General Modeling, Fabrication & Measurement Report

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On the Modeling of Semiconductor Optical Amplifiers

This report gives an overview of the model that can be used to simulate semiconductor optical amplifiers (SOAs) based on indium-phosphide, or III/V in general, gain material. Specific trade-offs and considerations for the application of an SOA in a coupled opto-electronic oscillator (COEO) or mode-locked laser (MLL), as studied in the E-Phi project, will be discussed.

1 Rationale

Laser diodes and SOAs are typically modeled using the semiconductor rate equations. In literature a huge variety of sets of rate equations can be found, each incorporating a particular selection of physical properties, effects and time scales and each tailored to a particular application. This means that most of the SOA models found in literature do not have general validity and cannot be used unless a detailed study of the application is done first.

Obviously one can take or construct the most detailed version of the SOA model out of the full scope of available models and use that for every application, but the simulation time would become prohibitively long in most cases. An important consideration is to find an optimum trade-off between shortening the simulation time, and hence a lower model complexity, and increasing the physical accuracy, and hence – often – higher complexity.

In this report I will present an overview of the trade-offs that I would consider and recommend for using the rate equations to model the SOA in an MLL or COEO.

2 General considerations

The most important considerations are time-scales involved and optical power levels. In the COEO optical power levels will be on the order of 10 – 20 mW average, not counting start-up spikes, and pulse durations will be around 1 – 10 ps at 20-GHz operation. Heating will be an important factor at higher injection currents, but is not part of this discussion.

The following effects will be included:

- Optical field envelope $A(x,t)$ propagating through the SOA in both directions;
- Time and position dependent carrier density $N(x,t)$ inside the SOA;
- Carrier injection by means of an injection current I and an injection efficiency parameter η_i;
- Finite carrier lifetime τ;
- Photon generation by means of a Langevin noise term and driven by the bimolecular recombination B. Only the part that is coupled into the optical mode is considered and expressed by β;
- Field amplification $G(x,t)$:
  - Saturation effects enter the equation by depletion of carriers $N(x,t)$;
  - Ultrafast gain compression due to carrier-heating (CH) and spectral hole-burning (SHB) enter the equation explicitly by ε_1. This approach is the so-called 'adiabatic

approach' [1], which means that the gain compresses 'instantaneous' with the optical power. Since the CH and SHB timescales are ~1 ps and ~100 fs typically, so this approach is valid for pulses down to 1-ps duration.

- Coupling between real and complex part of the index, i.e. the coupling between the gain and the index of refraction. This is fully described by the Kramers-Kronig relation, but implemented using the effective parameter α_N, the linewidth enhancement factor or Henry-factor.
- Optical passive losses α_{int} due to scattering and free-carrier absorption.[1]

The following effects can be considered when required:

- Two-photon absorption (TPA). Two photons are absorbed simultaneously, expressed by β_2, creating a carrier pair, hence increasing carrier density N. However these carriers do not contribute immediately to the gain, since they are at an elevated temperature and have to thermalize with the lattice temperature first, as expressed by ε_2.
- Ultrafast nonlinear refraction (UNR) or optical Kerr-effect, as expressed by the power-dependent part of the index of refraction n_2.
- The coupling between ultrafast gain variations, due to TPA, CH and SHB, and index, as expressed by α_T.

3 Model rate equations

In this section I first introduce the rate equations for the complex field and for the carrier density in 3.1. Hereafter, in 3.2, I will present an alternative expression where the complex field rate equation is split into two (real) rate equations for power and phase. Also in 3.2 an alternative expression for the carrier density equation is given by using the gain. It is up to the user to make a choice between the two sets presented in 3.1 and 3.2, or a combination thereof, since these sets are fully equivalent.
In 3.3 I present and extended version of the rate equations that includes the effects of UNR and TPA. This set can be used when high peak-powers are considered, e.g., for lasers that emit short pulses. The effects of gain dispersion, i.e. finite gain bandwidth, and chromatic dispersion have not been included in these rate equations yet. In 3.4 it is discussed how these effects can be added to the model.

3.1 Complex field rate equations

Starting point for this overview will be the rate equations as found in, e.g., [1,2,3,4], in various shapes. Here I show the propagation of the complex optical field envelope $A(x,t)$, in units of $W^{0.5}$. Both propagation directions can be described by this equation. The optical (center, reference) frequency ω_0 has been split off and can be added by multiplying the field by $\exp(i\omega_0 t)$.

[1] This loss term is actually carrier-dependent, but it is hard to separate this effect from the gain term, which is also carrier dependent. So it makes sense to keep α_{int} constant. An attempt to include FCA losses in this parameter is given in [1].

(1.1)
$$\frac{\partial A(z,t)}{\partial z} \pm \frac{1}{v_g}\frac{\partial A(z,t)}{\partial t} = \frac{1}{2}\frac{G(z,t)}{1+\varepsilon_1\left|A(x,t)\right|^2}A(z,t) - \frac{i}{2}\alpha_N G(z,t)A(z,t) - \frac{1}{2}\alpha_{int}A(z,t) + F_{SE}(z,t)$$

In this equation v_g is the group velocity, $G(z,t)$ is the optical gain according to

(1.2)
$$G(z,t) = \Gamma a_N \left(N(z,t) - N_{tr}\right) ,$$

in which Γ is the optical confinement factor, i.e. the overlap of the optical mode with the gain region, a_N is the differential gain and N_{tr} is the transparency carrier density[2]. Alternatively a logarithmic gain model can be used:

(1.3)
$$G(z,t) = \Gamma a_N N_{tr} \ln\left(\frac{N(z,t)}{N_{tr}}\right) .$$

Both models are equal at small offsets from N_{tr}. Although the logarithmic gain model is the best option for gain at a fixed wavelength, e.g., for SOAs, the linear gain model might be more valid for lasers, since the lasing wavelength moves with the gain peak and hence the gain vs. carrier density relation appears to be more like a linear relation. This has to be studied in more detail. However for lasers the gain is clamped, so one has to choose a convention and just stick to it.

The spontaneous emission (SE) coupled to the optical mode can be expressed by the Langevin noise term $F_{SE}(z,t)$ [3,5,6,7]

(1.4)
$$F_{SE}(z,t) = \sqrt{\frac{\hbar\omega\sigma\beta B\left[N(z,t)\right]^2}{\Delta z}} \cdot \xi \cdot e^{i\zeta} .$$

In here $\hbar\omega$ is the photon energy, BN^2 the SE generated per unit volume and per unit time, β is the part of the SE coupled to the optical mode, σ is the optical mode area, ξ is a Gaussian random variable with zero mean and a width of 1 and ζ is a random variable between 0 and 2π. For $\Delta z \rightarrow 0$ this noise term corresponds to white noise. In a numerical implementation Δz is finite and a null in the noise spectrum will appear at $1/\Delta t = v_g/\Delta z$ [6]. This null has to be far beyond the spectral noise bandwidth of interest. With typical values of Δt = 50 fs, as explained above, this null can be found at 2 THz, which is far beyond the bandwidth of interest for 20-GHz oscillators, as studied in the E-Phi project.

The carrier density $N(z,t)$ is described by the following differential equation, ignoring the effects of SE noise:

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[2] The transparency carrier density indicates the point where the gain region is pumped to a level of zero loss or gain. Passive losses α_{int} still cause absorption, so the SOA is not transparent at this level of carrier density. Moreover care should be taken with this parameter as its value depends on the gain model. Using a linear approximation of the gain, as is done here, would underestimate N_{tr} for values of N far from transparency.

(1.5) $$\frac{\partial N(z,t)}{\partial t}=\frac{\eta_i I}{qV}-\frac{N(z,t)}{\tau}-\frac{1}{\Gamma\hbar\omega\sigma}\frac{G(z,t)P(z,t)}{1+\varepsilon_1 P(z,t)} \; .$$

In here q is the elementary charge, V the volume of the active area. The carrier lifetime τ is an approximation and can be fully expressed as

(1.6) $$\frac{1}{\tau}=A+BN(z,t)+CN(z,t)^2 \; ,$$

with A the nonradiative recombination, B the radiative recombination and C the Auger recombination. The reason to work with an effective lifetime τ is that experimentally it is hard to determine, and fit, all of these parameters separately. The set (1.1) and (1.5) fully describes the field propagation through an SOA and it can be implemented in a discrete time domain model using steps of $\Delta z = v_g \cdot \Delta t$, with typical step sizes of $\Delta z \approx 5$ μm or $\Delta t \approx 50$ fs.

3.2 Rate equations expressed in power and phase

The complex field $A(z,t)$ can be converted to a photon density $S(z,t)$ using:

(1.7) $$S(z,t)=\frac{|A(z,t)|^2}{\hbar\omega_0\sigma v_g} \; .$$

It can also be converted to an optical power [W] and phase:

(1.8) $$A(z,t)=\sqrt{P(z,t)}\cdot e^{i\varphi(z,t)} \; .$$

Substituting this into eq. (1.1) and omitting the full expression for the position z and time t dependency leads to

(1.9) $$\frac{e^{i\varphi}}{2\sqrt{P}}\frac{\partial P}{\partial z}\pm\frac{1}{v_g}\frac{e^{i\varphi}}{2\sqrt{P}}\frac{\partial P}{\partial t}+i\sqrt{P}e^{i\varphi}\frac{\partial\varphi}{\partial z}\pm\frac{1}{v_g}i\sqrt{P}e^{i\varphi}\frac{\partial\varphi}{\partial t}= \\ \frac{1}{2}\frac{G}{1+\varepsilon_1 P}\sqrt{P}e^{i\varphi}-\frac{i}{2}\alpha_N G\sqrt{P}e^{i\varphi}-\frac{1}{2}\alpha_{int}\sqrt{P}e^{i\varphi}+F_{SE}(z,t)$$

This complex differential equation can be separated into the real and imaginary parts, leading to two separate differential equations for the power P and phase φ:

(1.10) $$\frac{\partial P}{\partial z}\pm\frac{1}{v_g}\frac{\partial P}{\partial t}=\frac{GP}{1+\varepsilon_1 P}-\alpha_{int}P+2\sqrt{P}\,\mathrm{Re}\left(F_{SE}(z,t)\right)$$

(1.11) $$\frac{\partial\varphi}{\partial z}\pm\frac{1}{v_g}\frac{\partial\varphi}{\partial t}=-\frac{1}{2}\alpha_N G+\frac{1}{\sqrt{P}}\mathrm{Im}\left(F_{SE}(z,t)\right)$$

Unit-length complex factors $e^{i\varphi}$ have been factored into the random and complex parameter $F_{SE}(z,t)$ at no loss of validity. It is important to note that the derivative of the phase with respect to time, $\partial\varphi/\partial t$, gives the chirp of the field, i.e. the instantaneous detuning of the optical frequency with respect to the carrier frequency ω_0.
The carrier rate equation (1.5) can be rewritten using the linear gain approximation (1.2) into:

$$\frac{\partial G}{\partial t} = \frac{G_0 - G}{\tau} - \frac{1}{E_{sat}}\frac{GP}{1+\varepsilon_1 P} \quad . \tag{1.12}$$

In this equation I have introduced expression for the saturation energy of the SOA E_{sat}, the small-signal gain G_0, and the injection current required for transparency I_0:

$$E_{sat} = \frac{\hbar\omega\sigma}{a_N} \quad , \tag{1.13}$$

$$G_0 = \Gamma a_N N_{tr}\left(\frac{I}{I_0} - 1\right) , \tag{1.14}$$

$$I_0 = \frac{qVN_{tr}}{\eta_i \tau} \quad . \tag{1.15}$$

3.3 Rate equations including TPA and UNR

If required these eqs. (1.10), (1.11) and (1.12) can be expanded to include TPA, UNR and the effect of CH and SHB on the phase according to the work in [1]:

$$\frac{\partial P}{\partial z} \pm \frac{1}{v_g}\frac{\partial P}{\partial t} = \frac{G - \varepsilon_2 P^2}{1+\varepsilon_1 P}P - 2\Gamma_2\beta_2\frac{1}{\sigma}P^2 - \alpha_{int}P + 2\sqrt{P}\,\mathrm{Re}\left(F_{SE}\left(z,t\right)\right) \;, \tag{1.16}$$

$$\begin{aligned} &\frac{\partial \varphi}{\partial z} \pm \frac{1}{v_g}\frac{\partial \varphi}{\partial t} = \\ &\quad -\frac{1}{2}\left[\alpha_N G - \alpha_T\frac{\varepsilon_1 GP + \varepsilon_2 P^2}{1+\varepsilon_1 P}\right] - \Gamma'_2\frac{\omega_0}{c}n_2\frac{1}{\sigma}P + \frac{1}{\sqrt{P}}\mathrm{Im}\left(F_{SE}\left(z,t\right)\right) \end{aligned} \;, \tag{1.17}$$

$$\frac{\partial G}{\partial t} = \frac{G_0 - G}{\tau} - \frac{1}{E_{sat}}\frac{G - \varepsilon_2 P^2}{1+\varepsilon_1 P}P + \Gamma_2\beta'_2 P^2 \;, \tag{1.18}$$

with the TPA coefficient β'_2 defined according to [2][3].

[3] Note that the original paper [1] has an error in the definition of β'_2.

(1.19) $$\beta'_2 = \frac{a_N \beta_2}{\hbar\omega_0\sigma^2}$$

Note that the specific confinement factors for TPA and UNR, Γ_2 and Γ'_2, can be lumped into the β_2 and n_2 parameters to create effective parameters. Also be aware that this approach may also be taken in literature, so one has to pay attention to the exact definitions used. Here the convention of the work in [1] is used. The parameter α_T takes the coupling between ultrafast gain variations and index change into account, much like α_N, but on the faster timescales.

3.4 Gain dispersion and chromatic dispersion

Up to now gain dispersion and chromatic dispersion have been ignored. These are linear operations and can be implemented in either time of frequency domain. The gain can be modeled using a Lorentzian gain function, such as in [8], or a full set of gain curves that are obtained from experiments. One numerical implementation of an, albeit rather arbitrary, IIR filter that agrees well with a discrete implementation of a travelling-wave time-domain simulation approach is given in [7].

(1.20) $$\left|H_{BW}(\omega)\right|^2 = \frac{(1-\eta)^2}{1+\eta^2-2\eta\cos\left[(\omega-\omega_0)\Delta t\right]} \; .$$

The implementation in the discrete time-domain is given by:

(1.21) $$A(z+\Delta z, t+\Delta t) = D\cdot A(z+\Delta z, t) + (1-D)A(z,t) \; ,$$

with $0 < \eta < 1$ controlling the filter width and D defined as:

(1.22) $$D = \eta\exp(i\omega_0\Delta t) \; .$$

This filter shows the required curvature around the gain peak, which is sufficient for the applications considered here, where the optical bandwidth (1 – 5 nm) is typically far smaller than the gain bandwidth (> ~30 nm, depending on material). More advanced filters can be implemented at will. It is to be noted that when changes per roundtrip are small, as is the case for high-speed mode-locked lasers, the gain dispersion can be implemented as a lumped element, much like presented in [2].

Frequency dispersive elements can be implemented in a similar way with the same considerations. An example is given in [2],

(1.23) $$H_D \exp\left[-\frac{i}{2}k''_{tot}(\omega-\omega_0)^2\right]$$

with k''_{tot} the total second order dispersion of the cavity. Higher order dispersion can be implemented in a similar way.

4 On the model parameters and parameter values

Unfortunately due to the large amount of rate equation models and the lack of standardization, care should be taken when parameter values are taken from literature references:

- Parameters are often indicated by different symbols in different papers.
- Rate equations can be expressed in the field *A*(*z*,*t*) [$W^{0.5}$], the power *P*(*z*,*t*) [W], the photon density *S*(*z*,*t*) [m^{-3}] or the field *A*(*z*,*t*) [$m^{-1.5}$], i.e., the square root of the photon density. Especially effective parameters, such as ε_1 and ε_2, have different values depending on the context.
- Different parameters, with a different physical meaning, can have the same symbol. An interesting example is gain saturation, which is incidentally expressed by ε in simple SOA models. Such models are not dynamic and cannot be used for the applications considered here.

Another important point to note is that most parameters are waveguide geometry dependent. One should keep in mind that to reach the set of equations as presented above, averaging and integration over the transverse directions and with the mode profile was done. This obviously also means that these equations only hold for a single transverse mode. A consistent set of parameter values valid for InP-based ridge waveguide SOAs can be found in the work in [2,9]. As for the moment we do not have a complete set yet for the hybrid silicon platform SOAs.

5 References

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