Noncooperative Optimization of Space-Time Signals in Ad hoc Networks

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Supported in part by NSF CCF-0429596 and the International Foundation for Telemetering

Presented at ITA 2006 UCSD
Outline

• Ad hoc network definition and space-time waveforms. Time-reversal relationships.
• Optimization problem: Minimize sum power under decoupled QoS constraint.
• Network duality and weak duality.
• Network Lagrangian: Greedy algorithms vs. taxation.
• IMMSE-TR as a noncooperative game with taxation.
• Extension to ST eigencoding. Waterfilling with generalized eigenvalues. IMMSE-TR for eigencoding.
• Simulation results.
Ad hoc Quasi-Synchronous Network

\[ (w_1, \tilde{g}_1) \]

\[ (w_{l(1)}, \tilde{g}_{l(1)}) \]

\[ r_i(m) = H^{0}_{i, l(i)}\tilde{g}_{l(i)}b_{l(i)}(m) + \sum_{l \neq i, l(i)}^{2} \sum_{q=0}^{2} H^{q}_{i, l} \tilde{g}_{l}b_{l}(m-q) + n_i(m) \]
Space-Time QS Waveforms

QS Timing Advance = \( \frac{r_{i,l(i)}}{c} \)

\[ g_i = Vec \left\{ \begin{bmatrix} g_i^M(N_s - 1) & g_i^M(N_s - 2) & \cdots & g_i^M(0) \\ g_i^{M-1}(N_s - 1) & g_i^{M-1}(N_s - 2) & \cdots & g_i^{M-1}(0) \\ \vdots & \vdots & \ddots & \vdots \\ g_i^1(N_s - 1) & g_i^1(N_s - 2) & \cdots & g_i^1(0) \end{bmatrix} \right\} \]
QS Channel Model

Space-Time Channels are Block Toeplitz

\[ H_{i,l(i)} = \]

\[
\begin{bmatrix}
H_{i,l(i)}(0) & H_{i,l(i)}(1) & H_{i,l(i)}(N_f - 1) & \cdots & 0 \\
0 & H_{i,l(i)}(0) & H_{i,l(i)}(1) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & H_{i,l(i)}(0)
\end{bmatrix}
\]

Proposition: ST channel transpose is reciprocal time-reverse.

\[ H_{i,l(i)}^{T} = H_{l(i),i}^{r} \]
Time-Reverse Relationships

\[
g_i = \begin{bmatrix}
    g_i(N_s - 1) \\
    g_i(N_s - 2) \\
    \vdots \\
    g_i(0)
\end{bmatrix}
\]

\[
g_i^r = \begin{bmatrix}
    g_i(0) \\
    g_i(1) \\
    \vdots \\
    g_i(N_s - 1)
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
    H(0) & H(1) & \cdots & H(N_f - 1) & 0 \\
    0 & H(0) & H(1) & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \cdots & 0 & H(0)
\end{bmatrix}
\]

\[
H^r = \begin{bmatrix}
    H(0) & 0 & 0 & \cdots & 0 \\
    H(1) & H(0) & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & H(N_f - 1) & \cdots & H(1) & H(0)
\end{bmatrix}
\]

\[
r = Hg \\
r^r = H^r g^r
\]
Optimization for QoS

- Minimize network sum power

\[ \tilde{g}_l = \sqrt{P_l} g_l \]

\[ \sum_{i=1}^{N} ||\tilde{g}_i||^2 \]

Subject to SNR constraint

\[ \Gamma_{l(i)} = \frac{P_i |w_{l(i)}^H H_{l(i),i} g_i|^2}{\sum_{l \neq i, l(i)} \sum_{q=-1}^{1} P_l |w_{l(i)}^H H_{l(i),l} g_l|^2 + \sigma_n^2} \geq \gamma_0 \]

Equivalent to decoupled capacity constraint.

\[ \log \left( 1 + \Gamma_{l(i)} \right) = r_{l(i)} \]
Summary of Ad hoc QS Network

1. Each node $i$ transmits a space-time waveform $\tilde{g}_i$

2. Each node $i = 1, 2, \ldots, N$ has a unique link node $l(i) \neq i$.

3. Spatial channels are symmetric, $H_{i,j}(p) = H_{j,i}^T(p)$.

4. Transmission is asynchronous, half-duplex.

5. Node $i$ defined by (receive, transmit) S-T beamformer pair $(w_i, g_i)$ and transmit power $P_i$.

6. SNR at node $l(i)$ defined by

$$\Gamma_{l(i)} = \frac{P_i |w_{l(i)}^H H_{l(i),i} \tilde{g}_i|^2}{\sum_{l \neq i, l(i)} \sum_{q=-1}^{1} P_l |w_{l(i)}^H H_{l(i),l} g_l|^2 + \sigma_n^2}$$
S-T MMSE Receive Beamformer

Optimal receive solution – easily implemented using LMS/RLS algorithm and preamble, ACK, CTS, RTS training sequences (e.g. in 802.11b/g)

\[
\begin{bmatrix}
    r_i^M(N_s - 1) \\
    r_i^{M-1}(N_s - 1) \\
    \vdots \\
    r_i^1(N_s - 1) \\
    r_i^M(N_s - 2) \\
    \vdots \\
    r_i^1(0)
\end{bmatrix}
\]

\[
w_i(n)^H = 
\begin{bmatrix}
    w_i^M(N_s - 1)^* w_i^{M-1}(N_s - 1)^* \ldots w_i^1(N_s - 1)^* w_i^M(N_s - 2)^* \ldots w_i^1(0)^*
\end{bmatrix}
\]
Optimization Using MMSE Beamformer

Optimal MMSE S-T receive beamformer

$$w_{i}^{opt} = R_{i}^{-1}H_{i,l(i)}\tilde{g}_{l(i)}$$

Lagrangian for optimization problem – Equivalent to capacity QoS constraint.

$$\sum_{i=1}^{N} ||\tilde{g}_{i}||^2 + \sum_{i=1}^{N} \left( \gamma_{0} - \tilde{g}_{i}^{H}H_{l(i),i}^{H}R_{l(i)}^{-1}H_{l(i),i}\tilde{g}_{i}^{H} \right)$$

Definition of MAI plus noise covariance

$$R_{i} = \sum_{k\neq i,l(i)}^{1} \sum_{q=-1}^{1} H_{i,k}^{q}\tilde{g}_{k}\tilde{g}_{k}^{H}(H_{i,k}^{q})^{H} + I$$
Selected Results on Beamforming and Eigencoding

- Point-to-Point MIMO: OFDM with beamforming is optimal in absence of MAI. (Raleigh and Jones 99)
- Spatial-only eigencoding in Ad hoc networks – greedy optimization performs poorly (Ye and Blum 03.)
- Iterative MMSE beamforming approximately minimizes sum power under decoupled capacity QoS constraint (Bromberg and Agee 03.)
- IMMSE yields minimum power solution under SNR constraints for multi-access channel (duality) (Viswanath/Tse 03, Visotsky/Madhow 99, Rashid-Farrokhi et. al. 98)
- Noncooperative beamforming games (Iltis 04, Oteri and Paulraj 05.)
Summary of Network Duality Results

• For TDD networks with symmetric channels, the optimum transmit beamformer is the conjugate of the MMSE receive beamformer (duality – Bromberg and Agee 03)

• For MISO multiuser networks, duality also holds and iterative MMSE beamforming yields optimal solutions (Rashid-Farrokhi et. al. 98)

• For Ad hoc MIMO networks (non-TDD), weak duality holds (Iltis/Kim/Hoang 05.) IMMSE satisfies the FONC for optimization using Lagrangian analysis.

\[ g_i = w_i^r,* \]

UCSB
FONC Theorem for Optimization

Theorem

The optimal normalized transmit ST vectors $g_i$, in either a QS or synchronous network, are given by the eigenvector equation

$$g_i = \lambda_i Q_i^{-1} H_{l(i)}^H R_{l(i)}^{-1} H_{l(i),i} g_i. \quad (1)$$

These vectors $g_i$ are stationary points of the Lagrangian and satisfy the KKT conditions, and $\lambda_i$ are the Lagrange multipliers. The pseudo-covariance matrix $Q_i \in C^{MN_s \times MN_s}$ is given by

$$Q_i = I + \sum_{k \neq i, l(i)} \sum_{q=0}^{2} \rho_{l(k)} (H_{l(k),i}^q)^H w_{l(k)} w_{l(k)}^H H_{l(k),i}^q. \quad (2)$$

Corollary: A greedy noncooperative algorithm cannot satisfy the FONCs.
Optimality of FDMA in Ad hoc Networks

Let the network defined by the matrices $\mathbf{H}_{l(i),j}^P$ be synchronous, so that $\mathbf{H}_{l(i),j}^P = \mathbf{0}$ for $p \neq 0$. Assume that the waveforms $\tilde{\mathbf{g}}_i$ are augmented by a cyclic prefix so that the $\mathbf{H}_{l(i),j}^0$ matrices become circulant. Then the optimal ST waveforms in the absence of MAI are given by

$$\tilde{\mathbf{g}}_i^{opt} = \sqrt{P_i}\mathbf{c}_{k_i} \otimes \mathbf{a}_i,$$

(1)

where $\mathbf{c}_k = [1 \exp(i2\pi k/N_s) \ldots \exp(i2\pi k(N_s - 1)/N_S)]^T$ is an FFT vector with frequency $k$. The ST waveform is normalized to $||\mathbf{c}_{k_i} \otimes \mathbf{a}_i||^2 = 1$. The minimum power $P_i$ yielding an SNR of $\gamma_0$ is $P_i = \gamma_0/\lambda_{\max}(\mathbf{H}_{l(i),i}^H \mathbf{H}_{l(i),i})$, where $\lambda_{\max}({A})$ is the maximum eigenvalue of matrix $A$. The optimal frequency $k_i$ and spatial signature $\mathbf{a}_i$ without MAI satisfy

$$k_i, \mathbf{a}_i = \text{arg max}_{k,\mathbf{a}} \mathbf{a}^H \hat{\mathbf{H}}_{l(i),i}^H(k)\hat{\mathbf{H}}_{l(i),i}(k)\mathbf{a},$$

(2)

where $||\mathbf{a}_i||^2 = 1/N_s$. Furthermore, when the decoupled optimum frequencies $k_i$ are unique so that $k_i \neq k_j$ for $j \neq i, l(i)$, then the FDMA solution is the network optimum of the problem.
IMMSE Algorithm for S-T Beamforming

Step 1: Transmit training sequence from 1, use LMS/RLS at node 2 = l(1) to approximate MMSE/MVDR S-T beamformer. (Assume zero misadjustment)

\[ w_{l(1)}(m + 1) = R_{l(1)}(g_{-i}^n)^{-1}H_{l(1),1}g_1(m) \]

Step 2: Set transmit beamformer at l(1) to c.c. time-reverse of normalized MVDR receive beamformer.

\[ g_{l(1)}(m+1) = \frac{(w_{l(1)}(m + 1)^*)^r}{\|w_{l(1)}(m + 1)\|} = \frac{1}{c}(R_{l(1)}(g_{-i}^n)^*)^{-r}(H_{l(1),1})^r(g_1^*(m))^r \]
IMMSE S-T

Step 3: Transmit training sequence from 2 to 1, use LMS/RLS and repeat MVDR beamformer computation.

\[ w_1(m + 1) = R_1(g_{-i}^n)^{-1}H_{1,l(1)}g_{l(1)}(m + 1) \]

Step 4: Replace \( g_1 \) by \((w_1^*)^r\). Substitution reveals power alg.

\[ g_1(m+1) = \frac{1}{c}(R_1(g_{-i}^n)^*)^{-r}(H_{1,l(1)}^*)^r(g_{l(1)}(m+1)^*)^r \]

\[ g_1(m+1) = \frac{1}{c}(R_1(g_{-i}^n)^*)^{-r}H_{l(1),1}^H R_{l(1)}(g_{-i}^n)^{-1}H_{l(1),1}g_1(m) \]

\[ g_1(\infty) = \max \text{eigenvector}(R_1^*)^{-r}H_{l(1),1}^H R_{l(1)}^{-1}H_{l(1),1} \]
Noncooperative Game Interpretation

Use Gauss-Seidel iterations with $g_i^{n+1}$ updated sequentially. Power update achieves $\gamma_0$ immediately.

\[
P_i^{n+1} = \frac{\Gamma_0}{(g_i^{n+1})^H H_{l(i),i} R_{l(i)} H_{l(i),i} (g_{-i}^n)^{-1} g_i^{n+1}} = \Gamma_0 P_i^n / \Gamma_{l(i)}
\]

Matrix pencil interpretation of IMMSE.

\[
g_i^{n+1} = \arg \max_{g_i} g_i^H H_{l(i),i} R_{l(i)} (g_{-i}^n)^{-1} H_{l(i),i} g_i / g_i^H R_i r_i^* (g_{-i}^n) g_i
\]
Noncooperative Game Interpretation

Equivalent utility function: \( P_i^{n+1}, g_i^{n+1} \) maximizes \( u_n() \) (Best response.)

\[
u_n(P_i, g_i, \tilde{g}_-^n) = \eta \left( \gamma_0 - P_i \gamma_i(g_i, \tilde{g}_-^n) \right)
+ \ln g_i^H H_{l(i),i}^H R_{l(i)}^{-1}(\tilde{g}_-^n) H_{l(i),i} g_i
- \ln g_i^H R_i^*(\tilde{g}_-^n)^r g_i,
\]

Normalized SNR for link \( i \) to \( l(i) \), \( \gamma_i(g_i, g_{-i}) \).

Interference tax
Total Inference Function for Spatial-Only Beamforming

\[ \text{TIF} = \tilde{g}_i^H R_i^* \tilde{g}_i + ||\tilde{g}_i||^2 \]

\[ \text{TIF} = \sum_{i} \sum_{l \neq i, l(i)} \tilde{g}_i^H H_{i,l}^* \tilde{g}_l^* \tilde{g}_l^T H_{i,l}^T \tilde{g}_i + 2 \sum_{i} ||\tilde{g}_i||^2. \]

\[ P_i P_l \left| w_l^H H_{l,i} g_i \right|^2 \]

Decomposition of TIF after Gauss-Seidel update of \( g_i^n \).

\[ \text{TIF}^n_i = 2(\tilde{g}_i^n)^H R_i^*(\tilde{g}_-i^n)\tilde{g}_i^n + f(g_-i^n) \]
Relation of IMMSE-TR to Lagrangian

Theorem

Let \( \{\tilde{g}_i\} \) be the IMMSE-TR solution for the ST waveforms and assume the network is synchronous. Then the \( \{\tilde{g}_i\} \) correspond to a stationary point of the Lagrangian where each \( \lambda_i \) is minimized and satisfies the KKT conditions. The resulting dual of the Lagrangian for any set of transmit beamformers satisfying \( g_i = w_i^r \star \forall i \) is

\[
P_{sum} = \sum_{i=1}^{N} ||\tilde{g}_i||^2 = \sum_{i=1}^{N} \lambda_i \gamma_0 - \sum_{i=1}^{N} \sum_{j \neq i, l(i)} |\tilde{g}_i^r \mathbf{H}_{i,j} \tilde{g}_j|^2. \tag{1}
\]

Hence, IMMSE-TR, where each \( \lambda_i \) is minimized can only correspond to the minimum of \( P_{sum} \) in the absence of MAI, i.e. when \( g_i^r \mathbf{H}_{i,j} g_j = 0 \).

Interpretation: IMMSE-TR satisfies FONC, but in general is not optimum.
IMMSE-TR and FONC

• Recall Theorem giving FONC.

\[ \mathbf{g}_i = \lambda_i \mathbf{Q}_i^{-1} \mathbf{H}_l^{H}(i) \mathbf{R}_l^{-1}(i) \mathbf{H}_l(i),i \mathbf{g}_i. \]

IMMSE-TR matrix pencil interpretation.

\[ \mathbf{g}_i^{n+1} = \arg \max_{\mathbf{g}_i} \frac{(\mathbf{g}_i^{H} \mathbf{H}_l^{H}(i),i \mathbf{R}_l(i)(\tilde{\mathbf{g}}_i^{n})^{-1} \mathbf{H}_l(i),i \mathbf{g}_i)}{\mathbf{g}_i^{H} \mathbf{R}_i^{r,*}(\tilde{\mathbf{g}}_i^{n})\mathbf{g}_i} \]

Satisfies FONC for smallest Lagrange multiplier satisfying KKT conditions.

\[ \mu_i \mathbf{g}_i = \mathbf{R}_i^{-1} \mathbf{H}_l^{H}(i) \mathbf{R}_l^{-1}(i) \mathbf{H}_l(i),i \mathbf{g}_i. \]
Power Efficiency

• Power efficiency: Power required to maintain power on link i to l(i) in absence of multiuser interference/ Actual power used in IMMSE.

\[ \eta = \frac{P_{su}^i}{P_i(n)} \]

• Similar to CDMA asymptotic efficiency for Gaussian MAI (Verdú 86, 98).

Maximum normalized SNR on link i to l(i).

\[ \lambda_{max} \left( H_{l(i),i}^H, H_{l(i),i} \right) \]

Power efficiency:

\[ \eta = \frac{\Gamma_0}{P_i^n \lambda_{max} \left( H_{l(i),i}^H, H_{l(i),i} \right)} \]
Simulation of IMMSE – Exact Eigenvector Solution

$M = 4$ elements, $N_s = 14$ samples, SNR target = 10 dB, three path fading channel with $1/r^2$ loss.

![Graphs showing Meters and Nyquist Sample with User Number]
Power Efficiency for Space-Time Waveforms

Cooperative Augmented Lagrangian Efficiency

IMMSE-TR Efficiency $\eta$

Greedy Algorithm

$N = 10$ Nodes
$N_s = 14$, $M = 4$
Power Efficiency

Cooperative augmented Lagrangian (centralized) always yields best performance.

IMMSE-TR always outperforms greedy optimization, very close to cooperative solution.

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>10</th>
<th>14</th>
<th>18</th>
<th>22</th>
<th>26</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMMSE-TR $\eta$</td>
<td>0.88</td>
<td>0.87</td>
<td>0.83</td>
<td>0.82</td>
<td>0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>Greedy $\eta$</td>
<td>0.84</td>
<td>0.83</td>
<td>0.73</td>
<td>0.73</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>Centralized $\eta$</td>
<td>0.89</td>
<td>0.87</td>
<td>0.84</td>
<td>0.83</td>
<td>0.70</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 1: Power efficiency of 3 algorithms averaged over 50 network instances
Extension to Space-Time Eigencoding

- Global optimization problem. Minimize sum power under assumptions (a) MAI is Gaussian  (b) Decoupled capacity constraint.

$$\text{minimize } \sum_{i=1}^{N} \text{tr}\{\tilde{G}_i H \tilde{G}_i\}$$

subject to for all $l(i)$

$$\log |I + \tilde{G}_i H_{l(i),i} R_{l(i)}^{-1} H_{l(i),i} \tilde{G}_i| = r l(i),$$
Noncooperative ST Eigencoding (Taxation)

Minimize $\text{tr}\{\tilde{G}_i^H R_i^{*,r} \tilde{G}_i\}$

Subject to

$$\log |\mathbf{I} + \tilde{G}_i^H H_{l(i)}^H R_{l(i)}^{-1}(\tilde{G}_{-i}) H_{l(i)} \tilde{G}_i| = r_{l(i)}$$

with $\tilde{G}_{-i}$ fixed.

Interference taxation due to minimization objective.

$$Tr \left\{ \tilde{G}_i^H R_i^{*,r} \tilde{G}_i^H \right\} =$$

$$Tr\{\tilde{G}_i^H \tilde{G}_i^H\} + Tr \left\{ \tilde{G}_i^H \sum_{k \neq i} H_{I,k}^r \tilde{G}_k^{r,*} \tilde{G}_k^r, T H_{l,k}^{r,T} \tilde{G}_i^H \right\}$$
Waterfilling using Generalized Eigenvalues

Noncooperative optimum.

\[ \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1} \mathbf{H}_{l(i),i} \mathbf{G}_i = (\mathbf{R}_{i}^{*,r}) \mathbf{G}_i \Lambda_i^G \]

Powers give in terms of generalized eigenvalues.

\[ \mathbf{P}_{i,k} = \left( \frac{\mu_i}{\Lambda_{i,k}} - \frac{1}{\Lambda_{i,k}} \right)^+ \]

Properties of generalized eigenvalues and eigenmatrices.

\[ \mathbf{G}_i^H \mathbf{R}_{i}^{*,r} \mathbf{G}_i = \Lambda_i^I \]

\[ \mathbf{G}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1} \mathbf{H}_{l(i),i} \mathbf{G}_i = \Lambda_i^S \]

\[ \Lambda_i^G = \Lambda_i^S (\Lambda_i^I)^{-1} \]
IMMSE-TR for ST Eigencoding

Consider matrix ST MMSE detector.

\[ \text{MMSE}_i = \arg \min_w E \{ \| b_{l(i)} - W_i^H r_i \|^2 \} \]

MMSE matrix form of solution

\[ W_i = R_i^{-1} H_{i,l(i)} \tilde{G}_{l(i)} M_i \]

\[ M_i = \left[ I + \tilde{G}_{l(i)}^H H_{i,l(i)}^H R_i^{-1} H_{i,l(i)} \tilde{G}_{l(i)} \right]^{-1} \]

IMMSE-TR – column-wise conjugation and time-reversal

\[ G_i = W_i^{r,*} \]
Relationship of IMMSE-TR to Noncooperative Optimum

Theorem:
Consider the IMMSE-TR algorithm where the transmit matrices satisfy $G_i = W_i^{r,*}$ for all nodes $i$. Then at least one fixed point of IMMSE-TR corresponds to the generalized eigenmatrix solution rewritten as

$$G_i = (R_i^{r,*})^{-1} H_{l(i),i}^H R_{l(i)}^{-1} H_{l(i),i} G_i (\Lambda_i^g)^{-1}. \quad (1)$$

Outline of proof: From definition of MMSE detector

$$W_i = R_i^{-1} H_{i,l(i)} G_{l(i)} P_{l(i)}^{1/2} M_i D_i$$

Use IMMSE-TR relationship

$$G_i = (R_i^{r,*})^{-1} H_{l(i),i}^H R_{l(i)}^{-1} \times H_{l(i),i} G_i P_i^{1/2} M_{l(i)} D_{l(i)} P_{l(i)}^{1/2} M_i^{r,*} D_i^{r,*}$$

But $D_i$ becomes diagonal if $G_i$ is a generalized eigenmatrix. Leads to (1) above.
Exact channel/covariance knowledge – eigenmatrix solution. $N_s = 6$, $M = 4$, $r = 3$ b/s/Hz, 14 nodes.
Power Efficiency – ST Eigencoding vs. OFDM

M = 4
N_s = 6
r = 3 b/s/Hz

Power efficiency vs. Iterations
Zero-infrastructure Access Point Router – Rank-1 dominant beampattern approximations for noncooperative ST eigencoding.
$N_s = 64$ (OFDM-type) \( M = 6 \) antennas, 10 b/s/Hz.
Conclusions

• Lagrangian optimization, noncooperative game theory and taxation closely related to IMMSE-TR.
• Implicit interference taxation in IMMSE-TR yields consistently better solutions than greedy optimization.
• ST eigencoding with taxation – leads to waterfilling algorithm based on generalized eigenvalues.

Questions:
– Can taxation be optimized? Use game theory concepts? (Problems with lack of ordered action set, supermodularity.)
– Can IMMSE-TR be modified to yield consistent eigenmatrix solutions for ST eigencoding?
– Robust algorithms for channel/covariance estimation?
– Implement in reconfigurable hardware?