

***Mixed-Signal IC Design Notes set 2:  
Fundamentals of Analog IC Design***

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# Transistor Circuit Design

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## ***This note set***

- reviews the basics
- starts at the level of a first IC design course
- moves very quickly

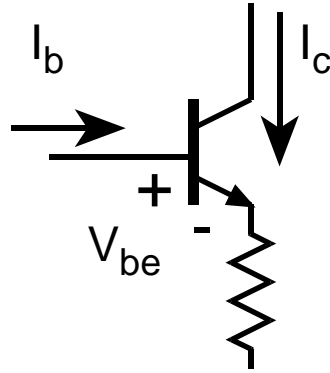
## ***This will***

- establish a common terminology
- accommodate capable students having minimal background in ICs.

# **DC models**

## **DC bias analysis**

# Large-Signal Model For Bias Analysis



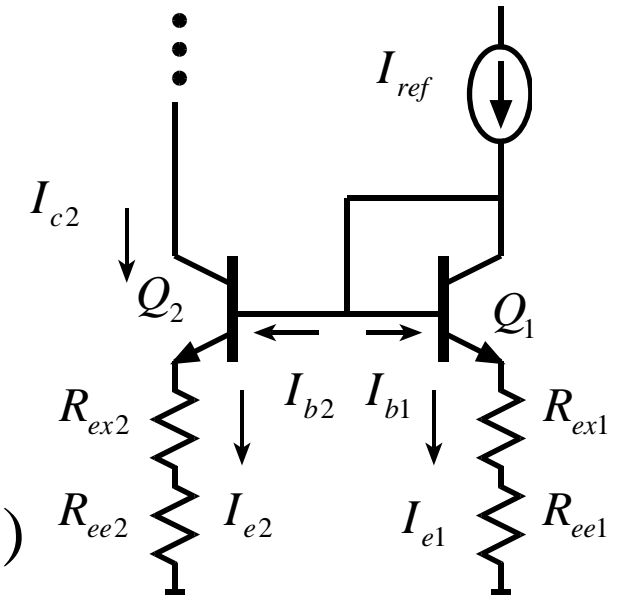
Provided that  $V_{ce} > 0$ ,

$I_c = I_s \exp(V_{be} / V_T)$  and  $I_b = I_c / \beta$ , where  $V_T = kT / q$

...note that  $V_{be}$  is specified internal to the emitter resistance  $R_{ex}$

The  $I_e R_{ex}$  drop is significant for HBTs operating at current densities near that required for peak transistor bandwidth.

# DC Bias Example: Current Mirror



We have  $V_{be1} + I_{e1}(R_{ex1} + R_{ee1}) = V_{be2} + I_{e2}(R_{ex2} + R_{ee2})$

and  $V_{be1} = V_t \ln(I_{c1} / I_{s1})$ ,  $V_{be2} = V_t \ln(I_{c2} / I_{s2})$

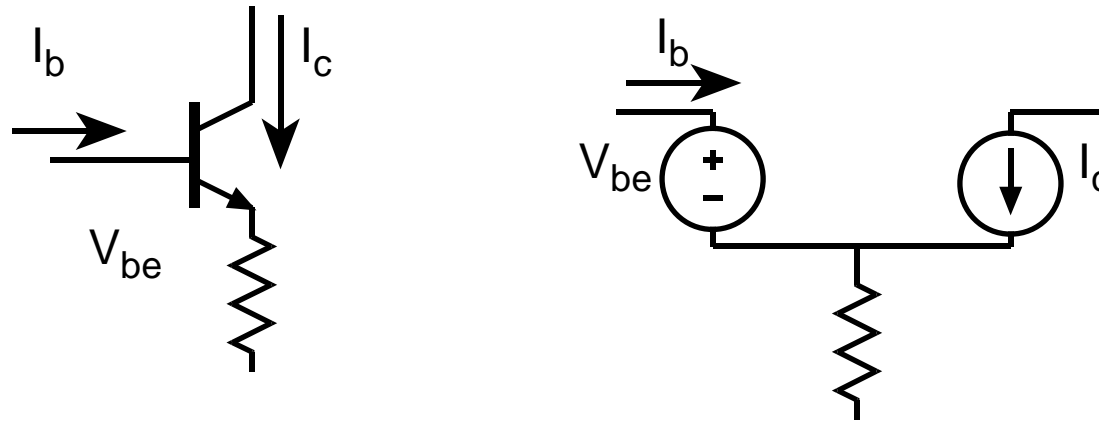
Assume that  $\beta \gg 1$ ,  $R_{ee2} = 2R_{ee1}$

& assume that  $A_{E1} = A_{E2}$  ( $A_E$  is the emitter area).

This implies  $R_{ex1} = R_{ex2} / 2$ , and  $I_{s1} = 2I_{s2}$ ,

from which we find  $I_{c2} = I_{c1} / 2$

# Simpler DC Model for Bias Analysis



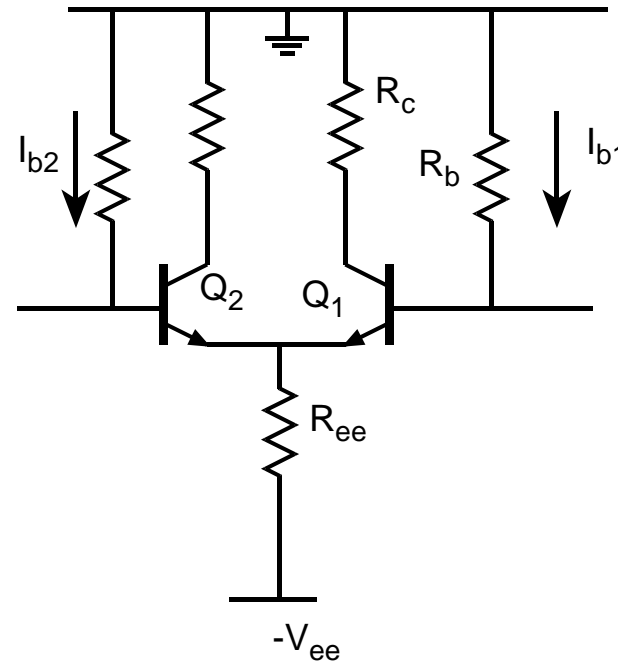
It is often sufficient in bias analysis to ignore the variation of  $V_{be}$  with  $I_c$  and instead take  $V_{be} = V_{be,on} = \phi$ .

$V_{be,on}$  depends upon current density and technology.

Biased at current densities within  $\sim 10\%$  of peak bandwidth bias,

$$V_{be,on} = \phi \sim \begin{cases} 0.9 \text{ V Modern Si/SiGe HBTs} \\ 0.7 \text{ or } 0.9 \text{ V InGaAs/InP HBTs} \\ 1.4 \text{ V GaAs/GaInP HBTs} \end{cases}$$

# Simple DC Bias Example



If we neglect the  $I_b R_b$  drops, then  $V_{b1} = V_{b2} = 0$  Volts.

Approximate  $V_{e1} = V_{e2} = -\phi \cong -0.9$  V (SiGe).

$$I_{c1} + I_{c2} = 2I_{c1} = (-V_{ee} - 0.9V) / R_{ee}$$

$$I_{c1} = I_{c2} = (-V_{ee} - 0.9V) / 2R_{ee}$$

# Efficiently Handling Base Currents In Bias Analysis

If  $I_b R_b$  drop is significant,

one can solve simultaneous equations :

$$I_{c1} + I_{c2} = 2I_{c1} = (-V_{ee} - \phi - I_{b1} R_b) / R_{ee}$$

where  $I_{b1} = I_{c1} / \beta$ ,

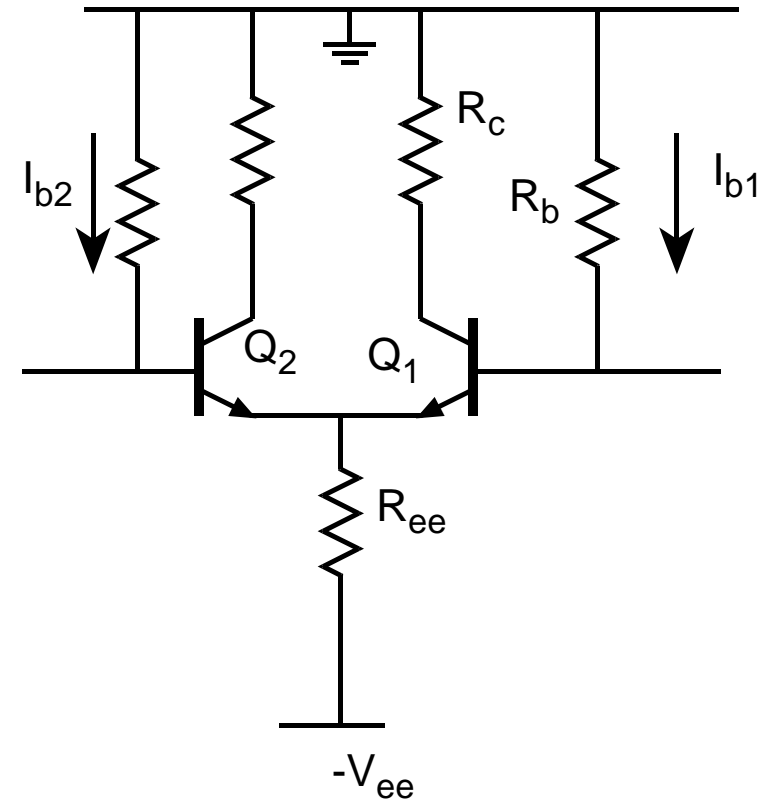
Quicker : find by iteration :

- 1) solve  $I_{c1} = (-V_{ee} - \phi) / 2R_{ee}$

- 2) solve  $I_{b1} \cong I_{c1} / \beta$

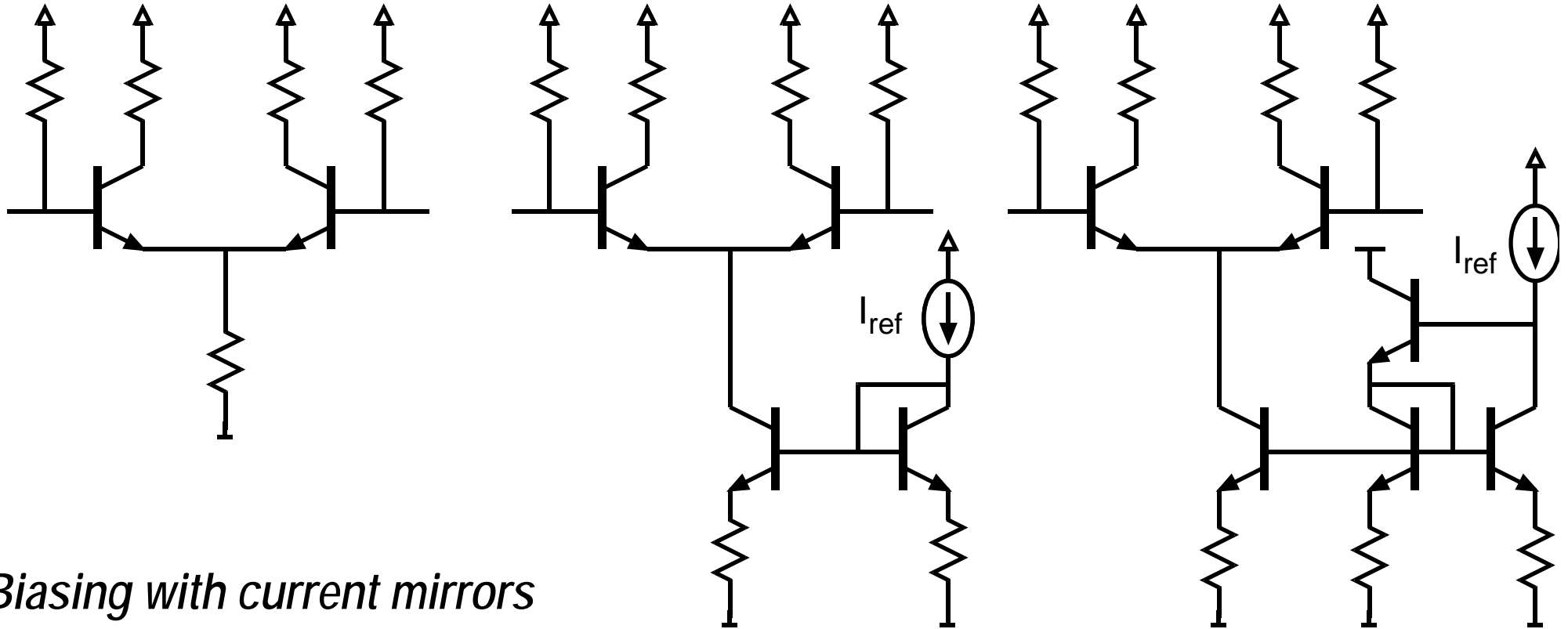
- 3) use this value of  $I_b$  to solve  $I_{c1} = (-V_{ee} - \phi - I_{b1} R_b) / 2R_{ee}$

Works because any well - designed circuit has DC bias only weakly dependent upon  $\beta$ .





# Typical Circuit Biasing in Mixed-Signal ICs



*Biasing with current mirrors*

*more precise---*

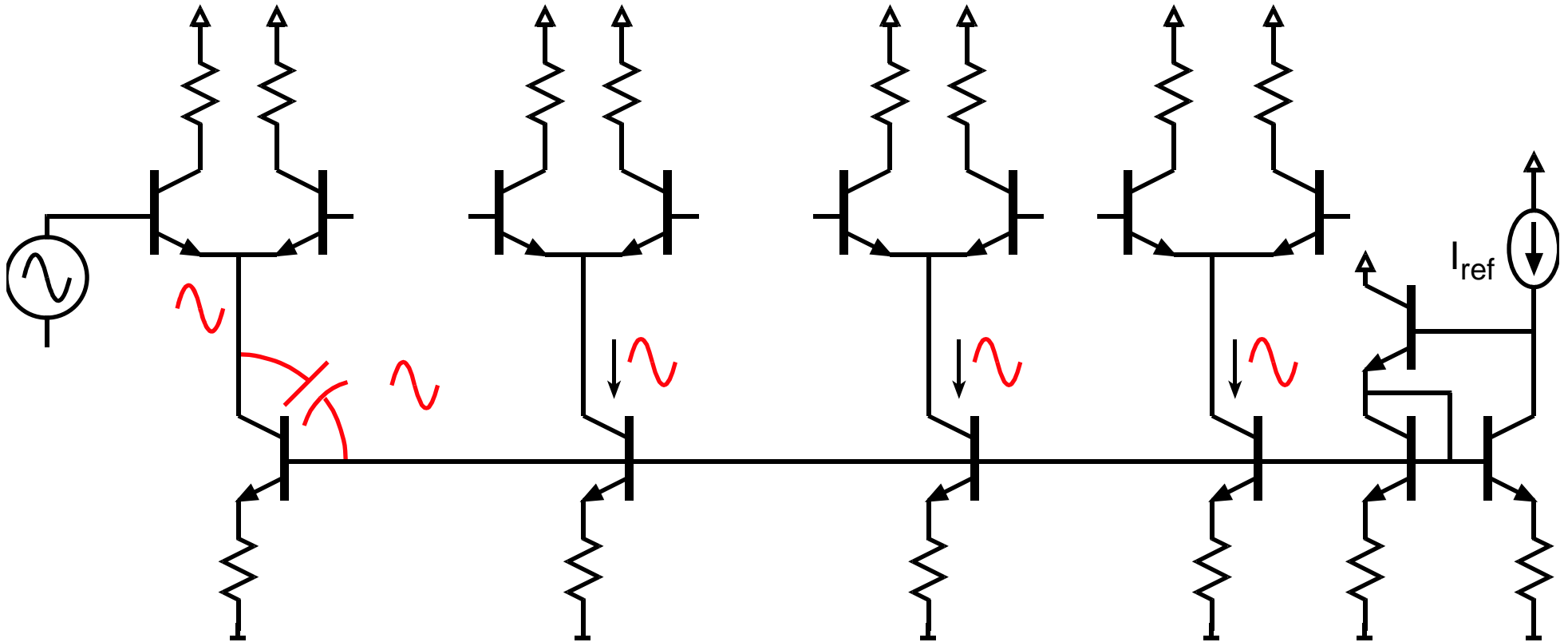
*often more compact*

*but loads emitter node with  $C_{cb}$  ---or worse*

*and provides path for stage-stage coupling*

*Resistive biasing--- resistively loads emitter node---lower DC precision*

# Problems with Current Mirror Biasing



*stage-stage coupling through  $C_{cb}$  and the shared current mirror reference...*

# Problems with Current Mirror Biasing

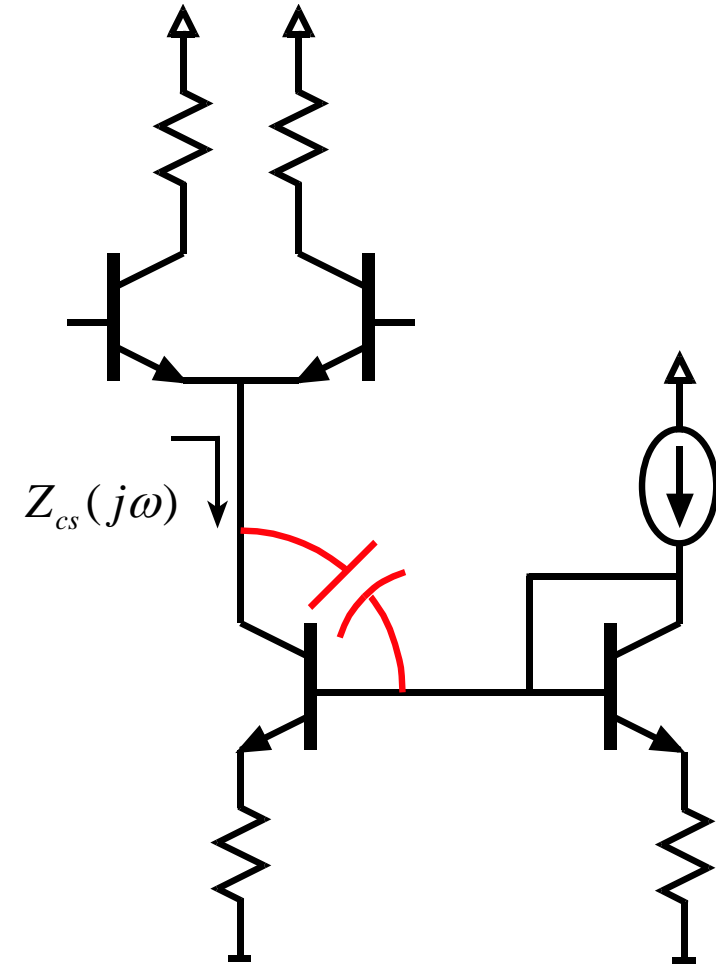
Because of  $C_{cb}$  and  $C_{be}$ , the current source output impedance  $Z_{cs}(j\omega)$  varies with frequency.

We will derive this in later lectures.

Implications :

increased common - mode gain at high frequencies

common - mode instability



# Avoiding Thermal Instability

Given that  $V_{be} = V_t \ln(I_{c2} / I_{s2}) = (kT / q) \ln(I_{c2} / I_{s2})$

...note that  $I_s \propto T^{3/2}$ , so at constant  $I_c$ ,

$$dV_{be} / dT = (k / q) \ln(I_{c2} / I_{s2}) - (kT / q)(dI_{s2} / dT)^{-1}$$

The device has some thermal resistance  $dT / dP = \theta$

(Kelvin/Watt). Current mirror is thermally unstable if

$$1 < \frac{dI_c}{dV_{be}} \cdot \frac{dV_{be}}{dT} \cdot \frac{dT}{dP} \cdot \frac{dP}{dI_c}$$

$$1 < \frac{1}{R_{ex} + kT / qI_e + R_{ee}} \cdot \frac{dV_{be}}{dT} \cdot \theta \cdot V_{ce}$$

Fast Bipolar ICs normally use  $I_e * R_{ee} \sim 300mV$  to

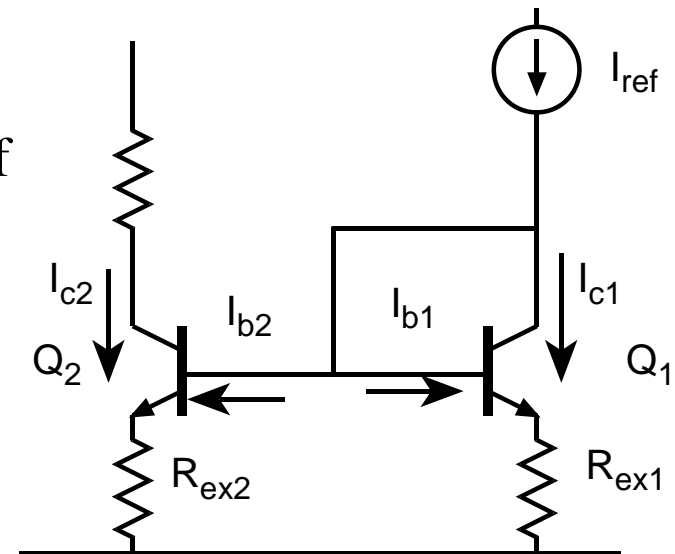
avoid thermal runaway. Thermal runaway can also result from

internal device temperature gradients. Large area devices must

therefore be avoided...use array of smaller devices with

ballasting. (process dependent)...UCSB HBT process puts

safe maximum device area  $\sim 10$  square microns at  $10^5 A / cm^2$  biasing

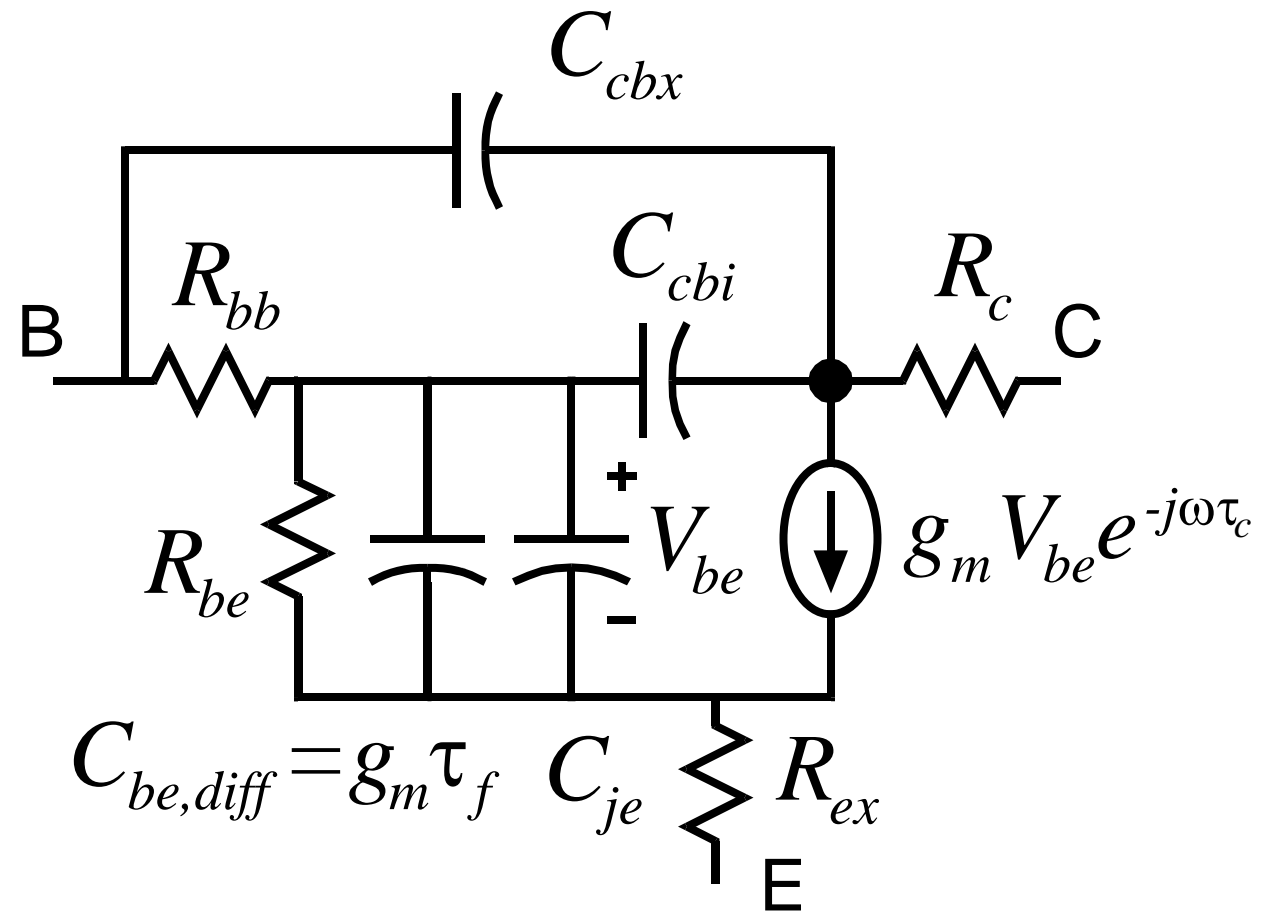


# **small-signal baseband analysis**

# Hybrid- $\pi$ Bipolar Transistor Model

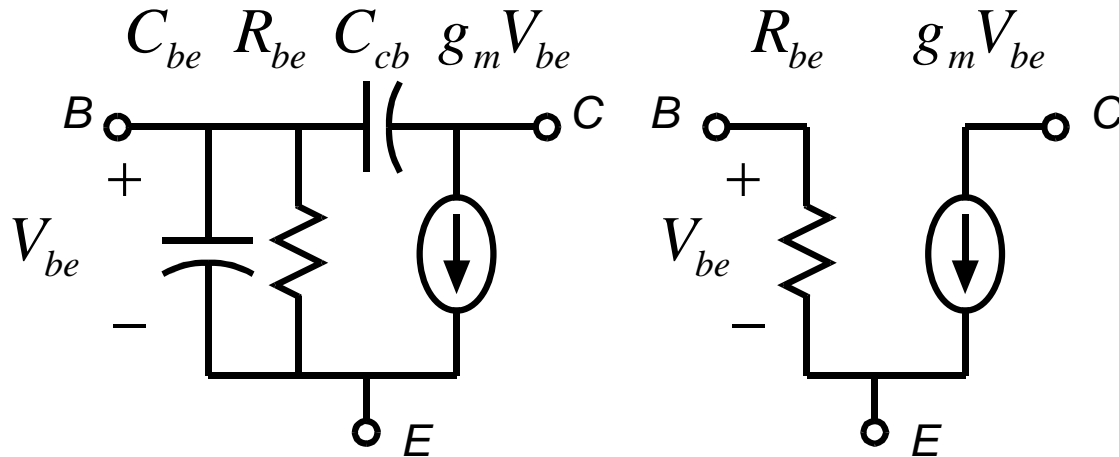
$$R_{be} = \beta / g_m$$

$$\tau_f = \tau_b + \tau_c$$



Accurate model, but too detailed for quick hand analysis

# Oversimplified Model for Quick Hand Analysis

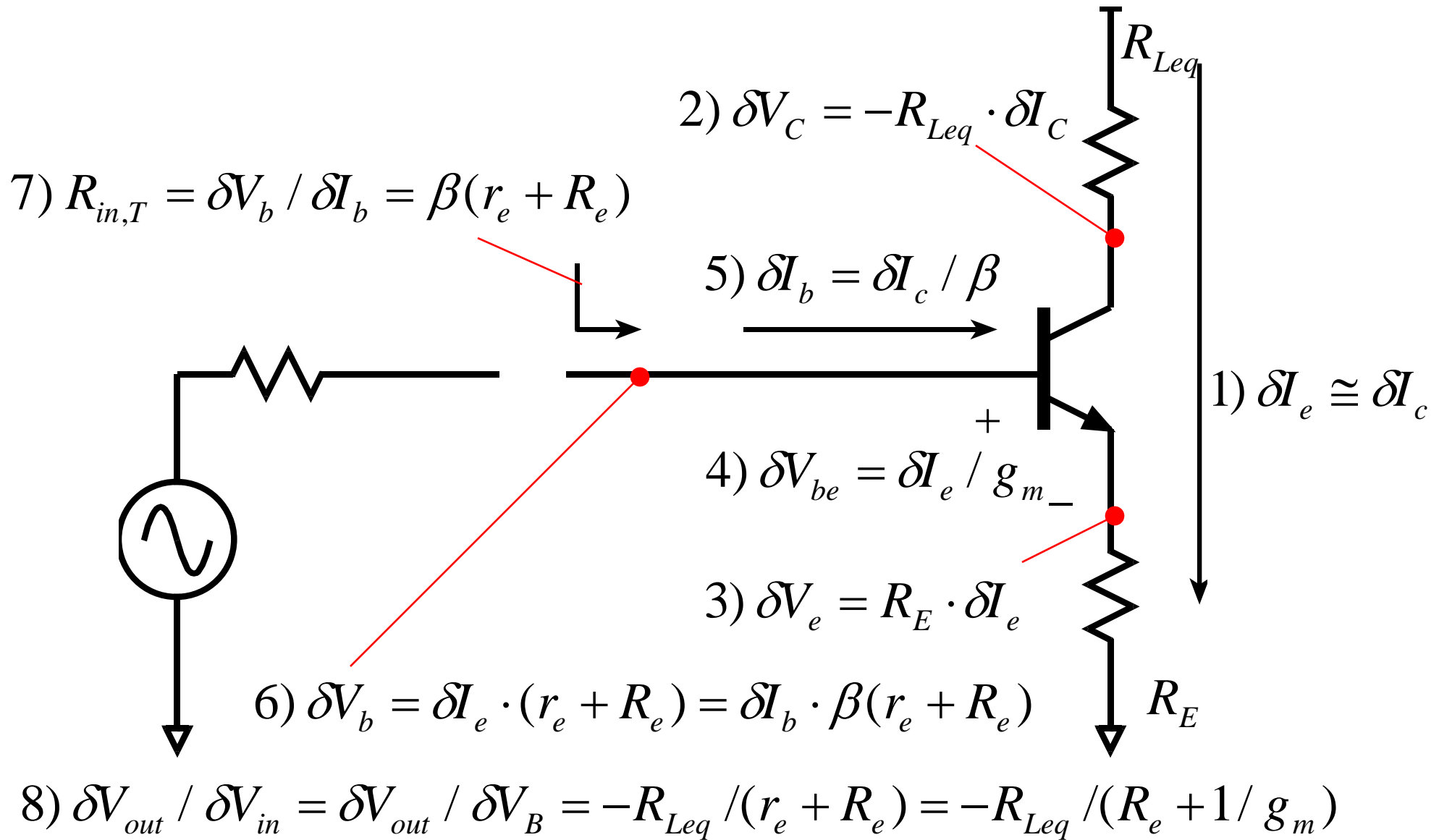


In most high-frequency circuits, the node impedance is low and  $R_{ce}$  is therefore negligible.

Neglecting  $R_{bb}$  in high-frequency analysis is a poor approximation but is nevertheless common in introductory treatments.

The "textbook" analyses which follow use this oversimplified model. These introductory treatments will later be refined.

# Common Emitter Stage: Basics

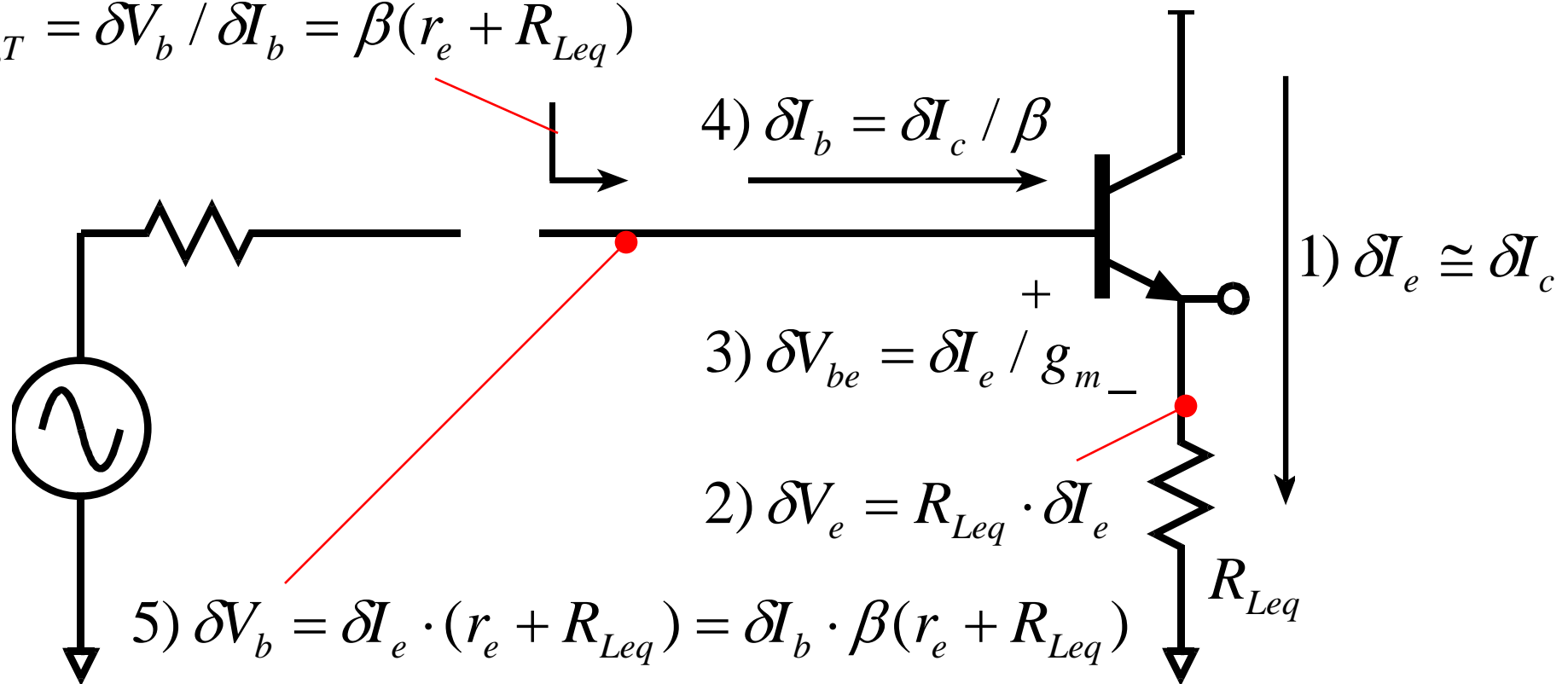


Gain is  $-R_{Leq} / (R_e + 1/g_m)$ ; Transistor  $R_{in}$  is  $\beta(r_e + R_E)$



# Emitter Follower Stage: Basics

$$6) R_{in,T} = \delta V_b / \delta I_b = \beta(r_e + R_{Leq})$$



$$7) \delta V_{out} / \delta V_{in} = \delta V_{out} / \delta V_E = R_{Leq} / (r_e + R_{Leq}) = R_{Leq} / (R_{Leq} + 1 / g_m)$$

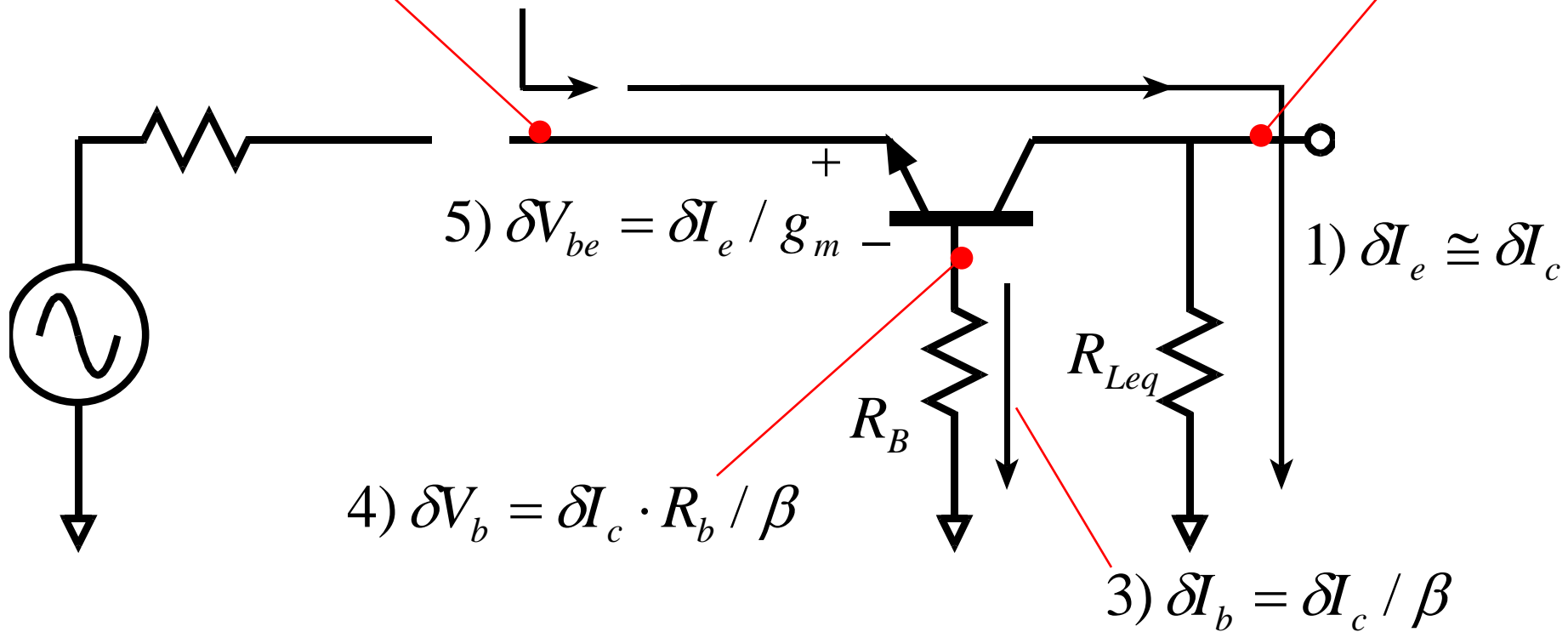
Gain is  $R_{Leq} / (R_{Leq} + 1 / g_m)$ ; Transistor  $R_{in}$  is  $\beta(r_e + R_E)$

# Common-Base Stage: Basics

$$6) \delta V_{in} = \delta I_e \cdot (r_e + R_b / \beta)$$

$$2) \delta V_{out} = R_{Leq} \cdot \delta I_c$$

$$7) R_{in,T} = \delta V_e / \delta I_e = r_e + R_B / \beta$$



$$5) \delta V_{be} = \delta I_e / g_m$$

$$1) \delta I_e \cong \delta I_c$$

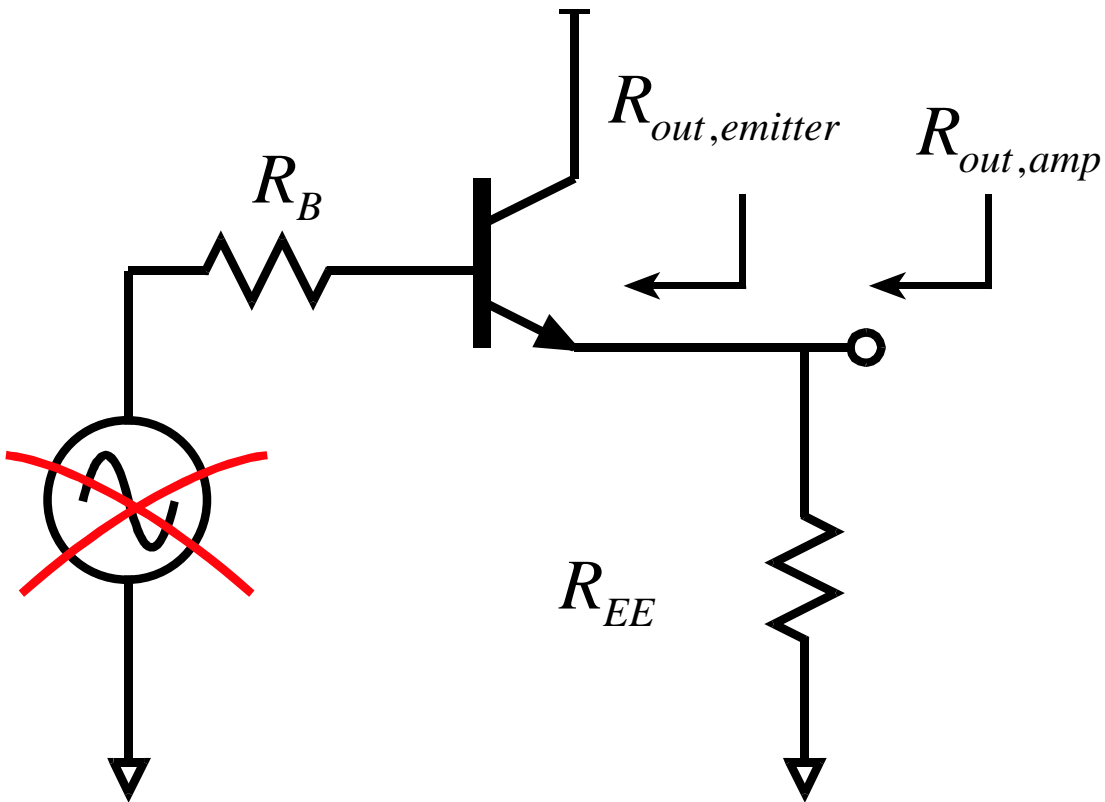
$$4) \delta V_b = \delta I_c \cdot R_b / \beta$$

$$3) \delta I_b = \delta I_c / \beta$$

$$7) \delta V_{out} / \delta V_{in} = R_{Leq} / (r_e + R_b / \beta)$$

Gain is  $R_{Leq} / (r_e + R_b / \beta)$ ; Transistor  $R_{in}$  is  $r_e + R_b / \beta$

# Emitter Follower Output Impedance

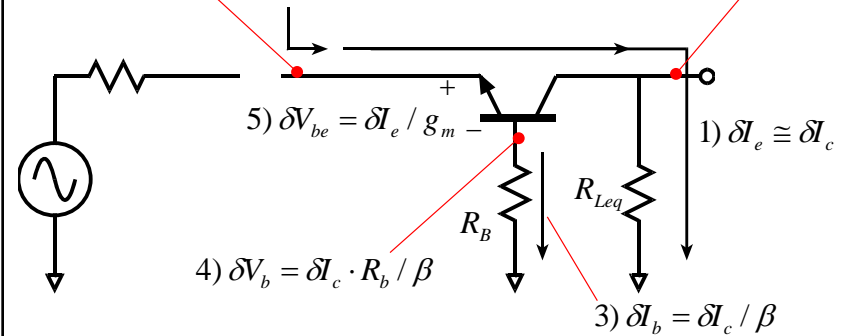


## Common-Base Stage: Basics

$$6) \delta V_{in} = \delta I_e \cdot (r_e + R_b / \beta)$$

$$2) \delta V_{out} = R_{Leq} \cdot \delta I_c$$

$$7) R_{in,T} = \delta V_e / \delta I_e = r_e + R_B / \beta$$



$$5) \delta V_{be} = \delta I_e / g_m$$

$$1) \delta I_e \cong \delta I_c$$

$$4) \delta V_b = \delta I_c \cdot R_b / \beta$$

$$3) \delta I_b = \delta I_c / \beta$$

$$7) \delta V_{out} / \delta V_{in} = R_{Leq} / (r_e + R_b / \beta)$$

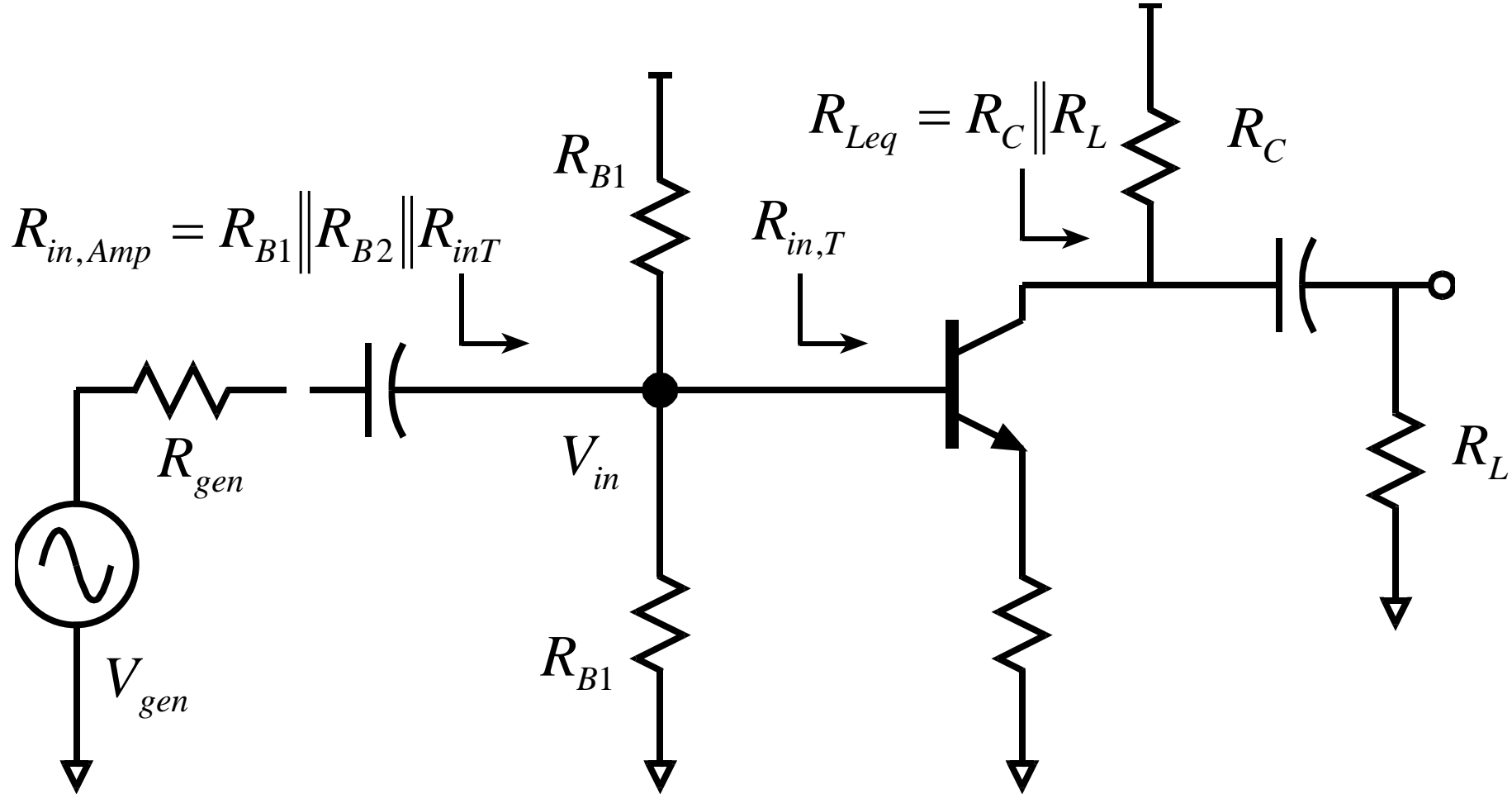
Gain is  $R_{Leq} / (r_e + R_b / \beta)$ ; Transistor  $R_{in}$  is  $r_e + R_b / \beta$

E.F. output impedance is same problem as C.B. input impedance

$$R_{out,emitter} = r_e + R_B / \beta = 1 / g_m + R_B / \beta$$

$$R_{out,amp} = R_{out,emitter} \parallel R_{EE}$$

# Including Bias Circuit Resistances



These are (trivially) added in parallel with the transistor terminal impedances to determine the net circuit impedances.

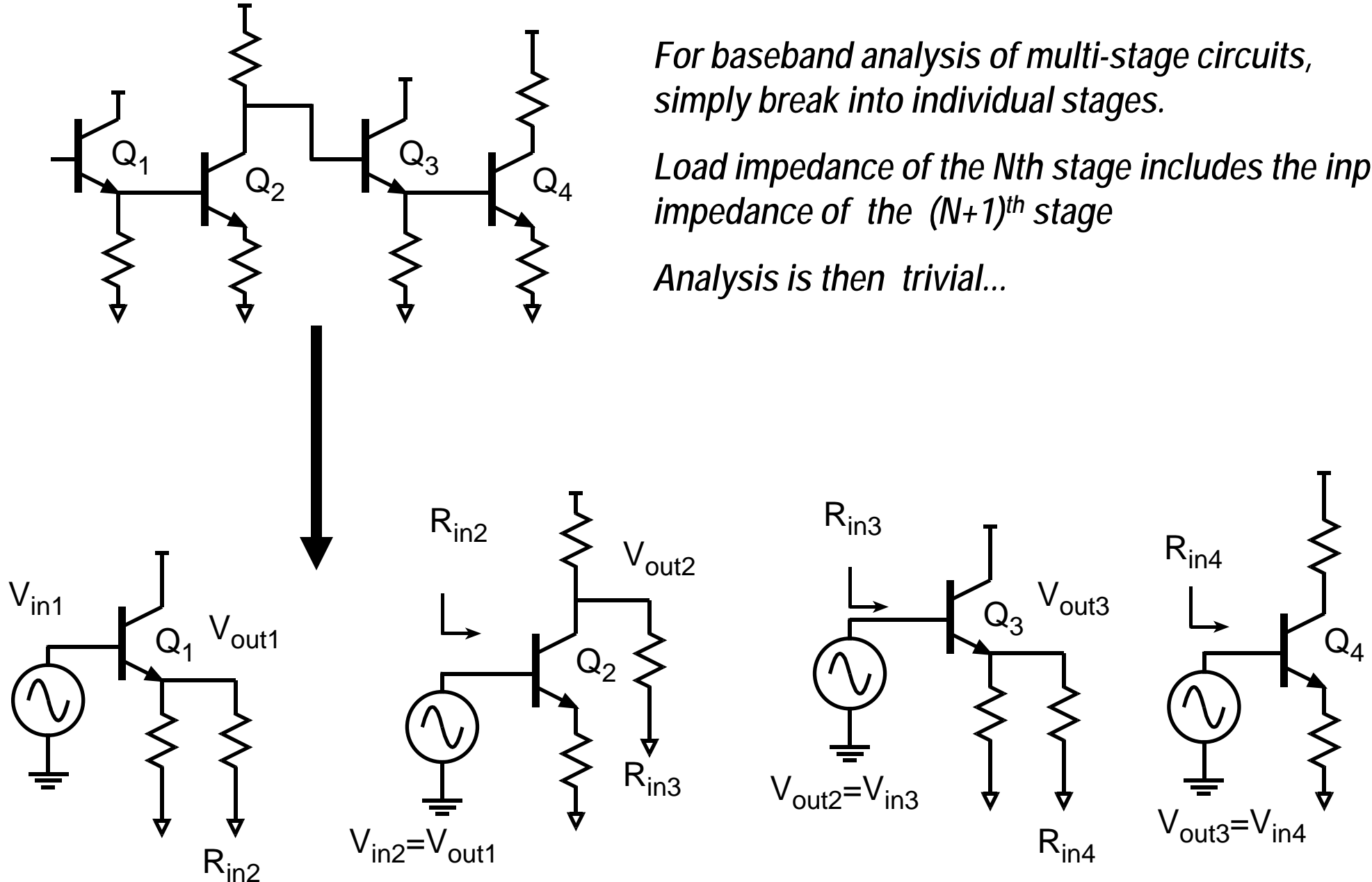
From which,  $V_{in} / V_{gen} = R_{in,amp} / (R_{in,amp} + R_{gen})$ , etc.

# Baseband Analysis Of Multistage Circuits

*For baseband analysis of multi-stage circuits,  
simply break into individual stages.*

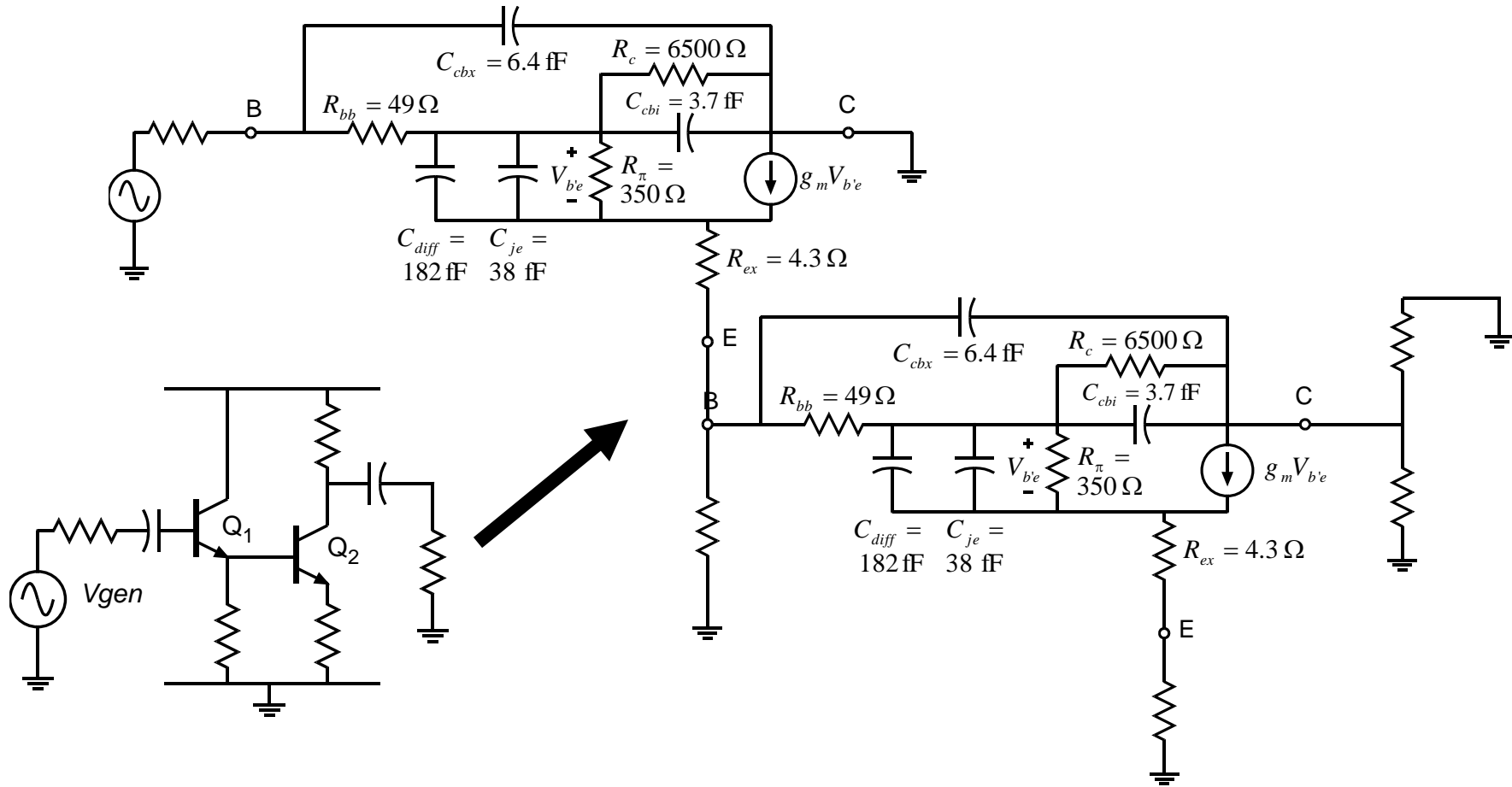
*Load impedance of the Nth stage includes the input  
impedance of the (N+1)<sup>th</sup> stage*

*Analysis is then trivial...*



# **small-signal baseband analysis**

# High-Frequency Analysis: The General Problem



Analyzing frequency response is difficult: cannot separate stage-by-stage

Method #1: nodal analysis: accurate, general, tedious.

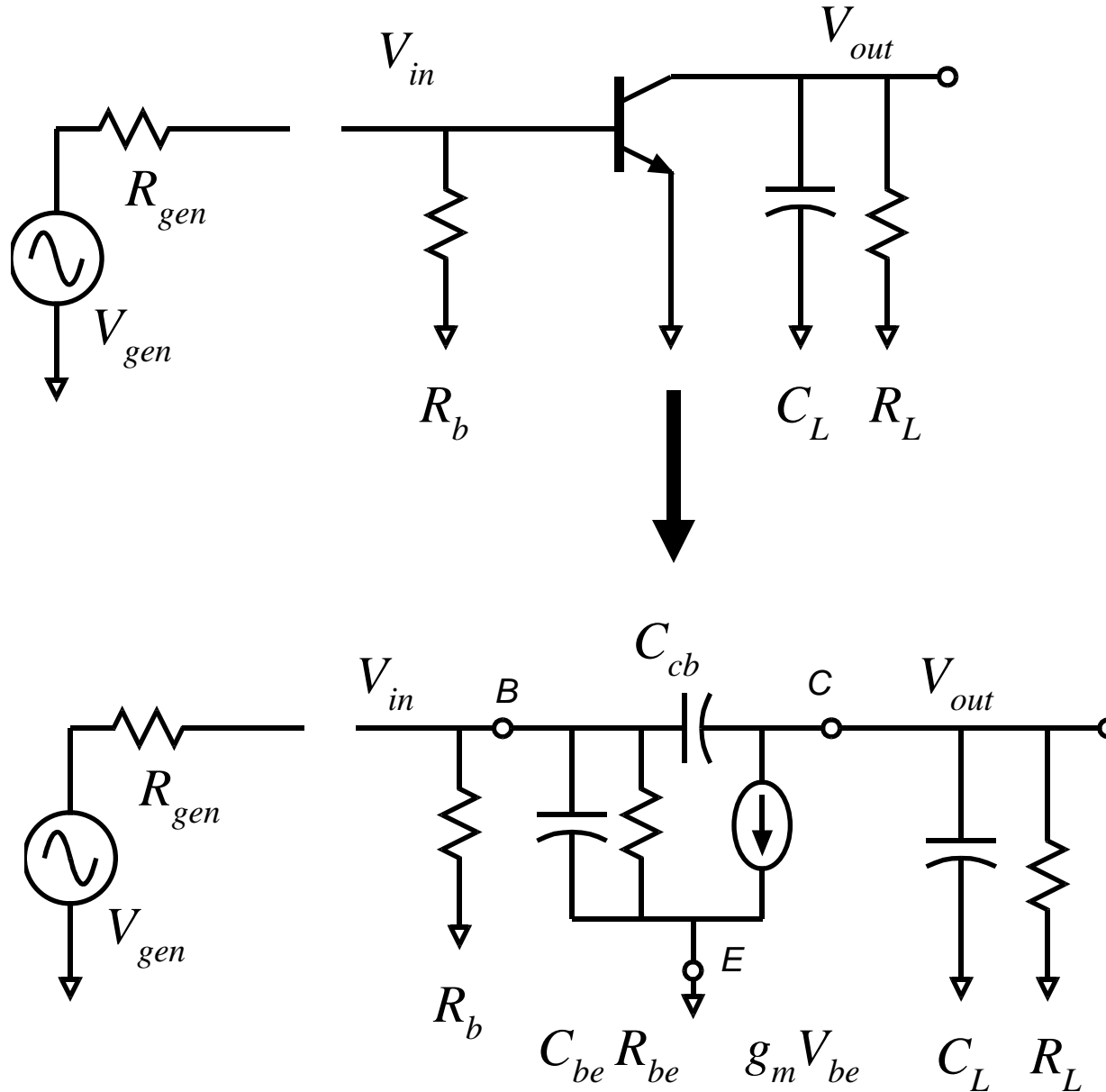
Method #2: method of time constants: accurate, limited applicability, quick & intuitive

# Nodal Analysis



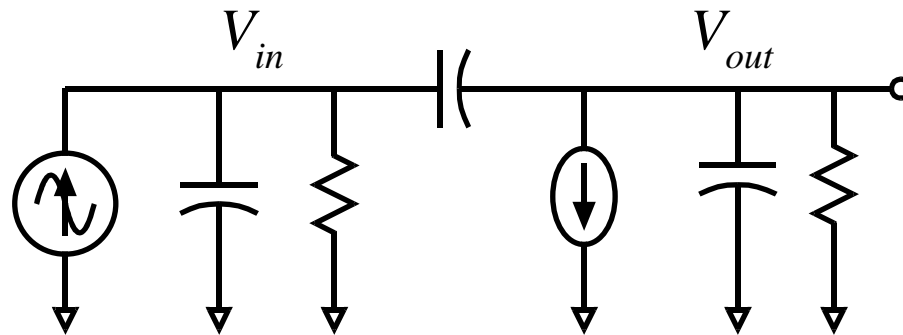
# Tutorial: Transfer Function Analysis: Nodal Analysis I

Simple & very familiar example : common - emitter amplifier.



# Tutorial: Transfer Function Analysis: Nodal Analysis II

Reduced circuit :



$$(R_i = R_{gen} \parallel R_{be} \parallel R_b)$$

$$I_i = V_{gen}/R_{gen} \quad C_{be} \quad R_i \quad g_m V_{be} \quad C_L \quad R_L$$

Step 1 : Write Nodal Equations from KCL

$$\begin{bmatrix} G_i + sC_{be} + sC_{cb} & -sC_{cb} \\ g_m - sC_{cb} & G_L + sC_L + sC_{cb} \end{bmatrix} \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} I_i \\ 0 \end{bmatrix}$$

# Tutorial: Transfer Function Analysis: Nodal Analysis III

## Step 2 : Solve Nodal Equations :

$$V_{out} / I_{in} = N(s) / D(s)$$

$$N(s) = \begin{vmatrix} G_i + sC_{be} + sC_{cb} & 1 \\ g_m - sC_{cb} & 0 \end{vmatrix} = -(g_m - sC_{cb})$$

$$D(s) = \begin{vmatrix} G_i + sC_{be} + sC_{cb} & -sC_{cb} \\ g_m - sC_{cb} & G_L + sC_L + sC_{cb} \end{vmatrix}$$

$$D(s) = (G_i + sC_{be} + sC_{cb})(G_L + sC_L + sC_{cb}) - (g_m - sC_{cb})(-sC_{cb})$$

## Step 3 : Organize in powers of $s$

$$\begin{aligned} D(s) = & G_i G_L \\ & + s(G_i C_L + G_i C_{cb} + G_L C_{be} + G_L C_{cb} + g_m C_{cb}) \\ & + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L + \cancel{C_{cb} C_{cb}} - \cancel{C_{cb} C_{cb}}) \end{aligned}$$

# Tutorial: Transfer Function Analysis: Nodal Analysis IV

Step 4 : Separate into dimensionless ratio - of - polynomials form, separating constants and gains from the transfer function...

$$\frac{V_{out}}{I_{in}} = \frac{V_{out}}{V_{gen} / R_{gen}} = \frac{N(s)}{D(s)}$$

$$= \frac{-(g_m - sC_{cb})}{\left( G_i G_L + s(G_i C_L + G_i C_{cb} + G_L C_{be} + G_L C_{cb} + g_m C_{cb}) \right.}$$

$$\left. + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L) \right)$$

$$\frac{V_{out}}{V_{gen}} = \frac{-(g_m - sC_{cb})R_i R_L / R_{gen}}{\left( 1 + s(R_L C_L + R_L C_{cb} + R_i C_{be} + R_i C_{cb} + g_m R_i R_L C_{cb}) \right.}$$

$$\left. + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L)R_i R_L \right)$$

# Tutorial: Transfer Function Analysis: Nodal Analysis V

note that 
$$\frac{R_i}{R_{gen}} = \frac{(R_{be} \parallel R_b \parallel R_{gen})}{R_{gen}} = \frac{(R_{be} \parallel R_b)}{(R_{be} \parallel R_b) + R_{gen}} = \frac{R_{in,Amp}}{R_{in,Amp} + R_{gen}}$$

SO...

$$\frac{V_{out}}{V_{gen}} = \left( \frac{R_{in,Amp}}{R_{in,Amp} + R_{gen}} \right) (-g_m R_L) \times \frac{-\left(1 - sC_{cb} / g_m\right)}{\left(1 + s(R_L C_L + R_L C_{cb} + R_i C_{be} + R_i C_{cb} + g_m R_i R_L C_{cb}) + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L) R_i R_L\right)}$$

$b_1$  (points to  $-g_m R_L$ )  
 $a_1$  (points to  $1 - sC_{cb} / g_m$ )  
 $a_2$  (points to  $s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L) R_i R_L$ )

$$\Rightarrow \frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \Big|_{mid-band} \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$$

# Tutorial: Transfer Function Analysis: Nodal Analysis VI

Step 5 : Find the roots (poles & zeros) of the polynomial

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \Big|_{mid-band} \frac{1 + b_1s}{1 + a_1s + a_2s^2} = \frac{V_{out}}{V_{gen}} \Big|_{mid-band} \frac{1 + b_1s}{(1 - s/s_{p1})(1 - s/s_{p2})}$$

what are efficient methods of finding the poles ?

# **Finding Poles from Transfer Functions**

# Finding Poles and Zeros

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Ratio - of - Polynomial Form :

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left. \frac{V_{out}}{V_{gen}} \right|_{\text{at mid-band}} * s^m \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$$

Poles and Zeros :

$$\frac{V_{out}(s)}{V_{gen}(s)} = \left. \frac{V_{out}}{V_{gen}} \right|_{\text{at mid-band}} * s^m \frac{(1 - s/s_{z1})(1 - s/s_{z2})(1 - s/s_{z3})\dots}{(1 - s/s_{p1})(1 - s/s_{p2})(1 - s/s_{p3})\dots}$$



# Finding Poles: Complex Poles

$$\frac{V_{out}}{V_{gen}} = k \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

If  $a_3 / a_2 \ll a_2$  then we can ignore the  $s^3$  at moderate frequencies and

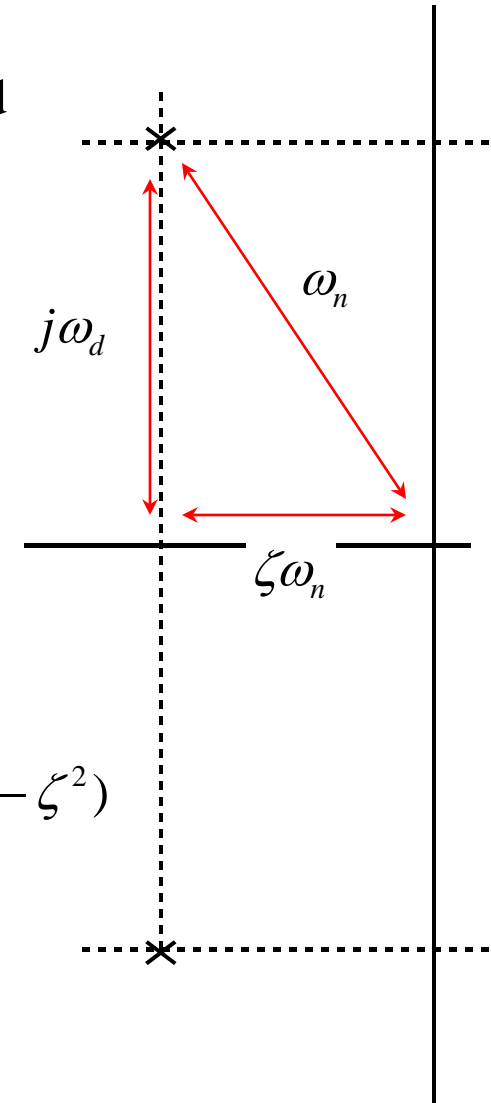
$$\frac{V_{out}}{V_{gen}} \approx k \left( \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + a_1 s + a_2 s^2} \right)$$

If the roots of this are complex, then

$$\frac{V_{out}}{V_{gen}} = k \left( \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + a_1 s + a_2 s^2} \right) = k \left( \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + (2\zeta / \omega_n) s + s^2 / \omega_n^2} \right)$$

$$\frac{V_{out}}{V_{gen}} = k \left( \frac{1 + b_1 s + b_2 s^2 + \dots}{\left( 1 - \frac{s}{-\zeta\omega_n + j\omega_d} \right) \left( 1 - \frac{s}{-\zeta\omega_n - j\omega_d} \right)} \right)$$

$$\omega_d^2 = \omega_n^2 (1 - \zeta^2)$$



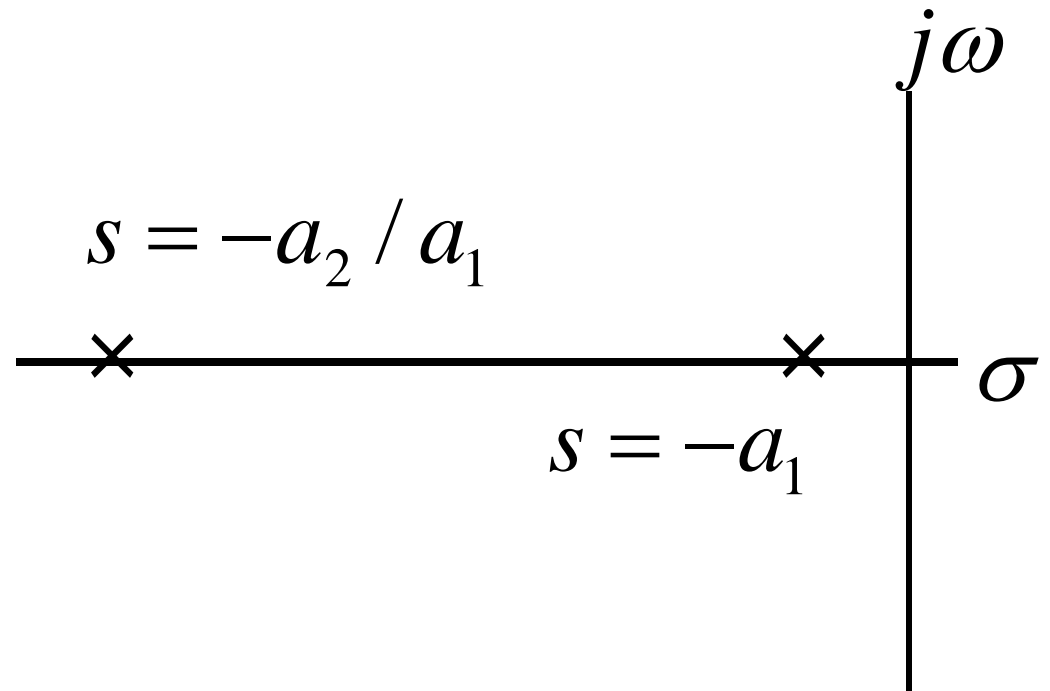
# Finding Poles: Separated Pole Approximation

If the roots are widely separated

e.g.  $(a_2/a_1) \ll a_1$ , then

$$\frac{V_{out}}{I_{in}} = k \left( \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2} \right)$$

$$\frac{V_{out}}{I_{in}} \cong k \frac{1 + b_1s + b_2s^2 + \dots}{(1 + a_1s) \left( 1 + \left( \frac{a_2}{a_1} \right) s \right)}$$

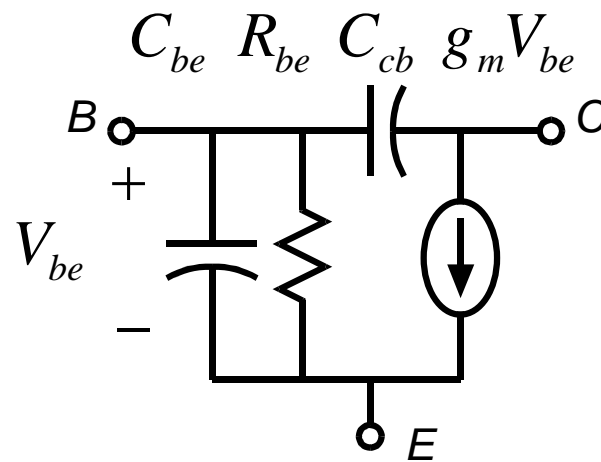


$a_1$  is the dominant pole.

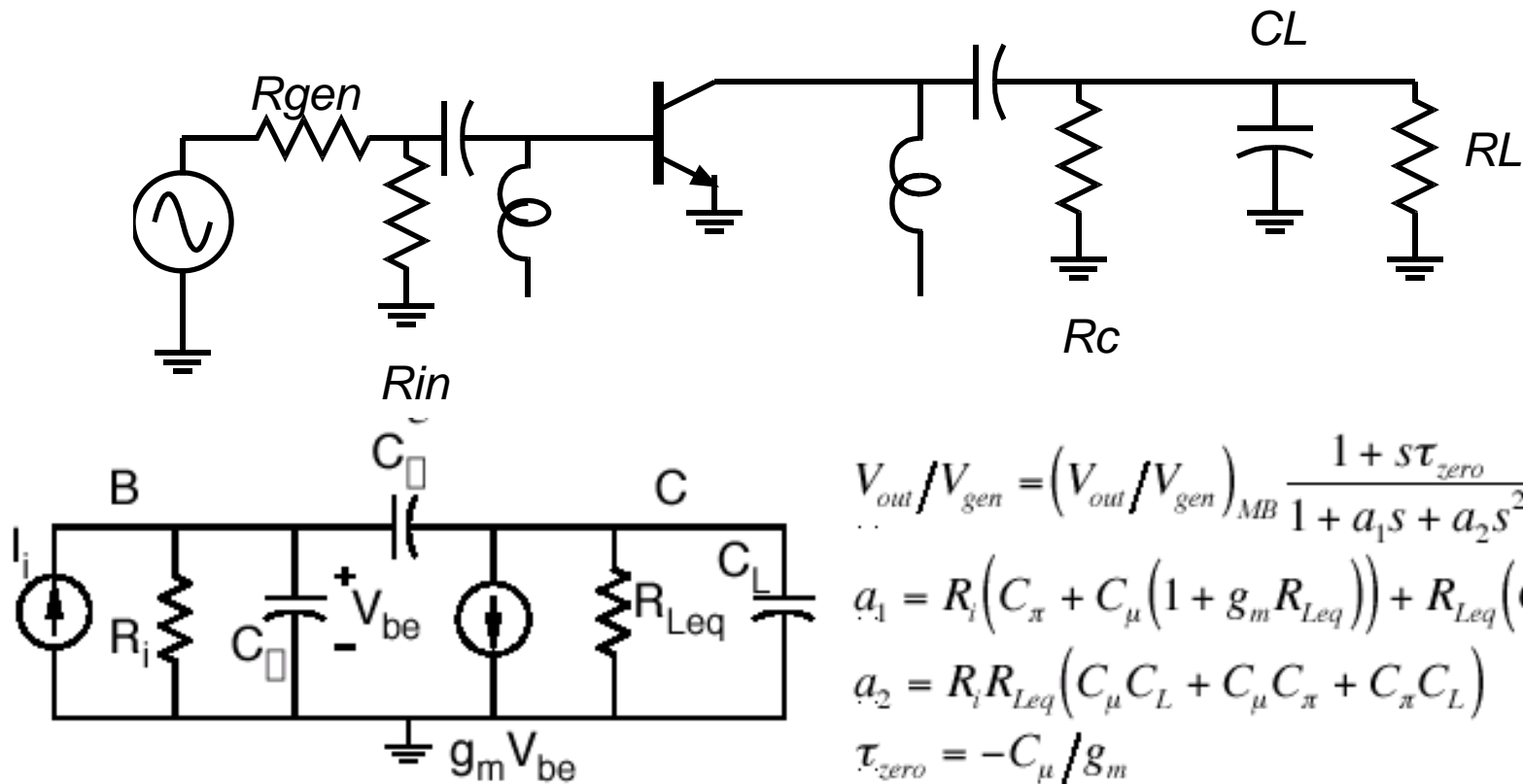
# **Introductory Circuit Design: summary**

# Gain Stages: Elementary Bandwidth Analysis

Using the oversimplified device model below, with  $C_{pi}$  denoting the sum of base-emitter depletion and diffusion capacitances, bandwidth of CE/CB/CC stages can be found....



# CE Stage: Elementary Bandwidth Analysis



$$V_{out}/V_{gen} = (V_{out}/V_{gen})_{MB} \frac{1 + s\tau_{zero}}{1 + a_1s + a_2s^2}$$

$$a_1 = R_i \left( C_{\pi} + C_{\mu} (1 + g_m R_{Leq}) \right) + R_{Leq} (C_{\mu} + C_L)$$

$$a_2 = R_i R_{Leq} (C_{\mu} C_L + C_{\mu} C_{\pi} + C_{\pi} C_L)$$

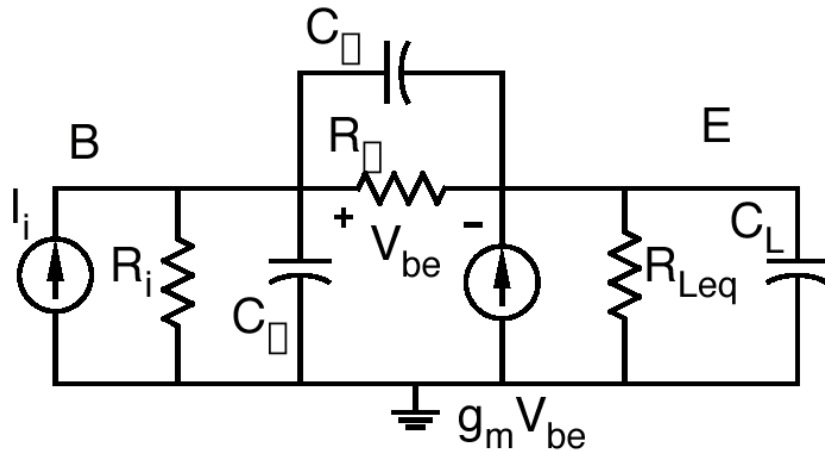
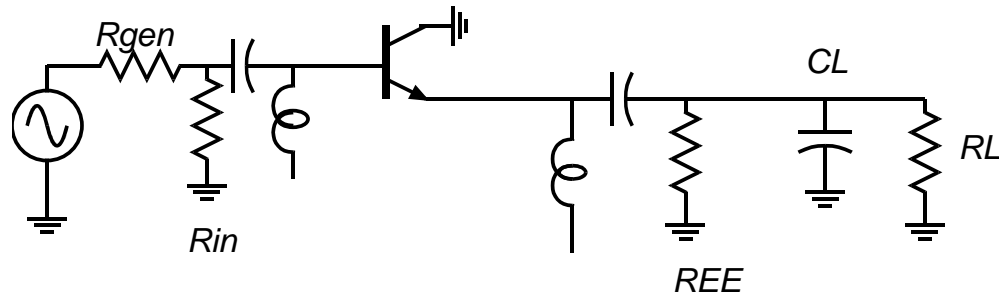
$$\tau_{zero} = -C_{\mu} / g_m$$

$R_i$  is the parallel combination of  $R_{gen}$ ,  $R_{in}$ , and  $R_{\pi}$

$R_{Leq}$  is the parallel combination of  $R_L$ ,  $R_C$ , and  $R_o$

Note in the dominant pole ( $a_1$ ) the miller-multiplication of the collector base capacitance

# CC Stage: Elementary Bandwidth Analysis



$$\frac{V_{out}}{V_{gen}} = \left( \frac{V_{out}}{V_{gen}} \right)_{MB} \frac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2}$$

given that  $A_{vmb} = \left( r_e / (r_e + R_{Leq}) \right)$ :

$$a_1 = C_\pi \left( R_\pi \left\| \left( r_e \parallel R_{Leq} + R_i (1 - A_{vmb}) \right) \right\| \right)$$

$$+ C_\mu \left( R_i \parallel \text{transistor input resistance} \right)$$

$$+ C_L \left( R_{Leq} \parallel \text{transistor output resistance} \right)$$

$$a_2 = \left( R_i \parallel \text{transistor input resistance} \right) \left( R_{Leq} \parallel r_e \right)$$

$$\times \left( C_\mu C_\pi + C_\mu C_L + C_L C_\pi \right)$$

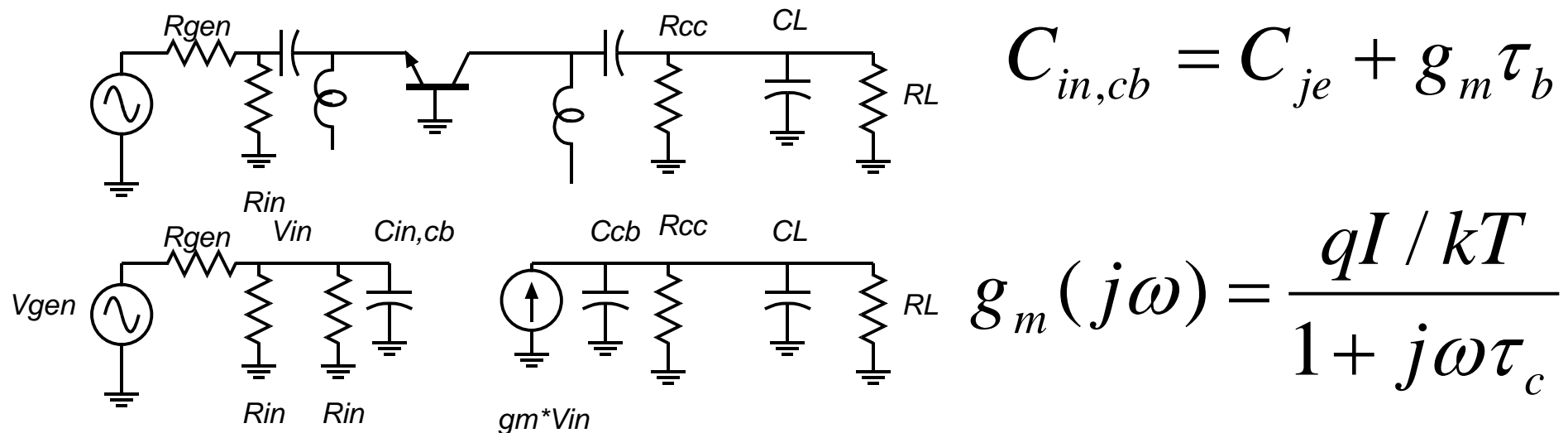
$$\tau_{zero} = g_m / C_\pi$$

$R_i$  is the parallel combination of  $R_{gen}$ , and  $R_{in}$ ,

$R_{Leq}$  is the parallel combination of  $R_{ee}$  and  $R_L$

Note that the frequency response is a mess. Given  $C_L$ , the transfer function very often has complex poles, and may show strong gain peaking, hence ringing in the pulse response.

# CB Stage: Elementary Bandwidth Analysis



Here we have a problem. To the extent that the CB stage is modeled by a very very simple hybrid-pi model (explicitly, with zero  $R_{bb}$ ), we find (by very simple analysis) very high bandwidth, with poles having time constants equal to  $\tau_b$ , to  $\tau_c$ , and to the product of the load resistance times  $(C_{cb} + C_L)$ .

## Note that

- 1) **Input capacitance is indeed as noted. Does not include effect of  $\tau_c$**
- 2) **Ignoring  $R_{bb}$  in CB stage analysis, while appealing for simplicity (e.g. undergrad classes) is quite unreasonable, as  $C_{cb}R_{bb}$  often dominates high frequency rolloff.** More regarding this later.

# **Method of Time Constants**



make revision for 2008----first before MOTC, give by summary without derivation the standard stage expressions.

then define MOTC, first and second order  
then show a 1-stage Darlington diff amp, and say caps to ground, caps between inputs and outputs.

Give expression for caps to ground

Give expression for caps between in and out of general block

then use this for CD stage  $C_{gs}$  only

then use this for CC stage  $C_{be}$  only

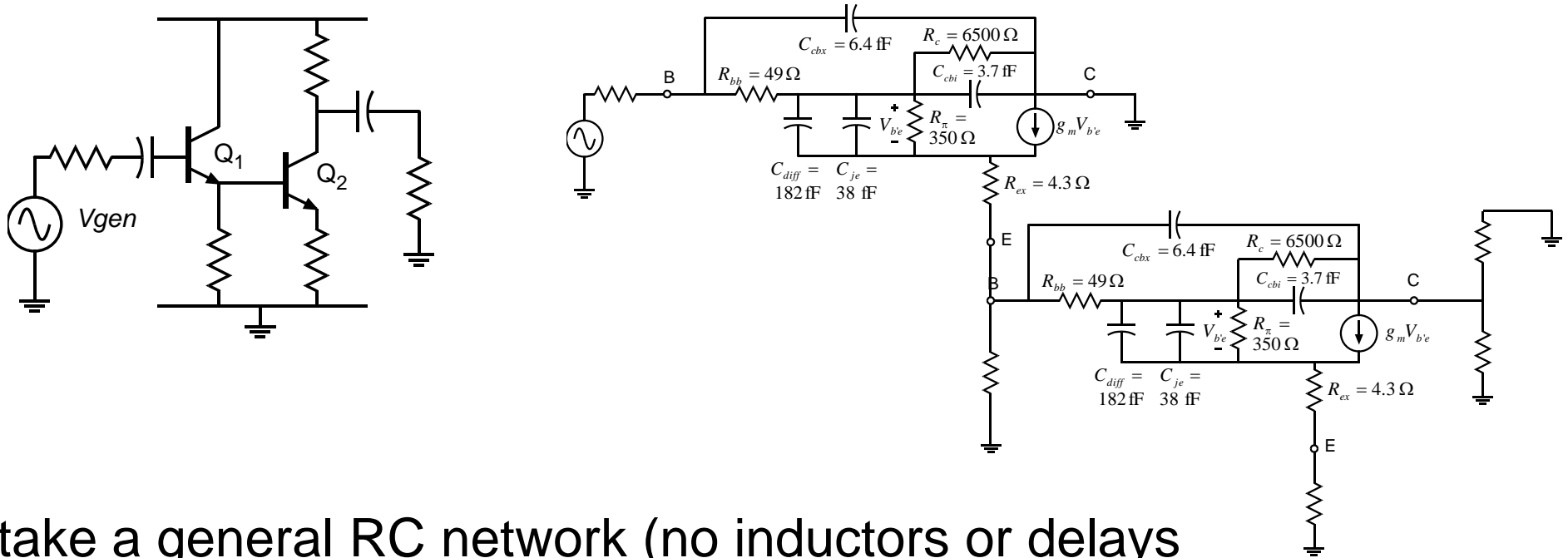
then do for CE stage  $C_{cb}$  only

then work the full Darlington diff amp

then show how CE (with degen) CB CC are same problem

then re-show stage relationship

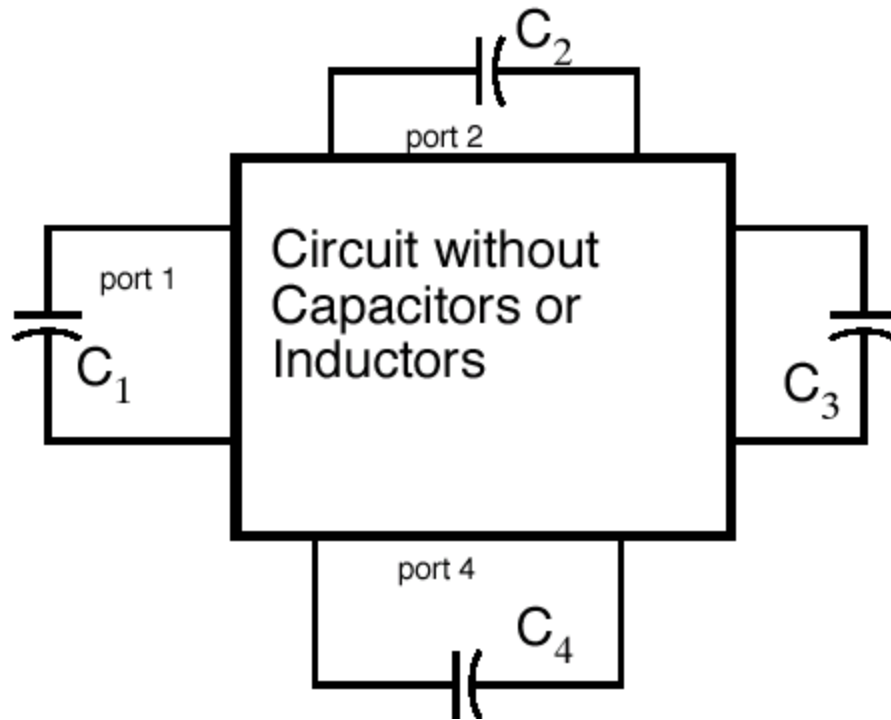
# Finding Bandwidth: Method of Time Constants



take a general RC network (no inductors or delays tau), and separate into 2 parts, network without capacitors, and the capacitors:

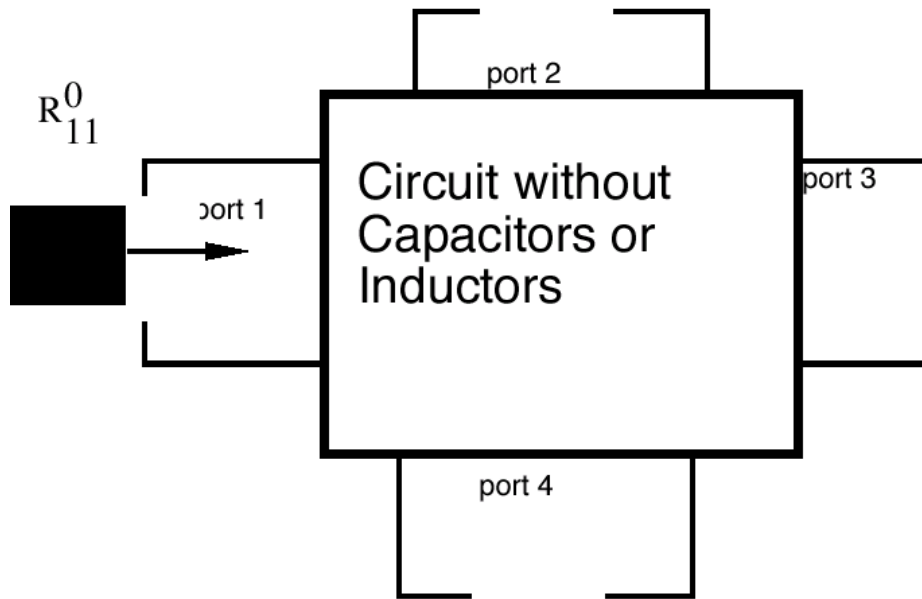
# MOTC: Separation into Capacitors & Resistive N-port

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The internal capacitor-free network is now frequency-independent. The MOTC method (not proven here) relies on results from n-port network theory

# MOTC: Open-Circuit Resistances



*work examples  
to illustrate this*

$R_{11}^0$  is the small signal resistance measured at port one with all other ports open - circuited. This is determined by applying a test voltage (or current) at the port and computing from this the resulting current (or voltage)

# MOTC: the Dominant Time Constant

---

$$\frac{V_{out}}{V_{gen}} = \left( \frac{V_{out}}{V_{gen}} \right)_{mb} \frac{1 + b_1s + b_2s^2 + \dots}{1 + a_1s + a_2s^2 + \dots}$$

$$a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 + R_{44}^0 C_4$$

The MOTC first - order time constants directly give us the dominant time constant  $a_1$  of the circuit. If (and only if) the secondary time constant  $a_2$  is negligible, the 3 - dB bandwidth is  $1 / 2\pi a_1$ . We must use the second - order (short - circuit) time constants to determine  $a_2$ .

# MOTC: Are We Saving Any Work ?

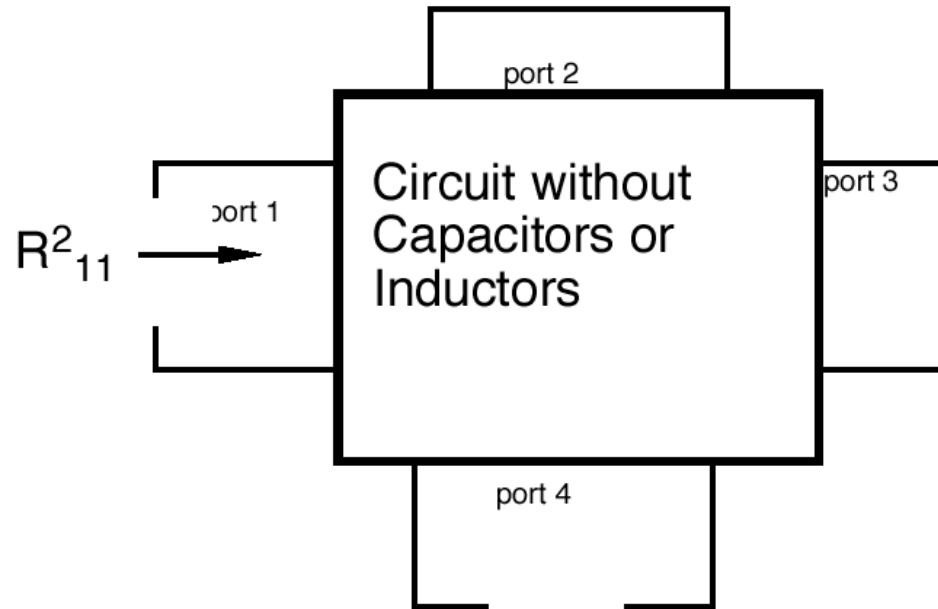
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Are we saving work relative to brute-force nodal analysis: MOTC would be of only moderate value if we had to calculate all the  $R_i$ 's each time. Fortunately, most terms involve quantities ***already found in midband stage analysis: input and output impedances, load impedances, etc.***

*work examples to illustrate this*

# MOTC: Short-Circuit Resistances

---



$R_{11}^2$  is the small signal resistance measured at port one with all other ports open - circuited, except for port 2, which is shorted

# MOTC: The Second-Order Time-Constant

---

$$\frac{V_{out}}{V_{gen}} = \left( \frac{V_{out}}{V_{gen}} \right)_{mb} \frac{1 + b_1 s + b_2 s^2 + \dots}{1 + a_1 s + a_2 s^2 + \dots}$$

$$a_2 = R_{11}^0 R_{22}^1 C_1 C_2 + R_{11}^0 R_{33}^1 C_1 C_3 + R_{11}^0 R_{44}^1 C_1 C_4 \\ + R_{22}^0 R_{33}^2 C_2 C_3 + R_{22}^0 R_{44}^2 C_2 C_4 + R_{33}^0 R_{44}^3 C_3 C_4$$

notethat  $R_{xx}^0 R_{yy}^x = R_{xx}^y R_{yy}^0$



# MOTC: An Example

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*work example...second order terms  
in either CE stage or CC stage....*

# MOTC: Working these Efficiently

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Because  $R_{xx}^y R_{yy}^0 = R_{xx}^0 R_{yy}^x$ , we always have 2 choices in finding each term in the MOTC. The trick is to work the problem so that as much as possible :

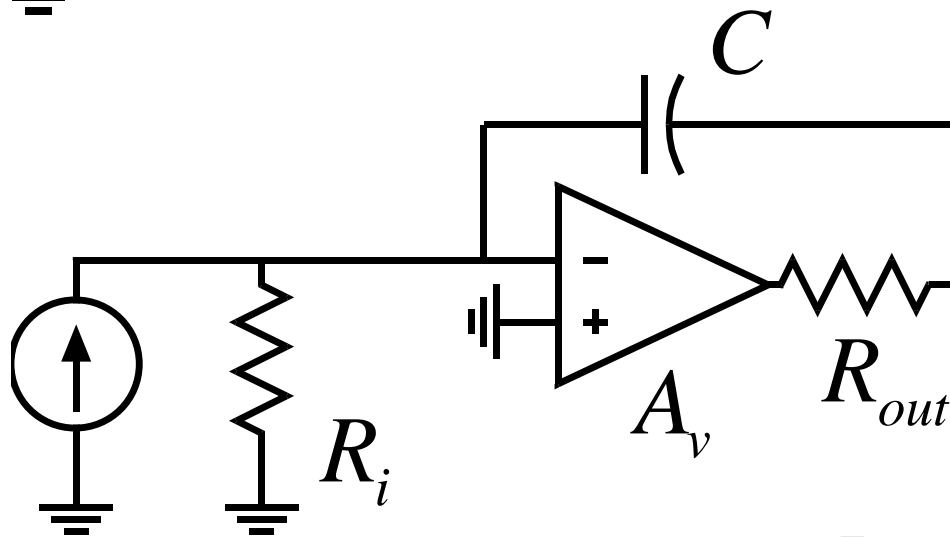
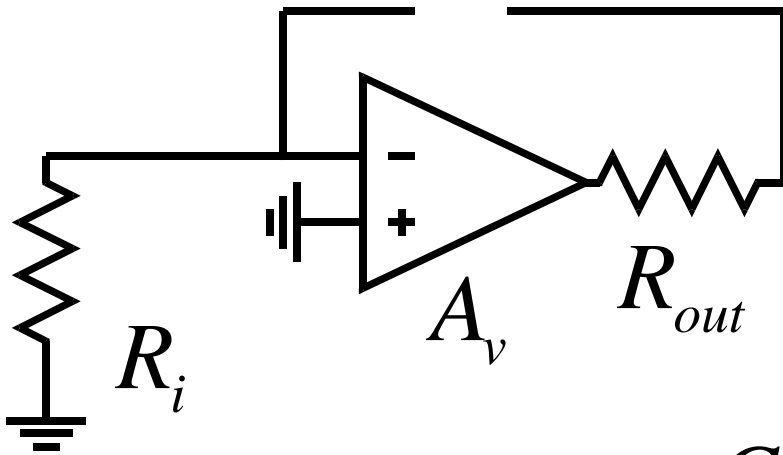
- 1) terms are related to input, output, load impedances
- 2) terms are ones found earlier, in  $a_1$  analysis.

There are 2 "funny" cases which arise so often that I will give them on the next 2 pages (note these are intimately related to the well-known Miller effect)

# MOTC and the Miller Effect

derive

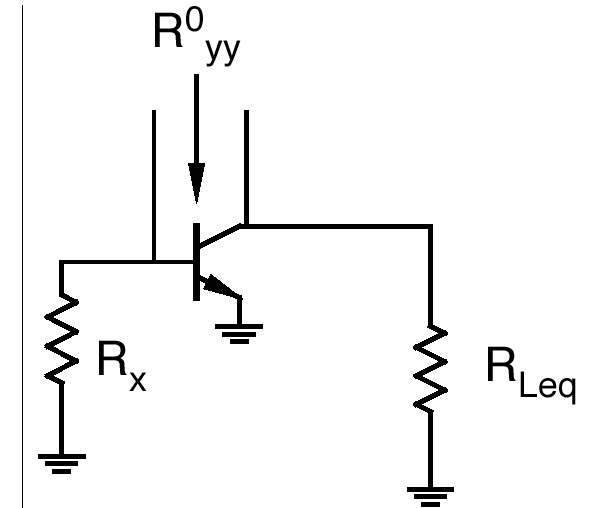
$$R_{xx}^y = R_i (1 + A_v) + R_{out}$$



$$a_1 = \tau = [R_i (1 + A_v) + R_{out}] \bullet C$$

# MOTC: port impedances between collector and base

*derive*



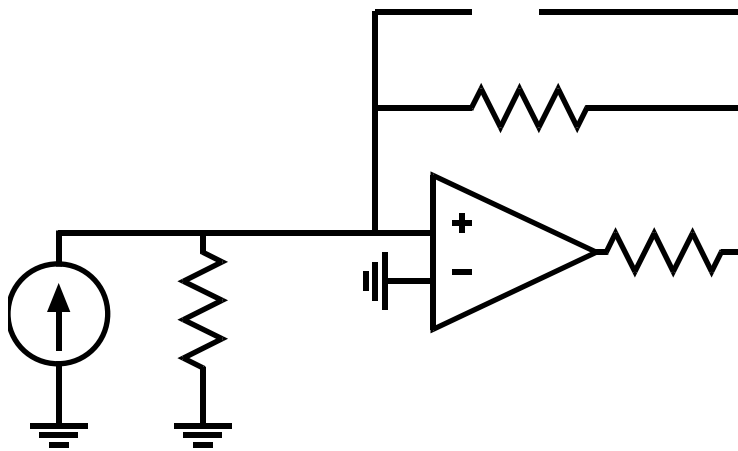
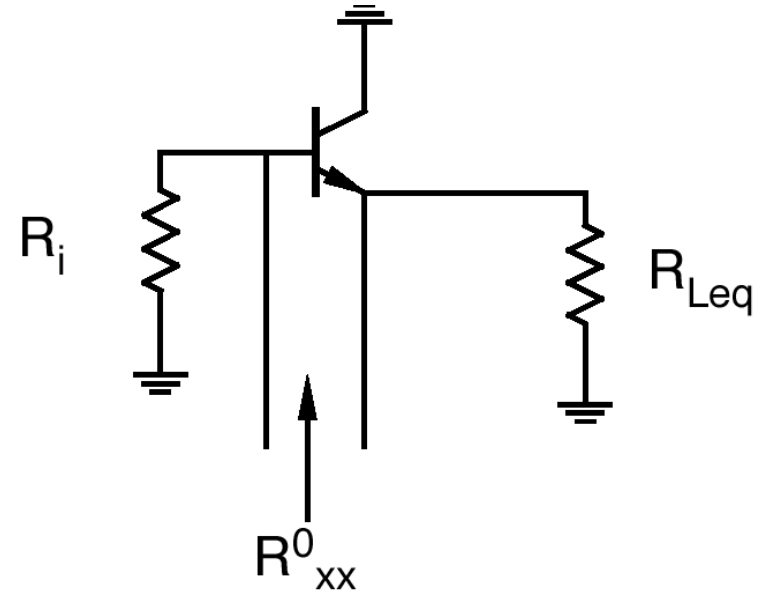
If we decide explicitly that  $R_x$  is to denote the parallel combination of any external circuit resistances and  $R_{be}$ , and that  $R_{Leq}$  similarly denotes the combined effect of external resistors and  $R_{ce}$ , then

$$R_{yy}^0 = R_x (1 + g_m R_{Leq}) + R_{Leq}$$

# MOTC: Port Impedances Between Emitter & Base

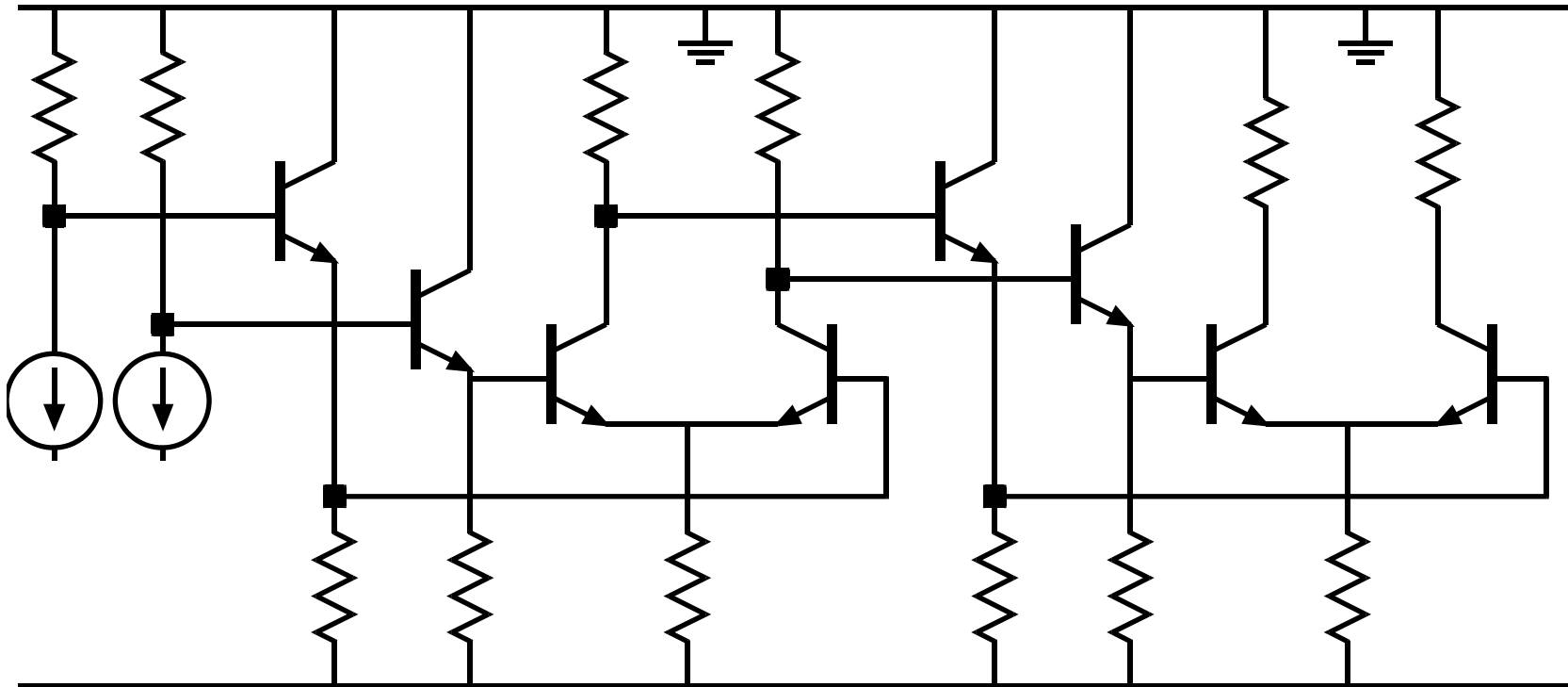
$$R_{xx}^0 = R_{\pi} \parallel \left( r_e \parallel R_{Leq} + R_i (1 - A_{vmb}) \right)$$

$$A_{vmb} = \left( R_{Leq} / (r_e + R_{Leq}) \right)$$

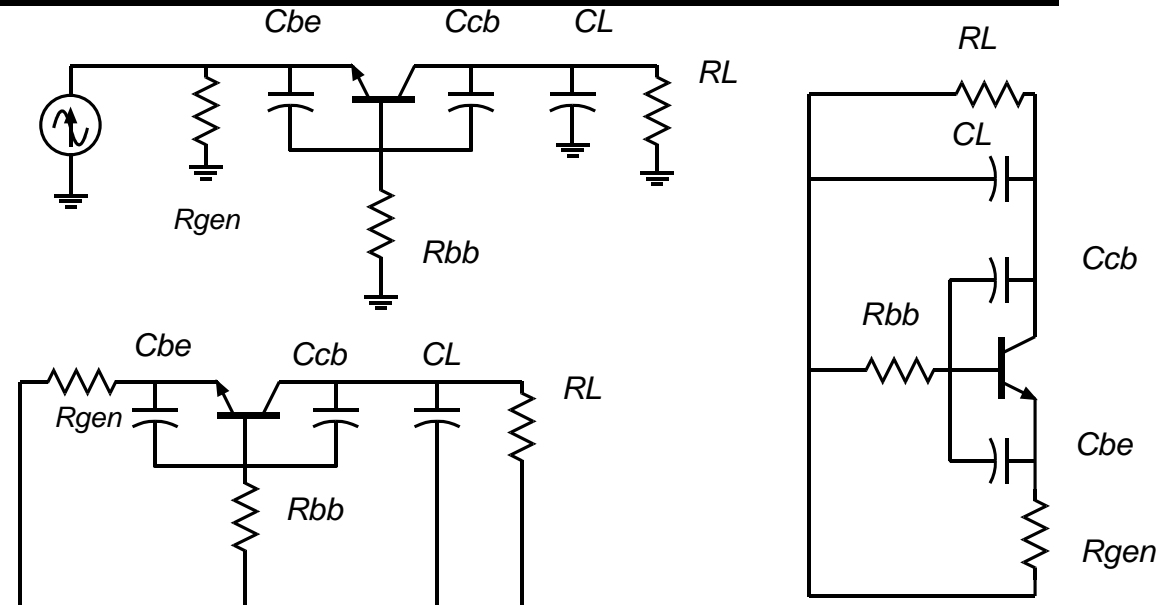


# MOTC: Multistage Example

*work on the board...*



# Common-base stage by MOTC



$$a_1 = C_{be} \left( R_{bb} \left( 1 - \frac{R_{gen}}{R_{gen} + r_e} \right) + r_e \parallel r_{gen} \right) \parallel r_{be}$$

$$+ C_{cb} \left( R_{bb} \left( 1 + \frac{R_L}{r_e + R_{gen}} \right) + R_L \right) + C_L R_L$$

$$a_2 = \dots$$

note that because  $a_1$ ,  $a_2$  are independent of location of input and output, the form is identical to that of CE stage.

Note also the Miller multiplication effect with  $R_{bb} C_{cb}$

# What's in next lecture ?

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Relating amplifier gains to S-parameters

Cascaded common-emitter amplifiers,  
gain-bandwidth and  $f_t$  limits

Resistive feedback amplifiers for higher bandwidths

The Cherry-Hooper (transconductance-transimpedance) design

Darlington stages, benefits, limits, and headaches

$f_t$ -doubler stages

Distributed amplifiers (briefly)

And later: input tuning for (1) improving  $S_{11}$ ,  $S_{22}$  and (2) improving bandwidth.

.....



**End**