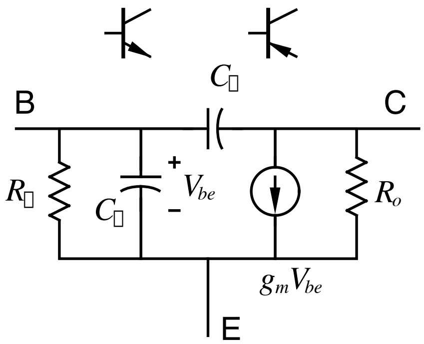
Basics: Transistor small-signal high-frequency models



Hybrid-pi model:

$C_\mu = C_{bc}$

$C_\pi = C_{be} = C_{\pi,depl} + C_{\pi,diff}$

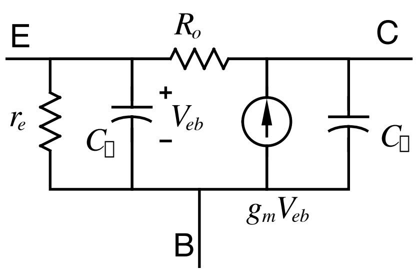
$C_{\pi,diff} = g_m \tau_f$

$f_\tau = g_m \big/ \left(2\pi\left(C_\pi + C_\mu\right)\right)$

$g_m = \alpha / r_e = I_c / V_T$

$R_\pi = (\beta + 1) r_e$

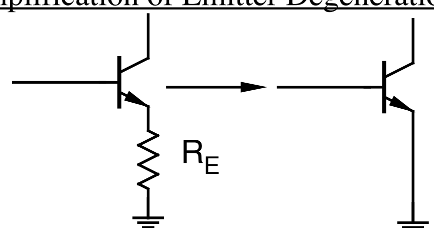
$R_o = V_A / I_C$



"T"-model

Makes common-base analysis much easier. Both these models are only approximate, being "good" up to f_t. Additionally $R_{bb'}$ is often an important parasitic (not discussed much in 137B.

Simplification of Emitter Degeneration



approximate only; check notes for bounds on validity

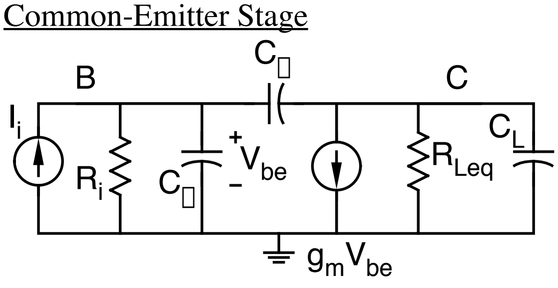
$\tilde{C}_\pi = C_\pi \left(r_e / \left(r_e + R_E\right)\right)$

$\tilde{g}_m = g_m \left(r_e / \left(r_e + R_E\right)\right) = \alpha / \left(r_e + R_E\right)$

$\tilde{C}_\mu = C_\mu$

$\tilde{R}_\pi = R_\pi \left(\left(r_e + R_E\right) / r_e\right) = (\beta + 1)\left(r_e + R_E\right)$

Common-Emitter Stage



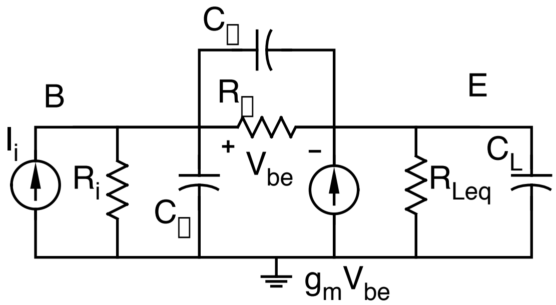
$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \dfrac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2}$

$a_1 = R_i\left(C_\pi + C_\mu\left(1 + g_m R_{Leq}\right)\right) + R_{Leq}\left(C_\mu + C_L\right)$

$a_2 = R_i R_{Leq}\left(C_\mu C_L + C_\mu C_\pi + C_\pi C_L\right)$

$\tau_{zero} = -C_\mu / g_m$

Emitter-Follower Stage



$$V_{out}/V_{gen} = \left(V_{out}/V_{gen}\right)_{MB} \frac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2}$$

given that $A_{vmb} = \left(r_e/\left(r_e + R_{Leq}\right)\right)$:

$$a_1 = C_\pi\left(R_\pi \| \left(r_e \| R_{Leq} + R_i\left(1 - A_{vmb}\right)\right)\right)$$
$$+C_\mu\left(R_i \| \text{transistor input resistance}\right)$$
$$+C_L\left(R_{Leq} \| \text{transistor output resistance}\right)$$
$$a_2 = \left(R_i \| \text{transistor input resistance}\right)\left(R_{Leq} \| r_e\right)$$
$$\times\left(C_\mu C_\pi + C_\mu C_L + C_L C_\pi\right)$$
$$\tau_{zero} = g_m/C_\pi$$

General Solutions of Problems: Nodal analysis (Know how to do this!)

1)Write the nodal equations (sum of the currents=0) at each circuit node, and put the resulting equations in matrix form (the Y's being various combinations of g_m's, 1/R's, and sC's):

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 = V_{in} \\ V_2 \\ V_3 \\ V_4 = V_{out} \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2) Use Cramer's rule to solve:

$$\frac{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & I_{in} \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ Y_{31} & Y_{32} & Y_{33} & 0 \\ Y_{41} & Y_{42} & Y_{43} & 0 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix}} = V_{out}$$

3) This comes out as:

$$\frac{V_{out}}{I_{in}} = ks^m \frac{c_0 + c_1 s + c_2 s^2 + \cdots}{d_0 + d_1 s + d_2 s^2 + \cdots}$$

,which is divided through to get:

(if present, m is the number of zeros, minus the number of poles, in the transfer function)

4)

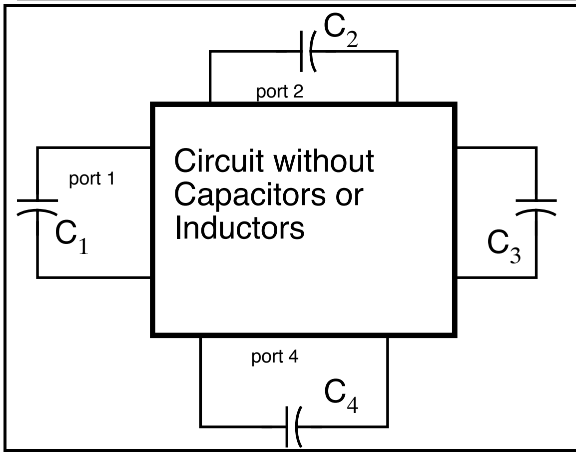
$$\frac{V_{out}}{I_{in}} = \left(\frac{V_{out}}{I_{in}}\right)_{mb} s^m \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$$

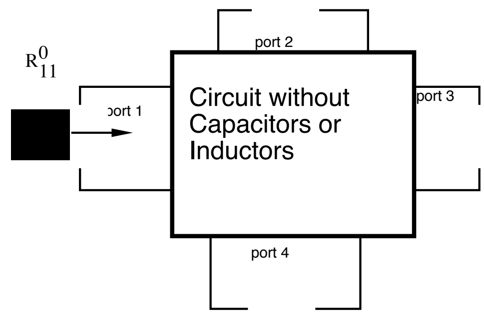
...and the poles and zeroes are found by factoring the numerator and denominator. The separated-pole approximation, if applicable, makes this factoring easy.

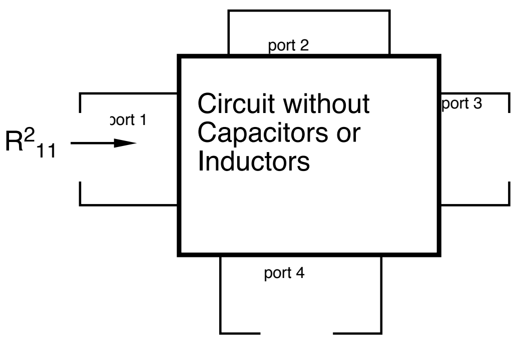
5) To find the *impulse response*, do a partial-fraction expansion and then take the inverse LaPlace transform.

6) To find the sinusiodal frequency response, set $s = j\omega$.

General Solutions of Problems: Method of Time Constants





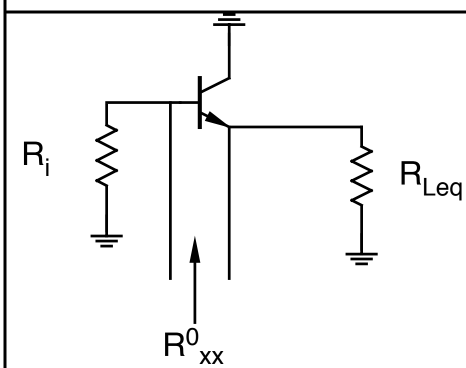


$$\frac{V_{out}}{V_{gen}} = \left(\frac{V_{out}}{V_{gen}}\right)_{mb} \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}$$

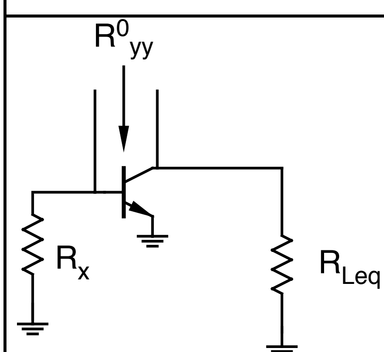
$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 + R_{44}^0 C_4$$

$$a_2 = R_{11}^0 R_{22}^1 C_1 C_2 + R_{11}^0 R_{33}^1 C_1 C_3 + R_{11}^0 R_{44}^1 C_1 C_4$$
$$+ R_{22}^0 R_{33}^2 C_2 C_3 + R_{22}^0 R_{44}^2 C_2 C_4 + R_{33}^0 R_{44}^3 C_3 C_4$$

notethat $R_{xx}^0 R_{yy}^x = R_{xx}^y R_{yy}^0$



$$R_{xx}^0 = R_\pi \left\| \left(r_e \| R_{Leq} + R_i \left(1 - A_{vmb} \right) \right) \right.$$
$$A_{vmb} = \left(R_{Leq} / \left(r_e + R_{Leq} \right) \right)$$



$$R_i = R_x \| R_\pi$$
$$R_{yy}^0 = R_i \left(1 + g_m R_{Leq} \right) + R_{Leq}$$