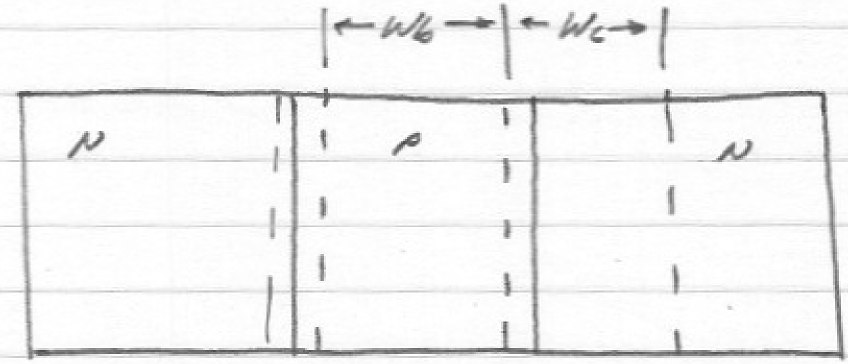
• Notes Set 9: Bipolar Transistor Noise

- BJT operation: DC and high frequency analyisis inclusive of diffusion and space charge effects
- Small-signal (noiseless) models. Addition of base diffusion noise to model.
- Full (correlated) base and collector diffusion noise.
- Simplified (normal, zero correlation) BJT model.

ECE ______ Notes set 9

Bipolar Transistor Noise

Lets first review BJT operation.



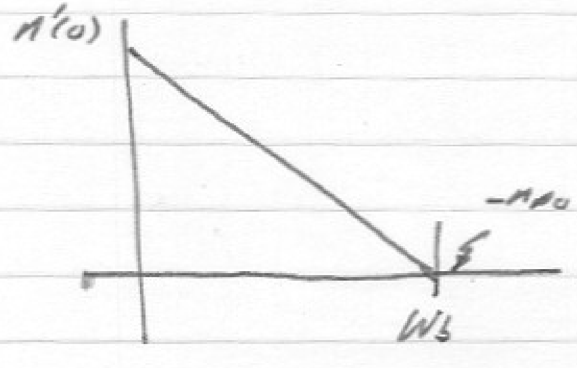
assuming $V_{be} \gg kT/q$ & $-V_{BC} \gg kT/q$

the minority carrier concentrations are, in the base,

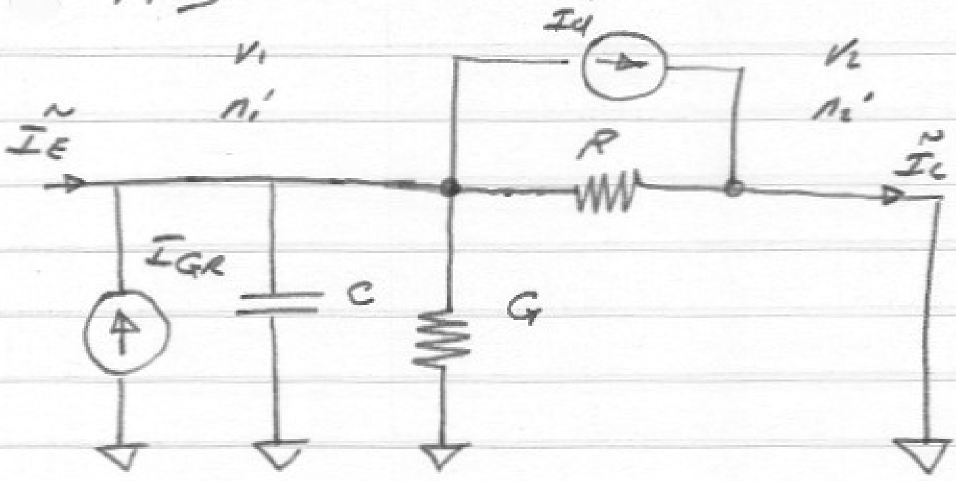
$$n'(0) = n_{po}\left(e^{qV_{be}/kT} - 1\right) \simeq n_{po}\, e^{qV_{be}/kT}$$

$$n'(w_b) = -n_{po} \cong 0$$

$(n' = n - n_{po})$



Apply the one-lump diffusion model:



$G = qA \frac{W_b}{2\tau}$ "V_1" $= N_1' = n_{p0}(e^{qV/kT} - 1) \cong n_{p0} e^{qV/kT}$

$C = qA \cdot \frac{W_b}{2}$ "V_2" $= N_2' = -n_{p0} \cong 0$

~~I_{GR}~~ $R = W_b / qAD_n$

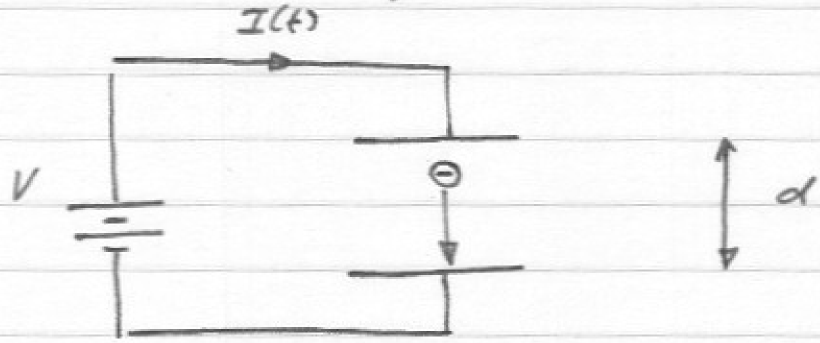
$I_{gr} \simeq 2q \left(q \frac{n_1' W_b A}{2\tau} \right)$

$I_d \cong 2q \left(q \frac{n_1 D_n}{W_b} \right)$

... all using the approximation $e^{qV/kT} >> 1$ ...

Note, that I have labelled the current leaving the physical base as $\tilde{I}_c$, not I_c. Why?

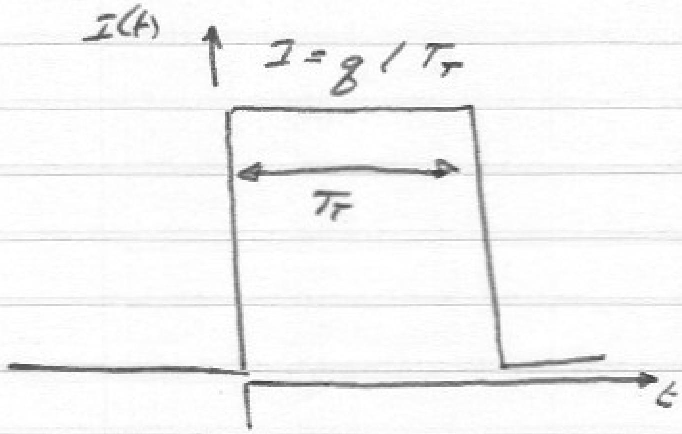
Because of space-charge transit time in the collector depletion layer.



with an electron moving between the plates of a capacitor of separation d, Energy balance ~~consideration~~ consideration between the electron electrostatic potential ($\int E \cdot dx$) and the circuit energy ($\int V \cdot I(t) dt$) results in

$$I(t) = q \cdot \frac{v}{d}$$ while the electron is between the plates.

so the current in the external circuit is:



$\tau_T = d/v$

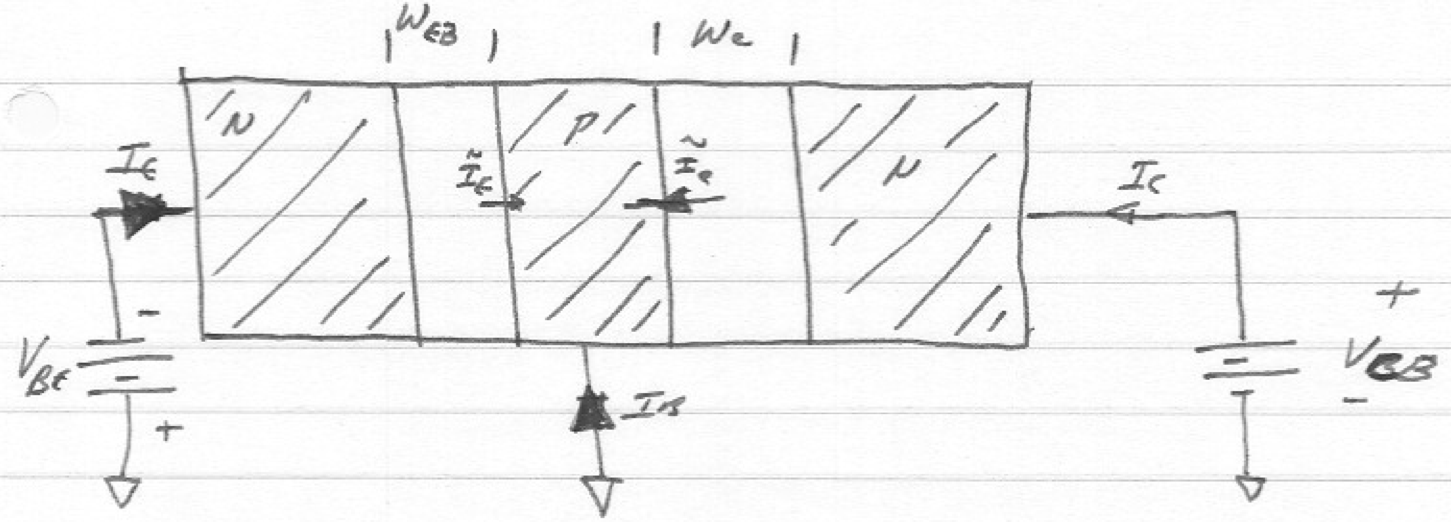
The width of this current pulse is d/v

The average delay of the current pulse is $d/2v$.

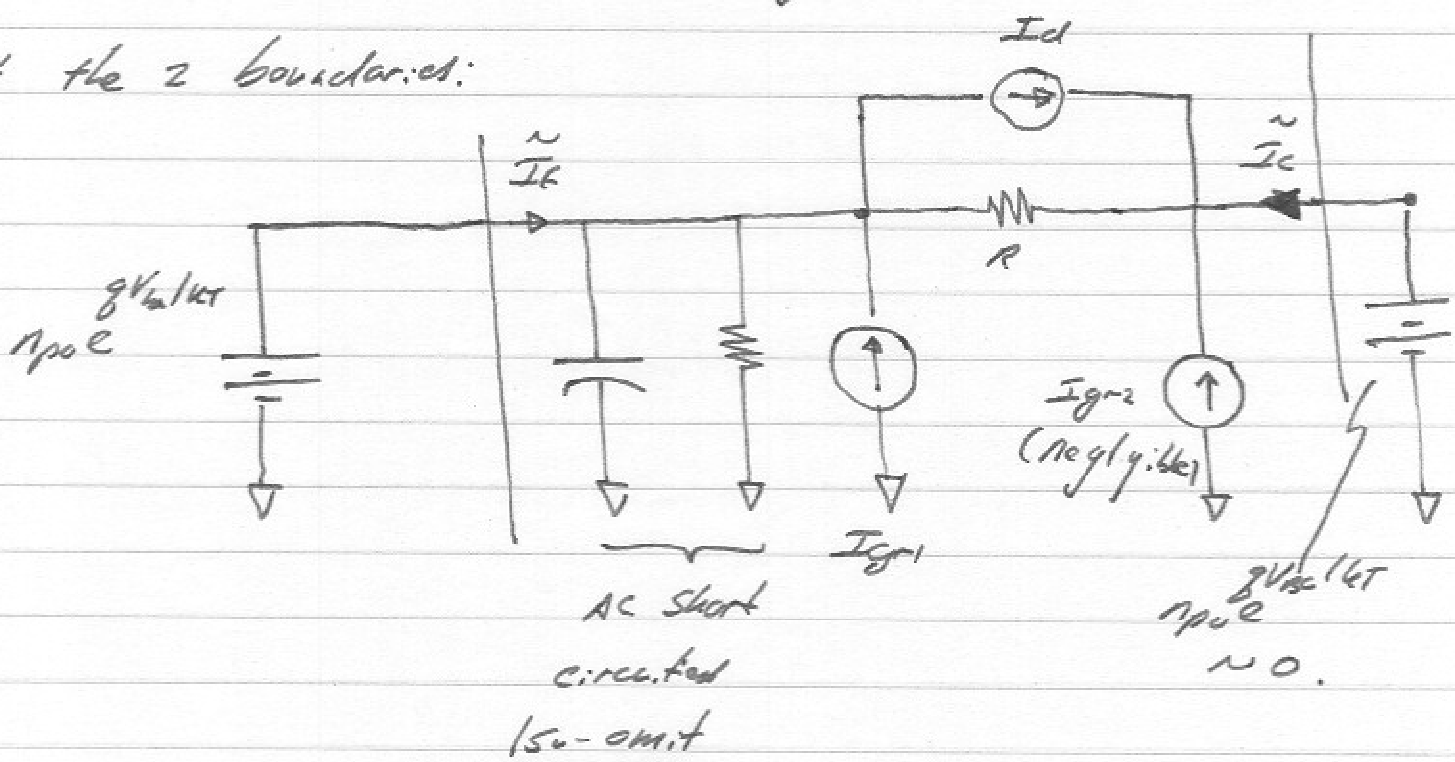
$\tau_c = \dfrac{d}{2v}$ is called the "collector transit time.

Now, i will caution you that some analyses, e.g. van der Ziel's use an base of arbitrary thickness and arbitrary doping profile. The theory for base & collector shot noise then becomes inordinately complex. Lets not do this. Instead use the thin base approximation.

Nevertheless, we will have to deal with the collector transit time. The approach I will take will be physical, not mathematical. The approach is nonstandard, but does reduce to Van-Der Ziel's expression.



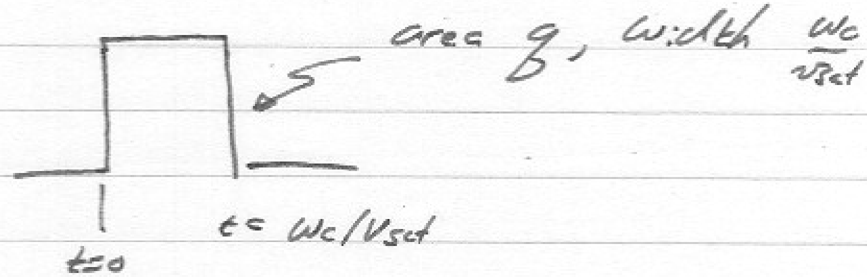
we are forcing V_{BE} & V_{CE} to be fixed dc. voltages. Therefore in our base-diffusion equivalent circuit model, we are forcing the electron concentrations at the 2 boundaries:



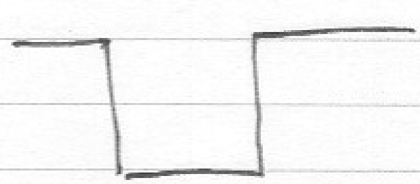
The key point is that $\tilde{I}_E$ & $\tilde{I}_C$ are the particle fluxes leaving the base at the emitter & collector edges, <u>not</u> the <u>total</u> <u>emitter</u> <u>&</u> <u>collector</u> <u>currents</u>. These differ by the space charge transit time effects.

an impulsive current $\overline{\tilde{I}_c}$ of 1 <u>electron</u>.

leads to a collector current



and a base current of opposite sign:

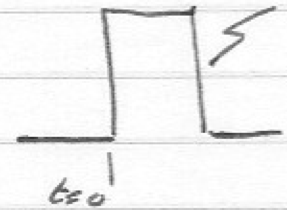


Similarly, an impulsive current

$\tilde{I}_E$ of 1 electron leads to an

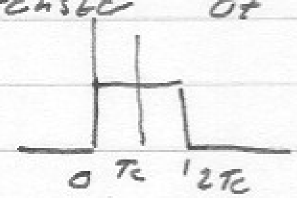
emitter current

area q, width W_{EB}/v_{sat}.



and a base current of opposite sign

The fourier transfer of a rectangle



Is $F(\omega) = e^{-j\omega T_c} \dfrac{\sin \omega T_c}{\omega T_c}$

and is approximated to first order in ωT_c by...

$$F(\omega) \cong 1 - j\omega T_c \cong \frac{1}{1 + j\omega T_c} \cong e^{-j\omega T_c}$$

The external currents are related to the particle fluxes by:

$$I_C = \frac{\tilde{I}_c}{1 + j\omega\tau_c} \qquad \tau_c = W_c/2v$$

$$I_E = \frac{\tilde{I}_E}{1 + j\omega\tau_E} \cong \tilde{I}_E \qquad \tau_E = W_e/2v$$

and

$$I_B = \tilde{I}_E(1 - j\omega\tau_E) + \tilde{I}_c(1 - j\omega\tau_c)$$

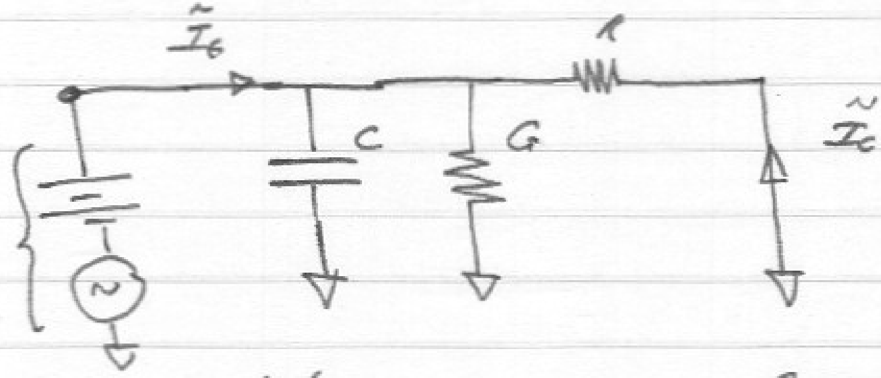
$$\cong \tilde{I}_E + \tilde{I}_c(1 - j\omega\tau_c)$$

We can now work the problem!

the approximation is made because

The emitter-base transit time is very small.

Small-signal characteristics:



$n_1 = n_{po} e^{qV_{be}/kT}$

$G = qAW_b/2\tau$

$C = qAW_b/2$

$R = \frac{W_b}{qAD_n}$.

Ignore G as it should be negligible and unnecessarily complicates the analysis...

$$\tilde{I}_E = \frac{qI_{EDC}}{kT}\left[1 + j\omega\tau_B\right] \cdot V_{be}$$

$\tau_b = W_b^2/2D_n$.

$$\tilde{I}_C = -q\frac{I_{EDC}}{kT} \cdot V_{be}.$$

note the sign.

These are the electron fluxes...

so the terminal currents are...

$$\tilde{I}_E \cong q\frac{I_{EDC}}{kT} \cdot V_{be}\left[1 + j\omega\tau_s\right]$$

$$\tilde{I}_C \cong -q\frac{I_{EDC}}{kT} V_{be}\left(\frac{1}{1+j\omega\tau_c}\right)$$

better approximation

$$= -q\frac{I_{Edc}}{kT} V_{be}\, e^{-j\omega T_c} \frac{\sin \omega T_c}{\omega T_c}$$

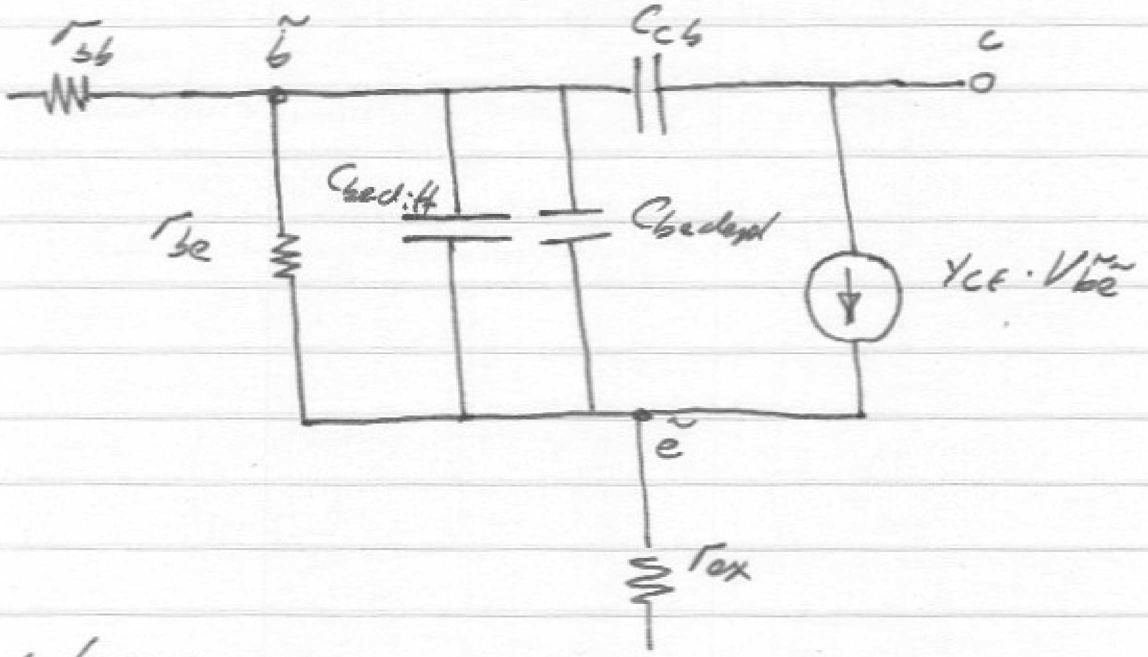
note the sign.

$$\tilde{I}_B = q\frac{I_{EDC}}{kT} \cdot V_{be}$$

$$\times\left[(1 + j\omega\tau_s) - (1 - j\omega T_c)\right]$$

$$= q\frac{I_{Edc}}{kT}(j\omega)(\tau_s + \tau_c) \cdot V_{be}.$$

We have just derived the Bipolar hybrid-π model:



where:

$$C_{be\,diff} = q \frac{I_{boc}}{kT} \cdot (\tau_b + \tau_c)$$

$$r_{be} = q \frac{I_B}{kT}$$ (I have added back the recombination term g)

and I have added the depletion capacitances $C_{be\,depl}$ & C_{cb} and the base & emitter contact resistances r_{bb} & r_{ex} ...

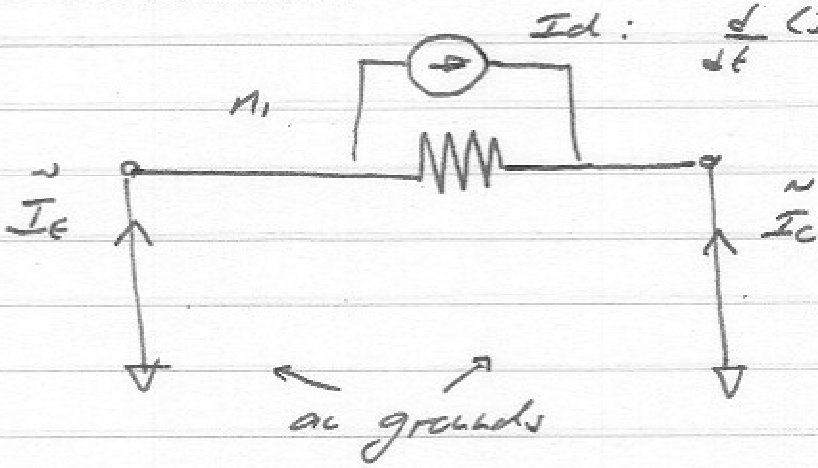
We can now derive a noise model:

Diffusion Noise:

$$I_d: \quad \frac{d}{dt}\langle I_d I_d^* \rangle = 2q \, \frac{q A D_n n_1}{W_b}$$

$$= 2q I_{DC \cdot E}$$

$$= 2q \, \overline{I_E}$$



$$\tilde{I}_C = -I_d \, , \qquad \tilde{I}_E = +I_d$$

$$I_C = -I_d \left(\frac{1}{1 + j\omega\tau_c} \right) = -I_d \,(1 - j\omega\tau_c)$$

$$I_B = I_D + (-I_d)(1 - j\omega\tau_c) = + j\omega\tau_c \cdot I_d$$

— ! all approximations to first order in $\omega\tau_c$! —

So this means that diffusion noise gives base & collector current fluctuations:

$$\frac{d}{df}\langle I_C I_C^* \rangle = 2q\,\overline{I_E}\left(\frac{1}{1+\omega^2 \tau_c^2}\right) \rightarrow 2q\,\overline{I_E}$$

$$\frac{d}{df}\langle I_B I_B^* \rangle = (\omega^2 \tau_c^2) \cdot 2q\,\overline{I_E}$$

&

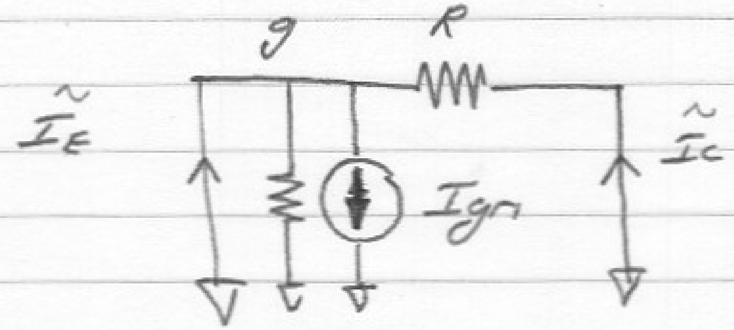
$$\frac{d}{df}\langle I_B I_C^* \rangle = j\omega\tau_c \cdot \left[-(1 + j\omega\tau_c)^*\right] 2q\,\overline{I_E}$$

$$= j\omega\tau_c * \left[-(1 + j\omega\tau_c)\right] 2q\,\overline{I_E}$$

$$= (-j\omega\tau_c - \omega^2\tau_c^2)\, 2q\,\overline{I_E}$$

$$= (-j\omega\tau_c) \cdot 2q\,\overline{I_E}$$

<u>recombination noise:</u>



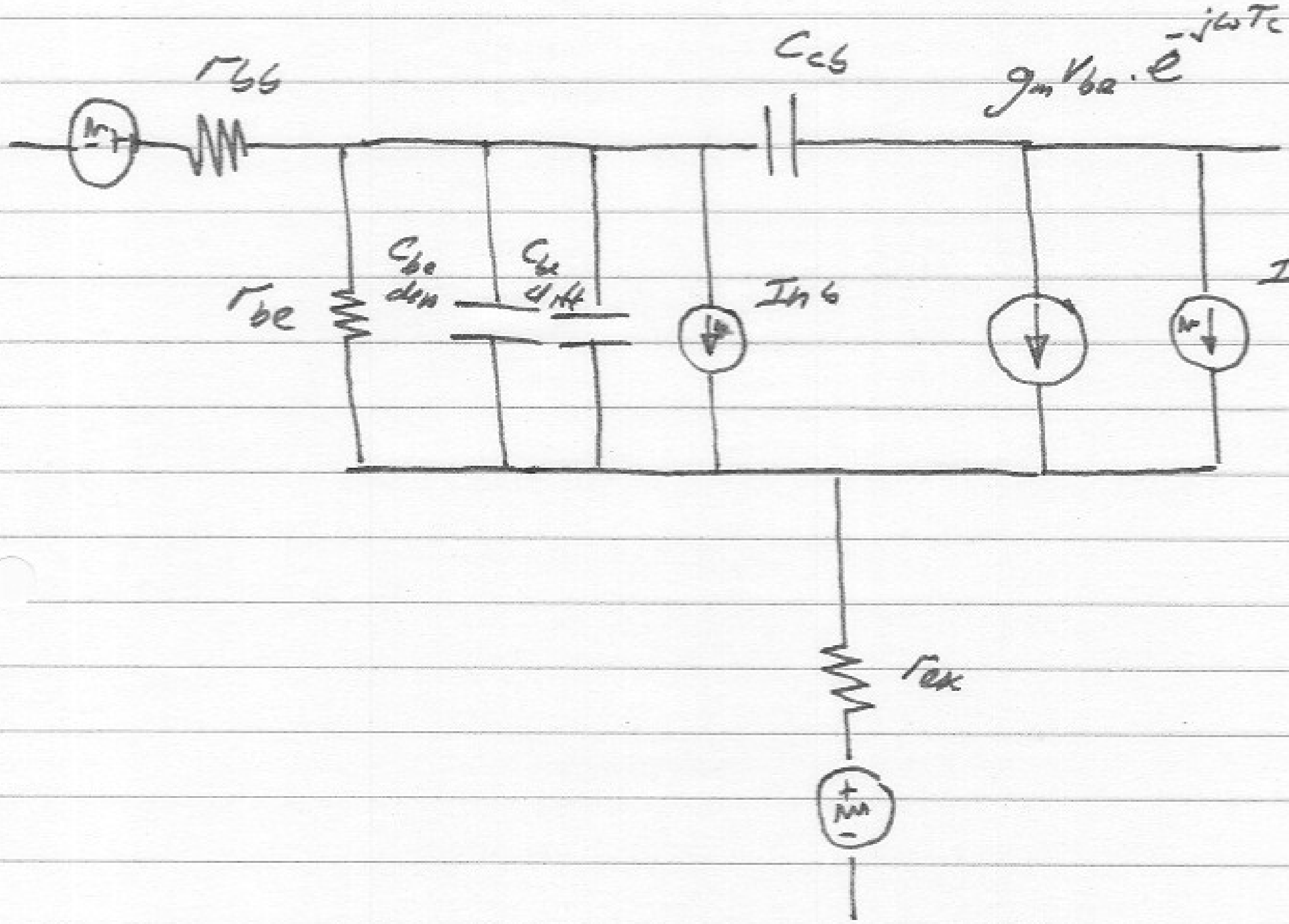
$$\frac{d}{df}\langle I_{gr} I_{gr}^{*}\rangle = 2q \; \frac{q W_b}{2\tau_n} \cdot n_1 \; = \; 2q \, \bar{I}_B \qquad !$$

Then $\tilde{I}_E = I_{gr}$, $\tilde{I}_c = 0$

So $I_b = \tilde{I}_b = I_g$

or $\frac{d}{df}\langle I_b I_b^{*}\rangle = 2qI_B$, $\frac{d}{df}\langle I_c I_c^{*}\rangle = 0$

So, we have now built our full ~~small~~ noise model of a bipolar:



elements are the usual small-signal parameters, r_{ex} & r_{bb} have, as indicated, thermal noise, and I_{nb} & I_{nc} are as given below:

$$\frac{d}{df}\langle I_{nc} I_{nc}^{*}\rangle = 2q\overline{I_E}$$

$$\frac{d}{df}\langle I_{nb} I_{nb}^{*}\rangle = 2q\overline{I_b} + (\omega^2 T_c^2)\cdot 2q I_E$$

$$\frac{d}{df}\langle I_{nb} I_{nc}^{*}\rangle = -(j\omega T_c)\cdot 2q\overline{I_b}$$

<u>now we have a full noise model!</u>

Cautions:

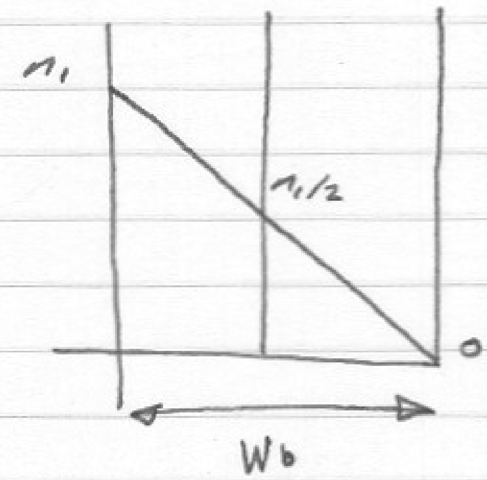
1) This model assumes

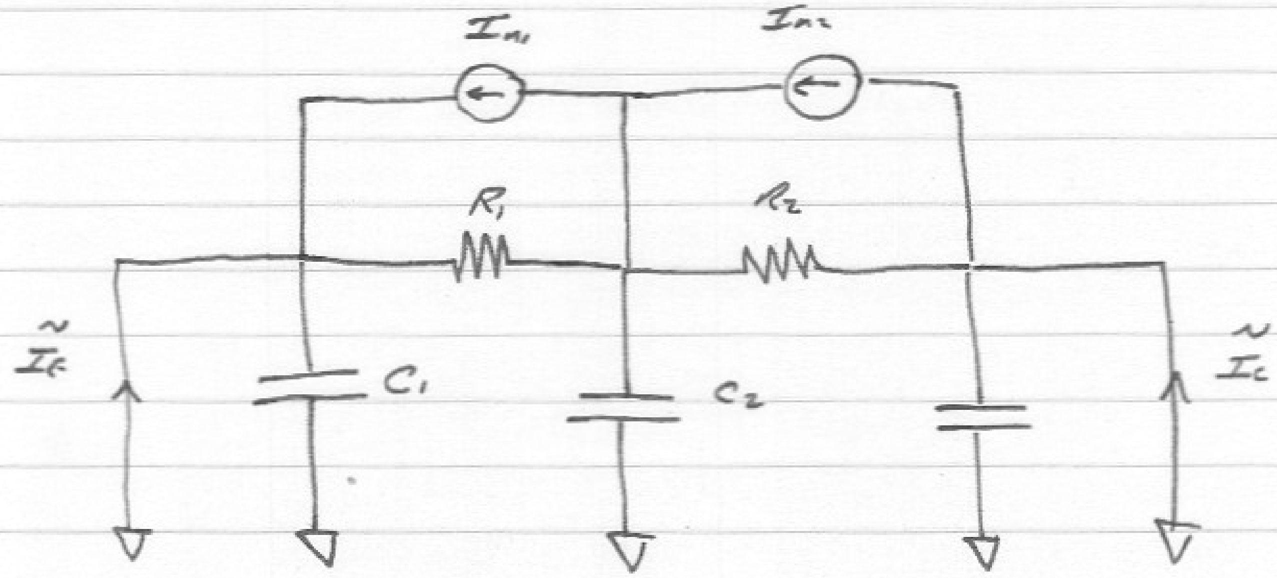
- narrow base with no built-in field
- first order in $\omega \tau_c$ & $\omega \tau_b$

2) "Shot noise" (diffusion noise) of reverse b→e hole injection can be added; answer is still $2q\bar{I}_b$

3) If we measure a dc base current $\bar{I}_b$, we *may* not get a base shot ~~not~~ noise $2q\bar{I}_b$, because $\bar{I}_b$ may be due to generation in the E-B depletion region, which need not give full $2qI$ noise...

There must be some degree of base noise coupling due to the base transit time:





$C_1 = q \cdot A \cdot W_b / 4 \qquad C_2 = q \cdot A \cdot W_b / 2$

$R_1 = R_2 = W_b / 2qAD_n$

I_{n1}: fluxes are $\delta \frac{D_n n_1/2}{W_b/2}$ & $\delta \frac{D_n n_1/2}{W_b/2}$

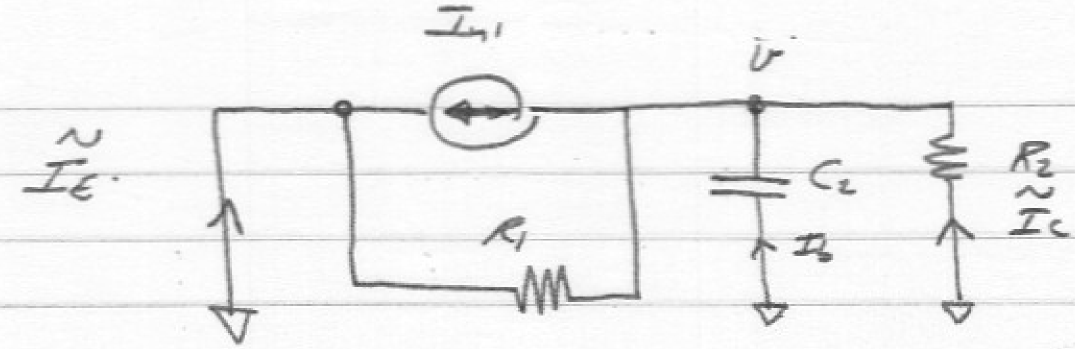
so $\frac{d}{df}\langle I_n I_{n1}^* \rangle = 2\delta \;\delta \frac{D_n n_1}{W_b/2} + 2\delta \;\delta \frac{D_n n_1/2}{W_b/2}$

$= 2\delta^* \;\delta \frac{D_n n_1}{W_b}\;[3]$

$= 3 \cdot (2\delta \bar{I}_b)$

$\underline{I_{n2}}$: fluxes are $\delta \frac{D_n \cdot n_1/2}{W_b/2}$

so $\frac{d}{df}\langle I_{n2} I_{n2}^* \rangle = 2\delta \;\delta \frac{D_n n_1}{W_b} = 2\delta \cdot \bar{I}_b$

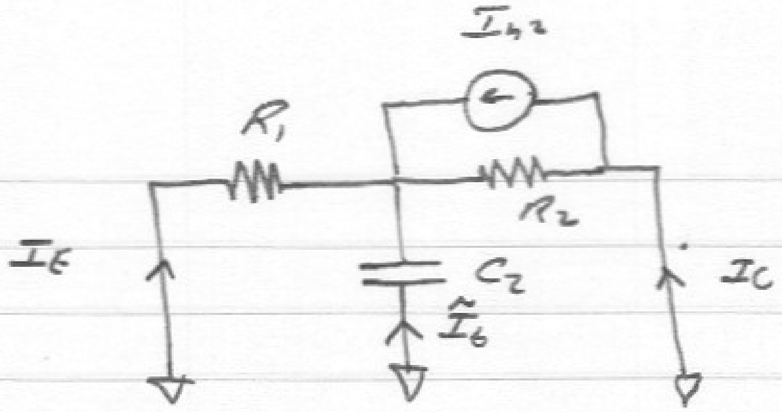


$$v = \frac{I_{n1}}{1/R_1 + 1/R_2 + j\omega C_2}$$

$$\tilde{I}_c = \frac{-v}{R_2} = -I_{n1}\left[\frac{R_1}{R_1+R_2}\ \frac{1}{1+j\omega C_2(R_1 \| R_2)}\right]$$

∴ base current is through C_2

$$\tilde{I}_{c2} = I_{n1} \cdot \frac{j\omega C_2 (R_1 \| R_2)}{1 + j\omega C_2 (R_1 \| R_2)} \cong I_{n1} \cdot j\omega C_2 \cdot (R_1 \| R_2)$$



$$\tilde{I}_{c2} = -I_{n2} \cdot \frac{j\omega C_2 (R_1 \| R_2)}{1 + j\omega C_2 (R_1 \| R_2)} \cong -I_{n2}\left[j\omega C_2 (R_1 \| R_2)\right]$$

but $C_2 (R_1 \| R_2) = \left(qA \frac{W_b}{2}\right)\left(\frac{W_b}{2qAD_n} \cdot \frac{1}{2}\right) = \frac{1}{4} \frac{W_b^2}{2D_n} = \frac{\tau_b}{4}$

so

$$\tilde{I}_b = (j\omega \tau_b / 4)\left[I_{n1} - I_{n2}\right]$$

$$\tilde{I}_c = \frac{I_{n1}}{2} + \frac{I_{n2}}{2}$$

$$-\tilde{I}_E = \tilde{I}_b + \tilde{I}_c = I_{n1}\left[\frac{1}{2} + \frac{j\omega\tau_b}{4}\right] + I_{n2}\left[\frac{1}{2} - j\omega\tau_b/4\right]$$

These again are the particle currents.

The terminal currents are $\tilde{I}_c \cong I_c$

$\tilde{I}_E \cong I_E$

$I_B \cong I_E + I_c(1 - j\omega\tau_c)$

Collector Current:

$$I_c \cong I_{n1}/2 + I_{n2}/2$$

Base Current

$$I_b = -\frac{I_{n1}}{2} - I_{n1}\frac{j\omega\tau_b}{4} - \frac{I_{n2}}{2} + I_{n2}\cdot\frac{j\omega\tau_b}{4}$$

$$+ \frac{I_{n1}}{2} + \cancel{\frac{I_{n2}}{2}} + \frac{I_{n2}}{2}$$

$$- I_{n1}\,j\omega\tau_b/2 \qquad - I_{n2}\,j\omega\tau_c/2$$

$$I_b = I_{n1}(-1)\left[j\omega\frac{\tau_b}{4} + \frac{j\omega\tau_c}{2}\right]$$

$$+ I_{n2}\left[j\omega\tau_b/4 - j\frac{\omega\tau_c}{2}\right]$$

now we can calculate the spectral densities:

$$\frac{d}{dt}\langle I_c I_c^*\rangle = \frac{1}{4}(3)\, 2q\, \bar{I}_E + \frac{1}{4}\cdot(1)\cdot 2q\, \bar{I}_E$$

$$= 2q\, \bar{I}_E \quad \text{as expected.}$$

$$\frac{d}{dt}\langle I_b I_b^*\rangle = 3\cdot 2q\, \bar{I}_E \left[\omega^2\right]\left[\left(\tau_b/4 + \frac{\tau_c}{2}\right)^2\right]$$

$$+ 2q\, \bar{I}_E \left[\omega^2\right]\left[\left(\tau_b/4 - \frac{\tau_c}{2}\right)^2\right]$$

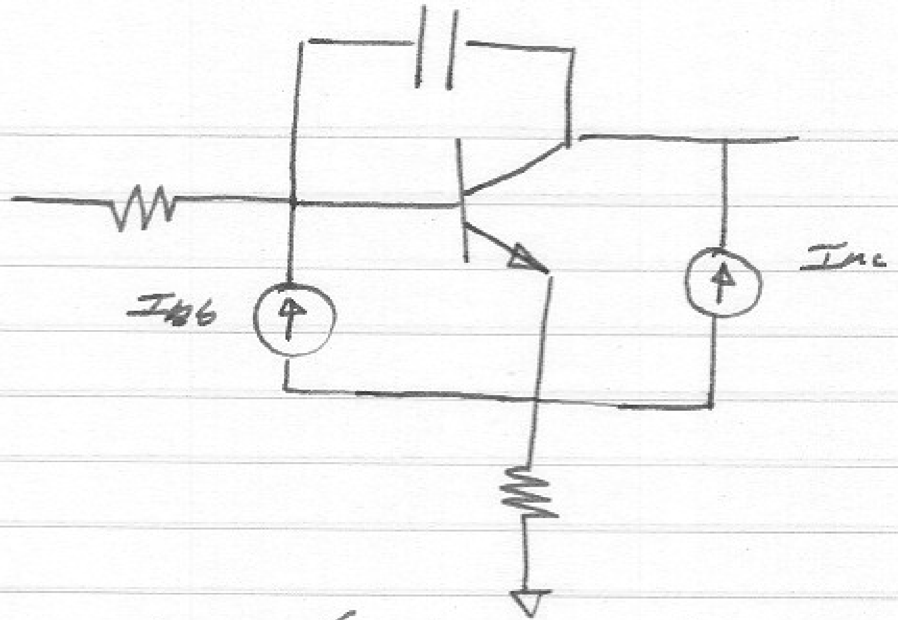
$$= 2q\, \bar{I}_E \left[\omega^2\right]\left[\tau_c^2 + \left(\frac{\tau_b}{2}\right)^2 + \tau_c\cdot\left(\frac{\tau_b}{2}\right)\right]$$

$$\frac{d}{dt}\langle I_b I_c^*\rangle = \left(-j\omega\frac{\tau_b}{4} - j\omega\frac{\tau_c}{2}\right)\, 3\cdot 2q\, \bar{I}_E \cdot \frac{1}{2}$$

$$+ \left(j\omega\frac{\tau_b}{4} - \frac{j\omega\tau_c}{2}\right)\cdot 2q\, \bar{I}_E \cdot \frac{1}{2}$$

$$= \left[-\frac{j\omega\tau_b}{4} - j\omega\tau_c\right] 2q\, \bar{I}_E$$

$$= -j\omega\left[\tau_b/4 + \tau_c\right]\cdot 2q\, \bar{I}_E$$



So we have found, to leading order in ω,

$$\frac{d}{df}\langle I_{nc} I_{nc}^{*}\rangle = 2q\,\overline{I_E}\,(1-\omega^2\tau_c^2)$$

$$\frac{d}{df}\langle I_{nb} I_{nb}^{*}\rangle = 2q\,\overline{I_b} + 2q\,\overline{I_E}\cdot\omega^2\left[\tau_c^2 + \tau_b^2/4 + \tau_b\tau_c/2\right]$$

$$\frac{d}{df}\langle I_{nb} I_{nc}^{*}\rangle = 2q\,\overline{I_E}\cdot[-1]\cdot j\omega\left[\tau_c + \tau_b/4\right]$$

Pause, & examine what we have done:

1) we have illustratated how one can include device transit delays within the noise model.

2) we note that base current will show an additional noise term $\sim O[\omega^2(T_b^2 + T_c^2)]$ times the output noise current

3) We note that this term is small except when $\omega(T_b + T_c) \sim 1$, and that the exact form of the term will depend on the base doping profile for graded-base Bipolar transistors.

We therefore conclude:

a) we know how to derive exact noise expression if we want to.

b) we can probably ignore the $\omega^2[T_b^2 + T_c^2]$ terms with acceptable accuracy in subsequent work.

$$\Downarrow$$

$$\frac{d}{df}\langle I_c I_c^* \rangle = 2q\bar{I}_c$$

$$\frac{d}{df}\langle I_b I_b^* \rangle = 2q\bar{I}_b$$

$$\frac{d}{df}\langle I_b I_c^* \rangle = 0$$

We will use this in subsequent circuit analyses.