Multiperiod virtual topology design in wavelength routed optical networks

P. Manohar, A. Padmanath, S. Singh and D. Manjunath

Abstract: A wavelength routed optical network is considered for which the traffic matrix and possibly the physical topology are different during different periods of a time horizon. The optimal virtual topology for each of the periods will be different. Rather than using the best topology in every period and incurring a possibly significant reconfiguration cost, it is necessary to consider the optimum sequence of virtual topologies that will minimise the sum of the operating and reconfiguration costs over the entire horizon. The presence of short-term cycles in the traffic pattern and the possible evolution of resources and demand in the network motivates this work. The approach adopted is to find a ranked set of virtual topologies for each period and then define a shortest path problem to obtain the optimum sequence. Examples on a 10 node section of the NSFNET topology are presented.

1 Introduction

Optical networks allocate bandwidth at the granularity of a lightpath, i. e. an all optical channel is set up between node pairs in the network. The collection of lightpaths is called the lightpath topology or virtual topology and it is overlaid on the physical topology. The optimum virtual topology is typically designed as follows: the traffic matrix indicating the volume of traffic flow between source destination pairs in the network is specified and each flow is allocated to an ordered sequence of lightpaths. The optimal virtual topology is typically obtained as a solution to a mixed integer linear programming (MILP) problem that minimises the average hop distance (number of lightpaths traversed by the flows) in the network [1], the maximum flow (congestion) in a lightpath [2–4] or the total number of lightpaths [5]. The MILP is defined by specifying two types of constraints: multicommodity flow equations determined by the physical topology and the traffic matrix, and resource constraints to limit the number of lightpaths emerging and terminating at a node to no more than the number of transmitters and receivers at the node. See [6] for a survey of the virtual topology design problem and algorithms.

An associated problem with the virtual topology design is reconfiguration. The virtual topology may need to be changed because of changes in the traffic matrix and/or the physical topology. There is a cost associated with reconfiguration due to disruption of service and reprogramming of the switches. The little literature that there is on reconfiguration considers minimising the cost of one time reconfiguration. In this paper, our interest is in multiple reconfigurations and we aim to minimise the total cost over all these reconfigurations. There are two motivations for this. First, the short-term cycle problem that arises from cyclical patterns in the traffic matrix like, for example, that seen in circuit switched telephony networks. The cycles are usually of 24-hour or 7-day duration. For optimal operation of the network it may be necessary to have different virtual topologies for different periods in a cycle. Second, the planning horizon problem. It is well known that bandwidth demand evolves fairly rapidly and so does the physical network topology. Thus, it is important to look at network reconfiguration over a longer time scale that includes multiple changes in the demand (traffic matrix) and supply (network topology in terms of physical links and transceivers). The projections for the traffic matrix and the roadmap for the physical topology are assumed to be known. Once again, the optimum virtual topology may be different at different times depending on the traffic matrix and the physical topology at these times.

An obvious way to handle the evolution of demand and supply, even if cyclical, is to periodically obtain the optimum virtual topology and then reconfigure the existing virtual topology into the new one by changing a set of lightpaths. Although obtaining the actual reconfiguration cost can be quite complex, in its simplest form, we may assume that it is an increasing function of the number of lightpaths that have to be added and removed and/or the number of switches that need to be reprogrammed. Thus, the total cost of operating the network will include the operating cost, which will be a function of the value of the objective function used in the virtual topology design, and the reconfiguration cost over the cycle in the short-term cycle problem or over the planning horizon. This leads us to formulate and solve the multiperiod virtual topology reconfiguration (MVTR) problem for wavelength routed optical networks. As far as we know, a multiperiod design problem for optical networks has not been considered before.

2 Previous work

We classify the manner in which reconfiguration is addressed in the literature into three types: (i) the 'known-

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target-objective' algorithm; (ii) the 'constrained-change' algorithm; and (iii) the 'known-target-topology' algorithm. In the known-target-objective algorithm [1] the value of the objective function for the optimum virtual topology for the new traffic matrix and the physical topology is obtained. Let this value be OPT_2. The virtual topology design problem is then reformulated with constraints that search for the target topology from among the virtual topologies whose objective function will be OPT_2. The objective function for this is to minimise the number of lightpaths that must be added or deleted. The assumption here is that there are possibly many 'optimal' virtual topologies and we should search from among these such that the reconfiguration cost is minimised. This method does not achieve a balance between finding an optimal new virtual topology and one that involves as little change from the old one as possible. It may be that the cost of change is traded for optimal operation under the new conditions.

The constrained-change method [7] provides a trade-off between the objective function of the operating virtual topology and the number of changes to the virtual topology. An upper bound, N_{range}, on the number of changes is specified. A list of possible new virtual topologies that are less than N_{range} changes from the current virtual topology is obtained from the current virtual topology such that it satisfies the N_{range} constraint. From among the topologies generated as above, the one with the best objective function is chosen as the target topology. This list of virtual topologies is obtained from heuristics based on the following intuition. If lightpaths are established between node pairs with high traffic, the routed traffic and hence the average weighted hop count is reduced. Let $t_{s,d}$ be the total traffic from source s to destination d and $h_{s,d}$ be the number of hops from s to d in the virtual topology. Define $h_{\text{val}}:= t_{s,d}\, h_{s,d}$. h_{val} is computed for each source destination pair. Lightpaths which are to be added to the old virtual topology to obtain the new virtual topology are selected in decreasing order of h_{val}. Lightpaths are added or deleted such that the resultant topology remains connected. After each addition of a lightpath, the virtual topology is checked to see if it satisfies the N_{range} constraint.

The third approach is the known-target-topology in which the target topology is known and the reconfiguring cost is minimised. This algorithm has been applied to broadcast-and-select optical networks [8] but not to wavelength routed networks.

The algorithms considered above are for one step reconfiguration and minimise the cost of reconfiguration only for that step. In another context, [9] considers adapting the virtual topology to changing traffic patterns. This too is a one step change algorithm. Our interest in this paper is to consider reconfiguration over multiple periods and to minimise the overall cost, the changeover cost and the operation cost. In the next Section, we describe the approach to multiperiod reconfiguration. We mention here that the computation costs of the algorithms described here are significant and they are not directly applicable for use as online algorithms. The algorithms proposed are essentially offline algorithms.

3 Method for multiperiod design

Multiperiod design is a well known operations research problem and has been applied to, among others, long-run multiple warehouse location problems [10, 11]. One way of solving the multiperiod design problem is by obtaining ranked solutions for each period and finding the optimal sequence of solutions to be used from dynamic programming methods. In the context of virtual topology design, the problem is defined as follows. Consider an optical network in which there are significant shifts in the traffic matrix and the physical topology, over n periods with the periods named T_1–T_n. Let TM_i and PT_i be the traffic matrix and the physical topology respectively in period T_i. We will assume that there is a cost of network operation during period T_i and that it is a function of the virtual topology used during the period. If the virtual topology is changed for period T_{i+1}, then it has to be reconfigured with some existing lightpaths deleted and new ones established. There is a cost associated with reconfiguration that is a function of the number of lightpaths that are changed and the number of switches that need to be reprogrammed. We need to choose the sequence of virtual topologies that will minimise the total cost of the network, the sum of the operating cost and the reconfiguration cost, over the n periods. In the short-term cycle problem, the total cost, as defined above, for the period from the beginning of a cycle to the beginning of the next is to be minimised.

To obtain the optimum overall sequence we proceed as follows. For period T_i, given the traffic matrix TM_i and the physical topology PT_i, we obtain the R_i best virtual topologies. We will assume that the operating cost with a given virtual topology is a function of the physical topology, the traffic matrix and the virtual topology. For each period T_i, we rank the virtual topologies in increasing order of the operating cost. Let $VT_{i,j}$ be the virtual topology in period T_i with rank j, $j=1,\cdots,R_i$, and $OC_{i,j}$ its operating cost. There will also be a changeover cost of going from topology $VT_{i,j}$ in period T_i to $VT_{(i+1),k}$ in period T_{i+1}. This changeover cost will be denoted by $CC_{(i,j)\to((i+1),k)}$, and will be a function of the $VT_{i,j}$, $VT_{(i+1),k}$, PT_i, PT_{i+1}, TM_i and TM_{i+1}. We will discuss the determination of a good value for R_i later in the section.

The ranked solutions for each of the n periods are as shown in Table 1. Column i corresponds to the solutions or period T_i and the entries in the column are the acceptable virtual topologies for T_i. The problem now reduces to finding the sequence $VT_{1,j_1}, VT_{2,j_2}, \cdots VT_{n,j_n}$ that optimises the overall cost over the periods T_1-T_n. The next step depends on whether the optimisation is for short-term cycles or over a planning horizon.

First consider the short-term cycle problem in which we want to return to the same virtual topology at the beginning of the next cycle. We can now state the problem of finding the shortest path in a graph G constructed as follows. Each $VT_{i,j}$, $i=1,\cdots n+1, j=1\cdots, R_i$ is a vertex in the graph. Note that the virtual topology $VT_{n+1,j}$ is the same as $VT_{1,j}$. An edge between $VT_{i,j}$ and $VT_{(i+1),k}$, $i=1,\cdots, n-1$, has a weight $C_{(i,j)\to((i+1),k)} = CC_{(i,j)\to((i+1),k)} + OC_{(i+1),k}$, the sum of the changeover cost to the new topology and the operating cost in that topology. The shortest path between a

Table 1: virtual topologies ranked according to the operating cost in each of the n periods

T_1	T_2	T_3	…	…	T_n
$VT_{1,1}$	$VT_{2,1}$	$VT_{3,1}$	…	…	$VT_{n,1}$
$VT_{1,2}$	$VT_{2,2}$	$VT_{3,2}$	…	…	$VT_{n,2}$
$VT_{1,3}$	$\vdots$	$VT_{3,3}$	…		$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	VT_{2,R_2}	$\vdots$			VT_{n,R_n}
VT_{1,R_1}		VT_{3,R_3}			

Note that not all columns contain the same number or entries virtual topology in the first period and the same virtual topology in the $(n+1)$th period is found using any of the well known shortest path algorithms for all the the virtual topologies in T_1. The least cost sequence of these R_1 sequences is selected as the optimum sequence of virtual topologies.

Now consider the planning horizon problem in which we need to minimise the sum of operating and changeover costs, over the planning horizon. Assume that the process of reconfiguration started with a virtual topology before T_1, say VT_o and we have a target topology to be achieved after the nth period T_n, say VT_f. We need to find the sequence of virtual topologies over the n periods such that the overall cost of transition from VT_o to VT_f is minimised. It is easy to see that this problem is identical to the short-term cycle problem except that we need the shortest path from VT_o to VT_f, where VT_f is different from VT_o. An example graph G for three periods with a possible best sequence is shown in Fig. 1.

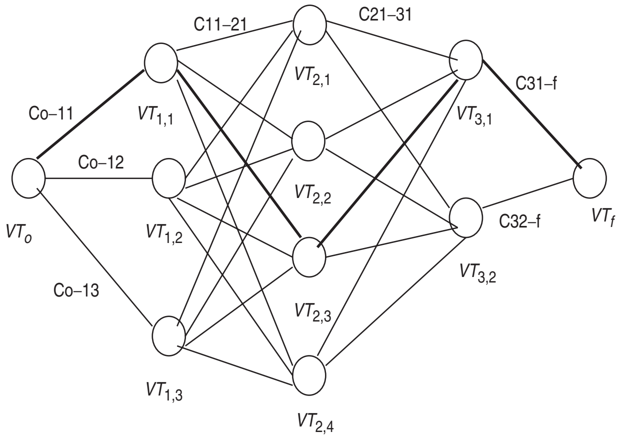

Fig. 1 *Example graph G for a three-period problem*
A possible best sequence of virtual topologies is shown by thick lines $\{VT_o, VT_{1,1}, VT_{2,3}, VT_{3,1}, VT_f\}$

In the method for multiperiod reconfiguration described above, there are two important issues. First, we need to select an appropriate value of R_i, $i=1,\cdots,n$. The following method is adapted from [10]. Let TC denote the unknown optimal total cost over the planning horizon. Define $TC^{\inf} \triangleq \sum_{i=1}^{n} OC_{i,1}$. Observe that $TC \geq TC^{\inf}$. This is because the changeover cost is not considered. Let TC^* denote an upper bound on TC. Observe that for each period T_i, i.e. for each column of Table 1, it is sufficient for the R_ith solution to satisfy $OC_{i,R_i} - OC_{i,1} \leq TC^* - TC^{\inf}$ and $OC_{i,R_i+1} - OC_{i,1} > TC^* - TC^{\inf}$. The computational cost of the algorithm depends on the R_i for each of the periods and the values of R_i depend on the quality of the upper bound on the total cost. Thus it is important to have as tight an upper bound as possible. This can be obtained by an iterative method as follows. Initially, the value of R_i is chosen as some predefined constant. Alternatively, we can obtain the virtual topologies for period T_i while $OC_{i,R_i} - OC_{i,1} \leq \theta$ where θ is a predefined constant. The optimal total cost from this limited set of virtual topologies gives a value for TC^*. Then, for each period, all virtual topology solutions that satisfy $OC_{i,R_i} - OC_{i,1} \leq TC^* - TC^{\inf}$ are obtained. The optimal total cost from this enhanced list of virtual topologies is an improved upper bound on T^*. This new T^* can be used to enhance the list of virtual topologies as before. This iterative procedure in improving T^* and calculating the ranked virtual topologies is repeated until we have $OC_{i,R_i} - OC_{i,1} > TC^* - TC^{\inf}$ for all the periods. Observe that the value of R_i can be different in each time period T_i.

The second issue is to obtain the ranked solutions for each column of Table 1. The path-add heuristics described in [7] may be extended to obtain the ranked list of topologies. Alternatively, a more formal method similar to the one outlined in [10] may be used. Recall that the virtual topology design is formulated as a MILP where, typically, the integer allocation variables are (0, 1) variables. We will assume that such is the case and let $Z=\{z_j; j=1,\cdots,J\}$ be this set of (0, 1) variables. Any feasible assignment of the (0, 1) variables can be represented by a vertex of the J-dimensional unit hypercube. Thus the optimal solution is also a vertex on the J-dimensional hypercube. To obtain a second rank solution, it is necessary to consider the hypercube without the optimal vertex, that is, the best rank solution along with the other constraints. This means that we need to use the same constraints as those used to obtain the optimal solution except that it is augmented such that only the 'optimal vertex' is excluded in the search. This is achieved by making a canonical cut on the unit hypercube [12] such that only the optimal vertex is excluded. This cut is performed by adding a constraint described below.

Let the best solution be denoted by Z^1. For this solution, the set A_1 and U_1 are defined as follows:

$$A_1 = \{j : z_j = 1 \text{ in the best solution } Z^1\}$$
$$U_1 = \{j : z_j = 0 \text{ in the best solution } Z^1\}$$
$$N(A_1) = \text{number of elements in } A_1$$

The 'optimal vertex' is removed by adding the constraint

$$\sum_{j\in A_1} z_j - \sum_{j\in U_1} z_j \leq N(A_1) - 1 \qquad (1)$$

The optimal solution to the MILP problem after adding this constraint is the second best solution. In this way the constraints can be added until the R_i best solutions are obtained for period T_i.

In a similar way, the third best solution can be obtained by cutting the hypercube to exclude the vertices corresponding to the first and second best solutions by adding another constraint of the form,

$$\sum_{j\in A_2} z_j - \sum_{j\in U_2} z_j \leq N(A_2) - 1 \qquad (2)$$

The sets A_2 and U_2 are defined for the second best solution, say Z^2, as follows:

$$A_2 = \{j : z_j = 1 \text{ in the second best solution } Z^2\}$$
$$U_2 = \{j : z_j = 0 \text{ in the second best solution } Z^2\}$$
$$N(A_2) = \text{number of elements in } A_2$$

4 Example

The multiperiod reconfiguration described above is illustrated by using a virtual topology design algorithm similar to that in [1]. The MILP to obtain the ranked list of virtual topologies is described below. The source and destination for the traffic will be denoted by s and d respectively, in either subscript or in superscript. The originating and terminating nodes for a lightpath will be i and j respectively, m and n will be the endpoints of a physical link used by a lightpath. The following inputs to the MILP are given for each period: the physical topology arc-incidence matrix $P=\{P_{m,n}\}$; the traffic matrix $TM=\{\Lambda_{s,d}\}$, where $\Lambda_{s,d}$ is the traffic demand from s to d expressed as a fraction of lightpath capacity; the maximum number of transmitters and receivers at node i, T_i and R_i; the maximum number of wavelengths per physical link, W. We will denote the number of nodes in the network by N. The following variables will be assigned by the MILP.

- $V_{i,j}$, the 0/1 indicator variable for a lightpath from i to j.
- $\lambda_{i,j}^{s,d}$ the traffic routing variable for the amount of traffic from node s to d over a lightpath between i and j.
- $p_{m,n}^{i,j}$, 0/1 indicator variable for routing lightpath between i and j over the physical link between m and n.

The objective function to be minimised is the total number of lightpaths in the network. The following MILP will give the $(K+1)$th best solution, after having obtained the best K solutions. This MILP is to be run R_i times for period T_i.

MILP-VTD: Minimise $\Sigma_{i,j}\ V_{i,j}$
subject to

- Transmitter and receiver constraints at the nodes:

$$\sum_j V_{i,j} \leq T_i \forall i; \sum_i V_{i,j} \leq R_j \forall j$$

- Multicommodity flow constraints on lightpath routing:

$$\sum_m p_{m,k}^{i,j} = \sum_n p_{k,n}^{i,j} \text{ if } k \neq i, j, m \neq j, n \neq i; \sum_n p_{i,n}^{i,j} = V_{i,j}; \sum_m p_{m,j}^{i,j} = V_{i,xj}$$

- Wavelengths per link constraint:

$$\sum_{i,j} p_{m,n}^{i,j} \leq WP_{m,n}$$

- Multicommodity flow constraints on the traffic variables $\lambda_{i,j}^{s,d}$:

$$\sum{}_i \lambda_{i,k}^{s,d} = \sum_j \lambda_{k,j}^{s,d} \text{ if } k \neq s, d, i \neq d, j \neq s; \ \sum_j \lambda_{s,j}^{s,d} = \Lambda_{s,d}; \sum_i \lambda_{i,d}^{s,d} = \Lambda_{s,d}.$$

- Lightpath capacity constraint and constraint to ensure that traffic can only flow through an existing lightpath:

$$\sum_{s,d} \lambda_{i,j}^{s,d} \leq V_{i,j}; \ \lambda_{i,j}^{s,d} \leq \Lambda_{s,d} V_{i,j}.$$

- Problem size reduction constraints to make the problem tractable. Obtain a number of alternative shortest paths between the source destination pairs on the physical topology and constrain the lightpaths to be routed only through nodes on these shortest paths. Lightpaths are allowed only over nodes present in these alternative shortest paths.
- Constraints to eliminate the K best solutions and obtain the $(K+1)$th best solution.

Obtain $A_k = \{(i, j): V_{i,j} = 1$ in the kth best solution, $U_k = \{(i, j): V_{i,j} = 0$ in the kth best solution, and $N(A_k)$ = number of elements in A_k. The kth, $k = 1, \cdots, K$, 'elimination' constraint is

$$\sum_{i,j \in A_k} V_{i,j} - \sum_{i,j \in U_k} V_{i,j} \leq N(A_k) - 1$$

An example with a 10-node section of the NSFNET is shown in Fig. 2. Bidirectional links between the nodes is assumed. The traffic matrix for each period of a six period cycle is generated as a random $N \times N$ matrix where each entry is uniformly distributed in [0, 1]. We use T_i, $R_i \leq 5$, $W \leq 5$. The operating cost of virtual topology $VT_{i,j}$ is assumed to be $\Sigma_{i,j}\ V_{i,j}$, the number of lightpaths in the

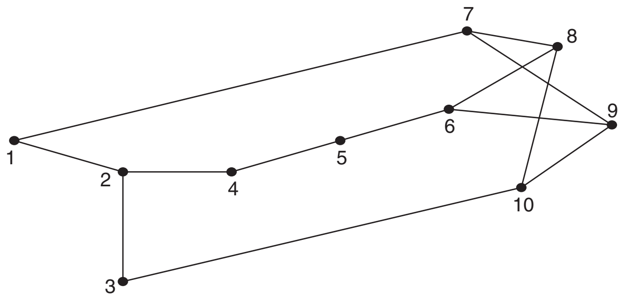

Fig. 2 *10-node section of NSFNET used in the numerical examples*

virtual topology. The reconfiguration cost of changing virtual topology $VT_{i,j}$ to $VT_{i+1,k}$ is arbitrarily taken to be 0.25 times the number of lightpaths added and removed to effect the change. The computational resources that we had access to were quite limited in terms of their ability to solve large MILP problems. Hence, we do not obtain the R_i solutions for each period as discussed above. Rather, we obtain ten ranks in each period. Figure 3 shows the optimal sequences of virtual topologies. We used AMPL modelling language [13] to specify the optimisation problems and the XPRESS-MP optimisation package to solve them.

Figure 3 shows the list of ranked virtual topologies obtained for each period. For this example we found two sequences of virtual topologies which minimise the cost over the cycle and these are shown in the Figure. Since our choice of the reconfiguration cost and the operating cost are quite arbitrary we will not discuss the numerical values of the operating, changeover or the total costs.

5 Summary and discussion

We have considered the problem of optimal virtual topology reconfiguration over multiple time periods where the time periods could corresponded to different stages in the evolution of the network (posed as a multiperiod planning horizon problem), or it could correspond to changes in traffic demands in a cyclical manner (posed as a multiperiod short-term cycles problem). Having classified the existing reconfiguration algorithms into three categories, we observed that these are essentially one-step reconfiguration algorithms that optimise the changeover costs. Choosing an optimal sequence of virtual topologies requires one to consider both the operational costs and the changeover and one needs to optimise over the entire sequence of virtual topologies rather than over just one step. Thus, we have presented a generalisation of the one-step reconfiguration problems that have been considered in the literature.

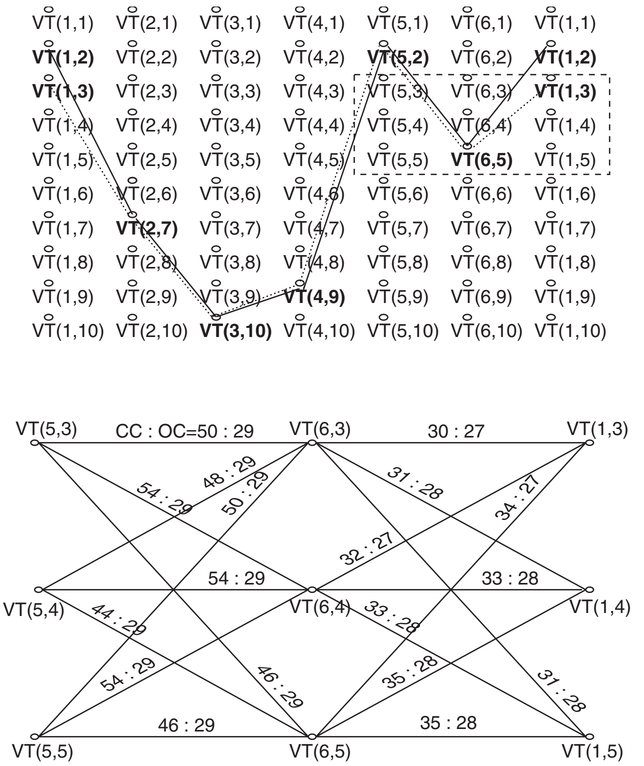

Fig. 3 *Set of virtual topologies for different periods along with two sequences of virtual topologies that are both optimal over the cycle*
The lower part of the Figure shows details of the operating and reconfiguration costs for the virtual topologies in the dashed box in the upper part of the Figure

The algorithms for multiperiod reconfiguration proposed in this paper can take into account changes in the demand for bandwidth and also changes in supply, e.g. the physical topology. We believe that the short-term cycles problem has important practical applications.

The solution of the MILP to obtain a virtual topology is computationally intensive. The method described in this paper requires one to obtain a number of MILP solutions for each period under consideration. Thus the computational effort in obtaining the optimal sequence is significant making this a suitable candidate for 'offline' rather than 'online' use.

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