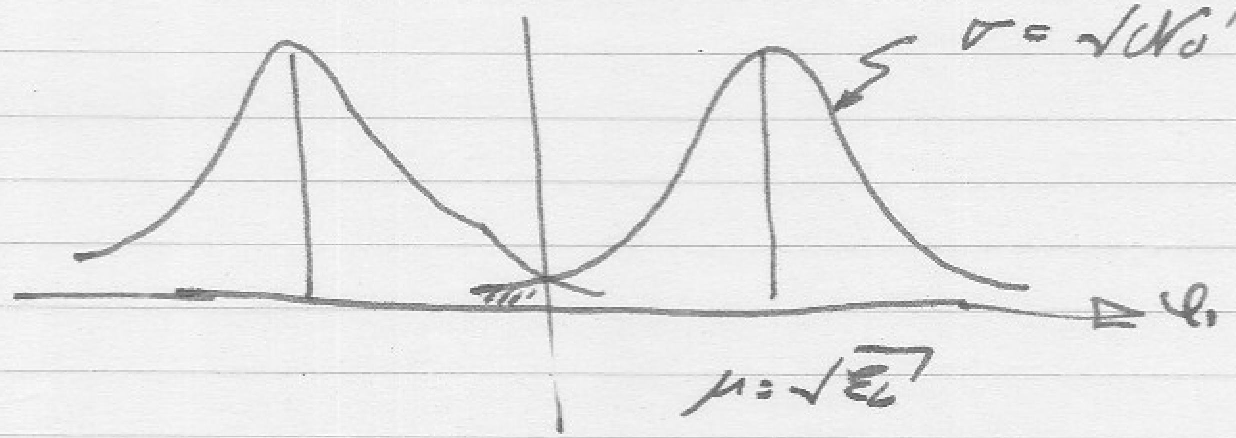


So now we can calculate sensitivities...

Binary Antipodal Modulation



Decision boundary is at the origin.

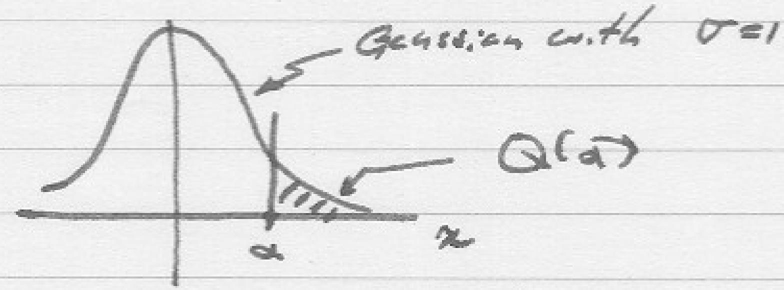


$P[\text{Error}] = P[\mathcal{E}]$

$= P[\text{ gaussian is } > \mu/\sigma \text{ standard deviations}]$

$$P[\mathcal{E}] = Q\left(\sqrt{E_b/N_0}\right)$$

where
$$Q(\alpha) \triangleq \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$



recall that

$$\frac{1}{\sqrt{2\pi}\,\alpha} e^{-\alpha^2/2}\left(1 - \frac{1}{\alpha^2}\right) < Q(\alpha) < \frac{1}{\sqrt{2\pi}\,\alpha} \cdot e^{-\alpha^2/2}$$

<u>specifically:</u> $Q(\alpha = 6) = 1.0 \cdot 10^{-9}$
probability of error.

so at 10^{-9} Ber, $\sqrt{\dfrac{E_b}{N_0}} = Q = 6$

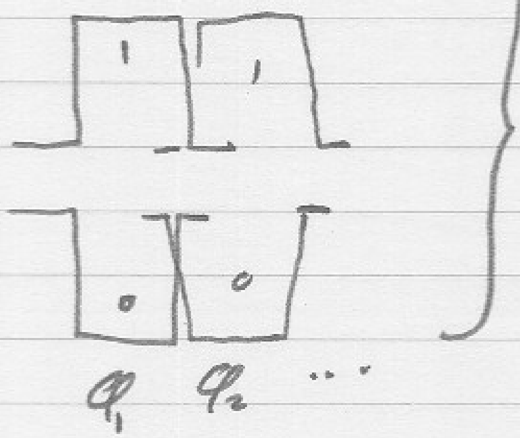
$$\boxed{E_{bit} = N_0 \cdot Q^2} \boxed{= KTF \cdot Q^2}$$

If we have a data rate $\underline{B}$ bits/sec

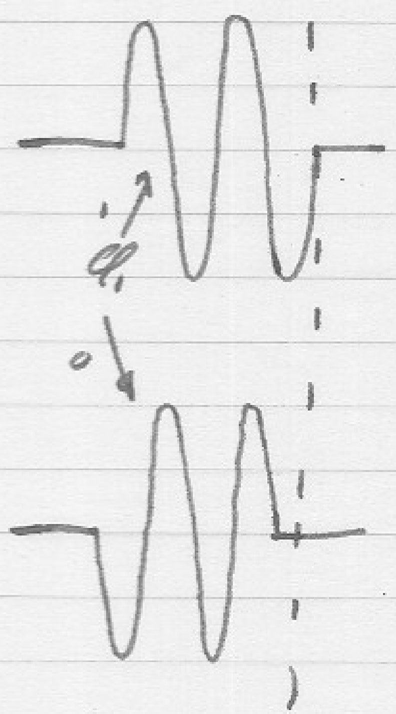
~~any~~ Then

$$\boxed{P_{rec} = N_0 B Q^2 = kTFB \cdot Q^2}$$

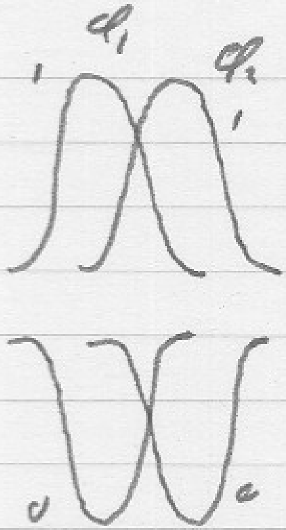
Note Emphatically that the signals & basis functions didn't matter as long as the bits are orthogonal & the noise white:



ok



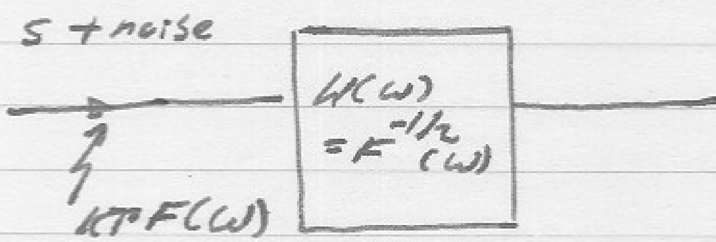
φ_2 is next time step.



not orthogonal

not ok:

If noise is not white, pass through whitening filter



← white noise
← signals no longer orthogonal.

we will skim over this in a minute.

Example Voyager at Saturn.

Given that Saturn is $\sim 10^{11}$ meters away.

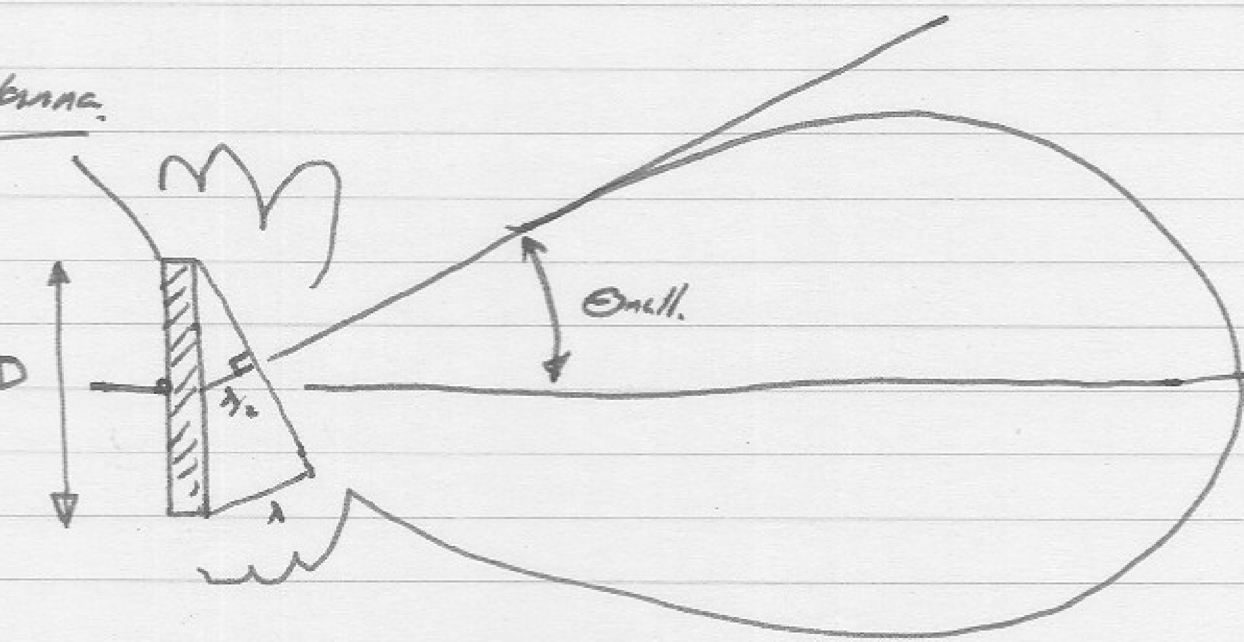
* Transmitter uses 1-m-diameter antenna
* Receiver uses 100-m-diameter antenna.

1 cm (30 GHz) signal.

Transmitter sends 1 watt signal, BAM format.

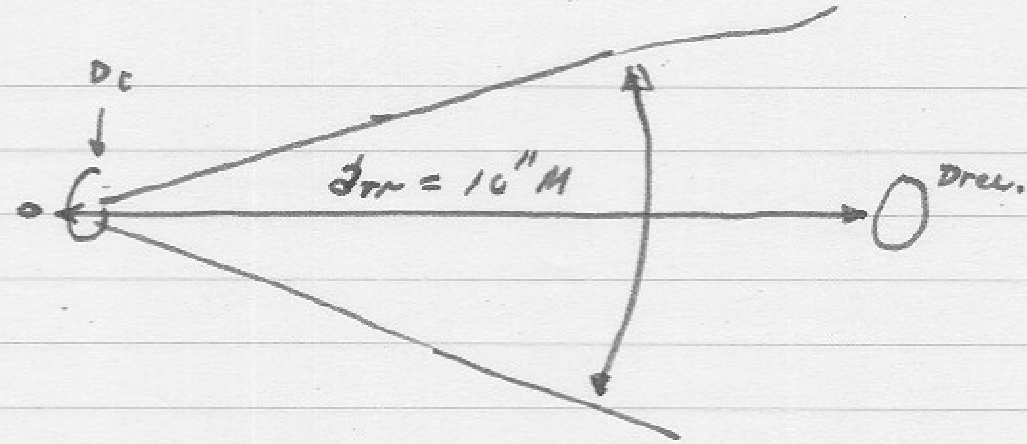
Receiver has 100 K effective temperature.

antenna



$$\Theta_{null} = \frac{\lambda/2}{D/2} = \frac{\lambda}{D}$$

antenna radiates into solid angle $4\left(\frac{\lambda}{D}\right)^2$



Beam diameter at rcvr: $\left(\frac{\lambda}{D_T}\right) \cdot d_{T-r}$

receiver diameter D_R

Power received $P_R = P_T \cdot D_R^2 \cdot \left(\frac{D_T}{\lambda}\right)^2 \frac{1}{d_{Tr}^2}$

$$\boxed{P_{Rec} = P_{Transmitter} \cdot \left(\frac{D_R D_T}{\lambda D_{TR}}\right)^2}$$

to a constant.

The accurate derivation gives the Friis transmission Formula:

$$\boxed{\frac{P_{rec}}{P_{trans}} = \frac{A_{trans} \cdot A_{rec}}{\lambda^2 \cdot D_{TR}^2}}$$

$A = \pi/4 \cdot D^2$

$$\frac{P_{rec}}{P_{trans}} = \left(\frac{\pi}{4}\right)^2 \frac{(1\,m \cdot 100\,m)^2}{(0.01\,m)(10^{11})^2}$$

$$\frac{P_{rec}}{P_{trans}} = 6.2 \cdot 10^{-15}$$

$$P_{rec} = 6.2 \cdot 10^{-15} \cdot 1\,W = \underline{\underline{6.2\ fW}}$$

For BAM:

$$P_{rec} = kTFB \cdot Q^2 \qquad Q = 6 \ @ \ 10^{-9} \ BER.$$

$$B = \frac{P_{rec}}{kTFQ^2} = \frac{P_{rec}}{kT_{eq}Q^2}$$

$$= \frac{6.2 \cdot 10^{-15} W}{1.38(10^{-23}) \ J/K \cdot 100K \cdot 36}$$

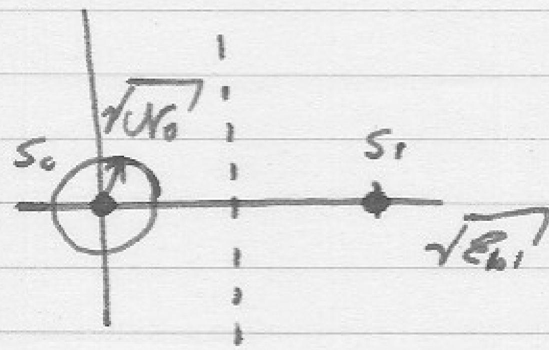
$$\boxed{B = 1.25(10^5) \ \text{Bits/sec.}}$$

We can transmit & receive slightly over 100 kB/sec.

☺

other Signal sets

on-off keying (OOK)



$E_{bit} = 2 E_{b1}$

$$\frac{\sqrt{E_{b1}}}{2\sqrt{N_0}} = Q$$

$$\frac{E_{b1}}{N_0} = 4Q \quad \Rightarrow \quad E_{bit} = 2 Q N_0 = 2 kTF\, Q$$

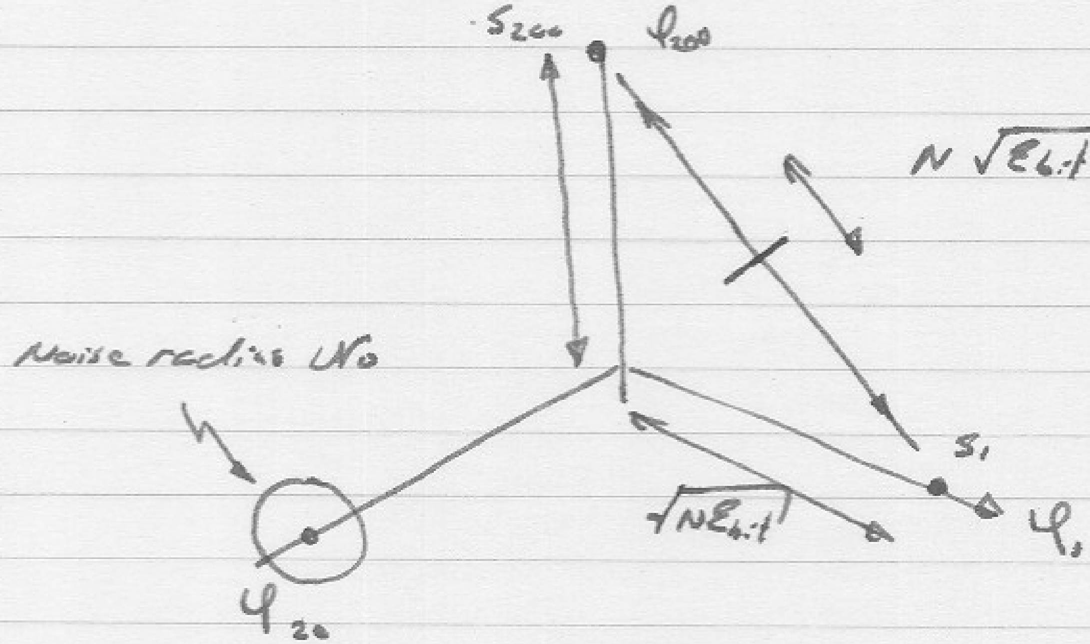
$$\boxed{P_{rec} = 2 \cdot kTF\, B\, Q}$$

This is 2:1 poorer than BAM, but sometimes (incoherent fiber optics) negative signals aren't possible...

Block orthogonal Coding – a though experiment.

we have N bits to send → 2^N combinations

→ chose 2^N basis functions φ_i, and send one of them with energy $N E_{bit}$



Error rate decreases with N bit

→ $E_{bit} \to \mathcal{N}_0 \cdot 2 \ln 2$, close to shannon limit.

... but bandwidth has grown unboundedly,