• Notes Set 7: First Look at Noise in Circuits

- pre-introduction of transistor noise models.
- circuit noise analysis.
- Total input referred noise voltage, total input referred noise current
- Short circuit input noise voltage. Open circuit input noise current,
- Noise figure, minimum noise figure and optimum source impedance.
- Friss Formula, available gain, noise measure, noise temperature.

ECE ______ Notes Set 7

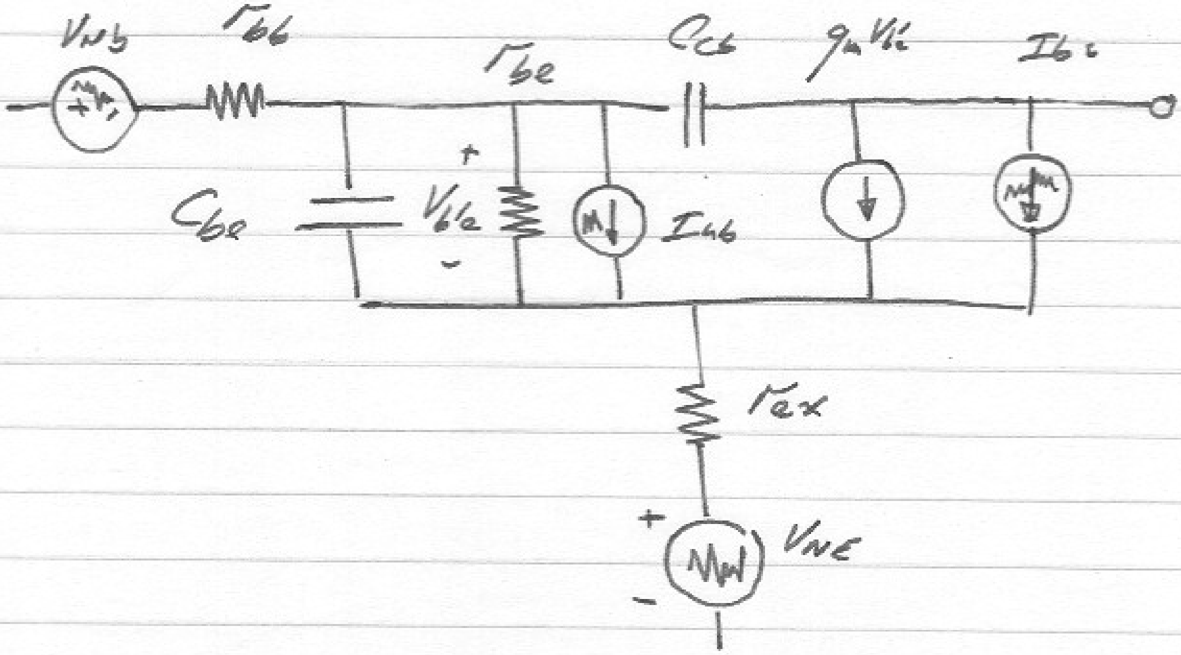
~~tt~~ Now do

First look at noise in circuits

we have not yet finished noise in devices, so why the diversion? Will want to talk about quantities like noise figure, etc, when we do talk about device noise.

So, will need to do initial treatment of noise in circuits now.

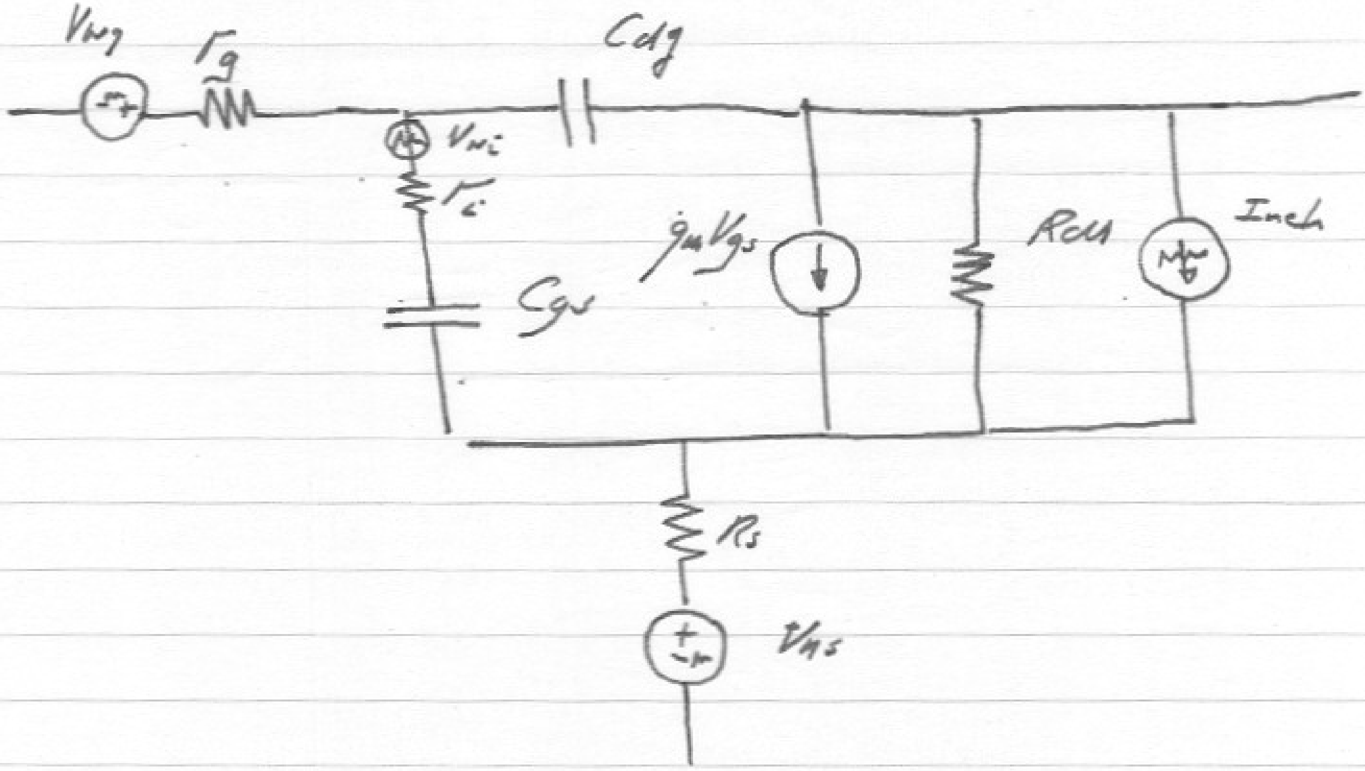
we will show later that bipolar transistors have noise models like so:



where V_{nb} & V_{NE} are the thermal noise generators of r_{bb} & r_{ex}

& I_{nb} & I_{nc} are shot noise (later) generators for the base & collector DC currents.

similarly, we will show that a FET can often be modelled like so:

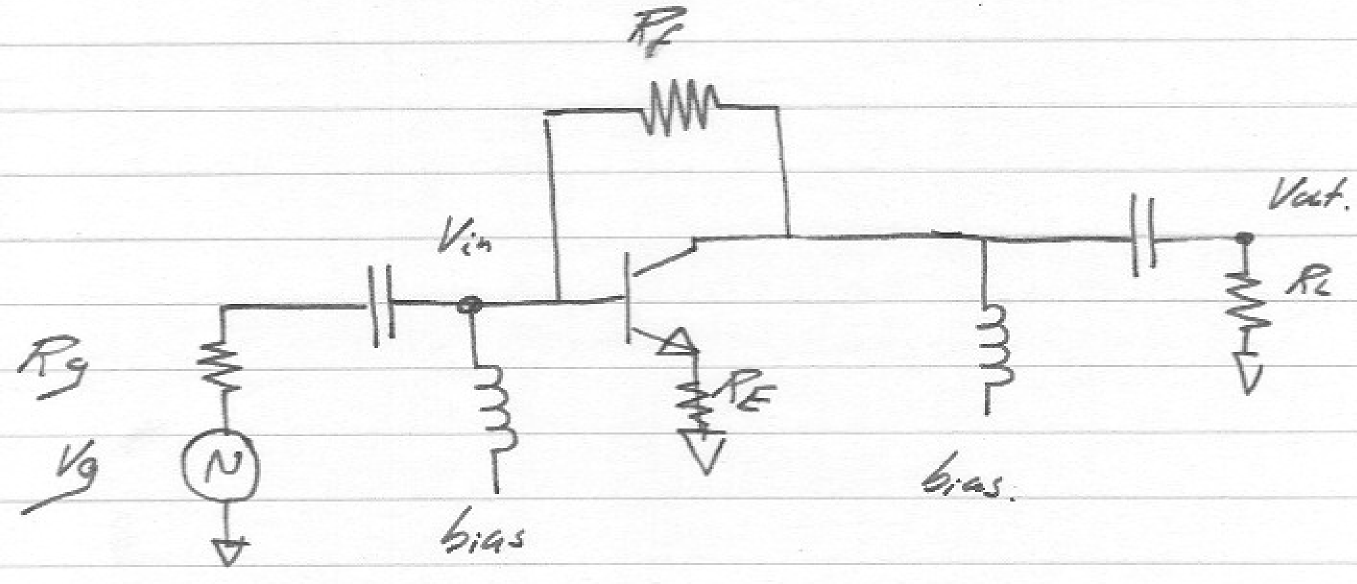


where V_{ng}, V_{ns}, are thermal noise generators for R_s & R_g.

I_{nch}, & V_{ni} are not strictly equilibrium noise sources & have perhaps a small correllation.

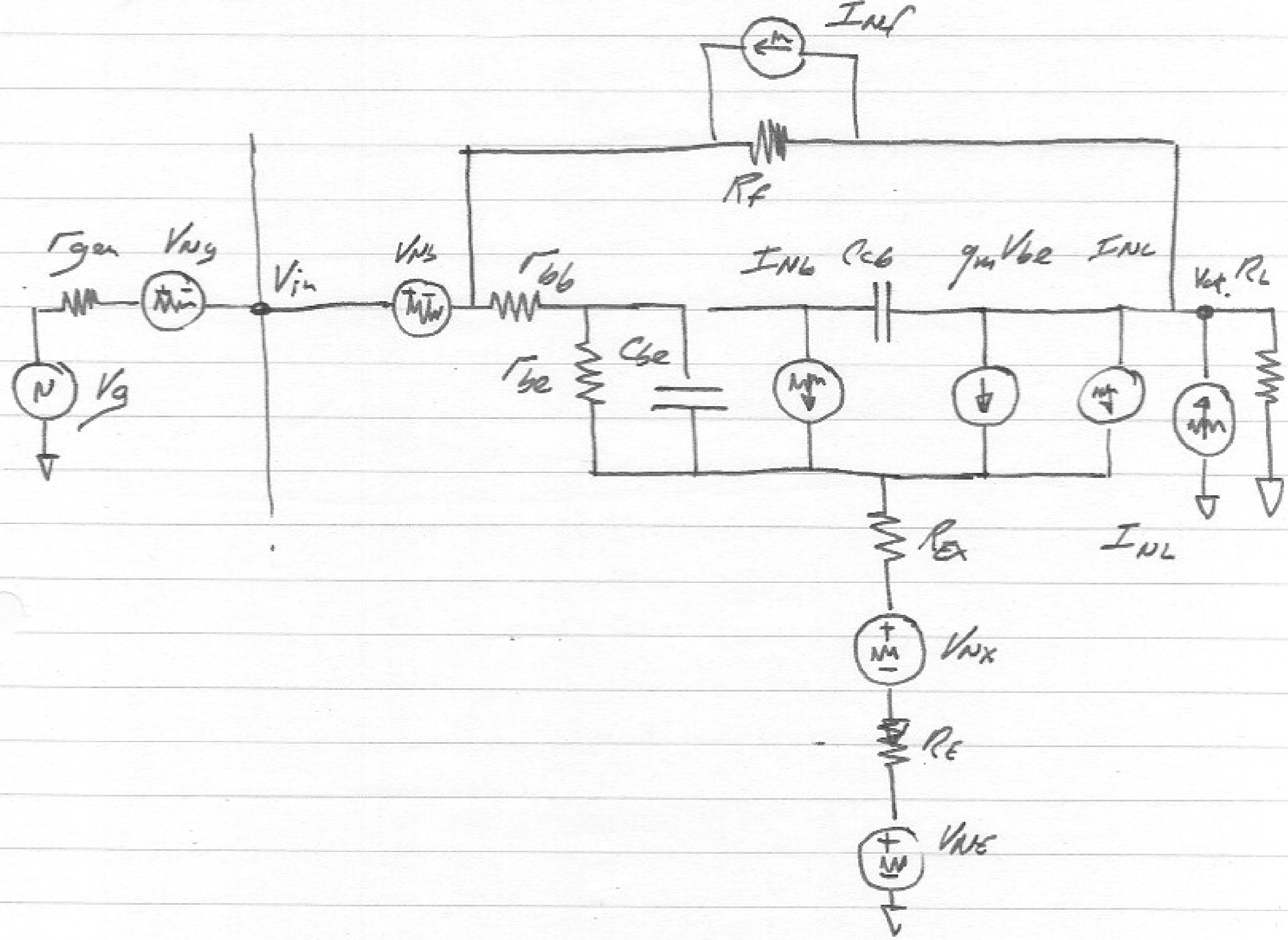
Again, we will <u>develop</u> these noise models soon. I need some device models now to support the circuit examples which follow.

<u>Introduction to noise analysis in IC circuits:</u>



The noise model of this circuit is as below:

- small-signal model shown -



V_{NX}	thermal	noise	of	R_{BX}
V_{NE}	"	"	"	R_E
V_{Ng}	"	"	"	R_{gen}
I_{Nf}	"	"	"	R_f
I_{NL}	"	"	"	~~I_{NL}~~ R_L
V_{Nb}	"	"	"	R_{bb}.

I_{Nb}	shot	noise	of	base	current
I_{NC}	"	"	"	collector	"

---

2 Noise sources are of particular significance:

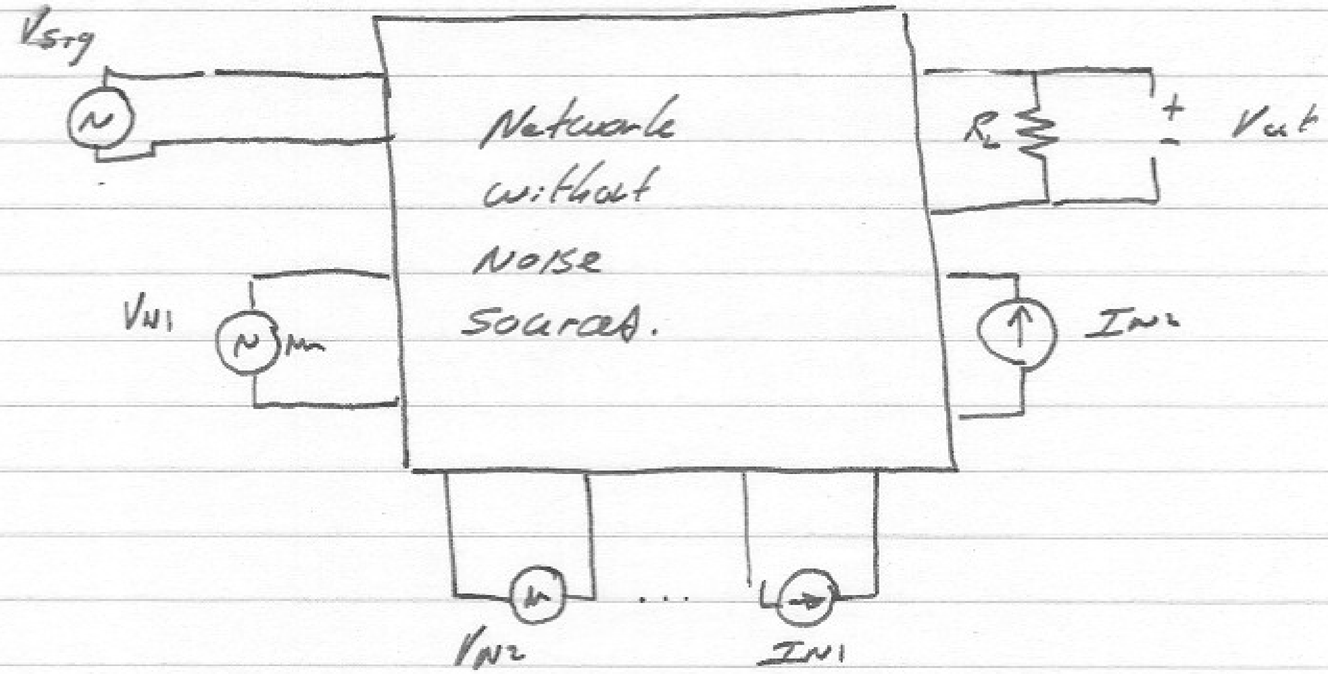
V_{Ng}: belongs to the generator.

~~#~~ I_{NL}: belongs to the load

Again, It is not our purpose here today to learn how to solve this problem ~~effic~~ efficiently, or to make performace optimizations.

Instead, right now we are setting up some circuit noise definitions which we will need shotly...

General Model:



Here,

$$V_{out} = A V_{sig} + A_1 V_{N1} + A_2 V_{N2} + \ldots$$

$$+ Z_1 I_{N1} + Z_2 I_{N2} + \ldots$$

this relationship is found by arduous circuit analysis. The first term is output signal, the remainder is output noise.

so, the ouput noise voltage is

$$V_{N,out} = A_1 V_{N1} + A_2 V_{N2} + \cdots$$
$$+ Z_1 I_{N1} + Z_2 I_{N2} + \cdots$$

now form

$$\frac{\partial}{\partial f} \langle V_{Nout} V_{Nout}^{*} \rangle$$

by multiplying the above by its complex conjugate

So

$$\frac{\partial}{\partial f} \langle V_{Nout} V_{Nout}^* \rangle$$

$$\|A_1\|^2$$

$$= \frac{\partial}{\partial f} \langle V_{N1} V_{N1}^* \rangle \cdot A_1 A_1^*$$

$$+ \frac{\partial}{\partial f} \langle V_{N2} V_{N2}^* \rangle \cdot A_2 A_2^*$$

$$+ \cdots$$

$$+ Z_1 Z_1^* \cdot \frac{\partial}{\partial f} \langle I_{N1} I_{N1}^* \rangle$$

$$+ Z_2 Z_2^* \frac{\partial}{\partial f} \langle I_{N2} I_{N2}^* \rangle$$

$$+ \cdots$$

$$\left\{ \begin{array}{l} + A_1 Z_1^* \frac{\partial}{\partial f} \langle V_{N1} I_{N1}^* \rangle + Z_1 A_1^* \frac{\partial}{\partial f} \langle I_{N1} V_{N1}^* \rangle \\ + \cdots \end{array} \right.$$

these (N^2-N) cross terms are zero if the noise sources are uncorrelated.

☆ So we have (after substantial effort)

calculated $\frac{\partial}{\partial f} \langle V_{nout} V_{nout}^* \rangle$

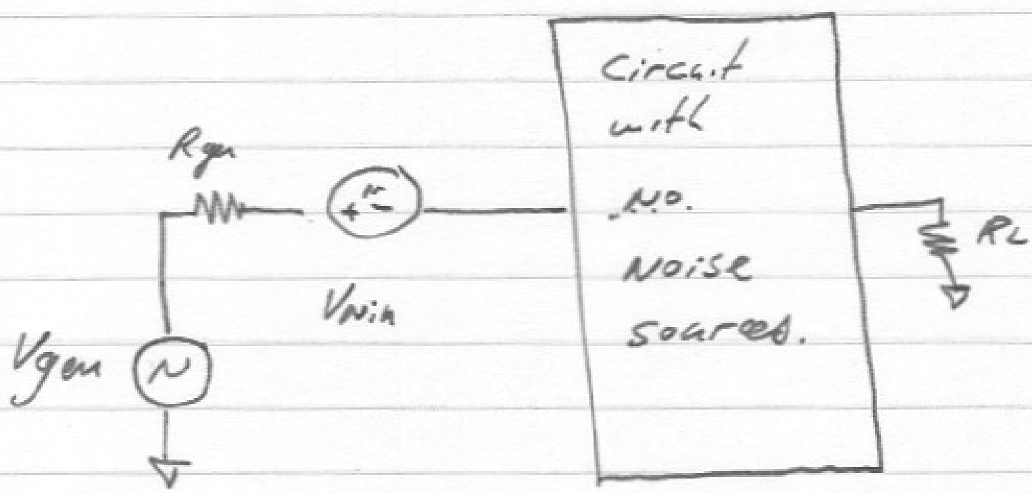
☆ We also have determined that

$$V_{out}\Big|_{signal} = A \underline{V}_{gen}.$$

So we can lump all the noise generators together into a single, fictitious, <u>input</u> referred <u>total</u> <u>noise voltage</u>

$$\frac{\partial}{\partial f} \langle V_{N,in} \cdot V_{Nin}^* \rangle \triangleq \frac{1}{AA^*} \frac{\partial}{\partial f} \langle V_{Nout} , V_{Nout}^* \rangle$$

The picture is as follows:



Vnin lumps together

all amplifier noise terms

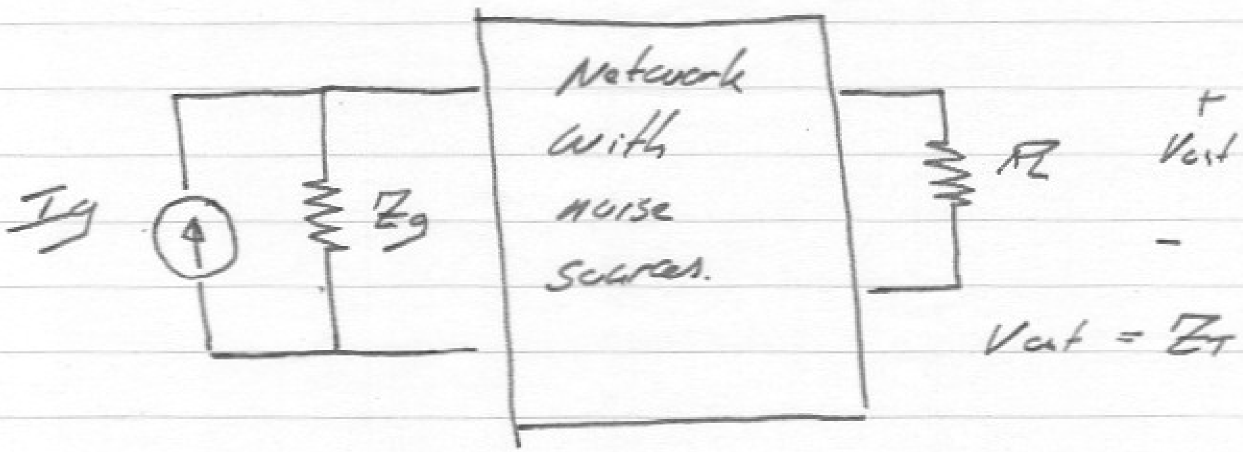
the generator noise

the noise of the load.

Further, Vnin as given depends explicitly on Rgen.

[This description may or may not be useful...]

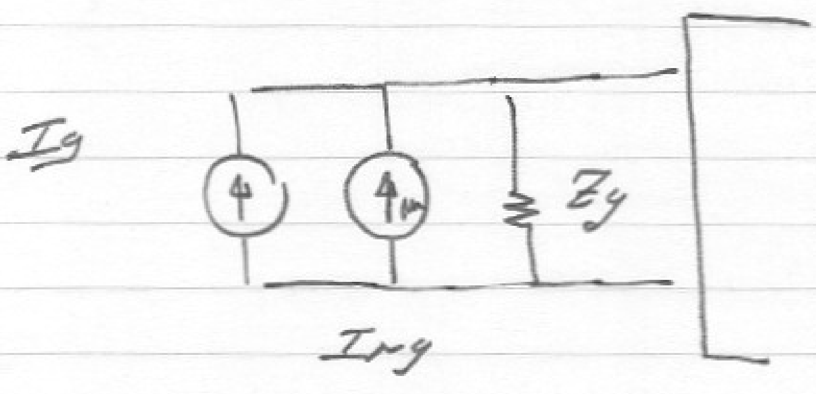
In a similar way, if the input signal source is a current source:



$$V_{out} = Z_T \cdot I_g$$

We can define an input referred noise current:

$$\frac{\partial}{\partial f} \langle I_{Nin} I_{Nin}^{*} \rangle = \frac{1}{Z_T Z_T^{*}} \frac{\partial}{\partial f} \langle V_{Nout} V_{Nout}^{*} \rangle$$



Again note that:

* $I_{N,in}$ defined above is the total input-referred noise current

* $I_{N,in}$ includes the noise of the generator, amplifier, and load.

* $I_{N,in}$ depends explicitly on the generator impedance.

$I_{N,in}$, the total input noise current will be useful when the generator is a current & when we have a known & specified generator impedance.

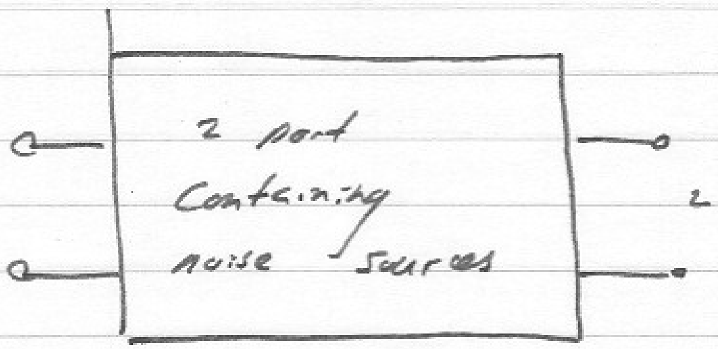
$V_{N,in}$: similar applicability, but when generator is a voltage...

We will also want descriptions of circuit noise which are useful over ~~the~~ range of possible generator impedances.

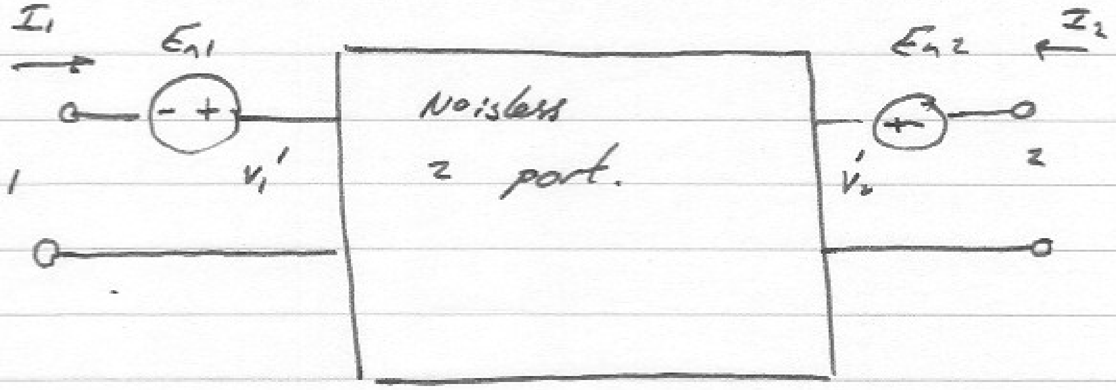
This model should therefore not include the generators' noise.

~~Cleaning up from last time:~~

From thevenins formula, a 2-port with many noise sources:



can be modelled as a sourceless network with open-circuit noise voltages at each port. These will be possibly correllated:



If the noiseless 2-port has:

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

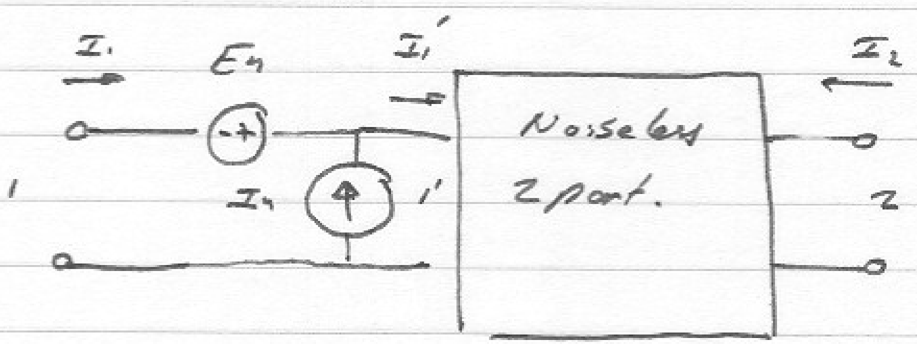
and

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} - \begin{bmatrix} E_{n1} \\ E_{n2} \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 - E_{n1}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 - E_{n2}$$

Can we instead model the 2-port as below:



This would give

$$V_1 = Z_{11}(I_1 + I_n) + Z_{12} I_2 - E_n$$

$$V_2 = Z_{21}(I_1 + I_n) + Z_{22} I_2$$

which is the same if we set

$$I_n = -\frac{E_{n2}}{Z_{21}}$$

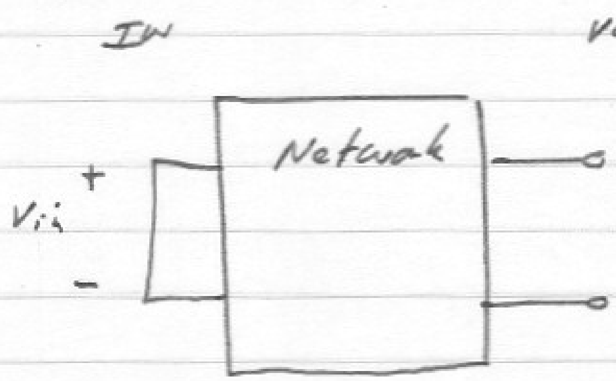
and

$$E_n = E_{n1} + \frac{Z_{11}}{Z_{21}} \cdot E_{n2}$$

∴ This proves that the (E_n, I_n) model is completely general for a 2-port.

To ~~do~~ create this model, look at a range of generator impedances:

<u>First short the input</u>



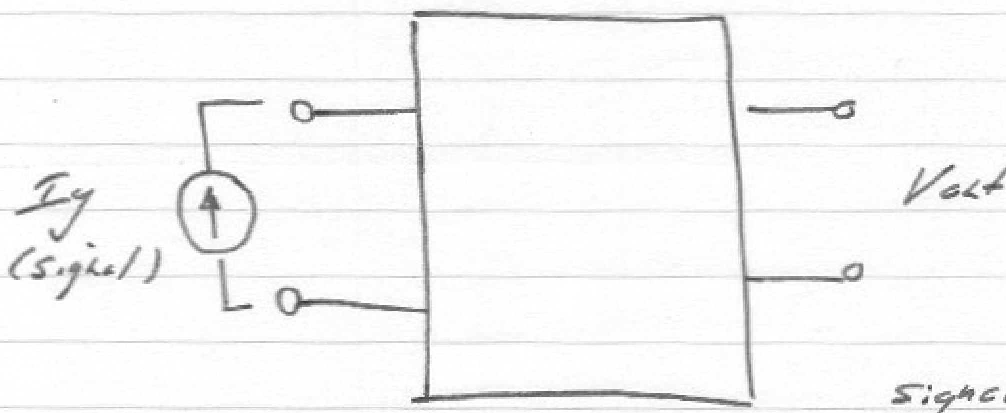
- and measure or calculate V_{Nout}

Divide by the voltage gain from input to output: $V_{out} = A_v \cdot V_{in}$ to get

$$\frac{\partial}{\partial f}\langle V_N V_N^* \rangle = \frac{1}{A_v A_v^*}\left[\frac{\partial}{\partial f}\langle V_{Nout} V_{Nout}^* \rangle \Big|_{\substack{\text{input}\\ \text{open}}} \right]$$

This is called the equivalent <u>short</u> <u>circuit</u> input <u>noise</u> <u>voltage</u>

Then make the input an open:



signal: $V_{out} = Z_T I_g$

... and measure or calculate $V_{N out}$.

Divide by the gain from input to output to get

$$\frac{\partial}{\partial f} \langle I_N I_N^* \rangle = \frac{1}{Z_T Z_T^*} \left[\frac{\partial}{\partial f} \langle V_{N out} V_{N out}^* \rangle \Big/_{\text{input shorted.}} \right]$$

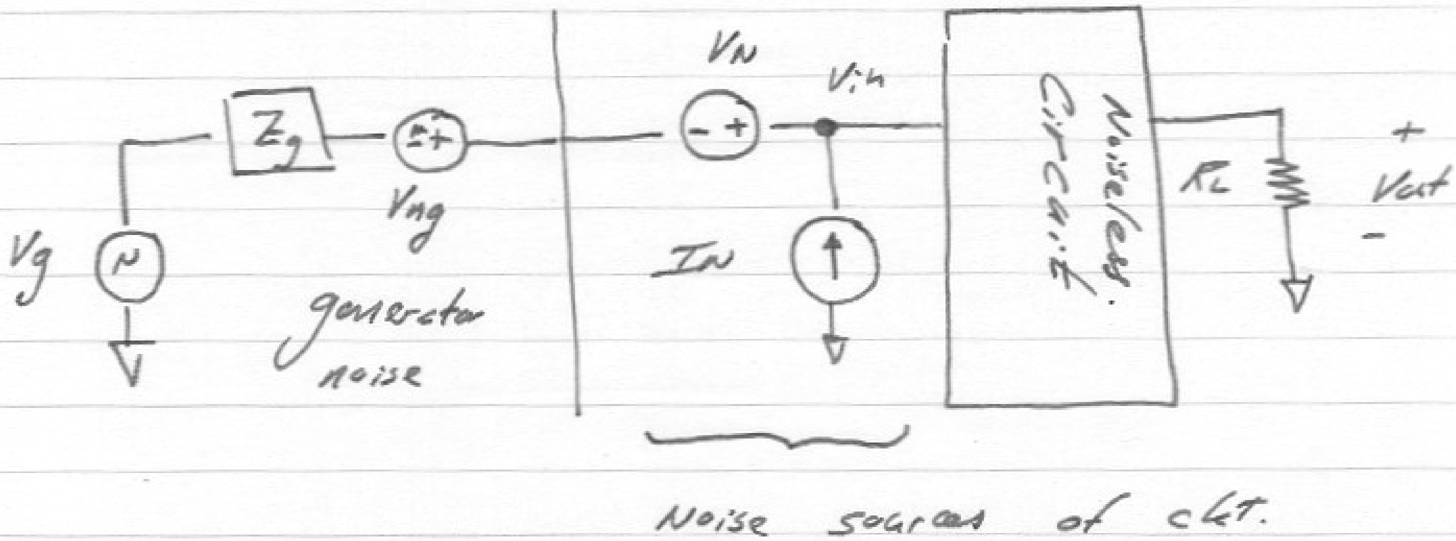
This is called the equivalent open circuit input noise current.

We will also find that these two quantities (noise generators) are correllated.

With the tools I have given you thus far, finding this correllation would require some inefficient method, such as applying a 1Ω generator impedance.

We will cover better methods later.

so, in general we can model the amplifier noise as below:



required parameters:

$$\frac{\partial}{\partial f}\langle V_N V_N^*\rangle,\ \frac{\partial}{\partial f}\langle I_N I_N^*\rangle,\ \frac{\partial}{\partial f}\langle V_N I_N^*\rangle$$

<u>We can now calculate the total input noise voltage</u>

$$V_{in} = \{ \underbrace{V_g}_{\uparrow \text{ Signal}} + \underbrace{\overbrace{V_{Ng}}^{\text{generator noise.}} + V_N + I_N Z_g}_{\substack{\text{Total input referred} \\ \text{Noise voltage} \\ V_{NT}}}] \cdot \frac{Z_{in}}{Z_{in} + Z_{gen.}}$$

So, the total input referred noise voltage is:

$$\frac{\partial}{\partial f} \langle V_{NT} V_{NT}^* \rangle = \frac{\partial}{\partial f} \langle V_N V_N^* \rangle$$

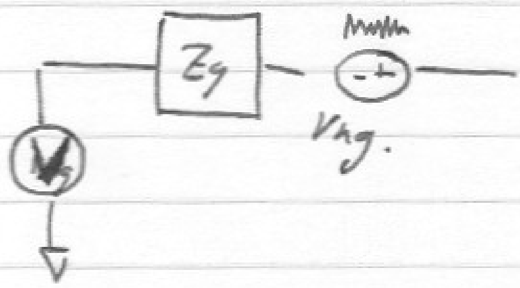
$$+ Z_g Z_g^* \frac{\partial}{\partial f} \langle I_N I_N^* \rangle$$

$$+ 2 \cdot \mathrm{Re} \left[\frac{\partial}{\partial f} \langle V_n I_n^* \rangle \cdot Z_g^* \right]$$

which shows explicitly the dependence upon Z_g.

Noise Figure

A measure of the change in signal/noise ratio incurred in using an amplifier:



Available signal power from generator is

$$P_{AV,S} = \frac{V_g V_g^*}{4 \cdot \mathrm{Re}\{Z_g\}}$$

Available noise power from generator is:

$$\frac{\partial}{\partial f} \langle P_{AV,N} \rangle = kT = \frac{\partial}{\partial f} \langle V_{ng} V_{ng}^* \rangle \cdot \frac{1}{4 \cdot \mathrm{Re}\{Z_g\}}$$

The signal / Noise ratio of the generator is:

$$\frac{P_{AV,S}}{\left[\frac{\partial}{\partial f}\langle P_{AV,N}\rangle\right]} = \frac{S}{N}\Big/_{gen}$$

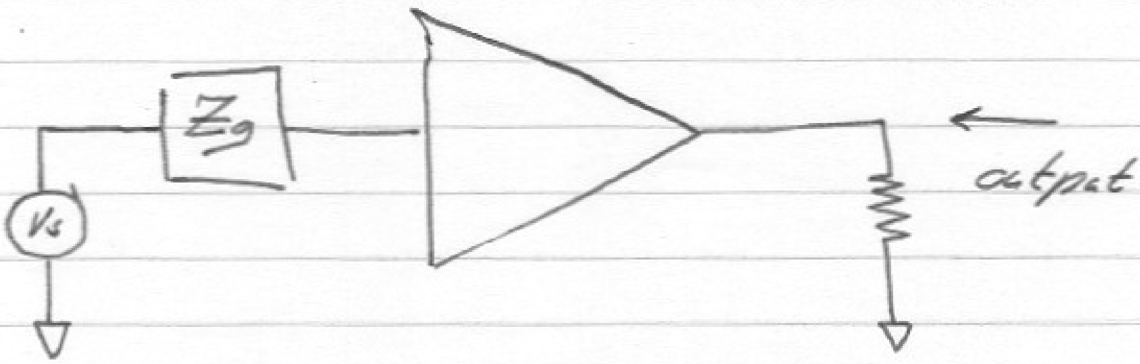
units are $\frac{W}{W/Hz} \rightarrow Hz$

!

[usually stated as dB in a 1Hz noise bandwidth

"S/N' = 30 dB (1Hz)"]

Connect the signal Generator to an amplifier:



there will be some output signal power $P_{out,S}$

& some output noise power spectral density: $\frac{\partial}{\partial f}\langle P_{out,N}\rangle$

The output S/N ratio is:

$$\left.\frac{S}{N}\right|_{out} = \frac{P_{out,S}}{\frac{\partial}{\partial f}\langle P_{out,N}\rangle} \quad Hz.$$

The Noise figure is:

$$F = \left(\frac{S}{N}\Big|_{generator} \right) \Big/ \left(\frac{S}{N}\Big|_{out} \right)$$

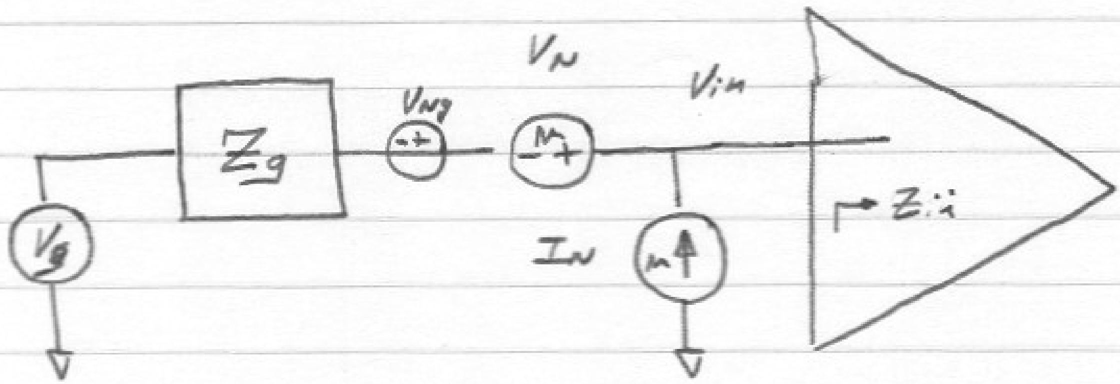
with the generator being a thermal noise source at temperature T.

Noise figure is usually set up so that the noise of the load is not included in the calculations.

Noise figure is a prime measure of the loss in signal quality due to the amplifier.

Lets calculate the noise figure:

start from $V_N \cdot I_N$ description:



$$V_{in} = \underbrace{\left(V_g + V_{Ng} + V_N + Z_g I_N\right)}_{V_{Th}} \cdot \frac{Z_{in}}{Z_{in} + Z_g}.$$

$$V_{TH} = \underbrace{V_g}_{\text{Signal}} + \underbrace{V_{Ng}}_{\text{gen noise}} + \underbrace{V_N + Z_g I_N}_{\text{amplifier noise.}}$$

Av. signal power $= V_s V_s^* \,/\, 4\,\mathrm{Re}\{Z_g\}$

Av. Noise power, generator $= kT = \left[4\,\mathrm{Re}\{Z_g\}\right]^{-1} \cdot \frac{d}{df} \langle V_{Ng} V_{Ng}^* \rangle$

Total Noise power:

$$\frac{d\langle P_{nT}\rangle}{df} = \left[\frac{\partial}{\partial f}\langle V_{ng}V_{ng}^{*}\rangle + \frac{\partial}{\partial f}\langle V_N V_N^{*}\rangle + \frac{\partial}{\partial f}\langle I_N I_N^{*}\rangle Z_g Z_g^{*} + 2\,Re\left[Z_g^{*}\frac{\partial}{\partial f}\langle V_N I_N^{*}\rangle\right]\right]$$

$$\times\ \left[4\,Re[Z_g]\right]^{-1}$$

Since I have input referred all quantities,

I need look at only the ratio of generator to total noise:

$$\boxed{F = 1 + \frac{\frac{\partial}{\partial f}\langle V_n V_n^{*}\rangle + Z_g Z_g^{*}\frac{\partial}{\partial f}\langle I_N I_N^{*}\rangle + 2\,Re\left[Z_g^{*}\frac{\partial}{\partial f}\langle V_n I_n^{*}\rangle\right]}{4\cdot K\cdot T\ Re[Z_g]}}$$

This Noise figure is clearly a function of

- the amplifier $\left[\frac{\partial}{\partial f} \langle V_n V_n^* \rangle , \frac{d}{df} \langle V_n I_n^* \rangle \right.$

  $\& \ \frac{d}{df} \langle I_n I_n^*$

- and the generator impedance Z_g.

... this noise figure will vary with generator impedance Z_g.

[There is a minimum noise figure & an optimum source impedance]

what are these?

If we write $Z_g = R_g + jX_g$, then

$$4kT(F-1) = \frac{1}{R_g}\frac{\partial}{\partial f}\langle V_n V_n^*\rangle + R_g \cdot \frac{\partial}{\partial f}\langle I_n I_n^*\rangle$$

$$+ \frac{X_g^2}{R_g}\cdot\frac{\partial}{\partial f}\langle I_n I_n^*\rangle$$

$$+ 2\cdot \mathrm{Re}\left[\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right]$$

$$+ 2\frac{X_g}{R_g}\cdot \mathrm{Im}\left[\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right]$$

Taking derivatives, WRT X_g, F is minimized

when

$$X_{g_{opt}} = -\mathrm{Im}\left[\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right] \Big/ \frac{\partial}{\partial f}\langle I_n I_n^*\rangle$$

<u>which leaves the noise figure as:</u>

$$4kT(F-1) = \frac{1}{R_g}\left[\frac{\partial}{\partial f}\langle V_n V_n^*\rangle - \frac{\left[\mathrm{Im}\cdot\frac{\partial}{\partial f}\langle VI^*\rangle\right]^2}{\frac{\partial}{\partial f}\langle II^*\rangle}\right]$$

$$+ R_g\left[\frac{\partial}{\partial f}\langle I_n I_n^*\rangle\right]$$

$$+ 2\,\mathrm{Re}\cdot\left[\frac{\partial}{\partial f}\langle VI^*\rangle\right]$$

Take derivatives WRT R_g to find:

$$\left(R_g\big|_{opt}\right)^2 = \frac{\left[\frac{\partial}{\partial f}\langle V_n V_n^*\rangle - \frac{\left[\mathrm{Im}\left[\frac{\partial}{\partial f}\langle VI^*\rangle\right]\right]^2}{\partial/\partial f\,\langle II^*\rangle}\right]}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle}$$

$$= \frac{\frac{\partial}{\partial f}\langle V_n V_n^*\rangle}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle} - \left[\frac{\mathrm{Im}\left(\frac{\partial}{\partial f}\langle VI^*\rangle\right)}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle}\right]^2$$

And the minimum noise figure is:

$$4kT(F-1) =$$

$$2 \cdot \sqrt{\frac{\partial}{\partial f}\langle v_n v_n^* \rangle \cdot \frac{\partial}{\partial f}\langle I_n I_n^* \rangle - \left[Im\left(\frac{\partial}{\partial f} \cdot \langle v I^* \rangle \right) \right]^2}$$

$$+ \; 2 \cdot Re \left[\frac{\partial}{\partial f} \langle v I^* \rangle \right]$$

~~[I am am not sure what I have accomplished here.]~~

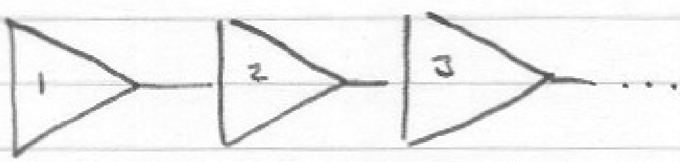
~~and I think I have a sign error on~~ $\cancel{2 \cdot Re \left[\frac{\partial}{\partial f} \langle v I^* \rangle \right]}$

Friss Formula for noise figure:

Available gain: power gain of an amplifier with the output matched to the load.

$$G_A = \frac{P_{AVA}}{P_{AVG}}.$$

Noise of a cascade of amplifiers:



$$F = 1 + F_1 + \frac{F_2 - 1}{G_{AV1}} + \frac{F_3 - 1}{G_{AV1} G_{AV2}}$$

where the noise figures of the individual amplifiers are computed given a generator impedance of the prior state. The available gain of the amplifier in question is also calculated using the generator impedance of the prior stage.

Noise Measure

One peculiarity of noise figure is that any device has poorer noise figure than a wire. Clearly F is not a good figure of merit, as minimizing F is accomplished by never amplifying.

Define $(1+M)$, the noise measure, as the noise figure of an ∞ ~~g~~ chain of amplifiers...

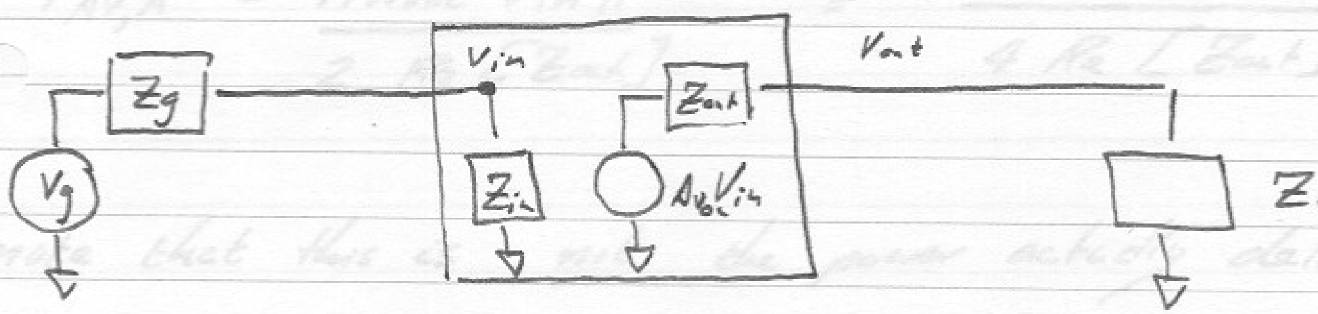
$$F\Big|_{\substack{\infty \\ \text{chain}}} = 1 + \frac{F-1}{1-1/G_{av}} = 1 + M$$

$F_\infty = 1 + M$ is a better measure of quality for low-gain devices.

Again, The Friss Noise formula uses the available gain:

$$G_A = \frac{\text{Power Available from amplifier}}{\text{Power Available from generator.}}$$

Lets derive G_A for a simple unilateral amplifier:



Avoc is the open-circuit voltage gain.

Power available from generator

$$P_{AV\_g} = \frac{V_g V_g^*}{4\,Re[Z_g]}$$