Mixed Signal IC Design Notes set 7: Electrical device noise models.

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Topics

Math: distributions, random variables, expectations, pairs of RV, joint distributions, mean, variance, covariance and correlations. Random processes, description, stationarity, ergodicity, correlation functions, autocorrelation function, power spectral density.

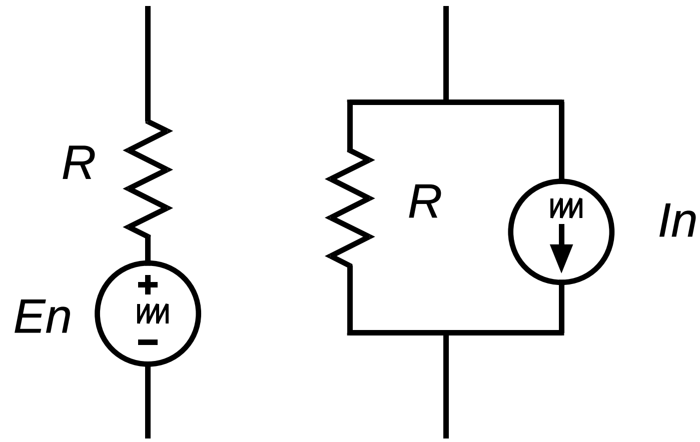
Noise models of devices: thermal and "shot" noise. Models of resistors, diodes, transistors, antennas.

Circuit noise analysis: network representation. Solution. Total output noise. Total input noise. 2 generator model. En/In model. Noise figure, noise temperature. Signal / noise ratio.

Thermal Noise

$$\frac{d\langle E_n E_n^* \rangle}{df} = 4R * \left[\frac{hf}{2} + \frac{hf}{\exp(hf/kT) - 1} \right]$$

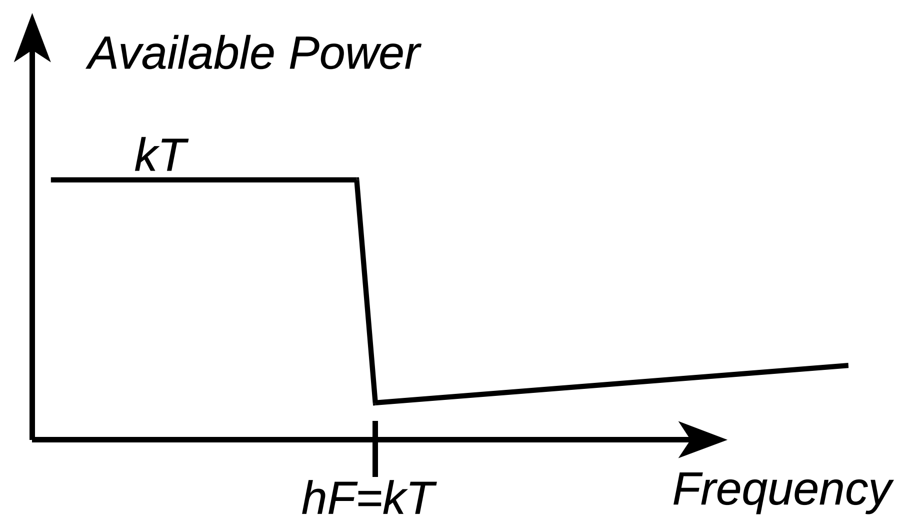
$$\frac{d\langle I_n I_n^* \rangle}{df} = \frac{4}{R} * \left[\frac{hf}{2} + \frac{hf}{\exp(hf/kT) - 1} \right]$$



For $hf << kT$ these become

$$\frac{d\langle E_n E_n^* \rangle}{df} = 4kTR$$

$$\frac{d\langle I_n I_n^* \rangle}{df} = \frac{4kT}{R}$$

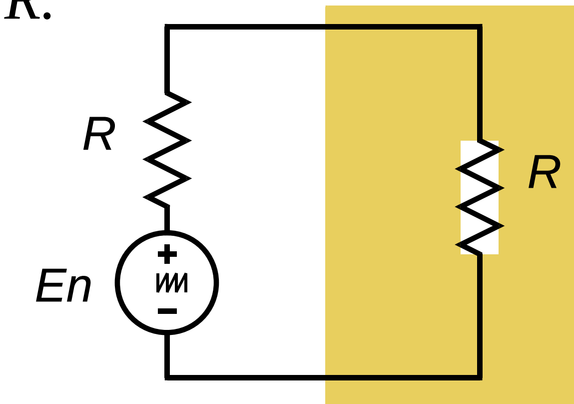


Available Thermal Noise Power

Maximum power transfer : load R matched to generator R.

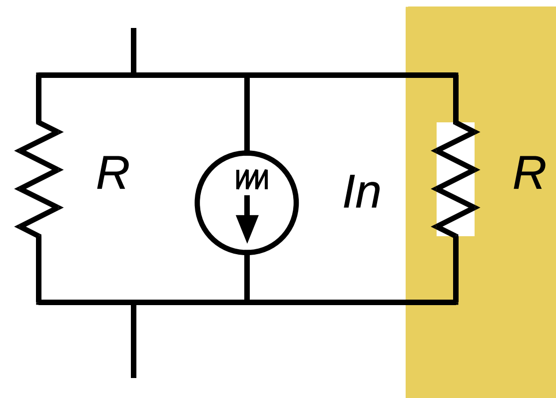
With matched load, voltage across load is $E_N / 2$

With matched load, current through load is $I_N / 2$



Given that

$$\frac{d\langle E_n E_n^* \rangle}{df} = 4kTR \text{ or } \frac{d\langle I_n I_n^* \rangle}{df} = \frac{4kT}{R} \Rightarrow \frac{d\langle P_{load} \rangle}{df} = kT$$



P_{load} is the maximum (the available) noise power, hence

$$\frac{d\langle P_{available,noise} \rangle}{df} = kT$$

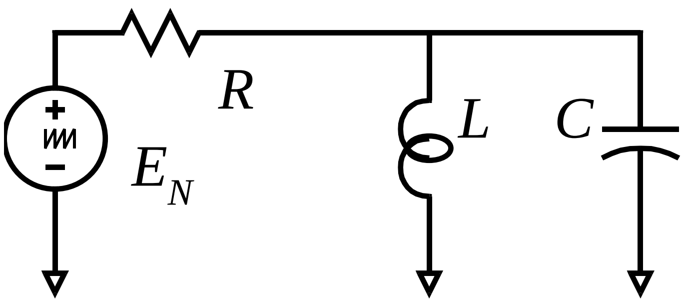
All resistors have equal available noise power.

Any compononent under thermal equilibrium (no bias) follows this law.

Rough Argument for Thermal Noise Expression

Each degree of freedom in a system at thermal equilibrium with a reservoir must have expected energy $kT/2$.

Thermal equilibrium is obtained via the resistor dissipation : circuit power $\rightarrow$ heat
thermal noise : heat $\rightarrow$ circuit power



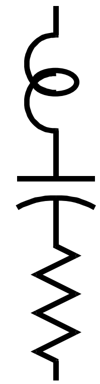
A filter of 1 Hz bandwidth, observed for 1 second, has two degrees of freedom ($\sin(2\pi f_o)$ & $\cos(2\pi f_o)$), hence the total available power in this bandwidth must be $kT\Delta f$.

Noise from any impedance under thermal equilibrium

For any component or complex network under thermal equilibrium
(no energy supply)

$$\frac{d\left\langle P_{available,noise}\right\rangle}{df} = kT$$

$$\Rightarrow \frac{d\left\langle E_n E_n^*\right\rangle}{df} = 4kT\,\mathrm{Re}(Z) \text{ or } \frac{d\left\langle I_n I_n^*\right\rangle}{df} = 4kT\,\mathrm{Re}(Y)$$



This follows from the 2nd law of thermodynamics.

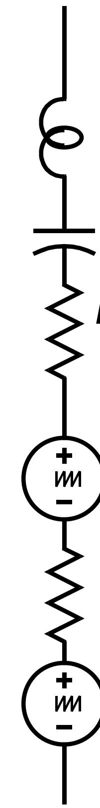
This allows quick noise calculation of complex passive networks

This allows quick noise calculation of antennas.

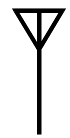
Biased semiconductor devices are NOT in thermal equilibrium.

Noise from an Antenna

$$\frac{d\langle P_{available,noise}\rangle}{df} = kT \Rightarrow \frac{d\langle E_n E_n^* \rangle}{df} = 4kT\,\mathrm{Re}(Z)$$



The antenna has both Ohmic and radiation resistances.

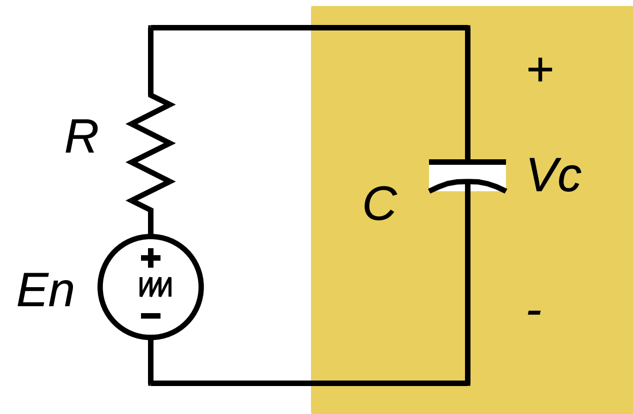


The Ohmic resistance has a noise voltage of spectral density $4kT_{ambient}R_{Ohmic}$, where $T_{ambient}$ is the physical antenna temperature

By the 2nd law, the radiation resistance has a noise voltage of spectral density $4kT_{field}R_{rad}$, where T_{field} is the average temperature of the region from which the antenna receives signal power

Inter - galactic space is at 3.8 Kelvin....

Noise on a capacitor



From

$$\frac{d\langle E_n E_n^*\rangle}{df} = 4kTR \quad \text{or} \quad \frac{d\langle I_n I_n^*\rangle}{df} = \frac{4kT}{R}$$

We find that

$$\frac{d\langle V_c V_c^*\rangle}{df} = \left(\frac{1}{1+j2\pi fRC}\right)\left(\frac{1}{1+j2\pi fRC}\right)^* \frac{d\langle E_n E_n^*\rangle}{df}$$

$$= \left(\frac{1}{1+4\pi^2 f^2 R^2 C^2}\right)\frac{d\langle E_n E_n^*\rangle}{df}$$

So the mean stored Capacitor energy is

$$(1/2)C\langle V_c V_c\rangle = \int_0^\infty \left(\frac{1}{1+4\pi^2 f^2 R^2 C^2}\right)\frac{d\langle E_n E_n^*\rangle}{df}df = kT/2$$

This also follows directly from the Boltzmann law.

Shot noise

The proof (not given) extends upon our earlier discrete - time shot - noise calculation.

If, given a DC current I, the arrival of each electron is statistically independent of every other electron,

then the current has a noise power spectral density at lower frequencies of

$$\frac{d\left\langle I_n I_n^* \right\rangle}{df} = 2qI$$

Most DC currents in circuits are *not* a statistically independent flow of electrons.

The electron motion in a resistor generates local fields which influence the flow of all other electrons. Classical resistors do not exhibit shot noise

Shot Noise Example: Heavily Attenuated Light

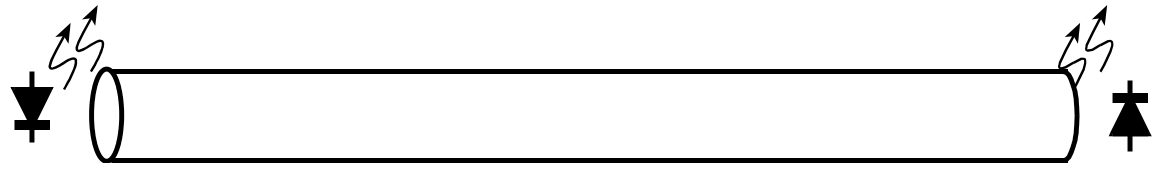
An optical fiber is illuminated by a noiseless source of optical power P_{in} and produces a flux of photons per unit time of $P_{in}/h\upsilon_{optical}$

The fiber has attenuation α, hence the received optical power is $F_{in} = P_{out} = \alpha P_{in}$ and the received flux of photons is $F_{out} = P_{out}/h\upsilon_{optical}$. Becuase each photon passes through the fiber with probablity α, the process again has shot noise with

$$\frac{d\left\langle P_{out}P_{out}^{*}\right\rangle}{df} = 2h\upsilon P_{out}$$

This produces on a photodetector with quantum efficiency η a photocurrent $I_{ph} = (\eta q/h\upsilon)P_{out}$ with a shot noise of

$$\frac{d\left\langle I_{out}I_{out}^{*}\right\rangle}{df} = 2qI_{out}$$



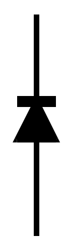
Shot noise Example: Reverse Biased Schottky Diode

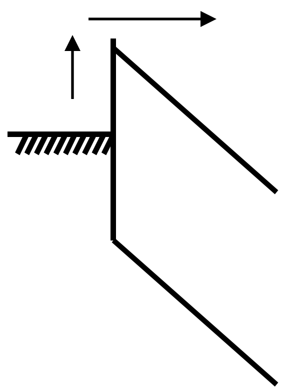
In the metal, electrons have a Fermi - Dirac energy distribution.
Some will have sufficient energy to cross over the barrier.

This produces reverse leakage current.

These events are (almost) independent, hence the resulting current has a noise spectral density of

$$\frac{d\left\langle I_{leak} I_{leak}^{*} \right\rangle}{df} = 2qI_{leak}$$

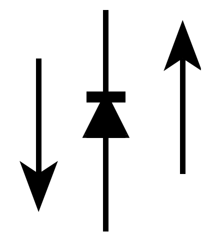




Shot noise* in PN junctions

The diode current is

$$I_{diode} = I_s\left(e^{qV/kT} - 1\right) = I_s e^{qV/kT} - I_s = I_{forward} + I_{reverse}$$



Both the forward and reverse currents have shot noise, hence

$$\frac{d\left\langle I_{diode} I^*_{diode} \right\rangle}{df} = 2qI_{forward} + 2qI_{reverse} = 2q\left(I_{diode} + 2I_s\right)$$

Under strong forward bias, $d\left\langle I_{diode} I^*_{diode} \right\rangle / df = 2qI_{diode}$

under strong reverse bias, $d\left\langle I_{diode} I^*_{diode} \right\rangle / df = 2qI_s$

Under zero bias, $d\left\langle I_{diode} I^*_{diode} \right\rangle / df = 4kT / r_{diode}$, as required by the 2nd law.

* Van der Zeil derives noise in PN junctions from the thermal noise of carrier diffusion.
The noise current spectral density thus calculated is equal to that of shot noise.

Shot noise and PN junctions: another model

For a strongly forward biased junction

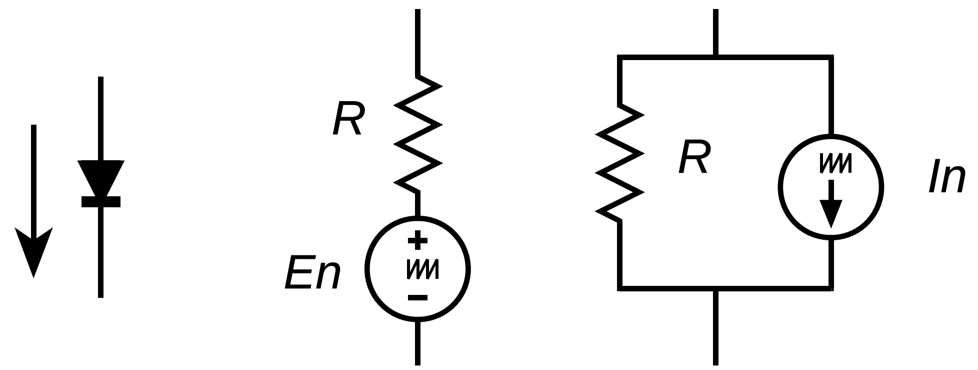
$$\frac{d\left\langle I_{diode} I_{diode}^{*}\right\rangle}{df} = 2qI_{diode} = 2kT / r_{diode} \text{ where } r_{diode} = kT / qI_{diode}$$

or

$$\frac{d\left\langle V_{diode} V_{diode}^{*}\right\rangle}{df} = 2kTr_{diode}$$

hence

$$\frac{d\left\langle P_{diode}\right\rangle}{df} = kT / 2$$

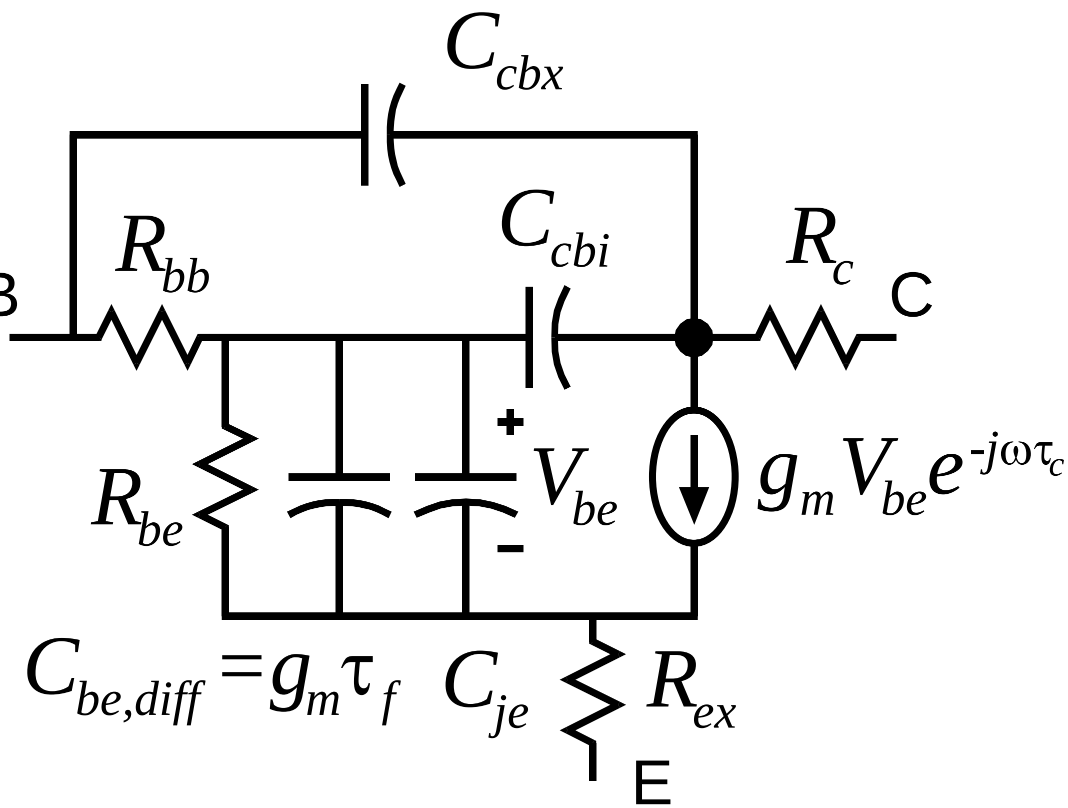


A biased diode has noise 1/2 that of a resistor of equal small-signal impedance.

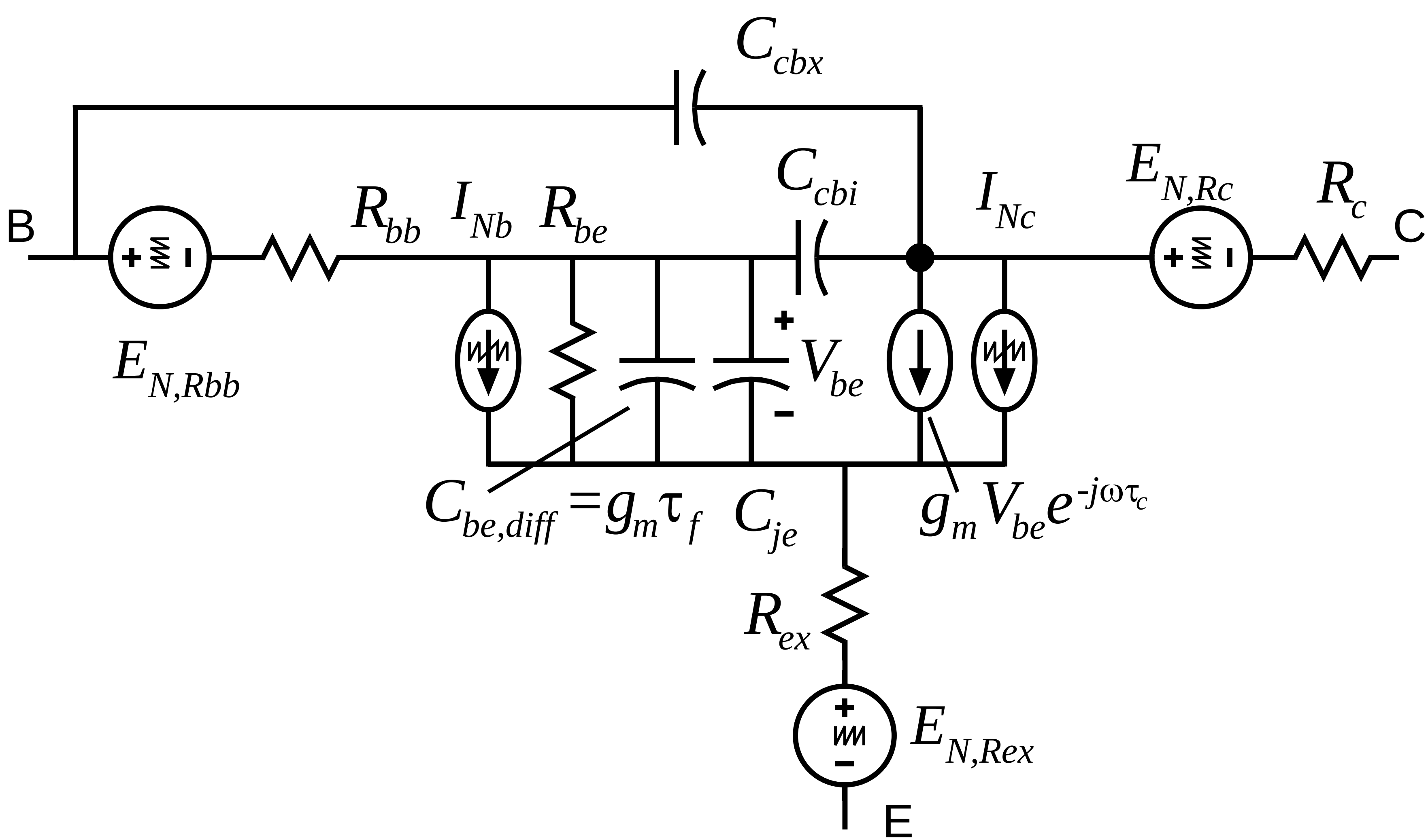
The factor of 2 arises from one-way current flow.

* Van der Zeil derives noise in PN junctions from the thermal noise of carrier diffusion. The noise current spectral density thus calculated is equal to that of shot noise.

Bipolar Transistor Model---without Noise



Bipolar Transistor Model---with Noise



Bipolar Noise Model

Collector shot * noise

$$\frac{d\left\langle I_{nc} I_{nc}^{*}\right\rangle}{df} = 2qI_c = 2kT/r_e = 2kTg_m$$

Base shot * noise

$$\frac{d\left\langle I_{nb} I_{nb}^{*}\right\rangle}{df} = 2qI_b = 2kT/r_{be}$$

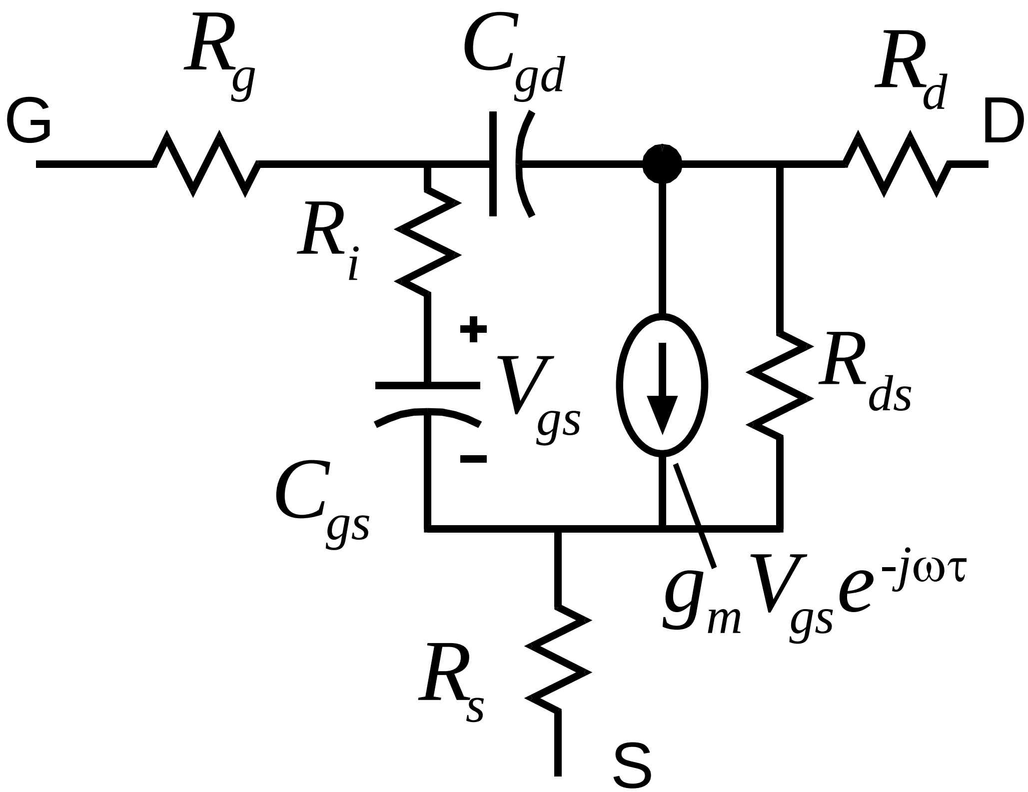
There is a slight correlation of I_{nb} and I_{nc} (a cross-spectral density) when $2\pi f(\tau_b + \tau_c)$ approaches 1. We will ignore this small effect.

The physical resistors (R_{bb}, R_{ex}, R_c) have thermal noise of spectral density $d\left\langle V^2\right\rangle / df = 4kTR$

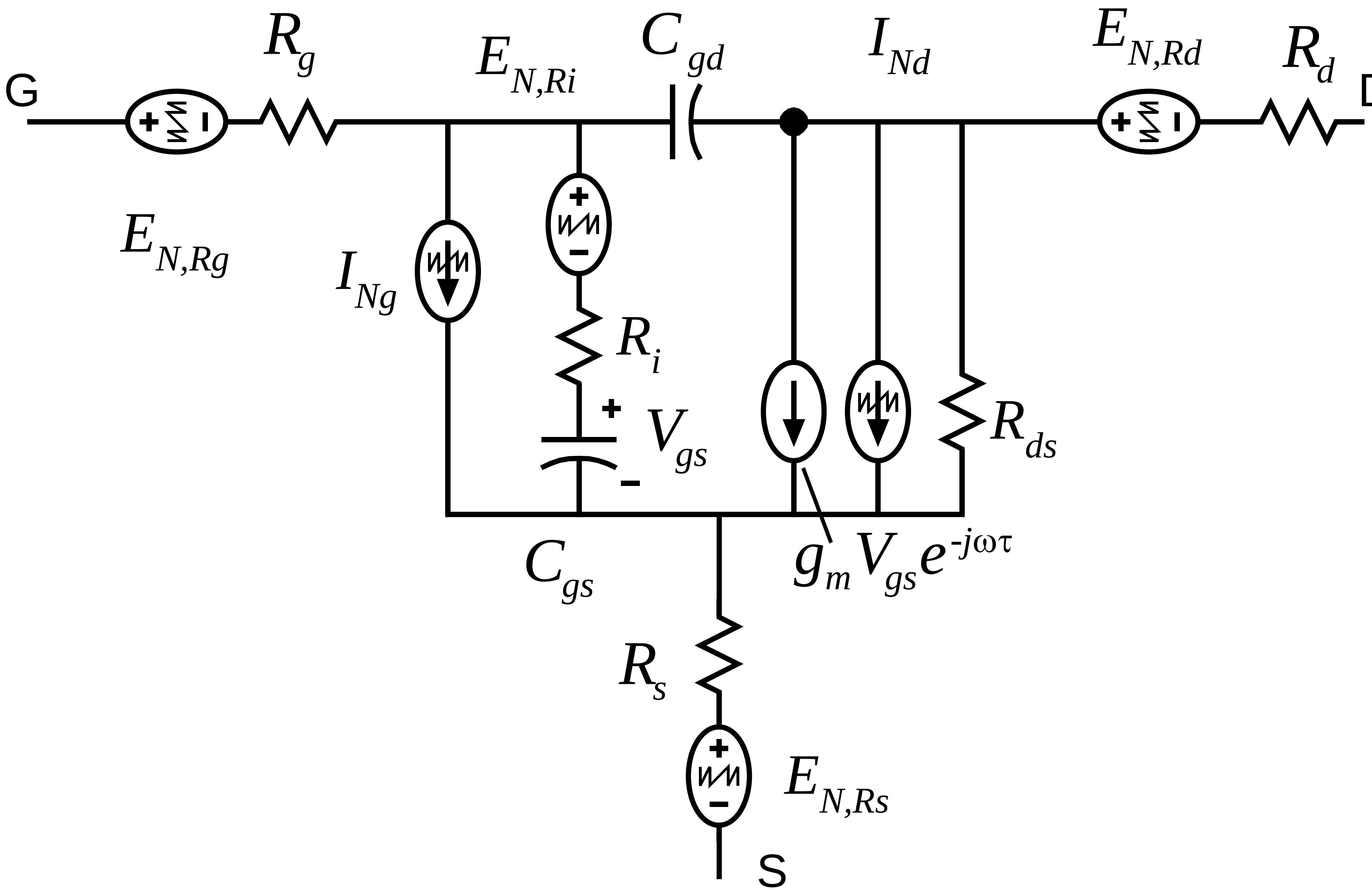
R_{be} and $r_e = 1/g_m$ are not physical resistors.
The noise of R_{be} and r_e are the base and the collector shot * noise generators.

* Van der Zeil derives noise in PN junctions from the thermal noise of carrier diffusion. The noise current spectral density thus calculated is equal to that of shot noise.

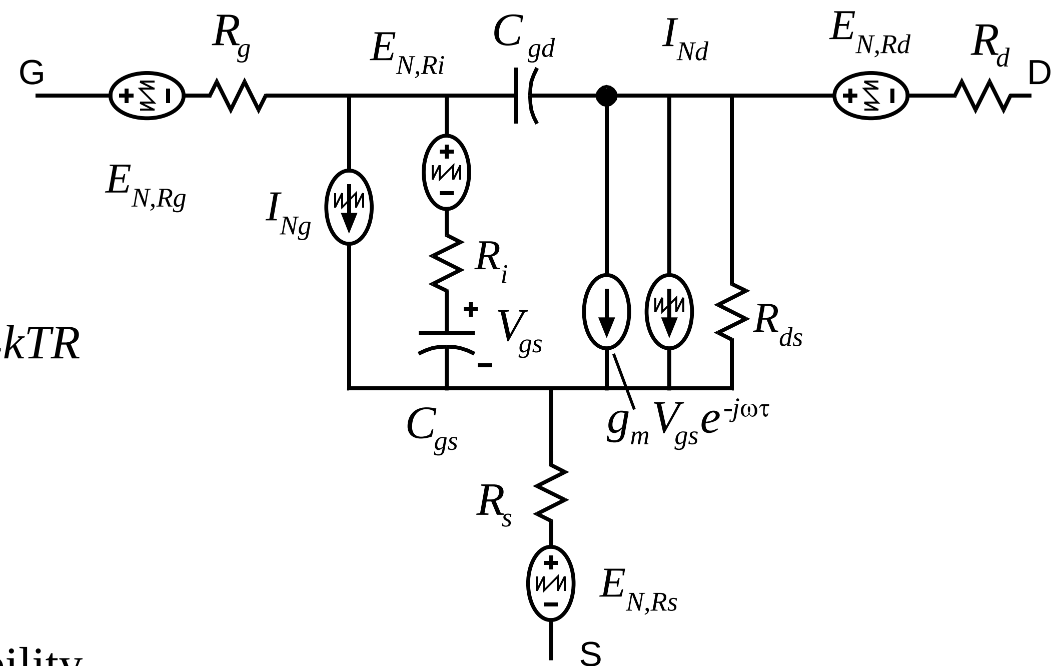
FET Small-Signal Model



FET Noise Model



FET Noise model



R_g, R_s, R_d are physical resistances $\Rightarrow d\langle V^2\rangle / df = 4kTR$

I_{nd} is the thermal noise of the channel current

$d\langle I_{nd} I_{nd}^*\rangle / df = 4kT\Gamma g_m$

$\Gamma = 2/3$: gradual-channel FET under constant mobility

$\Gamma \sim 1-1.5$: highly scaled FET under high-field conditions

R_i arises from the channel: $E_{N,Ri}$ and I_{nd} have small correlation

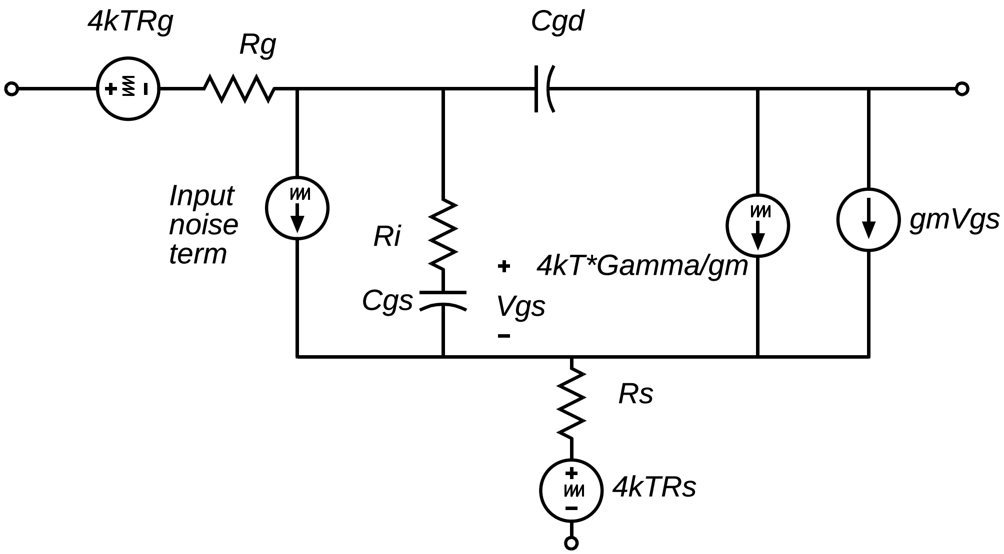
Effect is negligible $\rightarrow$ approximate as uncorrelated.

$d\langle E_{N,R_i} E_{N,R_i}^*\rangle / df \cong 4kTR_i$

I_{ng} is the shot noise of the gate leakage current: $d\langle I_{ng} I_{ng}^*\rangle / df = 2qI_{gate}$

R_{ds} is not a physical resistor - no associated noise generator.

Alternate FET Noise model



You will also see this in the literature. There is no new physics here at all.

We have just applied the formula that for a general impedance

$$\frac{d\left\langle I_n I_n^* \right\rangle}{df} = 2qI_{gate} + 4kT\,\mathrm{Re}(Y) = 2qI_{gate} + 4kT * \left(2\pi f C_{gs}\right)^2 R_i$$

...to get the input in terms of a current.

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Math: distributions, random variables, expectations, pairs of RV, joint distributions, mean, variance, covariance and correlations. Random processes, description, stationarity, ergodicity, correlation functions, autocorrelation function, power spectral density.

Noise models of devices: thermal and "shot" noise. Models of resistors, diodes, transistors, antennas.

Circuit noise analysis: network representation. Solution. Total output noise. Total input noise. 2 generator model. En/In model. Noise figure, noise temperature. Signal / noise ratio.