

Sidestepping the Rayleigh limit for LoS spatial multiplexing: a distributed architecture for long-range wireless fiber

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Abstract—While existing wireless systems rely on rich scattering environments to obtain spatial multiplexing gains, such gains are also available for line-of-sight (LoS) point-to-point links if the range is small enough. For fixed antenna separation (limited by node form factor) and carrier frequency (limited by available electronics), the range up to which multiple degrees of freedom are available is limited by the Rayleigh criterion. In this paper, we propose a distributed architecture for sidestepping this criterion, leading to spatial multiplexing gains at ranges much larger than those dictated by node form factor and carrier frequency constraints. Although the antenna spacings at the transmitter and the receiver are not large enough to support spatial multiplexing, the use of relays spread out over a larger area enables synthesis of a full rank MIMO channel. We focus specifically on the design of air-to-ground wireless fiber links over ranges of tens of kilometers, which have the following features in addition to relay-enabled spatial multiplexing: (a) the small carrier wavelength enables large beamforming gains even for nodes of compact form factor, (b) the large available bandwidths enable multiGigabit speeds for each spatially multiplexed data stream.

I. INTRODUCTION

In this paper, we present an architecture for LoS wireless links at speeds and ranges comparable to optical fiber (10-100 Gbps over 10-50 km), utilizing spatial multiplexing in addition to the large bandwidths available at millimeter (mm) wave carrier frequencies. Conventional wisdom says that the full rank MIMO matrix required for spatial multiplexing requires a rich scattering environment. It is known, however, that spatial multiplexing can also be obtained in LoS environments provided the transmit and receive antennas are spaced far enough apart, where the required antenna spacing increases with the range and decreases with the carrier frequency. The required spacing is related to the Rayleigh criterion from diffraction-limited optics, which also matches results from information-theoretic analysis on the number of available degrees of freedom [1]. Whether or not such Rayleigh spacings can be realized depends on form factor constraints. For existing WiFi and cellular systems at 1-5 GHz, these constraints imply that

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LoS spatial multiplexing is not possible for typical ranges of interest (hence the conventional wisdom on rich scattering being required). However, advances in radio frequency integrated circuit (RFIC) and packaging techniques are easing access to higher carrier frequencies, for which LoS spatial multiplexing starts becoming feasible and attractive. For example, recent work [1] shows that LoS (and near-LoS) spatial multiplexing can be supported in indoor mm wave links with ranges of the order of 10 m, with Rayleigh spacings attainable for transceiver form factors typical of consumer electronics devices. We are now interested in increasing the link range by several orders of magnitude (to tens of kilometers) while still employing mm wave frequencies, so that Rayleigh spacings are no longer attainable with reasonable form factor constraints despite the high carrier frequency. We therefore propose a distributed architecture which sidesteps these constraints.

The idea of using relay-based MIMO for addressing rank deficiency is known, and can be summarized in our present context as follows. The transmitter sends N_t data streams from N_t different antennas. The receiver also has N_r antennas ($N_r \geq N_t$), and can separate out these spatially multiplexed streams if the channel matrix between the transmit and receive antennas has rank at least equal to the number of data streams. If the MIMO matrix is rank deficient, we can employ $K \geq N_t$ independent nodes serving as intermediate receivers, or relays: these nodes each receive a linear mixture of the transmitted data streams, and forward them to the receiver. We are interested in scenarios where the natural spacing of these relays is enough that the $N_t \times K$ channel between the transmitter and the relays (viewed as a single virtual node) has rank at least N_t , as does the $K \times N_r$ channel between the relays and the receiver. The effective $N_t \times N_r$ MIMO matrix between the transmitter and receiver is the cascade of these channels, and therefore has rank N_t , permitting the receiver to demultiplex the data streams. Thus, the use of relays permits spatial multiplexing in LoS settings in which the transmit and receive antenna spacings do not satisfy the Rayleigh criterion.

While the basic principle of creating spatial degrees of freedom in a near-LoS environments using a distributed architecture is quite general, several additional considerations are involved in the design of the envisioned mm wave “wireless fiber” links. First, attaining sufficient link budget for a long-

range mm wave link requires highly directive links at both ends; to realize such directivity adaptively, each “antenna” in the preceding description may actually be an electronically beamsteered array. Thus, the proposed architecture includes beamsteering between the transmitter and relays, and between the relays and the receiver. In particular, the receiver employs $N_r = K$ subarrays, each steering a beam at a different relay, which greatly simplifies the MIMO geometry compared to less directive settings in which signals from the relays mix at the receiver. Second, attaining optical speeds of 40 Gbps and beyond requires signaling over large bandwidths of the order of 5 GHz or more, in addition to the use of spatial multiplexing, hence the challenge of signal processing for such large bandwidths must be addressed in terms of both analog and digital design. In particular, given the challenge of analog-to-digital conversion (ADC) at high bandwidths, we advocate digitally controlled analog processing both for forwarding at the relay and spatial demultiplexing at the receiver.

Related work: The Rayleigh criterion was explored for LoS MIMO in [1], and these results guide our design. A mm-wave LoS spatial multiplexing link was prototyped in [2]. The receiver in our system could use exactly the same architecture and algorithms, since it is oblivious to the presence of the relays. Important theoretical capacity results for half-duplex 2-hop MIMO relay networks and the canonical relay channel have been outlined in [3], [4]. The capacity for full-duplex AF MIMO relay networks was derived in [5]. Several papers proposing the use of relays as active scatterers in rank-deficient LoS MIMO channels have been published in recent years [6]–[10], but the proposed system differs from these in its geometry (which is significantly different because of the use of mm wave frequencies, beamforming, and a clear separation between the transmitter to relay, and relay to receiver links) and in terms of hardware design considerations (which are affected by the requirement of RF beamforming for large arrays and the high data rates).

II. LOS MIMO

We review some basic results on LoS spatial multiplexing [1] that we will build on. The inner product between any two columns \mathbf{h}_{*k} and \mathbf{h}_{*l} of the channel matrix \mathbf{H} is given by [1]

$$\langle \mathbf{h}_{*k}, \mathbf{h}_{*l} \rangle = \left(\frac{\lambda}{4\pi R} \right)^2 \frac{\sin(\pi N(l-k) \frac{d_T d_R}{\lambda R})}{\sin(\pi(l-k) \frac{d_T d_R}{\lambda R})} \quad (1)$$

where N is the number of degrees of freedom, λ is the wavelength, d_T and d_R are transmit and receive array lengths, and R is the distance between the transmit and the receive array. This inner product is driven to zero when

$$d_T d_R = n \cdot \frac{R\lambda}{N} \quad (2)$$

where n can be any positive integer except multiples of N . This coincides with the so-called Rayleigh criterion from diffraction-limited optics.

A key contribution in [1] was to show that, if we constrain the size of the transmit and receive antenna arrays, the number

of degrees of freedom is limited by the Rayleigh criterion, regardless of the number of antenna elements, but that a larger number of elements can be exploited to provide beamforming gains. This motivates the array of subarrays architecture shown in Figure 1.

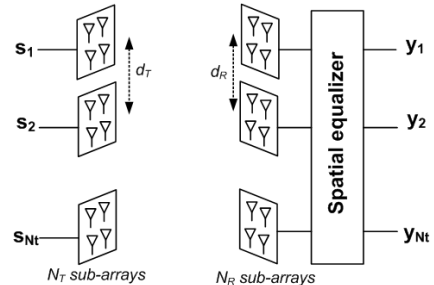


Fig. 1. Mm-wave LoS transmitter and receiver architecture.

The $N = N_t$ different data streams are transmitted over a N_t -element array, where the distance between the array elements is chosen based on (2). Each element of the transmit array is composed of a subarray of $\lambda/2$ -spaced elements. Each subarray provides beamforming gain, and the N_t -element array of subarrays provides spatial multiplexing. The receiver has an identical architecture. Each of the N_r subarrays provides (receive) beamforming gain, and the output of the N_r -element array of subarrays after beamforming is passed through a spatial equalizer to demultiplex the N_t transmitted streams.

III. RELAY-AIDED LOS MIMO

Let us consider a concrete application of the proposed architecture to synthesize a long-range air-to-ground “wireless fiber” link. The transmitter is an aircraft, possibly an unmanned aerial vehicle (UAV), communicating with a ground-based receiver over a range up to 50 km. The required data rates (10-100 Gbps) are obtained by using spatial (and possibly polarization) multiplexing, along with the large available bandwidths at mm-wave frequencies (e.g., E-band frequencies from 70-90 GHz). As in Section II, we employ an array of subarrays architecture at the transmitter and the receiver: the transmitter uses spatial multiplexing to send N_t data streams, one from each subarray, while the receiver has N_r subarrays for performing demultiplexing. Each subarray must itself provide a large beamforming gain in order to overcome path loss: this can be achieved by using mechanically steerable dish antennas, or electronically steerable arrays with a very large number (~ 1000) of elements with moderate directivity. However, for the large ranges of interest, it is not possible to satisfy the Rayleigh criterion with reasonable form factor constraints: the $N_t \times N_r$ MIMO channel matrix directly between transmitter and receiver is ill-conditioned.

In order to create spatial degrees of freedom, we introduce a set of amplify-and-forward relays that receive the signal from the transmitter over the long link and forward it to the receiver over a short link, as shown in Figure 2. Each relay

TABLE I
SYSTEM PARAMETERS

Parameter	Value
Long link distance	$50 \cdot 10^3$ m
Short link distance	100 m
N_t	2
d_t	1 m
N_r	2 - 6
d_r	0.05 m
Center frequency	73.5 GHz

has two beamformers, a receive beamformer providing a large gain (e.g., using a mechanically steerable array or a 1000-element electronically steerable array) over the long link from the transmitter, and a transmit beamformer providing a smaller gain over the short link to the receiver. Each subarray of the receiver directs its beam towards a different relay, so that the short link can thus be seen as a wire-like link between each relay and the corresponding receive subarray. We assume that the relays are spread randomly over a region of diameter d_{\max} .

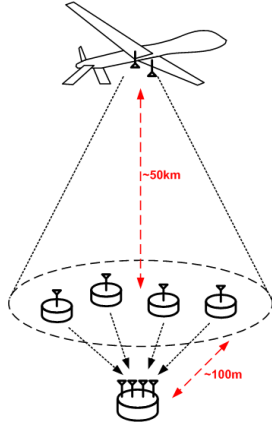


Fig. 2. Architecture of the long-range wireless fiber setup.

In order to obtain insight into the effect of relay placement on MIMO capacity, we consider for simplicity a transmitter and receiver with subarrays arranged along a single direction, labeled the x-axis, and relays placed randomly along this axis. The transmitter is located at coordinates $(0, 50 \cdot 10^3)$, the relays are at y-coordinate 0, and the receiver is located at coordinates $(0, -100)$. The number of subarrays and distances between the subarrays are given in Table I. From (2), we can compute, for two transmitters and two relays, that the optimal distance according to the Rayleigh criterion is 102 m for the nominal link parameters. However, we wish to be flexible about relay placement, allowing for mobile nodes, for example, and about the desired link range, hence exact placement of relays according to the Rayleigh criterion is out of the question. Instead, we assume that the number of relays is larger than the number of transmitted streams, and explore (in Section V) the effect of their number and geographical spread on the MIMO matrix.

IV. TRANSCIEVER ARCHITECTURE

We describe here some specific design choices in order to provide a concrete flavor of the issues involved.

As shown in Figure 3, the transmitter consists of an N_t -element array, where each element is a beamforming subarray. Our link budget calculations show that the transmit subarrays need to have a vertical and horizontal angular aperture of 0.5° . For a distance of 50 km, this angular aperture covers an area of at least 400 m diameter. As shown in Section V, it suffices to disperse relays randomly over a region of diameter around 200 m to provide the required MIMO matrix rank. Thus, each transmit subarray can direct its beam in the same direction, and the direction can be chosen so as to cover all of the relays, requiring only a coarse knowledge of the locations of the relays. This allows significant flexibility in terms of relay deployment and mobility. The large beamforming gains required for our link budget could be obtained by using mechanically steerable dish antennas, but electronically steerable arrays with a sufficiently large number (~ 1000) of less directive elements can also provide the desired gains. The latter is attractive not only in terms of adaptation speed and reduced mechanical wear, but also in terms of distributing the output power requirements over a large number of elements, thus greatly easing the task of power amplifier design. Of course, there are significant challenges in electronically adapting such large arrays, since we must employ RF beamforming, and precise control of the amplitude and phase for each element is difficult. However, adaptation techniques have been recently developed [11] for such arrays even when the phase for each element is coarsely quantized to as few as four values ($\{0, \pm \frac{\pi}{2}, \pi\}$), and it is straightforward to use these techniques, along with coarse knowledge of the relay locations, to adapt the transmit subarrays.

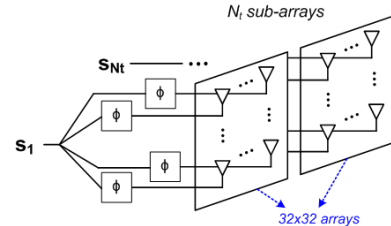


Fig. 3. Architecture of the transmitter node.

The relays employ analog FDD, receiving the signal from the transmitter over the long link in the 71-76 GHz band, and forwarding it to the receiver over the short link in the 81-86 GHz band. The relay nodes are composed of two distinct subarrays, as shown in Figure 4: a thousand-element (32×32) subarray to receive the signal over the long link, and a smaller subarray (nominally 4×4) to forward the signal over the short link after amplification. The receive subarray for each relay is identical to a subarray at the transmitter, and uses a similar approach for its adaptation (requiring coarse tracking of the transmitter's location). The transmit subarray for a relay node has a horizontal and vertical angular aperture of 20° , which

means that for a 100 m separation to the receiver, a relay's beam covers an area of 35 m diameter. Thus, in order to direct its beam toward the receiver, each relay must know the position of the receiver with moderate accuracy (easily achieved using GPS).

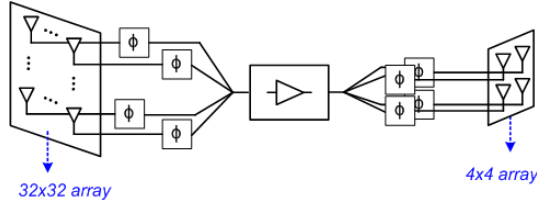


Fig. 4. Architecture of a relay node.

The receiver employs an N_r -element array, where each element is a 4×4 subarray. The beam of each 4×4 subarray is directed towards a separate relay node. The approach for adapting each receive subarray is analogous to that for adapting each relay's transmit subarray. After the receive subarrays have formed the beams, we can use a number of architectures for spatial demultiplexing. A particularly attractive architecture, prototyped in [2] for a 60 GHz testbed, performs analog spatial demultiplexing to reduce the dynamic range prior to A/D conversion and per-stream demodulation.

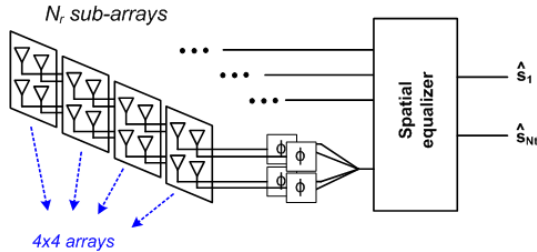


Fig. 5. Architecture of the receiver node.

A. Link budget for the long link

A critical part of the design is the link budget for the long link (see Table II). We consider QPSK modulation with a rate 13/16 LDPC code designed for high-speed applications [12]. While the link budget is described in Table II, we provide the following high-level comments. The propagation loss over 50 km is very large (189 dB, including free space and atmospheric losses). Other "diverse" losses in the budget include receiver packaging losses (1.5 dB), implementation loss (2.5 dB), beam aiming loss (1 dB) and path obstruction loss (5 B). For our coded modulation scheme and the given noise figure, the minimum received power, or "receiver sensitivity," can be calculated to be -60.44 dBm. Even with very large beamforming gains at both ends (52 dB each), the transmit power required is about 35 dBm. If we allow for packaging losses of, say 7.5 dB, at the transmitter, the net power output required is 42.5 dBm, which is truly challenging to produce at mm wave frequencies. However, if we use a 32×32 array at the transmitter, we can spread this requirement among 1024

TABLE II
LINK BUDGET PARAMETERS FOR THE LONG LINK

Parameter	Symbol	Value
Bit Rate per stream	-	10 Gbit/s
Carrier frequency	f_c	73.5 GHz
Bandwidth	B	5 GHz
Transmitter gain	G_t	52 dB
Receiver gain	G_r	52 dB
Path loss	PL	$\lambda^2 / (4\pi R)^2 = 164$ dB
Atmospheric loss	AL	$R \cdot 0.5$ dB/km = 25 dB
Diverse losses	L	10 dB
Packaging loss at Tx	-	7.5 dB
Required E_b/N_0	-	5.9 dB
Receiver noise figure	F	7.5 dB
Thermal noise	k_T	-173.83 dBm/Hz
Modulation format	-	QPSK, rate 13/16 BRCM LDPC code

transmit elements, so that we can use a 12.5 dBm power amplifier for each element, which is in fact realizable with silicon RFIC technologies. Thus, while hardware design for such large arrays is a challenging enterprise, it is well worth undertaking because of the ability to synthesize very high power and very high directivity using low-cost semiconductor processes.

B. Thousand-element subarray gain

The link budget showed that 50 km communications can only be achieved if we can get a subarray gain of 52 dB at the transmitter and at the relay. The directivity of a subarray is given by

$$D = \frac{4\pi A_{\text{eff}}}{\lambda^2} = \frac{4\pi}{\theta_{\text{HPBW}} \phi_{\text{HPBW}}} \quad (3)$$

where D is the directivity of the subarray, A_{eff} is the effective aperture of the subarray and θ_{HPBW} and ϕ_{HPBW} are the vertical/horizontal half-power beamwidths (HPBW) of the subarray. If we consider square subarrays, then $\theta_{\text{HPBW}} = \phi_{\text{HPBW}}$. To achieve a 52 dB subarray gain, the vertical/horizontal HPBW must be 0.5° . The vertical gain of a subarray is equal to

$$\theta_{\text{HPBW}} = \theta_{\text{HPBW}}^{\text{ant}} / L \quad (4)$$

where $\theta_{\text{HPBW}}^{\text{ant}}$ is the HPBW of a single antenna element, and L is the number of antenna elements along the vertical axis of the subarray. The horizontal HPBW is defined similarly. For 32 elements in the vertical and horizontal direction of the subarray, $\theta_{\text{HPBW}}^{\text{ant}}$ is equal to 16° , or a directivity of 22 dB. Such antenna elements can easily be designed with passive antenna structures.

C. Link budget for the short link

Given the link budget stress on the long link, we would like the short link to be transparent (i.e. the noise of the receiver should be negligible compared to the amplified noise of the relay). The path loss for a 100 m distance is 110 dB (with almost no atmospheric losses), and as with the long link, we budget for 10 dB diverse losses. By using 4×4 subarrays with similar antenna elements as for the long link, the transmit and receive subarray gains are equal to 34 dB. In order to ensure

that the short link becomes transparent, the relay must amplify the signal with a gain of 60-70 dB. One possible approach is an all-mm-wave design, where two cascaded amplifiers are used at the relays. Another is to downconvert the received signal to baseband, amplify it, and then upconvert back to mm-wave.

V. MIMO RANK AND SYSTEM PERFORMANCE

The available spatial degrees of freedom in our system depend on the rank of the effective channel from the UAV's transmit array to the ground-based receive array. If we denote $\mathbf{H}_1 \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{H}_2 \in \mathbb{C}^{N_r \times N_r}$ as the transmitter-relay and relay-receiver channel matrices, respectively, the effective composite channel is $\mathbf{H} = \mathbf{H}_2 \mathbf{H}_1$. The use of mm wave carrier frequencies simplifies this effective channel significantly: each of the N_r relays beamsteers towards the receiver, and the receiver has one array steering its beam for each relay, so that the link between each relay and the receiver is highly directional. That is, \mathbf{H}_2 is a approximately diagonal, so that the rank of the composite channel is effectively equal to that of the "long link" channel matrix \mathbf{H}_1 .

We consider a simple model to develop insight: N_r relay nodes independently and identically distributed (iid) along a line of length d_{\max} . The long-range LoS links from transmitters to relays have the same path loss, as the variations in length among them are extremely small due to the very large ratio of range to relay separation. Thus, we may model the entries of \mathbf{H}_1 as constant magnitude. When the relay separation is large enough (greater than the value of d_R satisfying the Rayleigh criterion in (2)), then these small variations in length are bigger than the carrier wavelength, and produce significant phase differences, which is what we hope creates a MIMO matrix with rank N_t .

A. Theoretical analysis

For "large enough" spacing between relays, we can model the phases of the entries of \mathbf{H}_1 as i.i.d., uniformly distributed over $[0, 2\pi]$. Recent results on random matrices with sub-Gaussian entries then provide bounds on the extreme singular values $\sigma_{\min}, \sigma_{\max}$ of such matrices: with probability at least $1 - 2 \exp(-cx^2)$

$$\sqrt{N_r} - C\sqrt{N_t} - x \leq \sigma_{\min} \leq \sigma_{\max} \leq \sqrt{N_r} + C\sqrt{N_t} + x \quad (5)$$

where C and c are absolute positive constants that depend only on the size of the sub-gaussian norm of the rows of \mathbf{H}_1 . Roughly speaking, (5) says that for a fixed number of data streams N_t , by increasing the number of receive array elements/relays N_r the normalized extreme singular values $\frac{\sigma_{\min}}{\sqrt{N_r}}, \frac{\sigma_{\max}}{\sqrt{N_r}}$ move closer to unity, thereby reducing their ratio. Thus, a MIMO rank of N_t is attained with high probability if we use a large enough number of relays dispersed over a large enough area.

For two spatially multiplexed streams $N_t = 2$ as considered in this paper, we can provide a more detailed characterization. Letting \mathbf{h}_1 and \mathbf{h}_2 denote the columns of \mathbf{H}_1 , note that \mathbf{h}_i is the response of stream i at the relays, $i = 1, 2$. Assuming linear separation at the receiver, at moderately high SNR, the

performance is characterized by the normalized correlation $\rho = \frac{\langle \mathbf{h}_1, \mathbf{h}_2 \rangle}{\|\mathbf{h}_1\| \|\mathbf{h}_2\|}$. The effective signal degradation due to zero-forcing interference suppression is then given by $\eta = 1 - |\rho|^2$. Assuming, without loss of generality, that the elements of $\|\mathbf{h}_i\|$ have unit amplitude, we have $\|\mathbf{h}_i\|^2 = N_r$, so that

$$\rho = \frac{1}{N_r} \mathbf{h}_2^H \mathbf{h}_1 = \frac{1}{N_r} \sum_{k=1}^{N_r} e^{j(\theta_{k1} - \theta_{k2})}.$$

For large enough N_r , we can model ρ as $CN(0, \frac{1}{N_r})$ using the central limit theorem (CLT), so that $|\rho|^2$ is an exponential random variable with mean $\frac{1}{N_r}$. Actually, the CLT kicks in quite quickly, and provides an analytical approximation for the CDF of the SNR after linear separation which matches quite well with simulations for $N_r = 4, 6$. For $N_r = 2$, the CLT gives a poor approximation, but the distribution of $|\rho|^2$ can be exactly characterized (details omitted).

B. Numerical results

Monte Carlo simulations were performed with the parameters in Tables I and II. The chosen performance metrics are the statistics of the channel condition number and the per-stream SNR after spatial demultiplexing.

In the first set of simulations, d_{\max} was fixed to 200 m (i.e., the interval over which the relays are randomly deployed is twice the value given by the Rayleigh criterion for $N_r = N_t = 2$ fixed receivers) and the number of relays/receive arrays N_r was either 2, 4, or 6. In Fig. 6 the 95% tail of the composite channel condition number is seen to decrease by around 20 dB by increasing N_r from 2 to 4, while further increasing N_r to 6 yields diminishing returns. Fig. 7 plots the CDF of the per-stream SNR with zero-forcing (ZF) spatial demultiplexing, comparing our analytical estimates with simulation results. The ZF SNR is compared against the benchmark of matched filtering (MF) ignoring the presence of the interfering stream, which serves as a performance upper bound (the distance between the ZF and MF SNRs is the noise enhancement due to channel inversion). We see that the improved channel rank and diversity gain for larger N_r translates to sizable SNR gains and lower ZF noise enhancement. A larger number of relays translates to significantly better tail behavior: setting the required per-stream SNR to 8.9 dB, the outage rates with $N_r \in \{2, 4, 6\}$ are $\{50\%, 4\%, 0.3\%\}$. Note that the analytical ZF SNR distributions (approximations provided via the CLT for $N_r = 4, 6$, and exactly characterized for $N_r = 2$) match the simulated SNR curves quite well, deviating by 0-0.5 dB for $N_r = 2, 4$ and markedly less than 0.1 dB for $N_r = 6$.

We now fix $N_r = 4$ and explore the effect of the geographic dispersion of the relays, considering three values of d_{\max} : 100, 200 and 300 m. Recalling that the Rayleigh limit is approximately 100 m, it is likely that the transmitters will see correlated spatial channels for $d_{\max} = 100$ m, while for d_{\max} of 200 or 300 m we expect less correlation and hence a better conditioned MIMO matrix. The simulation results in Fig. 8, which plot the channel condition number in dB, confirm this: inter-relay spacings below the Rayleigh limit of 100 m lead to

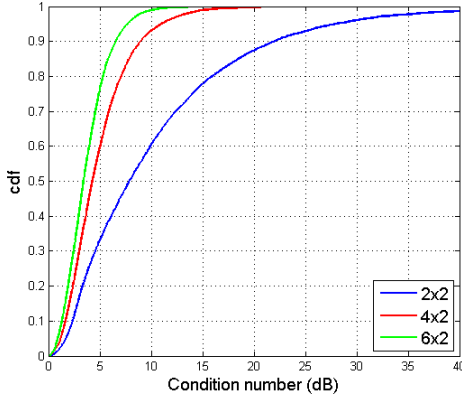


Fig. 6. Simulated condition number of the composite channel as a function of N_r for $d_{\max} = 200$ m.

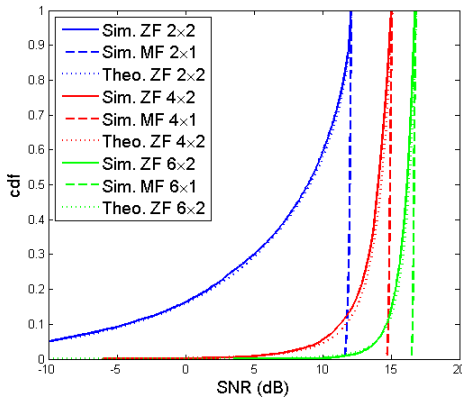


Fig. 7. Simulated and theoretical per-stream SNR for $d_{\max} = 200$ m as a function of N_r , for 2-stream ZF reception and 1-stream MF reception. The difference between pairs of ZF and MF curves is ZF noise enhancement.

poorly conditioned channels, whereas the condition numbers for 200 and 300 m exhibit similar, and satisfactory, behavior.

VI. CONCLUSIONS

The proposed distributed architecture achieves LoS spatial multiplexing over distances much larger than the Rayleigh limit at mm-wave carrier frequencies, by using amplify-and-forward relays randomly spread out close to the receiver. For two transmitted streams, our results indicate that by using as few as four relays, it is possible to synthesize a full rank MIMO matrix, and to demultiplex the streams linearly with limited degradation due to noise enhancement. There are a number of design guidelines in our paper study that represent interesting hardware challenges, including the realization of 1000-element subarrays providing sufficient gain for the long link between transmitted and relays, and relay design with an appropriate degree of isolation between the signal received from the transmitter and the signal transmitted to the receiver. A detailed characterization of the statistics of the MIMO matrix as a function of the number and geographical distribution

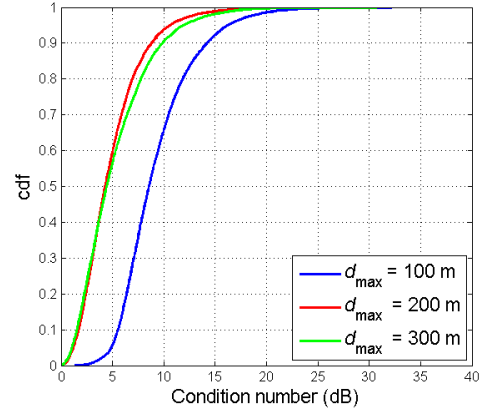


Fig. 8. Simulated condition number of the composite channel as a function of d_{\max} for $N_r = 4$.

of the relays, and the number of spatially multiplexed streams, is an important area for further work.

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