Iterative Joint Decoding and Sparse Channel Estimation for Single-Carrier Modulation

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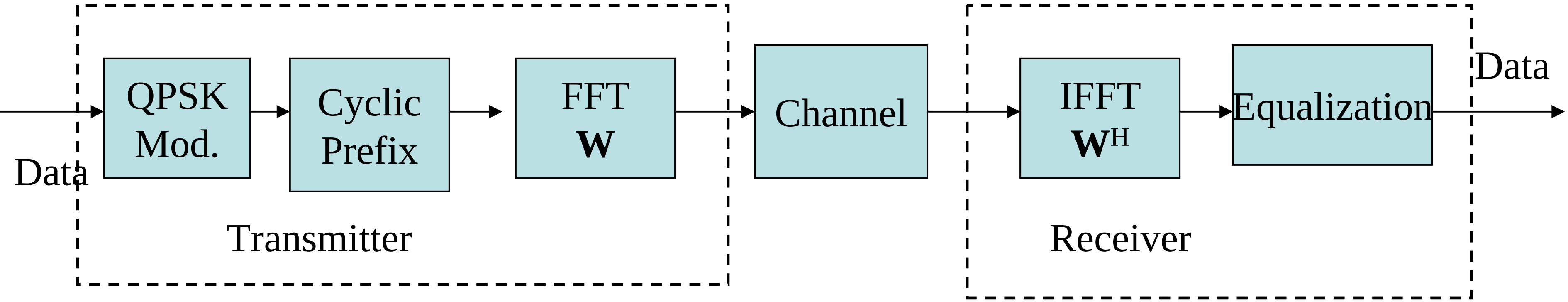
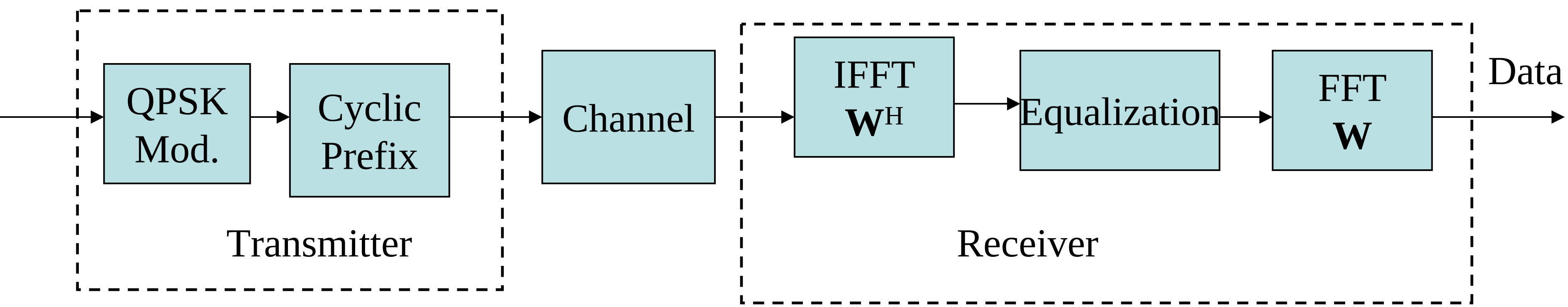
OFDM vs. SC-FDE Systems



OFDM



Single-Carrier Frequency Domain Equalization

OFDM vs SC-FDE

- OFDM
  - Achieves Shannon capacity on AWGN channel with frequency-selective fading. (Kalet IEEE Tans. Comm. 89, Chow, Cioffi, Bingham IEEE Trans. Com. 95) Requires transmit/receive CSI and optimal bit-loading.
  - Disadvantage – High peak-to-average power ratio (PAR) on the order of number of carriers.
- SC-FDE
  - Does not achieve Shannon capacity.
  - Much lower PAR than OFDM, can use constant-envelope waveform if necessary.
  - Performance of coded SC-FDE is similar to that of OFDM on mobile wireless channel. (Benvenuto and Tomasin, IEEE Trans. Comm. 02, Falconer, et. al. IEEE Comm. Mag. 02.)

Uncoded OFDM vs. SC-FDE-ZF Revisited

Received SC-FDE and OFDM waveforms after Nyquist sampling and vectorization. Channel is a convolution matrix.

$$\mathbf{r} = \mathbf{F}\mathbf{c} + \mathbf{n} \text{ SC-FDE}$$

$$\mathbf{r} = \mathbf{F}\mathbf{W}\mathbf{c} + \mathbf{n} \text{ OFDM}$$

$$\mathbf{F} = \begin{bmatrix} f_0 & f_1 & \cdots & f_{N_f-1} & \cdots & 0 \\ 0 & f_0 & \cdots & 0 & f_{N_f-1} & 0 \\ \vdots & & & & & \\ 0 & 0 & \cdots & 0 & 0 & f_0 \end{bmatrix}$$

ZF OFDM vs. SC-FDE Detection

$$\mathbf{W}^H\mathbf{F}\mathbf{W} = \mathbf{H} =$$
$$\mathrm{diag}\{H_{1,1}, H_{2,2}, .., H_{N_s,N_s}\}$$

$$\hat{\mathbf{c}} = \mathbf{H}^{-1}\mathbf{W}^H\mathbf{r}$$
$$= \mathbf{c} + \mathbf{H}^{-1}\mathbf{W}^H\mathbf{n} \qquad \text{OFDM}$$

$$\hat{\mathbf{c}} = \mathbf{W}\mathbf{H}^{-1}\mathbf{W}^H\mathbf{r}$$
$$= \mathbf{c} + \mathbf{W}\mathbf{H}^{-1}\mathbf{W}^H\mathbf{n} \qquad \text{SC-FDE}$$

Uncoded Error Rates OFDM-ZF vs. SC-FDE-ZF

OFDM

$$P_b = \frac{1}{N_d} \sum_{i=0}^{N_d-1} \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}|H'_{ii}|^2}\right)$$

SC-FDE

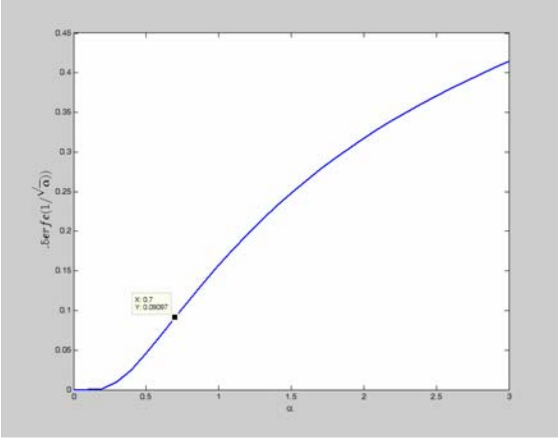
$$P_b = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0 \frac{1}{N_d} \sum_{i=0}^{N_d-1} \frac{1}{|H'_{ii}|^2}}}\right).$$

Uncoded Error Rates

Proposition 1 *Consider case (a) where* $|H'_{ii}|^2 < 3/(2\gamma_b)$ *for* $i = 0, \ldots, N_d - 1$*, where* $\gamma_b = E_b/N_0$*. In case (a), the zero-forcing SC-FDE BER is lower-bounded by the OFDM BER. Now for case (b), let* $|H'_{ii}|^2 > 3/(2\gamma_b)$ *for all* i*. Then the OFDM BER is lower bounded by the ZF SC-FDE result. The OFDM and SC-FDE system performance is equivalent when the channel is allpass.*

Uncoded Error Rates

Proof: Define $\alpha_i = \frac{1}{|H'_{ii}|^2}$. For case (a), we can show that $\text{erfc}(\sqrt{\gamma_b/\alpha_i})$ is concave in α_i in the region $\{\alpha_i\} \in [2\gamma_b/3, \infty)^{N_d}$. Then using Jensen's inequality



$$\frac{1}{N_d}\sum_{i=0}^{N_d-1}\frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma_b}{\alpha_i}}\right) < \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma_b}{\frac{1}{N_d}\sum_{i=0}^{N_d-1}\alpha_i}}\right). \tag{1}$$

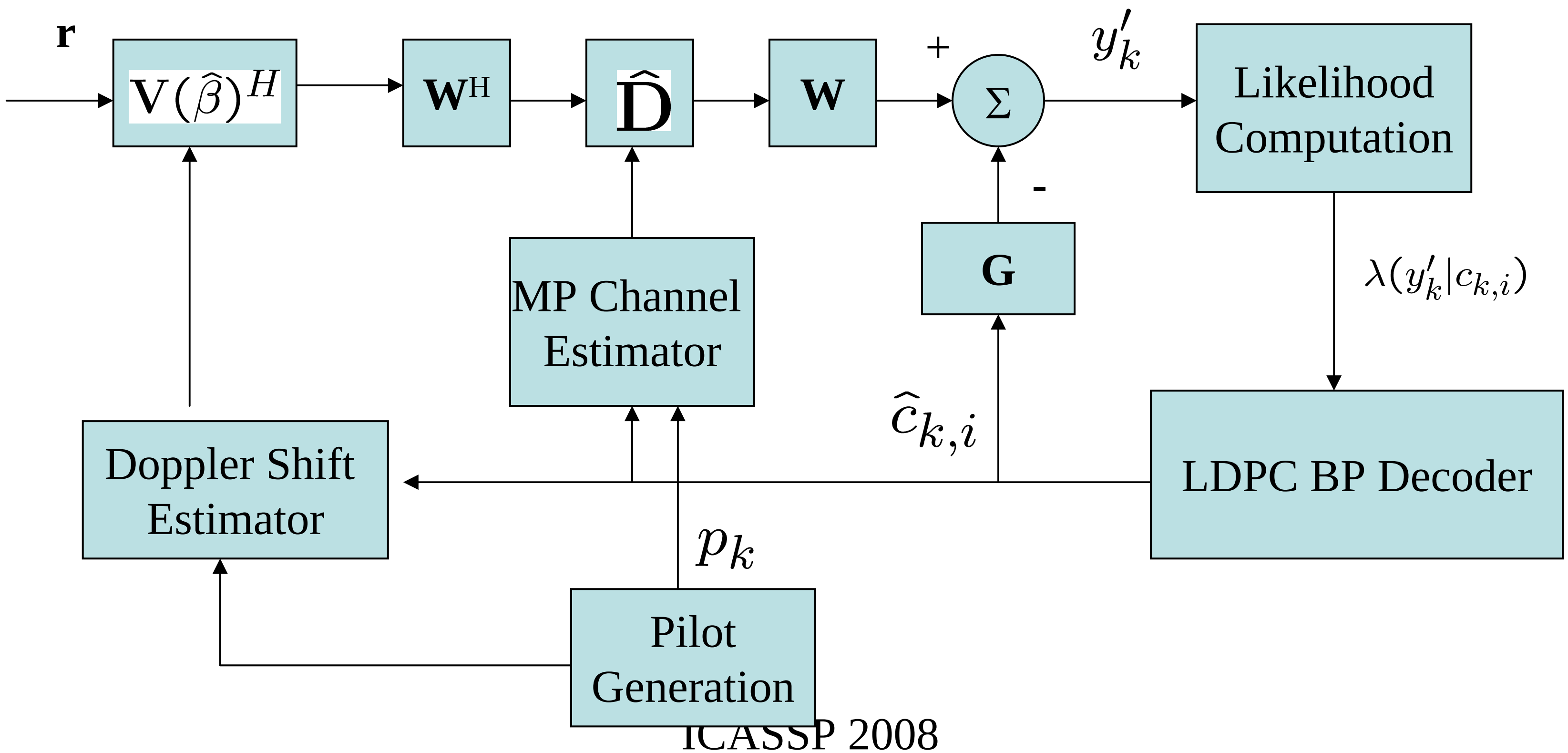
The erfc is convex in case (b), which reverses the inequality, so that the SC-FDE BER is less than OFDM

Iterative SC-FDE Receiver

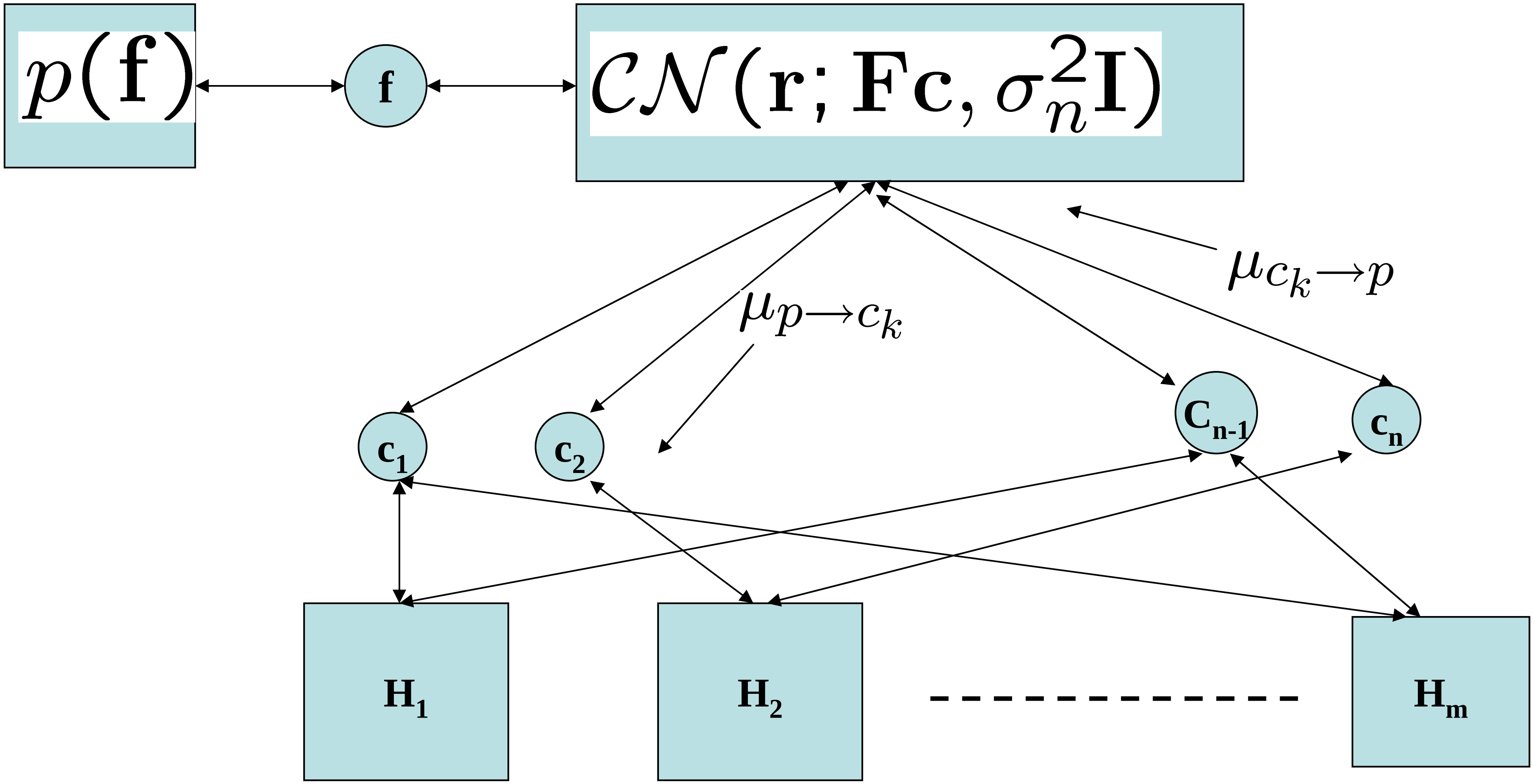
Incorporate Effect of Doppler Shift

$$\mathbf{r} = \mathbf{V}(\beta)\mathbf{F}\mathbf{c} + \mathbf{n}$$

$$\mathbf{V}(\beta) = \mathrm{diag}\{e^{i2\pi\beta f_c(N_d-1)T_s}, e^{i2\pi\beta f_c(N_d-2)T_s}, \ldots, 1\}$$



Turbo Equalization via Gaussian Approximation for Variable Messages



Turbo Equalization

•Turbo equalization has recently been justified via statistical physics methods (Nissila and Pasupathy IEEE TC July 07.)

•Measurement likelihood to code variable message. Marginalization under Gaussian approximation for the variable priors yields more direct justification for Turbo Equalization.

$$* \ \mu_{p \rightarrow c_k} = \int \mathcal{CN}(\mathbf{r}; \mathbf{F}_{(k)}\mathbf{c}_{(k)} + \mathbf{f}_k c_k, \sigma_n^2 \mathbf{I}) \mathcal{CN}(\mathbf{c}_{(k)}; \overline{\mathbf{c}}_{(k)}, \mathbf{Q}) d\mathbf{c}_{(k)}.$$

Means computed by decoder using total log-APPs

$$\overline{c}_{l,j} = \tanh(\lambda^e(c_{l,j})/2)$$

Proposition 1 *The density function to code variable message for SC modulation, when the decoder extrinsics are approximated as independent Gaussian, is given by*

$$
\begin{aligned}
*** \; \mu_{p\rightarrow c_k} &\propto \exp\left(-|c_k - \widehat{c}_k|^2/p_k\right) \\
\widehat{c}_k &= \frac{1}{\mathbf{f}_k^H \Sigma^{-1} \mathbf{f}_k} \mathbf{f}_k^H \Sigma^{-1} \left(\mathbf{r} - \mathbf{F}_{(k)} \overline{\mathbf{c}}_{(k)}\right) \\
\Sigma &= \mathbf{f}_k \mathbf{Q} \mathbf{f}_k^H + \sigma_n^2 \mathbf{I}, \; p_k = (\mathbf{f}_k^H \Sigma^{-1} \mathbf{f}_k)^{-1}.
\end{aligned}
$$

That is, the message is a Gaussian density with mean and covariance given by a MMSE Turbo equalizer.

Outline of Proof – Turbo Equalization

The integral (*) has a closed form solution identical to that for the Kalman filter innovations likelihood. The result is

$$** \ \mu_{p\to c_k} = \mathcal{CN}(\mathbf{r} - \mathbf{F}_{(k)}\bar{\mathbf{c}}_{(k)}; \mathbf{f}_k c_k, \mathbf{F}_{(k)}\mathbf{Q}\mathbf{F}_{(k)}^H + \sigma^2\mathbf{I}).$$

The density (**) is then readily manipulated to yield the density (***) in terms of c_k.

MMSE Frequency-Domain Turbo Equalization

- Problems with exact belief propagation (marginalization)
  - Closed-form marginalization w.r.t. both channel coefficients and code variables is not tractable.
  - MMSE Turbo Equalizer for exact code variable covariance **Q** cannot be computed in frequency domain.
- Solutions:
  - Design MMSE turbo equalizer using frequency-domain transformation of **r.**
  - Replace marginalization over channel coefficients by a "good" estimate of channel matrix **F.**
    - Use Matching Pursuits to estimate channel for target application (underwater acoustic communications.)

Frequency Domain Turbo Equalizer

Use offset-corrected received vector

$$\mathbf{r}' = \mathbf{W}^H \mathbf{V}(\widehat{\beta})^H \mathbf{r} \approx \mathbf{H}\mathbf{W}^H \mathbf{c} + \mathbf{n}',$$

Resulting MMSE equalizer is in the frequency domain and diagonal

$$D_{ii} = \frac{H_{ii}^*}{|H_{ii}|^2 + \frac{N_0}{T_s}}$$

$$\widehat{\mathbf{c}} = \mathbf{W}\mathbf{D}\mathbf{W}^H \mathbf{V}(\widehat{\beta})^H \mathbf{r}$$

Turbo Equalizer and Likelihood

MMSE detector output including effect of ISI

$$y_k = (\mathbf{W}\hat{\mathbf{D}}\mathbf{W}^H\mathbf{V}(\widehat{\beta})^H\mathbf{r})_k$$

$$\approx \frac{1}{N_d}\sum_{i=0}^{N_d-1}\frac{|\widehat{H}_{ii}|^2}{|\widehat{H}_{ii}|^2+\frac{N_0}{T_s}}c_k +$$

$$\sum_{j\neq k}(\mathbf{W}\mathbf{D}'\mathbf{W}^H)_{kj}(\widehat{c}_j+\tilde{c}_j)+n'_k.$$

Turbo Equalizer and Likelihood

ISI cancelled signal

$$y'_k = y_k - \sum_{j \neq k} (\mathbf{W}\hat{\mathbf{D}}'\mathbf{W}^H)_{kj} \hat{c}_j$$

$$\hat{c}_j = \tanh(\lambda^e(c_{j,1})/2) + j \tanh(\lambda^e(c_{j,2}/2)$$

Likelihood sent to decoder

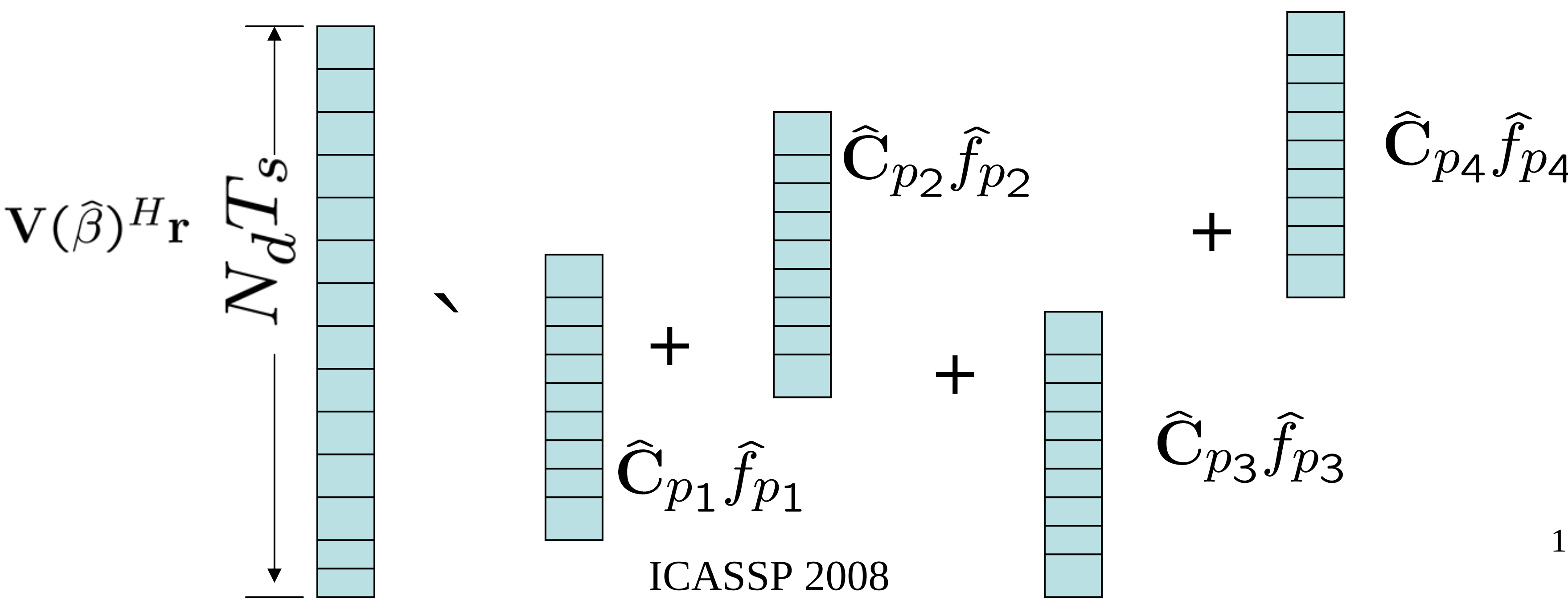
$$p(y'_k | c_k) = \mathcal{CN}\left(y'_k ; \frac{1}{N_d} \sum_{i=0}^{N_d - 1} \frac{|\hat{H}_{ii}|^2}{|\hat{H}_{ii}|^2 + \frac{N_0}{T_s}} c_k, \sigma_k^2) \right),$$

Turbo Channel Estimation

Given soft symbol decisions and pilots, the following convolution matrix representations are equivalent.

$$\mathbf{r} = \mathbf{V}(\beta)\mathbf{F}\hat{\mathbf{c}} + \mathbf{n} = \mathbf{V}(\beta)\hat{\mathbf{C}}\mathbf{f} + \mathbf{n}$$

Matching Pursuits – find sparse fit to **r** using columns of **C.**



Matching Pursuits – Sufficient Statistics Representation

Sufficient statistics

$$\mathbf{v}^1 = \widehat{\mathbf{C}}^H \mathbf{V}(\widehat{\beta})^H \mathbf{r} \qquad \mathbf{A} = \widehat{\mathbf{C}}^H \widehat{\mathbf{C}}$$

Successive least-squares channel estimates and cancellation

$$\mathbf{v}^k = \mathbf{v}^{k-1} - \mathbf{A}_{p_{k-1}} \widehat{f}_{p_{k-1}}$$

$$p_k = \arg \max_{l \neq \{p_1,\ldots,p_{k-1}\}} |\mathbf{v}_l^k|^2 / A_{p_l,p_l}$$

$$\widehat{f}_{p_k} = \mathbf{v}_{p_k}^1 / A_{p_k,p_k}.$$

Turbo Doppler Shift Estimation

Use Doppler shift estimator incorporating unconstrained LS channel estimates and decoder soft decisions plus pilots.

$$\widehat{\beta} = \arg\min_{\beta} ||\mathbf{r} - \mathbf{V}(\beta)\mathbf{C}\widehat{\mathbf{f}}_{LS}||^2$$

$$\widehat{\mathbf{f}}_{LS} = \left(\widehat{\mathbf{C}}^H\widehat{\mathbf{C}}\right)^{-1}\widehat{\mathbf{C}}^H\mathbf{V}(\beta)^H\mathbf{r}$$

$$\widehat{\beta} = \arg\max_{\beta} \mathbf{r}^H\mathbf{V}(\beta)\widehat{\mathbf{C}}(\widehat{\mathbf{C}}^H\widehat{\mathbf{C}})^{-1}\widehat{\mathbf{C}}^H\mathbf{V}(\beta)^H\mathbf{r}$$

UCSB

Simulation Parameters

Multipath spread < 25 msec.

Doppler spread < 1 Hz.

Packet length T_d = 105 msec.

Cyclic Prefis T_p = 25 msec.

Symbol rate 9600 sps.

Gallager LDPC code (1008,504)

3-ray Rayleigh fading channel, delays [0 17.2 22.2] msec.

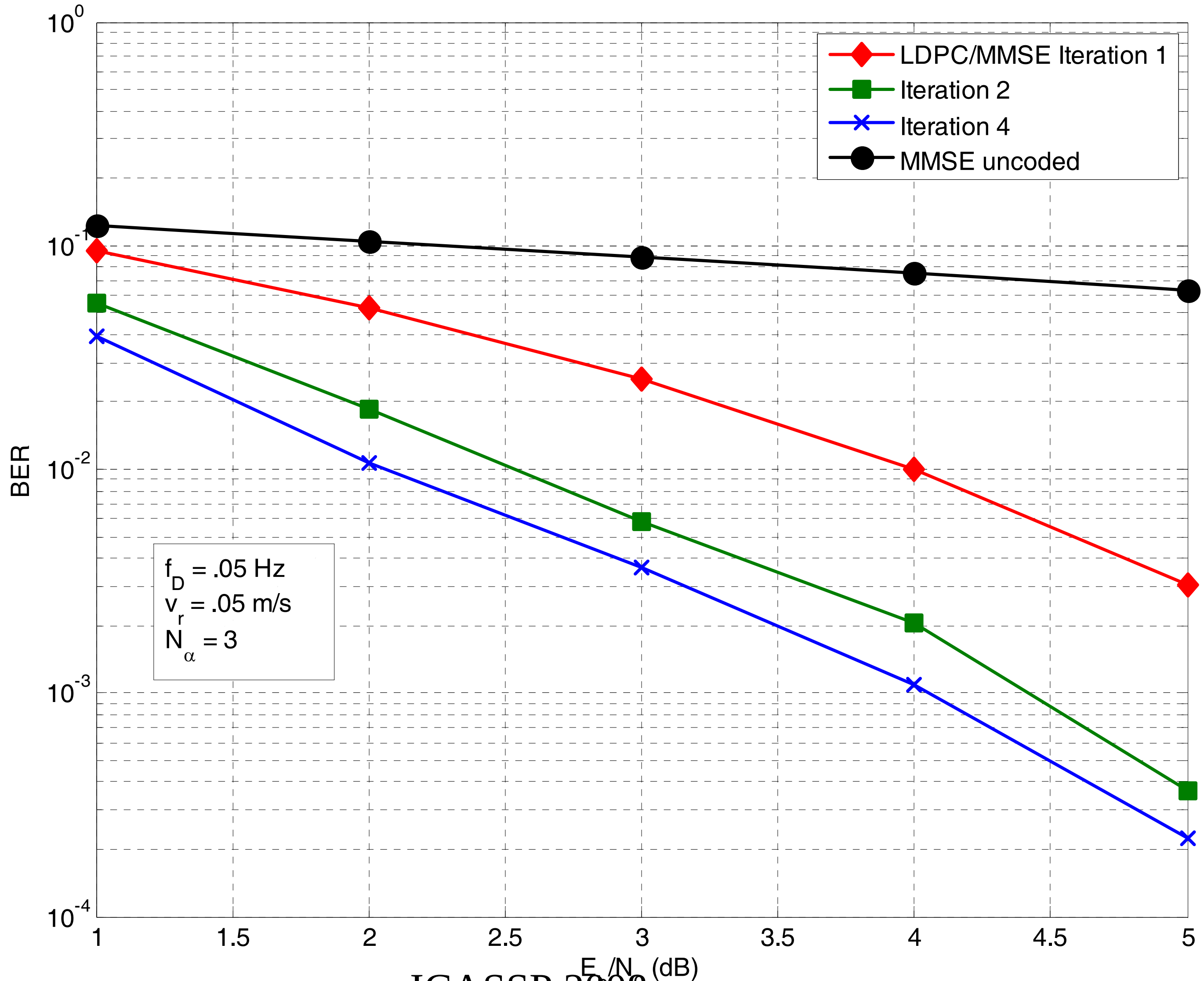
AR-1 models for channel coefficients

Doppler spreads .05 Hz and .1 Hz.

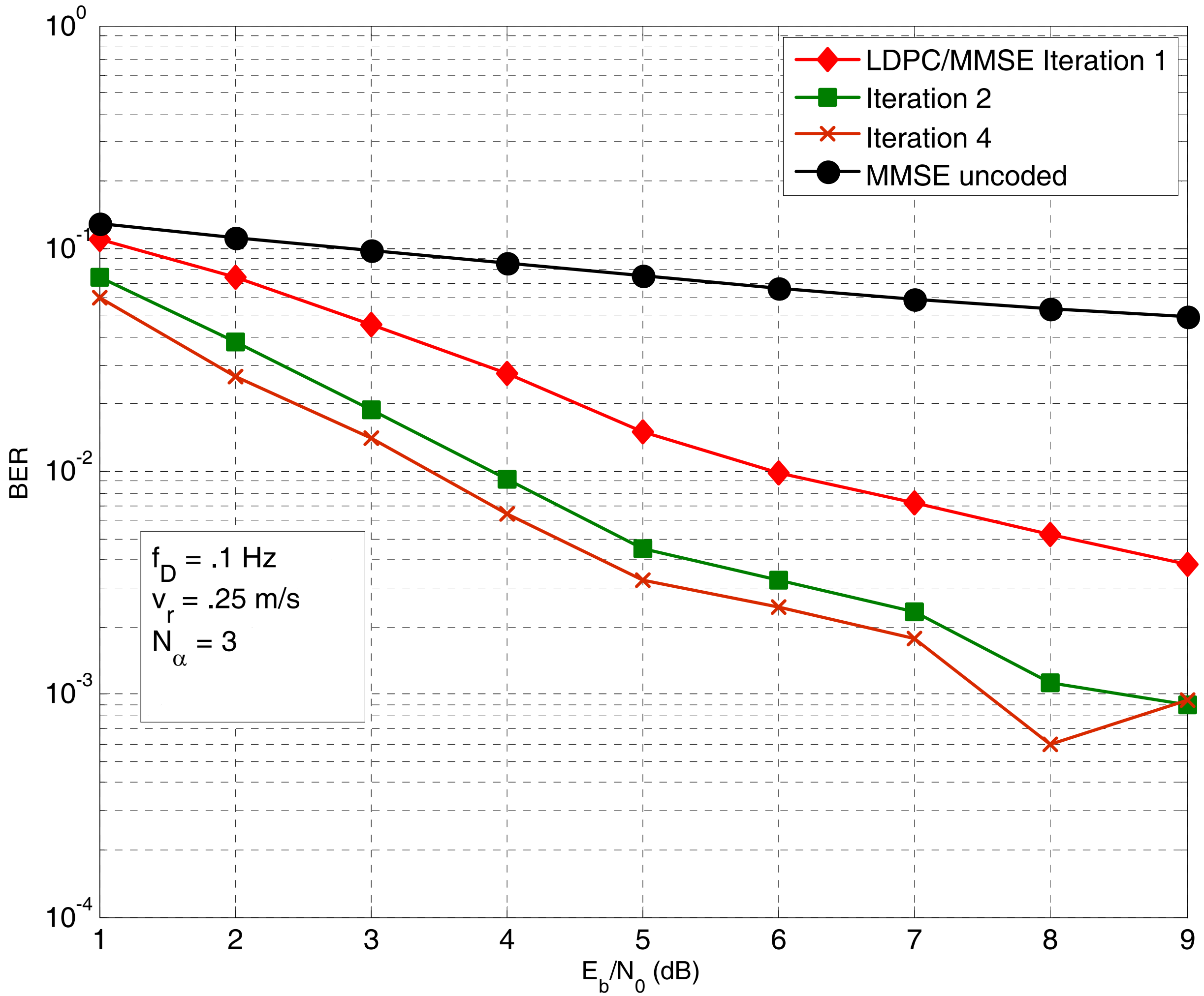
Doppler velocity .05 m/s and .25 m/s.

Pilot percentage – 50%.

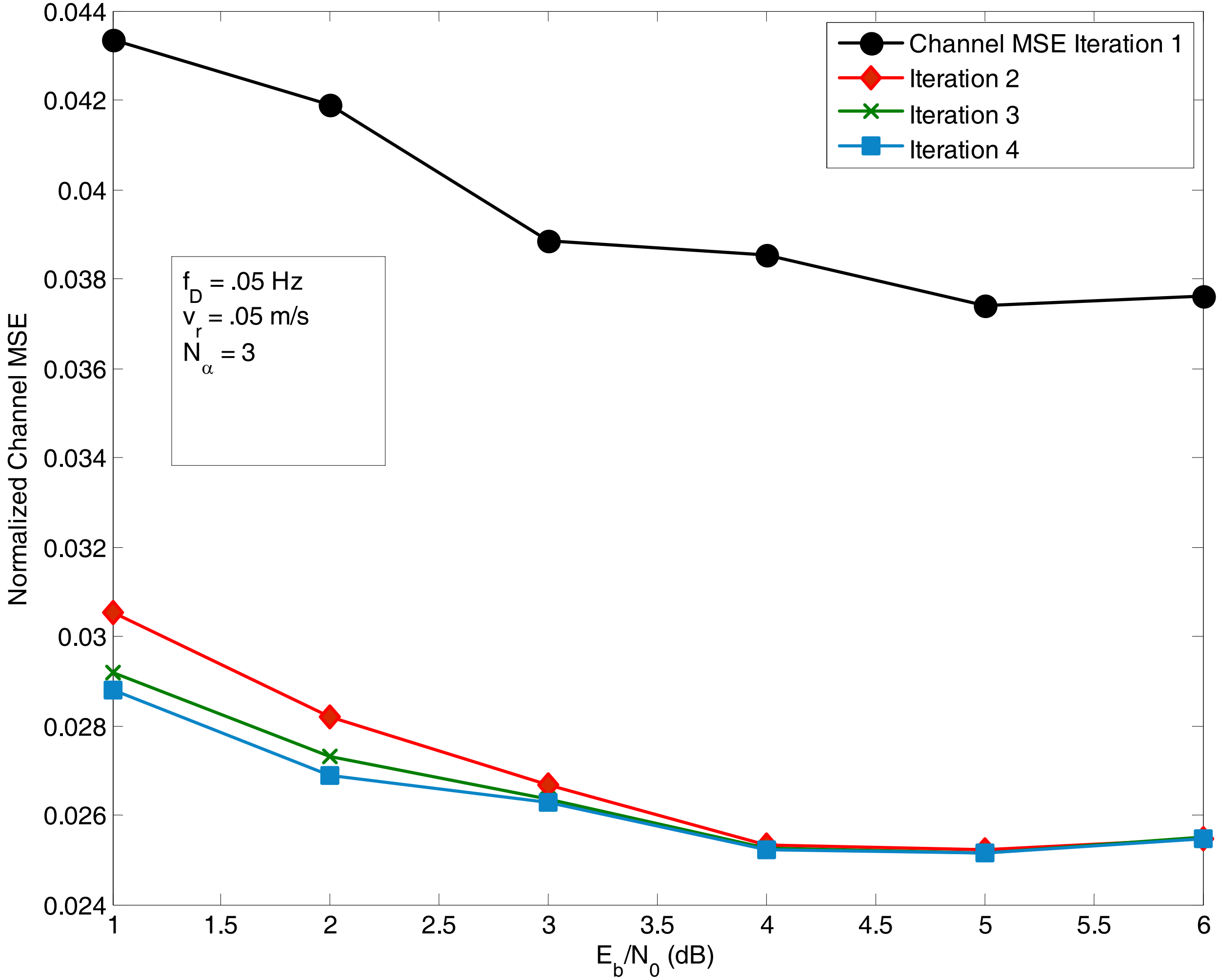
BER Results – Low Doppler Scenario



BER Results – Moderate Doppler Scenario



Channel Estimation Error – Low Doppler Scenario



Conclusions

- Uncoded SC-FDE performance can exceed OFDM in strongly Ricean channels. (Always exceeds OFDM for non-fading channels with effective $E_b/N_0 > 3/2$.)
- Direct justification for Turbo Equalization using Gaussian priors for code variables.
- Overall channel and Doppler estimator appear robust for underwater acoustic channel scenario.
- Adequate BER performance for UAC requires large percentage of pilots for SC-FDE. Need to use more accurate UAC channel models and implement in hardware/field test.