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Integrated Vision/Inertial Navigation System Design Using Nonlinear Filtering *

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Abstract

This paper addresses the problem of navigation system design for autonomous aircraft landing. New nonlinear filter structures are introduced to estimate the position of an aircraft with respect to a possibly moving landing site, such as a Naval vessel, based on measurements provided by airborne vision and inertial sensors.. By exploring the geometry of the navigation problem, the navigation filter dynamics are cast in the framework of linear parametrically varying systems (LPVs). Using this set-up, filter performance and stability are studied in an H_∞ setting by resorting to the theory of linear matrix inequalities (LMIs). The design of nonlinear, globally stable filters to meet adequate H_∞ performance measures is thus converted into that of determining the feasibility of a related set of LMIs and finding a solution to them, if it exists. This is done by resorting to widely available numerical tools that borrow from convex optimization techniques. The paper develops the mathematical framework that is required for integrated vision / inertial navigation system design and details a design example for an air vehicle landing on an aircraft carrier.

1 Introduction

This paper describes a solution to the problem of navigation system design for autonomous aircraft landing on board a moving platform, such as a Naval vessel. The main motivation for this work stems from the need to develop reliable, miniaturized advanced navigation systems to enable the safe operation of unmanned air vehicles. Economy considerations, together with strict requirements imposed in the course of some envisioned mission scenarios, all but dictate the need to use passive sensors only. Thus the emphasis on the integration of vision with other passive sensors such as altimeters and other inertial sensors installed on-board the aircraft.

For previous related work in this area, the reader is referred to [5, 8] and the references therein. Reference [8] describes a solution to the problem of estimating the ground velocity and position of an aircraft based on visual terrain information, whereas [5] focuses on the use of GPS and vision based systems for aircraft navigation. Both papers tackle the problem of navigation system design in the context of Extended Kalman Filters which lack stability and performance guarantees and require a complete characterization of process and observation noises, a task that may be difficult, costly, or not suited to the problem at hand. This issue is argued at great length in [4], who points out that in a great number of practical applications the filter design process is entirely dominated by constraints that are naturally imposed by the sensor bandwidths. In this case, a design method that explicitly addresses the problem of merging information provided by a given sensor suite over distinct, yet complementary frequency regions is warranted. Complementary filters have been developed to address this issue explicitly. See for example [4, 11] for a concise introduction to complementary filters and their applications.

Motivated by these observations, this paper proposes the use of special nonlinear filter structures - akin to those used in complementary filtering - to estimate the relative position and velocity of an aircraft with respect to a possibly moving landing site based on measurements provided by airborne vision and other inertial sensors. The paper builds on a key result introduced in [10], where a useful property of the so-called perspective projection map is derived and used in the development of a visual estimation system for dexterous manipulation. By using that result and exploring the geometry of the navigation problem at hand, the nonlinear filter dynamics are cast in the framework of linear parametrically varying systems (LPVs) [1]. Using this set-up, filter performance and stability are studied in an H_∞ setting by resorting to the theory of linear matrix inequalities (LMIs) [2]. The design of nonlinear, regionally stable filters to ensure stability and meet adequate H_∞ performance measures is thus converted into that of determining the feasibility of a related set of LMIs and finding a solution to them, if it exists. This is done by resorting to widely available numerical tools that borrow from convex optimization techniques. The paper develops the mathematical framework that is required for integrated vision/inertial navigation system design and details a design example for an air vehicle landing on an aircraft carrier.

The paper is organized as follows. Section 2 describes the class of integrated vision/inertial navigation systems that we consider and provides a rigorous mathematical formulation of the related filtering problems. Section 3 provides solutions to the problems posed in terms of linear matrix inequalities (LMIs). Section 4 includes a simple example illustrating application of the proposed techniques to a shipboard landing problem. Finally, section 5 contains the main conclusions and discusses theoretical and practical issues that deserve further consideration.

2 Problem Formulation.

This section describes the navigation problem that is the main focus of the paper and formulates it mathematically in terms of an equivalent filter design problem. For

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the sake of clarity we first introduce some required notation and review the kinematic relationships of an aircraft / ship carrier ensemble, where the former is equipped with a vision based system.

2.1 Notation

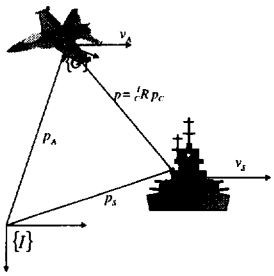

Figure 1: Coordinate Systems

Consider Figure 1, which depicts an aircraft equipped with a vision camera operating in the vicinity of the ship. Let $\{\mathcal{I}\}$ denote an inertial reference frame, $\{\mathcal{B}\}$ a body-fixed frame that moves with the aircraft, and $\{\mathcal{C}\}$ a a camera-fixed frame. The symbol $\{\mathcal{S}\}$ denotes a ship-fixed body frame. The following symbols will be used:

- $\mathbf{p}_B = [x_b\ y_b\ z_b]^T$ - position of the origin of $\{\mathcal{B}\}$ measured in $\{\mathcal{I}\}$ (i.e., inertial position of the aircraft).
- $\mathbf{p}_S = [x_s\ y_s\ z_s]^T$ - inertial position of the ship.
- $\mathbf{p}_{SB}$ (*abbv.* $\mathbf{p} = [x\ y\ z]^T$) - relative position of the ship with respect to the aircraft, resolved in $\{\mathcal{I}\}$.
- ${}^C\mathbf{p}_{SC}$ (*abbv.* $\mathbf{p}_C = [x_c\ y_c\ z_c]^T$) - relative position of the ship with respect to the aircraft, resolved in $\{\mathcal{C}\}$.
- $\mathbf{v}_B$ - linear velocity of the origin of $\{\mathcal{B}\}$ measured in $\{\mathcal{I}\}$ (i.e., inertial velocity of the aircraft).
- $\mathbf{v}_S$ - inertial velocity of the ship.
- ${}^B\mathbf{a}$ - linear acceleration of $\{\mathcal{B}\}$ with respect to $\{\mathcal{I}\}$, resolved in $\{\mathcal{B}\}$.
- Ω - angular velocity of $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$, resolved in $\{\mathcal{I}\}$.
- $\Lambda = [\phi\ \theta\ \psi]^T$ - vector of roll, pitch, and yaw angles that parameterize locally the orientation of frame $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$.

Given two frames $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$, ${}^A_B\mathcal{R}$ denotes the rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{A}\}$. In particular, ${}^I_C\mathcal{R}$ (abbreviated $\mathcal{R}$) is the rotation matrix from $\{\mathcal{C}\}$ to $\{\mathcal{I}\}$, parameterized locally by Λ, that is, $\mathcal{R} = \mathcal{R}(\Lambda)$.

2.2 Kinematic relations.

The rotation matrix $\mathcal{R}$ satisfies the orthonormality condition $\mathcal{R}^T\mathcal{R} = I$. Furthermore, [3]:

$$\dot{\mathcal{R}} = \mathcal{R}\mathcal{S}(\Omega), \tag{1}$$

where

$$\mathcal{S}(\Omega) := \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{2}$$

is a skew symmetric matrix, that is, $\mathcal{S}^T = -\mathcal{S}$. The matrix $\mathcal{S}$ satisfies the relationship $\mathcal{S}(a)b = a \times b$, where a, b are arbitrary vectors and x denotes the cross product operation. Furthermore, $||\mathcal{S}(\Omega)|| = ||\Omega||$.

We introduce the following assumption. **A1** - *The ship's inertial velocity* $\mathbf{v}_S$ *is constant* Now, from the above definitions, it follows that

$$\begin{aligned} &\mathbf{p}_S = \mathbf{p}_B +{}^I_C \mathcal{R}\mathbf{p}_C \Rightarrow \\ &\frac{d^2}{dt^2}\,{}^I_C\mathcal{R}\mathbf{p}_C = \frac{d^2}{dt^2}\mathbf{p}_S - \frac{d^2}{dt^2}\mathbf{p}_B \end{aligned} \tag{3}$$

and since $\frac{d^2}{dt^2}\mathbf{p}_S = 0$ (assumption A1) we obtain

$$\frac{d^2}{dt^2}({}^I_C R\mathbf{p}_C) = -\frac{d^2}{dt^2}\mathbf{p}_B \tag{4}$$

Equation (4) shows that aside from a change in sign, the relative acceleration of the ship with respect to the aircraft resolved in $\{\mathcal{I}\}$ is equal to the aircraft's inertial acceleration resolved in $\{\mathcal{I}\}$. However, in the case of strapdown inertial navigation systems widely in use today [14] the aircraft's inertial acceleration is usually given in $\{\mathcal{B}\}$. Therefore, since

$$\frac{d^2}{dt^2}\mathbf{p}_B ={}^I_B R\,{}^B\mathbf{a}$$

it follows that

$$\frac{d^2}{dt^2}({}^I_C R\mathbf{p}_C) = -{}^I_B R\,{}^B\mathbf{a}. \tag{5}$$

2.3 Process Model

We assume the image of the origin of $\{\mathcal{S}\}$ acquired by a camera installed on-board the aircraft is obtained using a simple pinhole camera model of the form [6]

$$\begin{bmatrix} u \\ v \end{bmatrix} = \pi_f(x_c, y_c, z_c) = \frac{f}{x_c}\begin{bmatrix} y_c \\ z_c \end{bmatrix}, \tag{6}$$

where f is the focal length of the camera and $[u\ v]^T$ are the image coordinates of $\mathbf{p}_c = [x_c\ y_c\ z_c]^T$ in the camera's image plane. We also make the following assumption. **A2** - $x_c > 0$, that is, *the ship is always located in front of the camera's image plane* . We further assume that **A3** - the rotation matrices ${}^I_B R$ and ${}^I_C R$ are available from the on-board attitude measurement system. This assumption is quite reasonable, considering the sophistication achieved by such systems today.

Suppose the aircraft is equipped with a barometric-based sensor that provides a measurement of the altitude of the aircraft with respect to the mean sea level. Then, using the relation $\mathbf{p} = ^I_C R\mathbf{p}_c$ and assuming that the aircraft is sufficiently away from the ship, so as to neglect the height h_s of the ship's deck above the mean sea surface we may assume that **A4:** $h_s = 0$. Thus we obtain that the altitude measurement equals

$$z = g(\mathbf{p}_c) = \sin\theta\, x_c - \cos\theta \sin\phi\, y_c + \cos\theta\cos\phi\, z_c.$$

In (2.3)ϕ and θ are the roll and pitch angles in the rotation matrix $^I_C\mathcal{R}$.

We now introduce the underlying design model that plays a fundamental role in this paper. Let $\mathbf{y} = [u\ v\ z]^T$. Then, the model that we consider can be written as

$$\mathcal{G} = \begin{cases} \frac{d}{dt}\mathbf{p} &= \mathbf{v} \\ \frac{d}{dt}\mathbf{v} &= -^I_B R(^B\mathbf{a}_m + \mathbf{w}_a) \\ \mathbf{y}_m &= h_{\phi,\theta}(\mathbf{p}_C) + \mathbf{w}_y, \end{cases} \tag{7}$$

where $h_{\phi,\theta} : \mathcal{R}^3 \to \mathcal{R}^3$ is defined by

$$[u\ v\ z]^T = h_{\phi,\theta}(\mathbf{p}_C) = [\pi_f^T\ \ g(\mathbf{p}_c)]^T$$

and $\mathbf{a}_m$ and $\mathbf{y}_m$ denote the measured values of $\mathbf{a}$ and $\mathbf{y}$, respectively, the measurements being corrupted by the process noises $\mathbf{w}_a$ and $\mathbf{w}_y$. We assume that the matrix $^I_B R$ can be measured accurately. In the sequel we adopt the deterministic set-up of H_∞ filtering [12] and assume that $\mathbf{w}_a$ and $\mathbf{w}_y$ are arbitrary functions in L_2.

2.4 Problem Definition

The problem that we consider in this paper consists of determining the relative position and relative velocity of an aircraft with respect to a landing site using vision and other on-board passive sensors. For the sake of clarity, we first tackle the simplified problem of designing a filter with no measurement noise in the model. This exercise is simple, yet it captures some of the key ideas used in the development that follows.

The additional notation that is required is introduced next. In what follows, we let $\hat{\mathbf{p}}$ and $\hat{\mathbf{v}}$ denote estimates of $\mathbf{p}$ and $\mathbf{v}$, respectively. In the camera frame, they are denoted by $\hat{\mathbf{p}}_C$, $\hat{\mathbf{v}}_C$. We assume that the orientation of camera frame $\{\mathcal{C}\}$ with respect to $\{\mathcal{I}\}$ is restricted through the set

$$\Lambda_C = \left\{\Lambda = [\phi\ \theta\ \psi]^T : |\phi| \le \phi_{max},\ |\theta| \le \theta_{max},\ |\psi| \le \psi_{max}\right\}. \tag{8}$$

Notice, ψ_{max}, for example, should be set to π. We further assume that the vectors $\mathbf{p}_C$ lie in the compact set

$$\begin{aligned} \mathcal{P}_C = \{&\mathbf{p}_C = [x_C\ y_C\ z_C], x_{min} \le x_C \le x_{max}, \\ &y_{min} \le y_C \le y_{max}, z_{min} \le z_C \le z_{max}\}. \end{aligned} \tag{9}$$

where $x_{min}, \ldots .z_{max}$ are determined from the geometry of the problem at hand. The set $\mathcal{P}_C$ can be determined as follows. First, compute $\mathcal{P}_C$ for a nominal orientation of the camera (usually inertial orientation). Determine the maximum range of camera orientation angles with respect to the nominal orientaion. Then compute $\mathcal{P}_C$ by allowing the angles to vary within these predetermined bounds.

Filter design will aim at ensuring that the estimates $\hat{\mathbf{p}}_C$ of $\mathbf{p}_C$ lie in a compact set

$$\begin{aligned} \hat{\mathcal{P}}_C = \{&\hat{\mathbf{p}}_C = [\hat{x}_C\ \hat{y}_C\ \hat{z}_C], |\hat{\mathbf{x}}_C - \mathbf{x}_C| \le dx, \\ &|\hat{\mathbf{y}}_C - \mathbf{y}_C| \le dx, |\hat{\mathbf{z}}_C - \mathbf{z}_C| \le dz\}. \end{aligned} \tag{10}$$

where dx, dy and dz are positive numbers, and $dx < x_{min}$.

F1: Regional Stability. *Consider the process model (7) and assume that $\mathbf{w}_a = \mathbf{w}_y = 0$. For a given $\hat{\mathcal{P}}_C$, find a number $\alpha > 0$, and a dynamical system (filter) $\mathcal{F}$ that operates on $\mathbf{y}_m$ and $\mathbf{a_m}$ to produce estimates $\hat{\mathbf{p}}$ of $\mathbf{p}$, and $\hat{\mathbf{v}}$ of $\mathbf{v}$, such that*

- $\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C$ *for any* $t > 0$,
- $\|\hat{\mathbf{p}} - \mathbf{p}\| + \|\hat{\mathbf{v}} - \mathbf{v}\| \to 0$ *as* $t \to \infty$.

provided that $\|(\hat{p}_C(0) - p_C(0), \hat{v}(0) - v(0))^T\| < \alpha$.

Notice that the problem described aims at finding a filter that complements the information available from the vision system / barometric pressure sensor with that available from the inertial sensors.

The problem **F1** focuses on the stability of the filter. The second filtering problem addresses the scenario where the performance of the filter in the presence of disturbances is considered.

F2: Regional Stability and Performance . *Consider the process model (7) where $\mathbf{w} = [\mathbf{w}_a\ \mathbf{w}_y]^T \in L_2$, $\|\mathbf{w}\|_2 \le 1$ and let the sets $\mathcal{P}_C$ and $\hat{\mathcal{P}}_C$ of allowable position vectors and allowable estimation vectors be given by (9) and (10), respectively. For given positive numbers $\gamma > 0$ and $\alpha > 0$, find a stable filter $\mathcal{F}$ that operates on $\mathbf{y}_m$ and $\mathbf{a}_m$ to obtain estimates $\hat{\mathbf{p}}$ of $\mathbf{p}$, and $\hat{\mathbf{v}}$ of $\mathbf{v}$ such that if $\|[(\hat{\mathbf{p}}(0) - \mathbf{p}(0))^T\ \ (\hat{\mathbf{v}}(0) - \mathbf{v}(0))^T]^T\| < \alpha$, the filter satisfies the following conditions for all $\|\mathbf{w}\|_2 \le 1$*

- $\hat{\mathbf{p}}_C(t) \in \hat{\mathcal{P}}_C$ *for all* $t \ge 0$.
- *If* $\mathbf{w} = 0$, *then* $\|\hat{\mathbf{p}}(t) - \mathbf{p}(t)\| + \|\hat{\mathbf{v}}(t) - \mathbf{v}(t)\| \to 0$ *as* $t \to \infty$.
- $\|T_{\mathbf{ew}}\|_{2,i} < \gamma$, *where* $\mathbf{e} := \hat{\mathbf{p}} - \mathbf{p}$ *is the estimation error and* $T_{\mathbf{ew}} : \mathbf{w} \to \mathbf{e}$.

Notice the technical requirement that an allowable set of position estimates $\hat{\mathcal{P}}_C$ be specified. As explained later, this requirement is essential to establishing the boundedness of a certain operator for all possible values of the estimates $\hat{\mathbf{p}}(t)$. In practice, the "size" of the allowable region $\hat{\mathcal{P}}$ plays the role of a design parameter.

3 Proposed Solution

This section describes the solutions to problems F1 and F2. First, however, we need the following basic results. Let H denote the Jacobian of $h_{\phi,\theta}$ with respect to $\mathbf{p}_C$. From the definition of $h_{\phi,\theta}$, it follows that

$$H(\mathbf{p}_C) = \begin{bmatrix} -\frac{f y_C}{x_C^2} & \frac{f}{x_C} & 0 \\ -\frac{f z_C}{x_C^2} & 0 & \frac{f}{x_C} \\ \sin\theta & -cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \tag{11}$$

It is easy to check that

$$\det(H) = \frac{f^2}{x_C^3} z.$$

Therefore, H is not invertible if and only if $z = 0$, which is only possible when aircraft is flying at sea level $h_{\phi,\theta} = 0$. This will never occur in practice. Therefore, $H(\mathbf{p}_C)$ is invertible for all admissible values of $\mathbf{p}_C$, ϕ and θ.

The next result is adopted from [10] and plays a key role in the development that follows.

Lemma 3.1 *Let $h_{\phi,\theta}$ be given by equation (7). Then*

$$h_{\phi,\theta}(\hat{\mathbf{p}}_C) - h_{\phi,\theta}(\mathbf{p}_C) = \Lambda(\hat{\mathbf{p}}_C, \mathbf{p}_C) H(\hat{\mathbf{p}}_C)(\hat{\mathbf{p}}_C - \mathbf{p}_C), \quad (12)$$

where H is given in equation (11), $\hat{\mathbf{p}}_C = [\hat{x}_c\ \hat{y}_c\ \hat{z}_c]^T$ and

$$\Lambda(\hat{\mathbf{p}}_C, \mathbf{p}_C) = \begin{bmatrix} \frac{\hat{x}_c}{x_c} & 0 & 0 \\ 0 & \frac{\hat{x}_c}{x_c} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Lemma 3.2 *Let $\phi : \mathcal{R}^6 \to \mathcal{R}^{3\times3}$ and $\phi_1 : \mathcal{R}^3 \to \mathcal{R}^{3\times3}$ be the operators defined by*

$$\phi(\hat{\mathbf{p}}_C, \mathbf{p}_C) = H^T(\hat{\mathbf{p}}_C)\Lambda(\hat{\mathbf{p}}_C, \mathbf{p}_C)H(\hat{\mathbf{p}}_C)$$

and $\phi_1(\hat{\mathbf{p}}_C) = H^T(\hat{\mathbf{p}}_C)H(\hat{\mathbf{p}}_C)$. Then

$$\phi(\hat{\mathbf{p}}_C, \mathbf{p}_C) > 0,\ \phi_1(\hat{\mathbf{p}}_C) > 0\ \ \forall \mathbf{p}_C \in \mathcal{P}_C \text{ and } \hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C.$$

The following result provides a solution to problem F1.

Theorem 3.3 *Define a filter*

$$\mathcal{F}_1 = \begin{cases} \dot{\hat{\mathbf{p}}} &= \hat{\mathbf{v}} + K_1 {}^I_C R H^T(\hat{\mathbf{p}}_C)\ (h_{\phi,\theta}(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \dot{\hat{\mathbf{v}}} &= -{}^I_B R\, {}^B\mathbf{a}_m + K_2 {}^I_C R H^T(\hat{\mathbf{p}}_C)\ (h_{\phi,\theta}(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \hat{\mathbf{p}} &= {}^I_C R \hat{\mathbf{p}} \end{cases} \quad (13)$$

Let $\hat{\mathcal{P}}_C$ be given. Suppose assumptions **A1- A4** *hold. Let $\alpha < \min\{dx, dy, dz\}$ be any positive number. Define $r_x = dx/x_{\min}$, and define $\epsilon = \min_{\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C} \lambda_{\min}(H^T(\hat{\mathbf{p}}_C)H(\hat{\mathbf{p}}_C))$. Suppose there exists a matrix P such that*

$$P > 0$$
$$F^T P + PF + \begin{bmatrix} -2(1-r_x)^2\epsilon & \\ 0 & 0 \end{bmatrix} < 0$$
$$P - \max\{\frac{1}{dx^2}, \frac{1}{dy^2}, \frac{1}{dz^2}\} > 0,$$
$$\frac{1}{\alpha^2} - P > 0$$

where $F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$. Let

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -P^{-1}(1-r_x)\begin{bmatrix} I \\ 0 \end{bmatrix}. \quad (14)$$

Then the filter $\mathcal{F}_1$ solves the filtering problem **F1**.

The solvability of the inequality (14) is addressed in Theorem 3.5. It shows that the inequality has a solution iff $\epsilon > 0$. Notice that the filter $\mathcal{F}_1$ complements the information available from the vision system / barometric pressure sensor with that available from the inertial sensors. This structure is similar to that found in complementary filters [4, 11], the practical relevance of which can hardly be overemphasized.

The next theorem provides solution to the filtering problem **F2**.

Theorem 3.4 *Let $\mathcal{P}_C$ and $\hat{\mathcal{P}}_C$ be given. Let r_x and α be positive number as in Theorem 3.3. Define*

$$\epsilon = \min_{\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C} \lambda_{min}\{\phi_1(\hat{\mathbf{p}}_C)\} \quad (15)$$

Suppose assumptions **A1-A4** *hold. For a given gain γ, suppose there exists a matrix $P = P^T \in \mathcal{R}^{6\times6}$ such that*

$$P > 0$$
$$\left[\begin{array}{c|c} F^T P + PF + \begin{bmatrix} \frac{I}{\gamma^2} - (1-r_x)^2\epsilon I & 0 \\ 0 & 0 \end{bmatrix} & PF^T \\ \hline FP & -I \end{array}\right] < 0, \quad (16)$$

$$P - 4\max(\frac{1}{dx^2}, \frac{1}{dy^2}, \frac{1}{dz^2}) > 0,$$
$$\frac{1}{\alpha^2} - P > 0$$

Let

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -P^{-1}\begin{bmatrix} (1-r_x)I \\ 0 \end{bmatrix}.$$

Define the filter

$$\mathcal{F}_2 = \begin{cases} \dot{\hat{\mathbf{p}}} &= \hat{\mathbf{v}} + K_1 {}^I_C R\, H^T(\hat{\mathbf{p}}_C)\ (h_{\phi,\theta}(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \dot{\hat{\mathbf{v}}} &= -{}^I_B R\, {}^B\mathbf{a}_m + K_2 {}^I_C R\, H^T(\hat{\mathbf{p}}_C)\ (h_{\phi,\theta}(\hat{\mathbf{p}}_C) - \mathbf{y}_m) \\ \hat{\mathbf{p}}_C &= {}^C_I R \hat{\mathbf{p}} \end{cases} \quad (17)$$

Then $\mathcal{F}_2$ solves the filtering problem F2 if

$$\|[(\hat{\mathbf{p}}(0) - \mathbf{p}(0))^T\ \ (\hat{\mathbf{v}}(0) - \mathbf{v}(0))^T]^T\| < \alpha.$$

The next theorem derives necessary and sufficient conditions under which (16) is satisfied.

Theorem 3.5 *Let F, γ and ϵ be defined in Theorem 3.4. Then $\exists\ P = P^T > 0$ such that*

$$\left[\begin{array}{c|c} F^T P + PF + \begin{bmatrix} \frac{I}{\gamma^2} - (1-r_x)^2\epsilon I & 0 \\ 0 & 0 \end{bmatrix} & PF^T \\ \hline FP & -I \end{array}\right] < 0$$
$$\iff$$
$$\frac{I}{\gamma^2} - (1-r_x)^2\epsilon I < 0.$$

Remark 3 Notice, that a similar argument as used in Theorem 3.5 can be used to show that the LMI

$$F^T P + PF - \begin{bmatrix} (1-r_x)^2\epsilon I & 0 \\ 0 & 0 \end{bmatrix} < 0$$

used in Theorem 3.3 has a solution $P > 0 \iff r_x < 1$.

Remark 4 Theorem 3.5 shows that the LMI (18) is feasible iff $\gamma^2 > \frac{1}{(1-r_x)^2\epsilon}$. Recall that

$$\epsilon = \min_{\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C} \{\lambda_{min}(H^T(\hat{\mathbf{p}}_C)H(\hat{\mathbf{p}}_C))\}.$$

Therefore, we obtain

$$\gamma^2 > \frac{1}{(1-r_x)^2} \max_{\hat{\mathbf{p}}_C \in \hat{\mathcal{P}}_C} \{\|(H^T(\hat{\mathbf{p}}_C)H(\hat{\mathbf{p}}_C))^{-1}\|\}.$$

This inequality imposes a lower bound on the achievable values of γ. The bound is similar to the classical Positional Dilution of Precision (PDOP) metric that is commonly used in navigation systems to determine a lower bound on the achievable error covariance as a function of geometry of the underlying navigation problem [7, 13, 14]. Using our notation, the classical PDOP can be written as

$$\text{PDOP} = \sqrt{\text{tr}(H^T(\mathbf{p}_C)H(\mathbf{p}_C))^{-1}}.$$

We therefore see that the new bound derived in this paper captures a worst case performance scenario. and the estimate of x_c increases the lower bound on the achievable γ, since

$$1 > (1 - r_x)^2 > 0.$$

Remark 5 The filters used in this paper borrowed from the structure of the nonlinear observer proposed in [10]. Both filters are designed for a process model that exhibits linear dynamics and nonlinear measurement equations. In view of this fact, one is naturally driven to ask the following question: why not simply solve the measurement equation to obtain estimate of $\mathbf{p_c}$ that can in turn drive a linear filter with a much simpler structure? This technique was, in fact, applied in an erlier version of the work reported in [10]. However, as pointed out by the authors the latency inherent to this approach led to unacceptable results. This stemmed from the fact that the estimate of $\mathbf{p_c}$ obtained by the nonlinear solver from the measurement equation and used by the linear filter represented a "delayed version" of the true position.

Furthermore, the algorithm used by the nonlinear solver requires inverting the Jacobian. In a noisy environment this may lead to excessive noise amplification. This problem is entirely avoided by the filters proposed in this paper as well as by the nonlinear observer in [10]. Finally, the gains used by every filter in this paper are of the form similar to the gains of optimal filters obtained for the linear time invariant (LTI) case. This is important, since in the LTI case even if the output matrix is invertible the optimal gain does not require inversion of this matrix.

4 Example

In this section we present simulation results for the filter $\mathcal{F}_2$. The LMIs included in Theorems 3.4 were implemented in Matlab. The gains obtained by solving these LMIs were used to simulate response of this filter to non-zero initial conditions and $\mathcal{L}_2$ measurement noise.

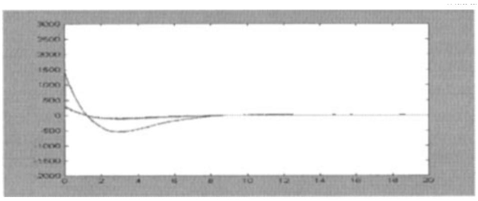

Figure 2: Filter $\mathcal{F}_2$: Position errors in meters along y-axis, time in sec along x-axis.

5 Conclusions

This paper addressed the problem of estimating the relative position and velocity of an aircraft with respect to a moving landing site such as a Naval vessel. The problem was cast in the LPV framework and solved using tools that borrow from the theory of Linear Matrix Inequalities. This approach resulted in nonlinear filtering structures that integrate vision with inertial measurements and have regional stability and performance guarantees. Employing inertial sensors has an additional benefit of offering robustness with respect to out-of-camera events and occlusions, whereby these sensors can be used by the filter to provide the estimates of the image coordinates of the ship to the image processing algorithms. Furthermore, it was shown that the worst case H_∞-based performance of these filters is bounded below by a quantity similar to the Positional Dilution of Precision used in the science of navigation to determine the impact of the geometry on the performance of a navigation system. Future work will include development of an example, where the proposed filters are applied to a realistic problem and the extension of these techniques to the multi-rate case.

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