PROBLEM 2.11

(a) \( h(t) = e^{-3t}u(t) \), non-zero for \( t \geq 0 \)
so the convolution integral is
\[ y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \]
\[ = \int_{-\infty}^{\infty} e^{-3\tau} (u(t-\tau-3) - u(t-\tau-5)) d\tau \]
at the same time \( u(t-\tau-3) - u(t-\tau-5) \) is non-zero
only in the range \( (t-5) < \tau < (t-3) \). So for \( t \leq 3 \) the integral is zero.

2 cases: for \( 3 < t \leq 5 \)
\[ y(t) = \int_{0}^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3)}}{3} = A \]
for \( t > 5 \),
\[ y(t) = \int_{t-3}^{t-5} e^{-3\tau} d\tau = \frac{(1 - e^{-3(t-5)})}{3} = B \]
So,
\[ y(t) = \begin{cases} 
0 & \text{for } -\infty < t \leq 3 \\
A & \text{for } 3 < t \leq 5 \\
B & \text{for } 5 < t \leq \infty
\end{cases} \]

(b) \( \frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5) \Rightarrow g(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5) \),
because the convolution of a signal with a \( \delta(t-a) \)
time shifts the signal by \( a \).

(c) \( g(t) \) is the derivative of \( y(t) \): \( y(t) = \frac{dy(t)}{dt} \)
which means that the differentiation property is preserved
in convolution.

PROBLEM 2.14

For \( h(t) \) to be the impulse response of a stable LTI system, it
must be absolutely integrable. In this case
\[ \int_{-\infty}^{\infty} |h_1(\tau)| d\tau = \int_{0}^{\infty} e^{-\tau} d\tau = 1 \]
and since \( e^{-t} \) is decaying and \( \cos(2t) \) bounded,
then \( e^{-t} \cos(2t) \) is exponentially decaying
for \( t > 0 \). So yes.
**Problem 2.21**

(a) \[ x(t) = e^{-a t} u(t) \]
\[ u(t) = e^{-\epsilon t} u(t) \]
\[ y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \]
\[ = \int_{-\infty}^{\infty} e^{-a \tau} e^{-\epsilon (t - \tau)} d\tau, \ t \geq 0 \]
\[ = \int_{-\infty}^{\infty} e^{(a - \epsilon) \tau} e^{-\epsilon t} d\tau, \ t \geq 0. \]

So for \( b \neq a \) \[ y(t) = \left[ e^{-\epsilon t} (e^{(a - \epsilon) t} - 1) / (b - a) \right] u(t) \]
for \( b = a \) \[ y(t) = te^{-\delta t} u(t) \]

**Problem 2.22(a),(b),(c)**

(a) \[ h(t) = e^{-\epsilon t} u(t - 2) \]
causal because \( h(t) = 0 \) for \( t < 0 \).
stable because \( \int_{-\infty}^{\infty} |h(t)| dt < \infty \)

(b) \[ h(t) = e^{-\epsilon t} u(t - 2) \]
non-causal because \( h(t) \neq 0 \) for \( t < 0 \).
unstable because \( \int_{-\infty}^{\infty} |h(t)| dt = \infty \)

(c) \[ h(t) = te^{-\epsilon t} u(t) \]
causal and stable, \( \int_{-\infty}^{\infty} |h(t)| dt = 1 \)

**Problem 2.40**

(a) \[ y(t) = \int_{-\infty}^{t} e^{-(t - \tau)} x(\tau - 2) d\tau \] rewrite it as \[ \int_{-\infty}^{t} e^{-(t - \sigma - \beta)} x(\sigma) h(\beta) d\sigma \]
which makes it look as if it is the convolution with the impulse response shifted. So \( h(t) = e^{-(t - 2)} u(t - 2) \)
So: the \( u(t - 2) \) comes from the upper bound of the integral.

(b)  
This signal is \( x(t) = u(t+1) - u(t-2) \)
So the convolution integral is \( y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \)
\[ \Rightarrow y(t) = \int_{2}^{\infty} e^{-(t - \tau - 2)} \left[ u(t - \tau + 1) - u(t - \tau - 2) \right] d\tau \]
We solve this through graphical convolution, see handout 3